Spectral quantities in thermal QCD

III: baryons

Gert Aarts

Swansea University
Prifysgol Abertawe

Baryons

correlators $$G^{\alpha\alpha'}(x) = \langle O^{\alpha}(x)\,\overline{O}^{\alpha'}(0)\rangle$$

examples: N, Δ, Ω baryons

$$O_N^{\alpha}(x) = \epsilon_{abc}\, u_a^{\alpha}(x)\left(d_b^T(x) C\gamma_5 u_c(x)\right)$$

$$O_{\Delta,i}^{\alpha}(x) = \epsilon_{abc}\left[2u_a^{\alpha}(x)\left(d_b^T(x) C\gamma_i u_c(x)\right) + d_a^{\alpha}(x)\left(u_b^T(x) C\gamma_i u_c(x)\right)\right]$$

$$O_{\Omega,i}^{\alpha}(x) = \epsilon_{abc}\, s_a^{\alpha}(x)\left(s_b^T(x) C\gamma_i s_c(x)\right)$$

with C charge conjugation matrix:

$$C^{\dagger}C = \mathbb{1} \qquad \gamma_{\mu}^T = -C\gamma_{\mu}C^{-1} \qquad C^T = -C^{-1}$$

action on fermionic operator:

$$\mathcal{C}O\mathcal{C}^{-1} = O^{(c)} = C^{-1}\bar{O}^T \qquad \mathcal{C}\bar{O}\mathcal{C}^{-1} = \bar{O}^{(c)} = -O^T C$$

Baryons

- essential difference with mesons: role of parity

$$\mathcal{P}O(\tau,\mathbf{x})\mathcal{P}^{-1} = \gamma_4 O(\tau,-\mathbf{x})$$

- positive/negative parity operators

$$O_\pm(x) = P_\pm O(x) \qquad\qquad P_\pm = \frac{1}{2}(1\pm\gamma_4)$$

- no parity doubling in Nature: nucleon ground state

positive parity: $m_+ = m_N \;\; = 0.939$ GeV

negative parity: $m_- = m_{N^*} = 1.535$ GeV

- thread: what happens as temperature increases?

Outline

baryons across the deconfinement transition:

- some basic thermal field theory
- lattice QCD – FASTSUM collaboration
- baryon correlators
- in-medium effects below T_c
- parity doubling above T_c
- spectral functions

FASTSUM: PRD 92 (2015) 014503 [arXiv:1502.03603 [hep-lat]]

+ JHEP 06 (2017) 034 [arXiv:1703.09246 [hep-lat]]

+ in preparation

Spectral decomposition: Positivity

$$\rho^{\alpha\beta}(x) = \sum \gamma_\mu^{\alpha\beta} \rho_\mu(x) + 1\!\!1^{\alpha\beta} \rho_m(x)$$

- take trace with γ_4, $P_\pm = (1\!\!1 \pm \gamma_4)/2$:

$$\rho_4(p) = \frac{1}{Z} \sum_{n,m,\alpha} \left(e^{-k_n^0/T} + e^{-k_m^0/T} \right) \frac{1}{4} \left| \langle n | O^\alpha(0) | m \rangle \right|^2 (2\pi)^4 \delta^{(4)}(p+k_n-k_m)$$

$$\rho_\pm(p) = \frac{\pm 1}{Z} \sum_{n,m,\alpha} \left(e^{-k_n^0/T} + e^{-k_m^0/T} \right) \frac{1}{4} \left| \langle n | O_\pm^\alpha(0) | m \rangle \right|^2 (2\pi)^4 \delta^{(4)}(p+k_n-k_m)$$

- $\rho_4(p), \pm\rho_\pm(p) \geq 0$ for all ω
- take trace with $1\!\!1$

$$\rho_m(p) = [\rho_+(p) + \rho_-(p)]/4$$

not sign definite

Charge conjugation

charge conjugation is a symmetry (at vanishing density):

- apply to correlator

$$G(x-x') = \langle \mathsf{T}_\tau \left[\mathcal{C}O(x)\overline{O}(x')\mathcal{C}^{-1}\right]\rangle = \left\langle C^{-1}\mathsf{T}_\tau\left[\left(O(x')\overline{O}(x)\right)^T\right]C\right\rangle$$

$$= C^{-1}G^T(x'-x)C$$

- use cyclicity $G(x'-x) = G(-\tau,\mathbf{x}'-\mathbf{x}) = -G(1/T-\tau,\mathbf{x}'-\mathbf{x})$
- this yields $G(\tau,\mathbf{p}) = -C^{-1}G^T(1/T-\tau,\mathbf{p})C$
- take the trace with $P_\pm$ (and use $\left(CP_\pm C^{-1}\right)^T = P_\mp$)

$$G_\pm(\tau,\mathbf{p}) = \mathrm{tr}P_\pm G(\tau,\mathbf{p}) = -\mathrm{tr}P_\pm C^{-1}G^T(1/T-\tau,\mathbf{p})C$$

$$= -\mathrm{tr}\left(CP_\pm C^{-1}\right)^T G(1/T-\tau,\mathbf{p}) = -\mathrm{tr}P_\mp G(1/T-\tau,\mathbf{p})$$

$$= -G_\mp(1/T-\tau,\mathbf{p})$$

Charge conjugation

charge conjugation symmetry (at vanishing density):

$$G_\pm(\tau,\mathbf{p}) = -G_\mp(1/T-\tau,\mathbf{p}) \qquad \rho_\pm(-\omega,\mathbf{p}) = -\rho_\mp(\omega,\mathbf{p})$$

- relates pos/neg parity channels

using $G_+(\tau,\mathbf{p})$ and $\rho_+(\omega,\mathbf{p})$

- positive- (negative-) parity states propagate forward (backward) in euclidean time
- negative part of spectrum of $\rho_+ \leftrightarrow$ positive part of ρ_-

