Lattice effective field theory methods for in-medium heavy quarkonium (I)

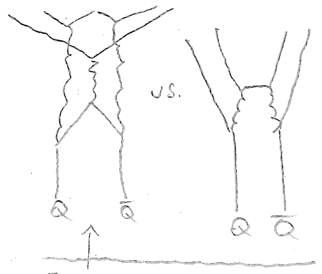
In the course of this lecture we will both explore the physics of and some of the theoretical tools to study a particular class of hadrons under extreme conditions. These mesons, christened heavy-quarkonium are composed of a heavy quark and antiquark. When referring to extreme conditions we mean temperatures as high as $T \sim \mathcal{O}(100\,\mathrm{MeV})$ similar to the conditions shortly after the Big Bang or found in relativistic heavy-ion collisions.

Let us recollect some details on <u>heavy quarkonium in vacuum</u>

Higgs contribution $m_{u/d} \sim 4\,\mathrm{MeV}$ $m_s \sim 95\,\mathrm{MeV}$ $m_c \sim 1.27\,\mathrm{GeV}$ $m_b \sim 4.6\,\mathrm{GeV}$ $m_t \sim 173\,\mathrm{GeV}$

$b\bar{b}$ Bottomonium $c\bar{c}$ Charmonium (see e.g. 1010.5827)

Heavy $Q\bar{Q}$ is a unique system, as it represents exceptionally stable bound states of strongly interacting particles with life times of $\tau^{-1} \sim \Gamma \sim \mathrm{keV}$. The reason is that decays of color neutral states are suppressed due to the OZI rule below the heavy-light threshold (D or B mesons.). Significant decay into di-leptons e^+e^-, $\mu^+\mu^-$



Decay via 1-gluon ↯ due to color
Decay via 2-gluons ↯ due to symmetries of WF

$\Rightarrow$ Long lifetimes means easier experimental access @ e.g. BELLE (KEKB), BABAR (SLAC), CLEO (FLAB), BESIII, LHCb (LHC) via elementary collision e.g. e^+e^-. High precision data available on masses, widths and decay channels available. Successful description via quark model
$[\frac{1}{2}\otimes\frac{1}{2} = 1 \oplus 0$ $S=0$ $PC=-+$ $S=1$ $PC=--$ S-wave $L=0$ $\begin{smallmatrix} S & 0^{-+} \\ T & 1^{--}\end{smallmatrix}$ $L=1$ $\begin{smallmatrix} 1^{+-} \\ \{0,1,2\}^{++}\end{smallmatrix}$ $]$

Current topic of interest exotic quarkonia with quantum numbers NOT available in the quark model: $\underset{1^{++}}{X(3872)}$ $Y(4260)$ $Z_c(3900)$ (proposals: meson molecule, tetraquarks)

Nomenclature similar to hydrogen $c\bar{c}$ $\underset{m_{J/\psi}=3.096\,\mathrm{GeV}}{J/\psi} = {}^3S_1(n=1)$ $\psi' = {}^3S_1(n=2)$ $\eta_c = {}^1S_0(n=1)$ $\chi_{c1} = {}^3P_1(n=1)$

$\Rightarrow$ Hierarchy of scales justifies non-relativistic treatment $M_Q \gg Mv \sim \frac{1}{r} \sim p \gg Mv^2 \sim E_b$

E_b $\underset{m_\Upsilon = 9.4603\,\mathrm{GeV}}{\Upsilon(b\bar{b})}$ 1.1 GeV $J/\psi(c\bar{c})$ 600 MeV $\chi_b(b\bar{b})$ 600 MeV $\Leftrightarrow$ $v^2 \approx 0.3\,(c\bar{c})$, $0.1\,(b\bar{b})$

Very successful modeling of bound states below threshold via Cornell-potential

$$V(r) = -\frac{\alpha}{r} + \sigma r = 1\ \begin{smallmatrix}\text{perturbative}\\ \text{gluon exchange}\end{smallmatrix} + \begin{smallmatrix}\text{non-perturbative}\\ \text{confinement.}\end{smallmatrix}\ \text{related to } \Lambda_{QCD}$$ Fig 1.

In addition lattice QCD simulations recovered the quarkonium spectrum with high accuracy and were able to even predict the masses of e.g. $\eta_b(2S)$ Fig 2.

While at $T=0$ we have a very good understanding of quarkonium bound states knowledge @ high temperatures is much less robust. Fig 3.