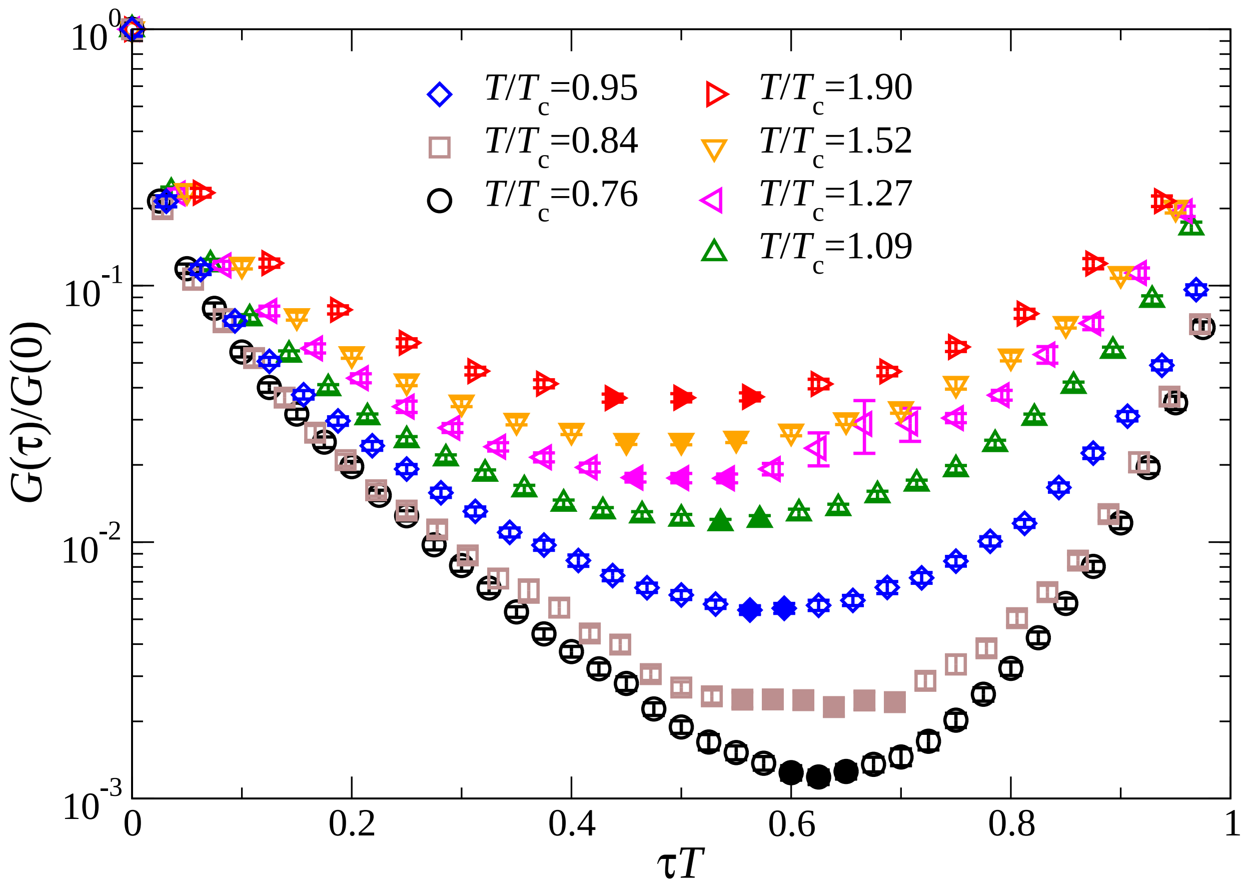
example: single state

$$G_+(\tau) = A_+e^{-m_+\tau} + A_-e^{-m_-(1/T-\tau)}$$

$$\rho_+(\omega)/(2\pi) = A_+\delta(\omega-m_+) + A_-\delta(\omega+m_-)$$

Nucleon correlators

- euclidean correlator $G_+(\tau)$



- not symmetric around $\tau = 1/2T$ below T_c
- more symmetric as temperature increases

Chiral symmetry

- propagator

$$G^{\alpha\beta}(x) = \sum_\mu \gamma_\mu^{\alpha\beta} G_\mu(x) + 1\!\!1^{\alpha\beta} G_m(x)$$

- chiral symmetry $\qquad \{\gamma_5, G\} = 0 \quad \Rightarrow \quad G_m = 0$
- hence

$$G_+(\tau, \mathbf{p}) = -G_-(\tau, \mathbf{p}) = G_+(1/T - \tau, \mathbf{p}) = 2G_4(\tau, \mathbf{p})$$

degeneracy of $\pm$ parity channels

$$\rho_+(p) = -\rho_-(p) = \rho_+(-p) = 2\rho_4(p)$$

- parity doubling
- in Nature at $T = 0$: no chiral symmetry/parity doubling

Parity and chiral symmetry

however, if chiral symmetry is unbroken ($m_q = 0$ and no SSB)

- degeneracy between pos/neg parity channels already at the level of the correlators

what happens at the confinement/deconfinement transition?

- SU(2)$_{\rm A}$ chiral symmetry restored
- expect degeneracies to emerge
- how does this affect mass spectrum?
- role of $m_s > m_{u,d}$?

Aside: nonzero chemical potential

- charge conjugation: baryon number changes sign
- also change $\mu \to -\mu$ to keep invariance
- relations:

$$G_\pm(\tau, \mathbf{p}; \mu) = -G_\mp(1/T - \tau, \mathbf{p}; -\mu)$$
$$\rho_\pm(-\omega, \mathbf{p}; \mu) = -\rho_\mp(\omega, \mathbf{p}; -\mu)$$

- single groundstate

$$G_+(\tau; \mu) = A_+(\mu)e^{-(m_+-\mu)\tau} + A_-(-\mu)e^{-(m_-+\mu)(1/T-\tau)}$$
$$-G_-(\tau; \mu) = A_-(\mu)e^{-(m_--\mu)\tau} + A_+(-\mu)e^{-(m_++\mu)(1/T-\tau)}$$

- chiral symmetry unbroken

$$G_+(\tau, \mathbf{p}; \mu) = -G_-(\tau, \mathbf{p}; \mu) = G_+(1/T - \tau, \mathbf{p}; -\mu) = 2G_4(\tau, \mathbf{p}; \mu)$$
$$\rho_+(p; \mu) = -\rho_-(p; \mu) = \rho_+(-p; -\mu) = 2\rho_4(p; \mu)$$

FASTSUM

- anisotropic $N_f = 2 + 1$ Wilson-clover ensembles

FASTSUM collaboration

GA (Swansea)
Chris Allton (Swansea)
Simon Hands (Swansea)
Seyong Kim (Sejong University)
Maria-Paola Lombardo (Frascati)
Sinead Ryan (Trinity College Dublin)
Don Sinclair (Argonne)
Jonivar Skullerud (Maynooth)

Ale Amato (Swansea->Helsinki->)
Davide de Boni (Swansea)
Wynne Evans (Swansea->Bern->)
Pietro Giudice (Swansea->Münster->)
Tim Harris (TCD->Mainz->Milan)
Benjamin Jäger (Swansea->ETH)
Aoife Kelly (Maynooth)
Bugra Oktay (Utah->)
Kristi Praki (Swansea)

This work

GA, Chris Allton, Simon Hands, Kristi Praki, Jonivar Skullerud

Davide de Boni, Benjamin Jäger

PRD 92 (2015) 014503, arXiv:1502.03603 [hep-lat]

JHEP 06 (2017) 034, arXiv:1703.09246 [hep-lat]

in preparation

FASTSUM ensembles

- $N_f = 2 + 1$ dynamical quark flavours, Wilson-clover
- many temperatures, below and above T_c
- anisotropic lattice, $a_s/a_\tau = 3.5$, many time slices
- strange quark: physical value
- two light flavours: somewhat heavy $m_\pi = 384(4)$ MeV

N_s	24	24	24	24	24	24	24	24
N_τ	128	40	36	32	28	24	20	16
T/T_c	0.24	0.76	0.84	0.95	1.09	1.27	1.52	1.90
N_{cfg}	140	500	500	1000	1000	1000	1000	1000
N_{src}	16	4	4	2	2	2	2	2

- tuning and $N_\tau = 128$ data from HadSpec collaboration

FASTSUM ensembles

anisotropic lattice $a_\tau < a_s$: why?

- many more time slices compared to isotropic lattice
- coarser spatial lattice/finer temporal lattice
- use fixed-scale approach:
  change temperature by changing N_τ, not cutoff
- ideally suited for spectroscopy:
  many time slices and many temperatures

downsides:

- nonperturbative tuning of anisotropic parameters
  "renormalisation of the speed of light"
- multi-dimensional parameter space

Baryon correlators

computed all octet and decuplet baryon correlators

$S = 0$:	N	Δ	
$S = -1$:	Λ	Σ	Σ^*
$S = -2$:	Ξ	Ξ^*	
$S = -3$:	Ω		

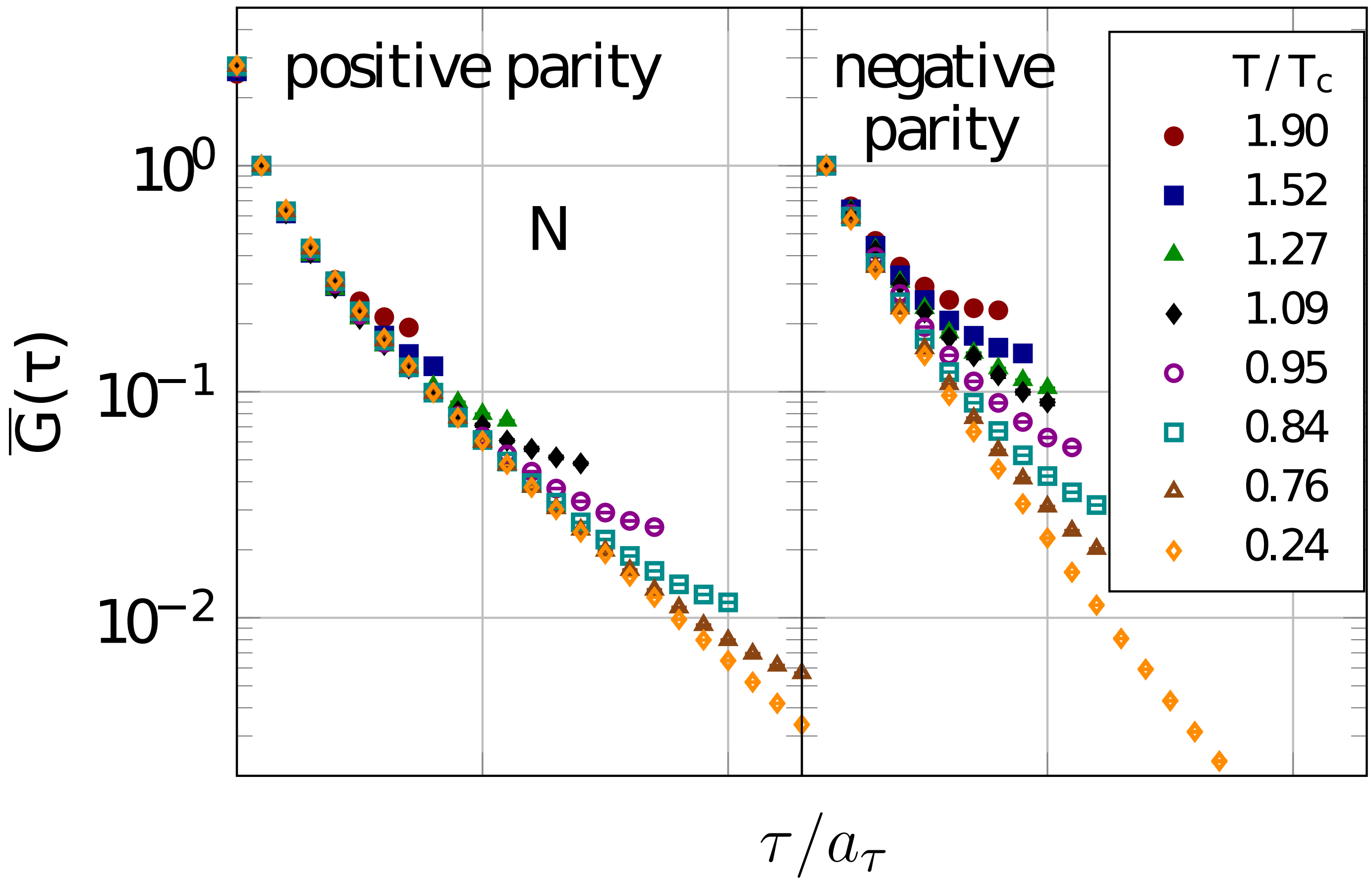
for each baryon: positive and negative parity channels

technical remarks

- studied various interpolation operators
- Gaussian smearing for multiple sources and sinks
- same smearing parameters at all temperatures