Our goal is TWOFOLD: On the one hand we want to understand how tightly bound $Q\bar{Q}$ states react to a heatbath with $T \lesssim E_b$, how do states dissociate

On the other hand we want to use that knowledge to investigate nuclear matter under extreme conditions, e.g. produced in a relativistic heavy-ion collision. (II)

Classic idea by Matsui & Satz: If in a heavy-ion collision deconfined matter (QGP) is produced it screens the color interaction between the $Q\bar{Q}$, which dissociates. Suppression of yields as sign of Quark-Gluon plasma formation

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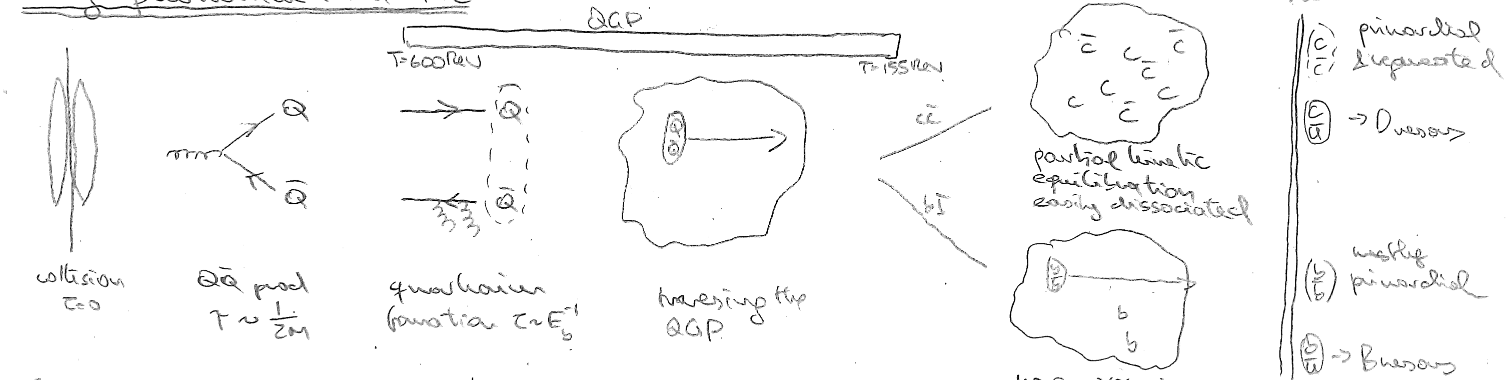
<u>Heavy quarkonium in a HIC:</u>



Common quantity $R_{AA} = \dfrac{\text{Yield}\,(A+A)}{\text{Yield}\,(p+p)\, N_{coll}}$

<u>Currently established observations in HIC:</u>

1. Quarkonia in HIC suppressed compared to pp $R_{AA} < 1$. Suppression hierarchically ordered with vacuum binding energy. $\boxed{\text{Fig. 5}}$
2. Different mechanisms for $b\bar{b}$ and $c\bar{c}$: with increasing beam energy $\sqrt{s_{NN}}$ $R_{AA}(b\bar{b}) \downarrow$ (primordial suppression) $R_{AA}(c\bar{c}) \uparrow$ increased regeneration. $\boxed{\text{Fig. 5&6}}$
3. J/ψ flows with the bulk matter, partial kinetic equilibration ($v_2 > 0$) $\boxed{\text{Fig. 7}}$

Current question of interest: Many different models with different underlying physics mechanisms can reproduce R_{AA} of ground state. What are more discriminating observables e.g. $J/\psi / \psi'$ ratio.

(see e.g. 1705.05810)

<u>Concrete questions to theory:</u>

1. How do masses and decay widths of $Q\bar{Q}$ change @ $T>0$
2. What is the real-time evolution of a $Q\bar{Q}$ in a HIC?

$\Rightarrow$ Since $T_{HIC} \lesssim 3T_c$ need nonperturbative methods to compute quarkonium correlation functions (Lattice QCD + effective field theory for heavy quarks)

$\Rightarrow$ Need numerical methods to extract spectral information (m, Γ) from correlators. (Bayesian inference)

$\Rightarrow$ Need to set up a potential description for in-medium $Q\bar{Q}$ similar to the $T=0$ case to implement real time evolution (EFT + Lattice + Bayesian inference)

III

You have already listened to dedicated lectures on lattice QCD, where the determination of meson properties is a central topic. So why should we use additional effective field theory techniques? $\Rightarrow$ Separation of scales

On the lattice: $\Lambda_{UV} \sim a^{-1} \gg M_{Q\bar{Q}} \gg E_{bind} > T \sim m_\pi \gg \Lambda_{IR} \sim L^{-1}$

T = 150 MeV
0,197 fm GeV = 1 $T^{-1} = 1{,}3$ fm

$m_{J/\psi}\, a < \frac{1}{2} \to a < 0{,}03$ fm $\quad m_\Upsilon \cdot a < \frac{1}{2} \to a < 0{,}01$ fm

Save box size 3 fm $\quad N_x = 100 \ldots 300$ (current realistic $48^3 \times 12 \ldots 24$ full QCD, quenched $92^3 \times 96$)

Let us have a look at non-interacting standard Wilson fermions: (Euclidean time)

$$S_{lat} = \sum_x \bar{\psi}(x) \left\{ m_0 + \frac{\Delta_\mu^+ + \Delta_\mu^-}{2}\gamma^\mu - a\frac{\Delta^-\Delta^+}{2} \right\} \psi(x)$$