Lattice correlators

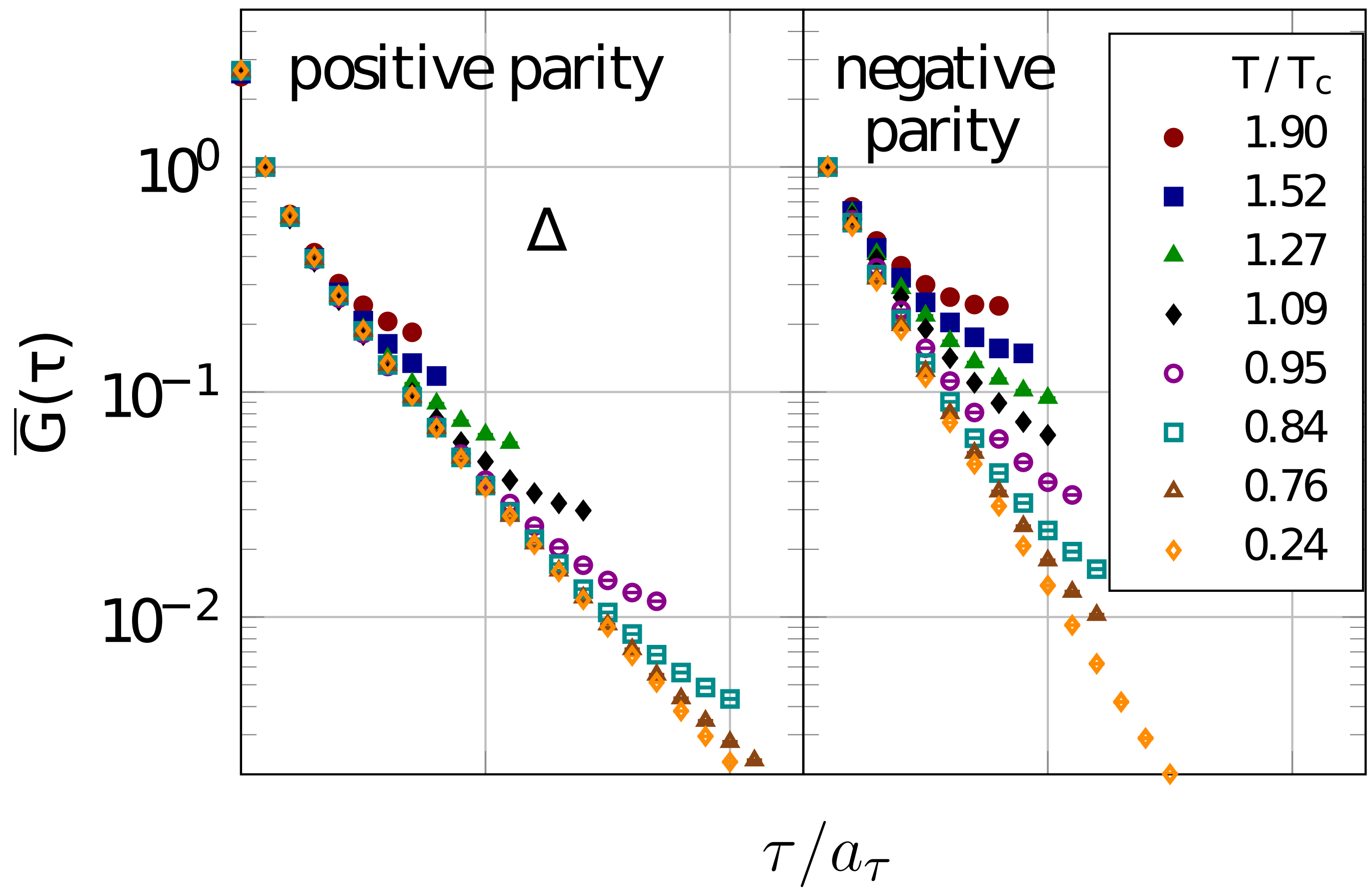
- nucleon



- pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Lattice correlators

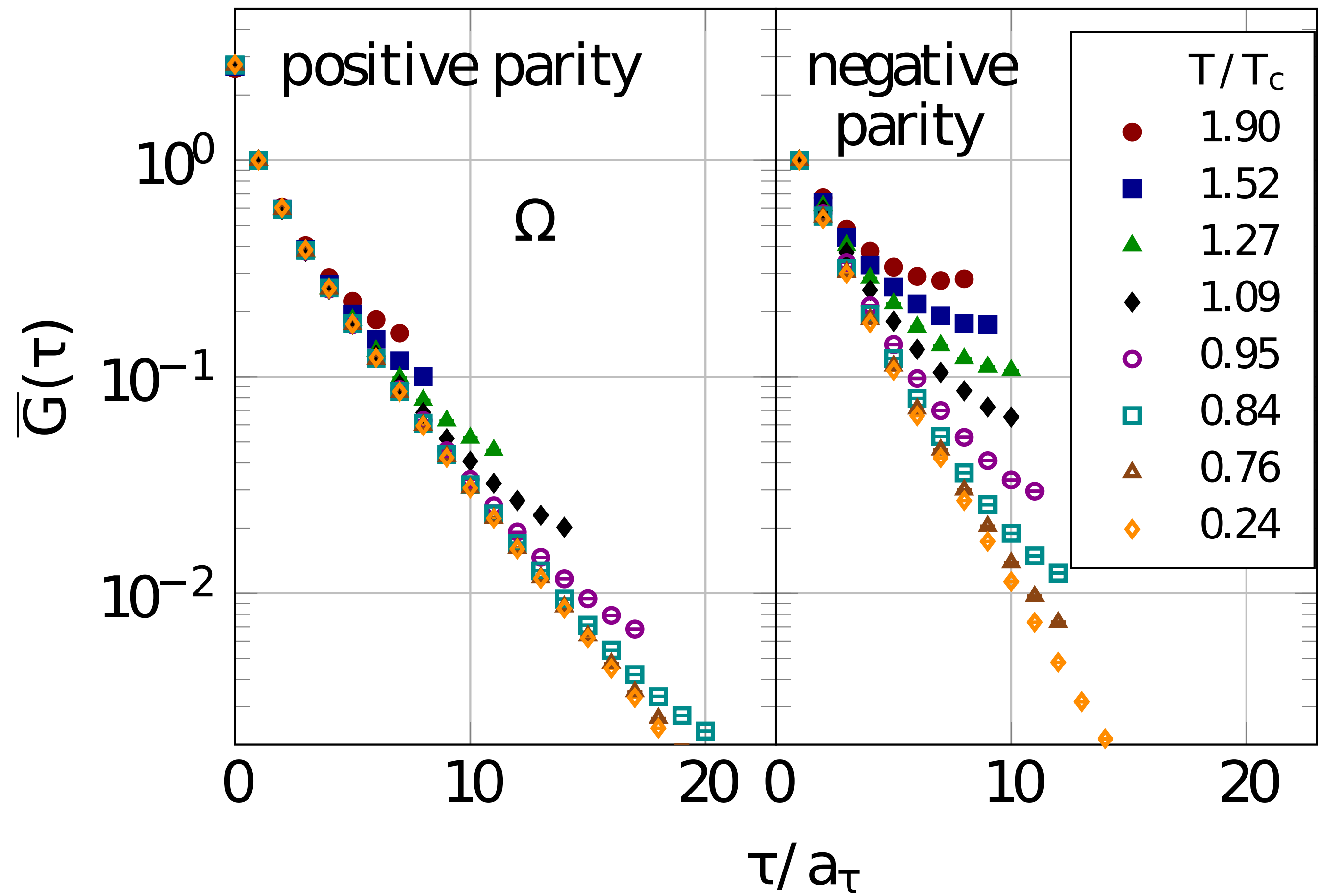
- Δ



- at low T pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Lattice correlators

- Ω



- at low T pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Baryons in the hadronic phase

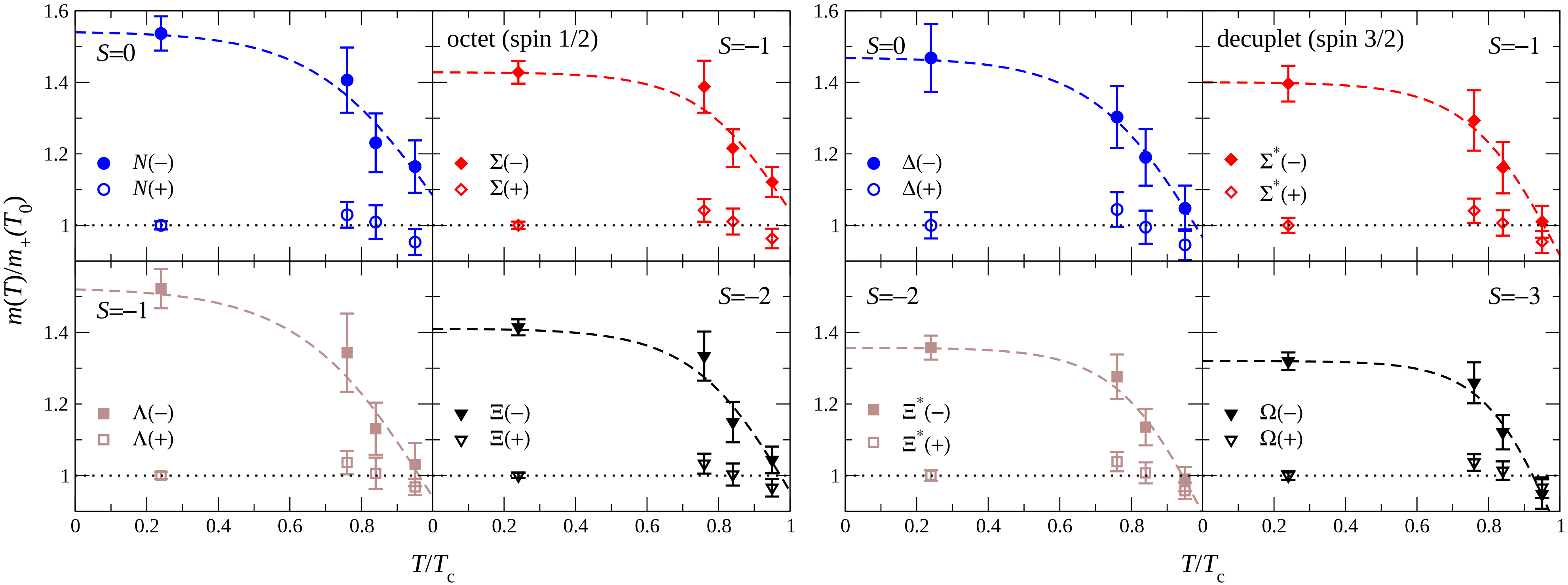
- determine masses of pos/neg parity groundstates
- in-medium effects

Masses of pos/neg parity groundstates (in MeV)

S	T/T_c	0.24	0.76	0.84	0.95	PDG ($T=0$)
0	m_+^N	1158(13)	1192(39)	1169(53)	1104(40)	939
	m_-^N	1779(52)	1628(104)	1425(94)	1348(83)	1535
	m_+^Δ	1456(53)	1521(43)	1449(42)	1377(37)	1232
	m_-^Δ	2138(114)	1898(106)	1734(97)	1526(74)	1700
-1	m_+^Σ	1277(13)	1330(38)	1290(44)	1230(33)	1193
	m_-^Σ	1823(35)	1772(91)	1552(65)	1431(51)	1750
	m_+^Λ	1248(12)	1293(39)	1256(54)	1208(26)	1116
	m_-^Λ	1899(66)	1676(136)	1411(90)	1286(75)	1405–1670
	$m_+^{\Sigma^*}$	1526(32)	1588(40)	1536(43)	1455(35)	1385
	$m_-^{\Sigma^*}$	2131(62)	1974(122)	1772(103)	1542(60)	1670–1940
-2	m_+^Ξ	1355(9)	1401(36)	1359(41)	1310(32)	1318
	m_-^Ξ	1917(27)	1808(92)	1558(76)	1415(50)	1690–1950
	$m_+^{\Xi^*}$	1594(24)	1656(35)	1606(40)	1526(29)	1530
	$m_-^{\Xi^*}$	2164(42)	2034(95)	1810(77)	1578(48)	1820
-3	m_+^Ω	1661(21)	1723(32)	1685(37)	1606(43)	1672
	m_-^Ω	2193(30)	2092(91)	1863(76)	1576(66)	2250

Baryons in the hadronic phase

masses $m_{\pm}(T)$, normalised with m_{+} at lowest temperature



in each channel:

- emerging degeneracy around T_c
- negative-parity masses reduced as T increases
- positive-parity masses nearly T independent

Baryons in the hadronic phase

findings

- positive-parity masses nearly T independent
- negative-parity masses reduced as T increases
- characteristic behaviour

$$m_-(T) = w(T,\gamma)m_-(0) + [1 - w(T,\gamma)]m_-(T_c)$$

with one-parameter transition function

$$w(T,\gamma) = \tanh[(1 - T/T_c)/\gamma]/\tanh(1/\gamma)$$

- small (large) $\gamma \Leftrightarrow$ narrow (broad) transition region

fits in each channel

- $0.22 \lesssim \gamma \lesssim 0.35$, mean $\gamma = 0.27(1)$
- $0.85 \lesssim m_-(T_c)/m_+(0) \lesssim 1.1$

Baryons and parity partners

- distinct temperature dependence in hadronic phase
- relevant for heavy-ion phenomenology?

Baryons and parity partners

- distinct temperature dependence in hadronic phase
- relevant for heavy-ion phenomenology?

model studies of the role of chiral symmetry

example: parity doublet model deTar & Kunihiro 89

- chiral invariant contribution m_0 equal for N and N^*
- mass splitting due to chiral symmetry breaking
- degeneracy emerges as chiral symmetry is restored
- $m_0 \sim 500 - 800$ MeV