$a\Delta_\mu^+ \psi = U_\mu \psi(x + a\mu) - \psi(x)$
$\tilde{p}_\mu = \frac{1}{a}\sin(p_\mu a) \quad p_\mu \in \frac{\pi}{N a}\nu \quad \nu \in \left(-\frac{N}{2}, \ldots, \frac{N}{2}\right]$
$\hat{p}^2 = \sum_\mu (\hat{p}_\mu)^2 \quad \hat{p}_\mu = \frac{2}{a}\sin\left(\frac{a p_\mu}{2}\right)$

at first $T=0$ infinite volume limit $L \to \infty$

$$= \int_{-\pi/a}^{\pi/a} d^4p\, \bar{\tilde{\psi}}(p) \left\{ m_0 + i\tilde{p}_\mu \gamma_\mu + \frac{a}{2}\hat{p}^2 \right\} \tilde{\psi}(p)$$

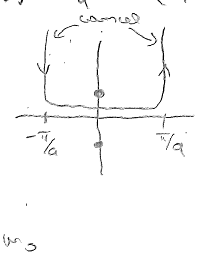
(see e.g. Montvay, Münster)

$\Rightarrow$ Propagator reads $$G_W(p) = \langle \bar{\psi}(-p)\psi(p) \rangle = \frac{\left(m_0 + \frac{a}{2}\hat{p}^2\right) - i\tilde{p}_\mu \gamma_\mu}{\left(m_0 + \frac{a}{2}\hat{p}^2\right)^2 + \tilde{p}^2} \quad \text{c.f.} \quad \frac{m - i\gamma p}{m^2 + p^2}$$

To read off the mass of the quark and dispersion relation, need to compute time-slice correlator (Montvay–Münster)

$$C(\tau, \vec{p}=0) = \frac{1}{V}\sum_x \langle \psi_{x,\tau} \bar{\psi}_{0,0} \rangle = \int_{-\pi/a}^{\pi/a} \frac{dk_4}{(2\pi)} e^{i\tau k_4}\, G(\vec{0}, k_4) \qquad \frac{\#}{\left(m_0 + \frac{2}{a}\sin^2\left(\frac{a p_4}{2}\right)\right)^2 + \frac{1}{a^2}\sin^2(p_4 a)}$$

Anticipate that the poles lie in the complex plane $E = -ip_4$



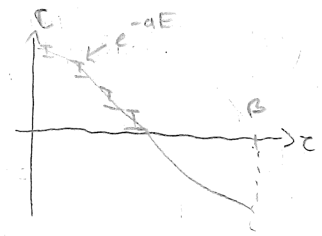
$$\left[m_0 - \frac{2}{a}\sinh^2\left(\frac{aE}{2}\right)\right]^2 - \sinh(Ea) = 0 \quad \Rightarrow \quad E_\pm = \pm a^{-1}\log(1 + a m_0)$$

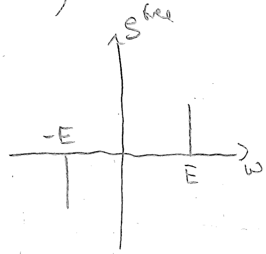
$$\approx m_0 - \underbrace{a\frac{m_0^2}{2}}_{\mathcal{O}(a) \text{ enhanced by } m_0} + \ldots$$

$$C(\tau, \vec{p}=0) = e^{-a E_1 |\tau|} + \delta_{\tau 0} \frac{1}{2(1 + a m_0)}$$

For finite Euclidean time extent: $\sum_{m=-\infty}^{\infty} e^{\frac{i m k_4}{N_\tau}} = \frac{1}{N_\tau}\sum_{\nu_4=0}^{N_\tau - 1} 2\pi\, \delta\left(k_4 - \frac{2\pi}{T}\nu_4\right)$
and anti periodic boundary conditions

$$= \sum_{m=-\infty}^{\infty} e^{-aE|\tau + m N_\tau|} = \frac{e^{-a\tau E} - e^{-aE(N_\tau - \tau)}}{1 + e^{-aEN_\tau}}$$





$$\Rightarrow \quad C(\tau, \vec{p}=0) = \int_{-\infty}^{\infty} d\omega\, \frac{e^{-\tau\omega}}{1 + e^{-\beta\omega}} \left(\underbrace{\delta(\omega - E)}_{\text{backward}} - \underbrace{\delta(\omega + E)}_{\text{forward}}\right)$$

In general for relativistic correlators $\left\{ \begin{matrix} - \text{Boson} \\ + \text{Fermion} \end{matrix} \right.$ $$G(\tau) = \int d\omega\, \frac{e^{-\tau\omega}}{1 \mp e^{-\beta\omega}}\, \rho(\omega)$$

$\Rightarrow$ <u>Practical problem</u>: We loose $N_\tau/2$ datapoints, since redundant information.
Makes extracting information difficult if realistic $N_\tau = 12 \ldots 24$
- Exponential decay with m_0 large requires extremely high statistics.