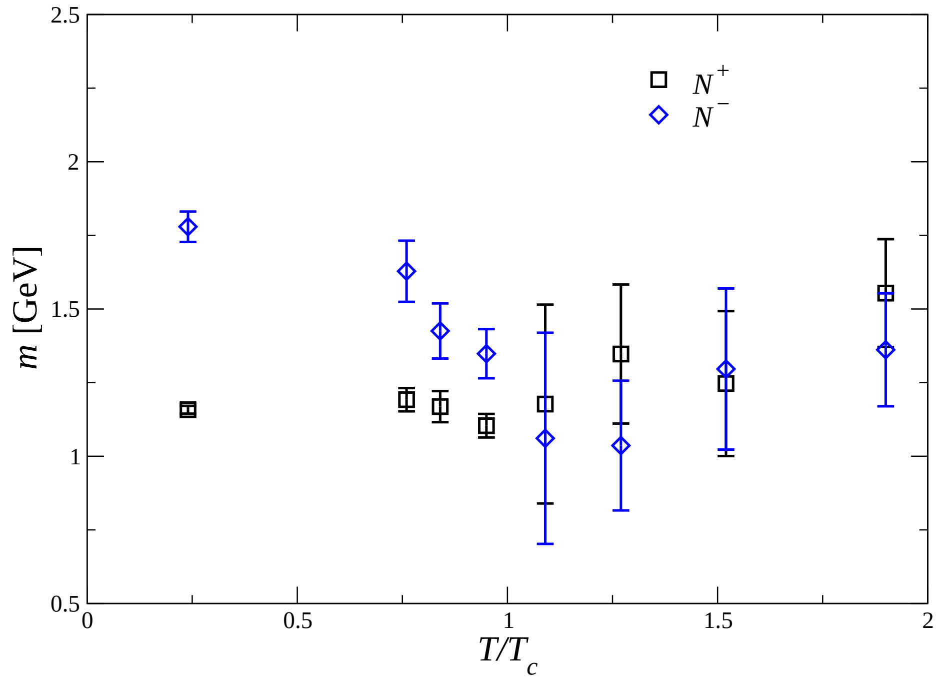
holographic predictions?

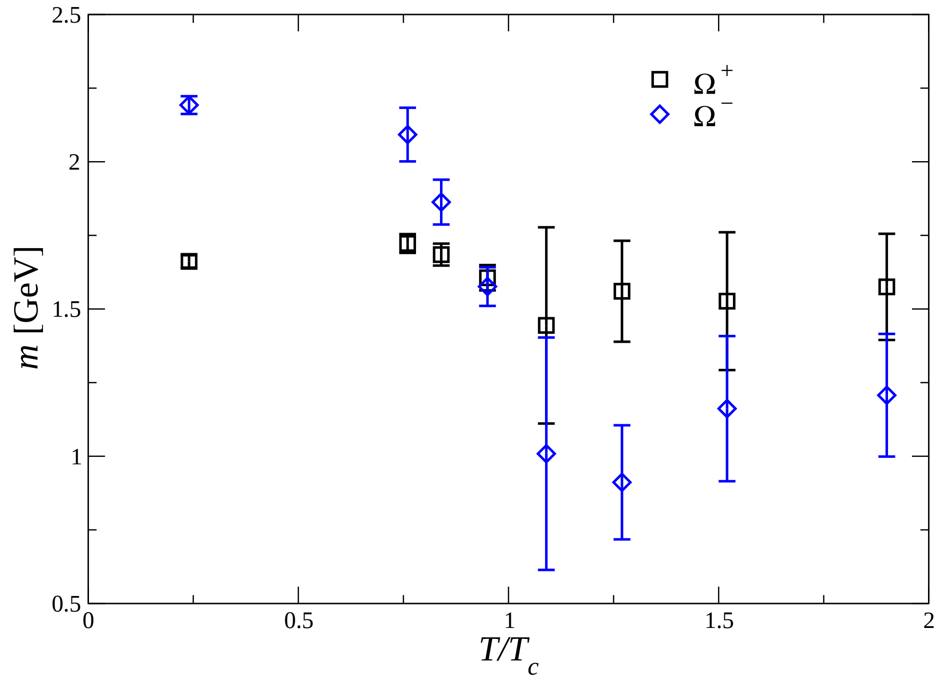
QGP: fate of light baryons

consider now the quark-gluon plasma

- no clearly identifiable groundstates: baryons dissolved

example: use conventional exponential fits





no clearly defined groundstates above T_c

QGP: fate of light baryons

- no clearly identifiable groundstates: baryons dissolved
- chiral symmetry restoration $\Leftrightarrow$ parity doubling
- study correlator ratio Datta, Gupta, Mathur et al 2013

$$R(\tau) = \frac{G_+(\tau) - G_-(\tau)}{G_+(\tau) + G_-(\tau)}$$

- no parity doubling and $m_- \gg m_+$: $R(\tau) = 1$
- parity doubling: $R(\tau) = 0$

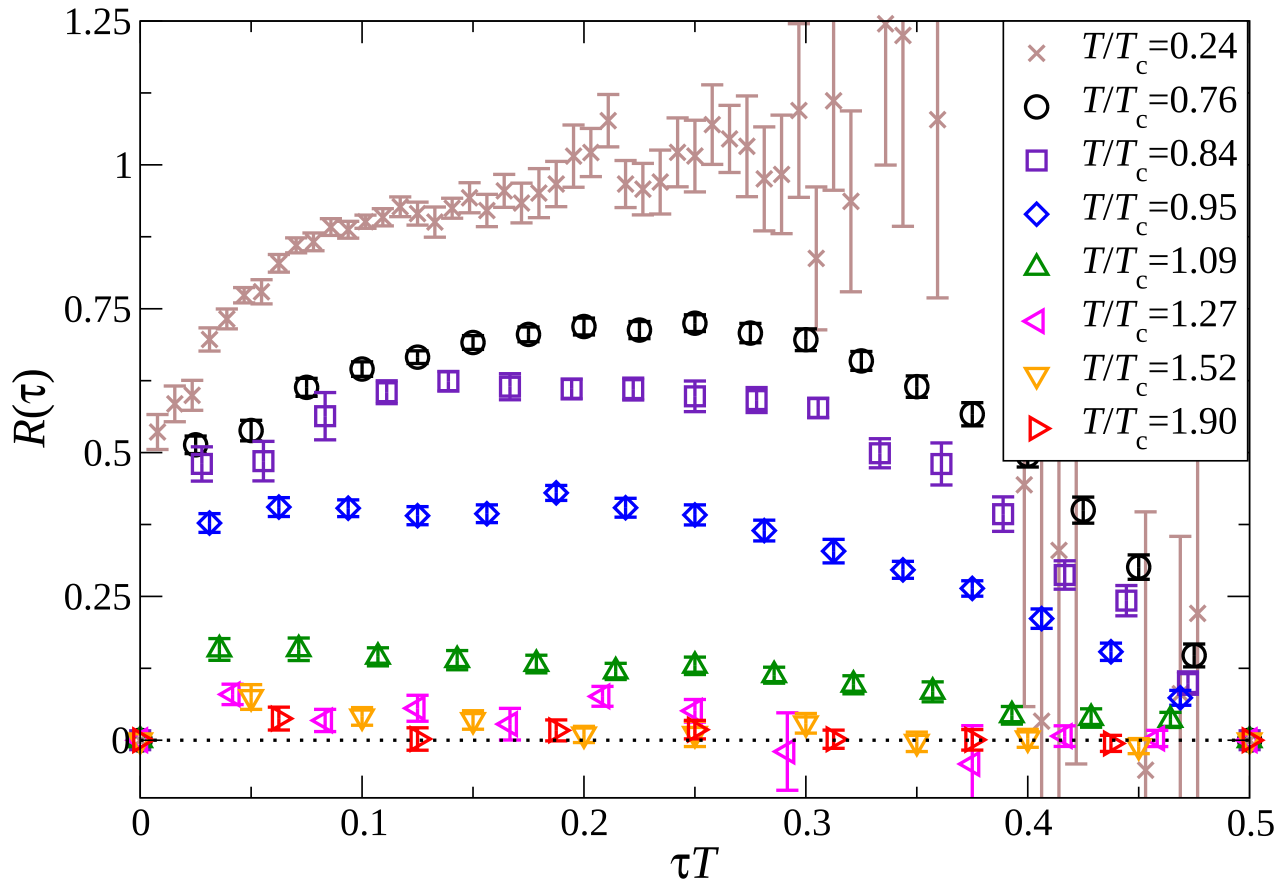
by construction: $R(1/T - \tau) = -R(\tau)$ and $R(1/2T) = 0$

- integrated ratio

$\Rightarrow$ quasi-order parameter

$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)}$$

Nucleon channel

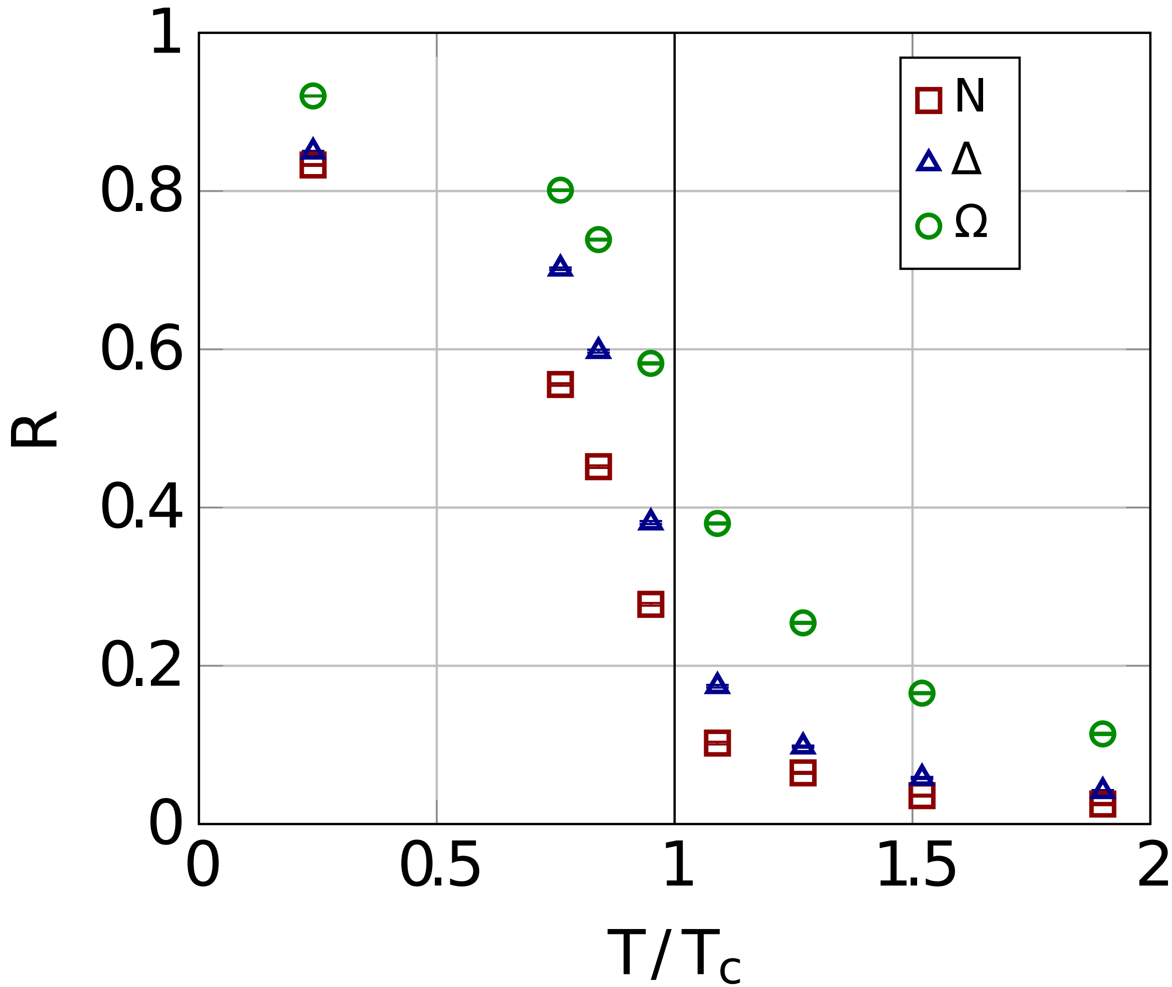


- ratio close to 1 below T_c, decreasing uniformly
- ratio close to 0 above T_c, parity doubling

Quasi-order parameter

- integrated ratio

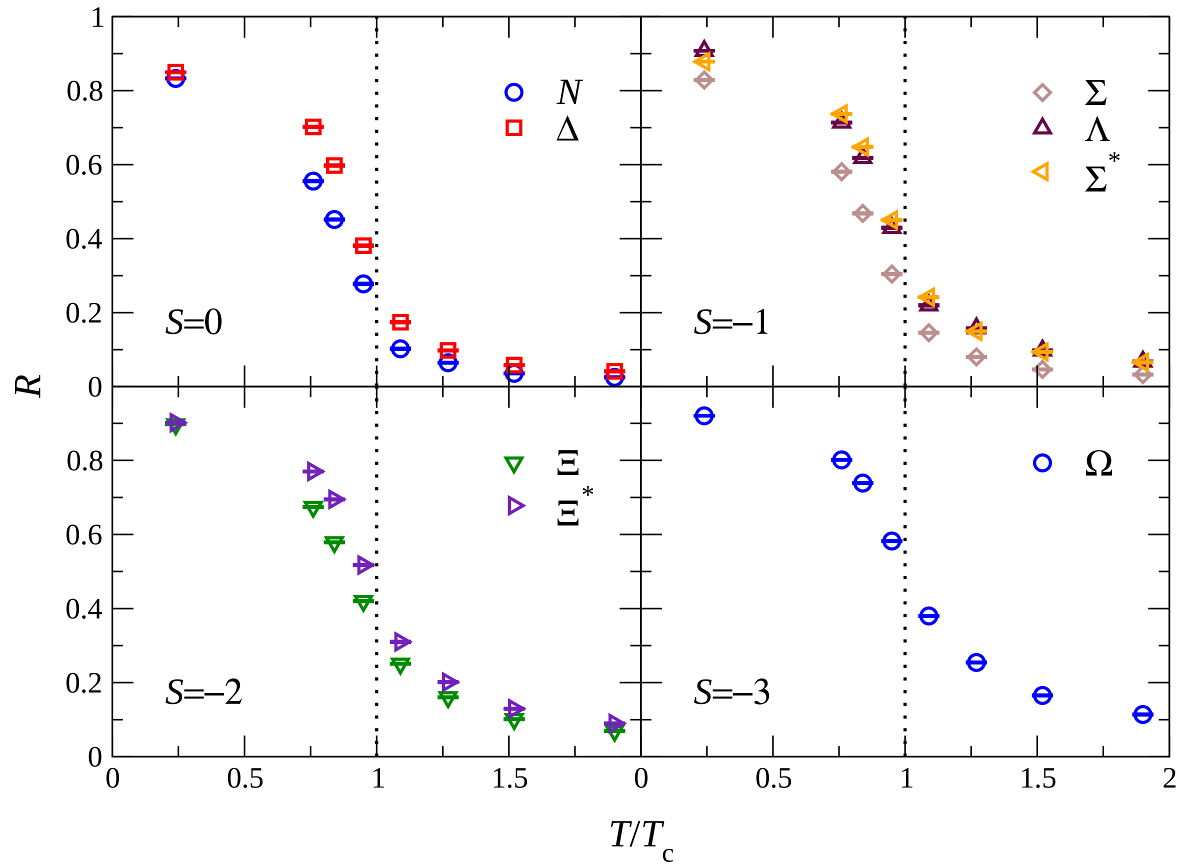
$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)}$$



- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: $m_q \neq 0$
- effect of heavier s quark visible

Quasi-order parameter

parity doubling in the QGP: $R \sim 1 \rightarrow 0$



- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: $m_q \neq 0$
- effect of heavier s quark visible

Parity doubling

- clear signal for parity doubling
  even with finite quark masses
- crossover behaviour, coinciding with transition to QGP
- visible effect of heavier s quark

lattice technical remark:

- Wilson fermions break chiral symmetry at short distances
  what about chiral lattice fermions?

Spectral functions

extract same information from spectral functions

$$G_{\pm}(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho_{\pm}(\omega) \qquad K(\tau,\omega) = \frac{e^{-\omega\tau}}{1+e^{-\omega/T}}$$

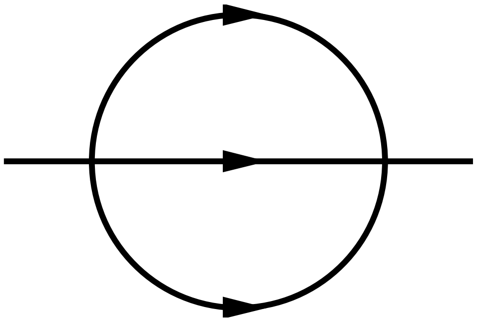
expectation at very high temperature

- compute baryon spectral functions at $g^2 \to 0$
- similar to computation of meson spectral functions

Karsch et al 03, GA & Martínez Resco 05

Free spectral functions

lowest order in perturbation theory



$$G(x) = \langle O(x)\overline{O}(0)\rangle \qquad O(x) \sim uu^T C\gamma_5 d(x)$$

two-loop diagram $\qquad (c = 4, i, m)$

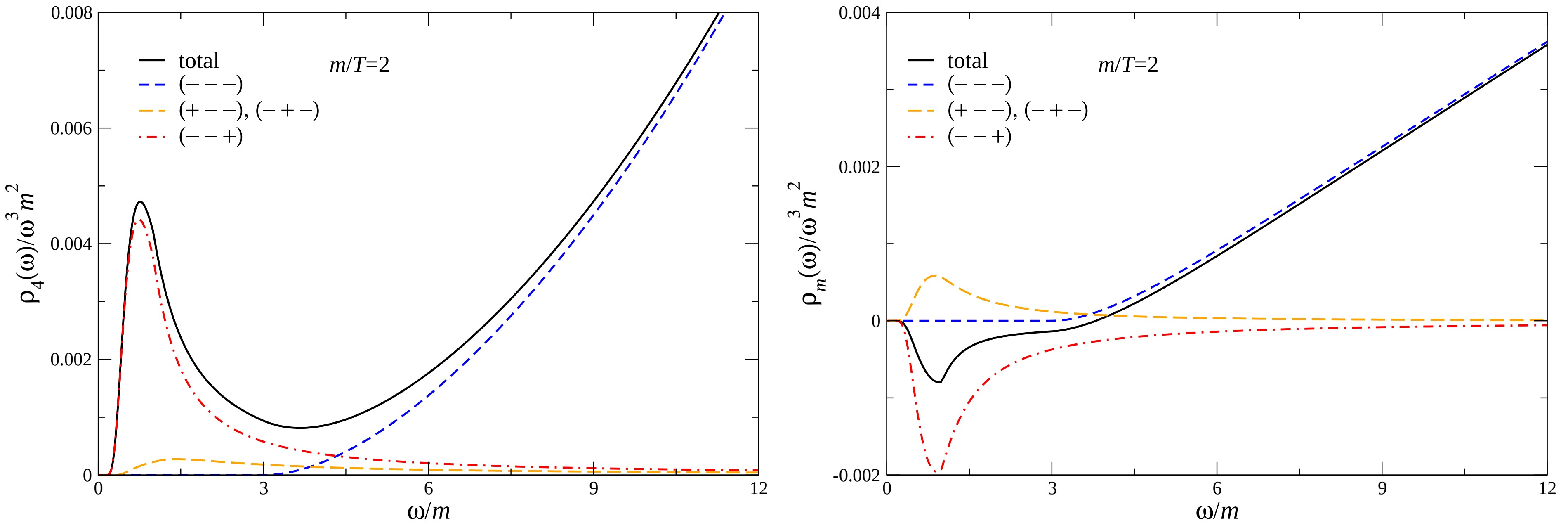
$$\rho_c(\omega) = 3\int_{\mathbf{k}_{1,2,3}} d\Phi_{123} \sum_{s_j=\pm} 2\pi\delta\left(\omega + \sum_j s_j\omega_{\mathbf{k}_j}\right)[\text{stat.}]f_c(\omega, s_i, \mathbf{k}_i)$$

with

$$d\Phi_{123} = \prod_{j=1}^{3} \frac{d^3k_j}{(2\pi)^3 2\omega_{\mathbf{k}_j}}(2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

$$[\text{stat.}] = n_F(s_1\omega_{\mathbf{k}_1})n_F(s_2\omega_{\mathbf{k}_3})n_F(s_3\omega_{\mathbf{k}_3}) + n_F(-s_1\omega_{\mathbf{k}_1})n_F(-s_2\omega_{\mathbf{k}_3})n_F(-s_3\omega_{\mathbf{k}_3})$$

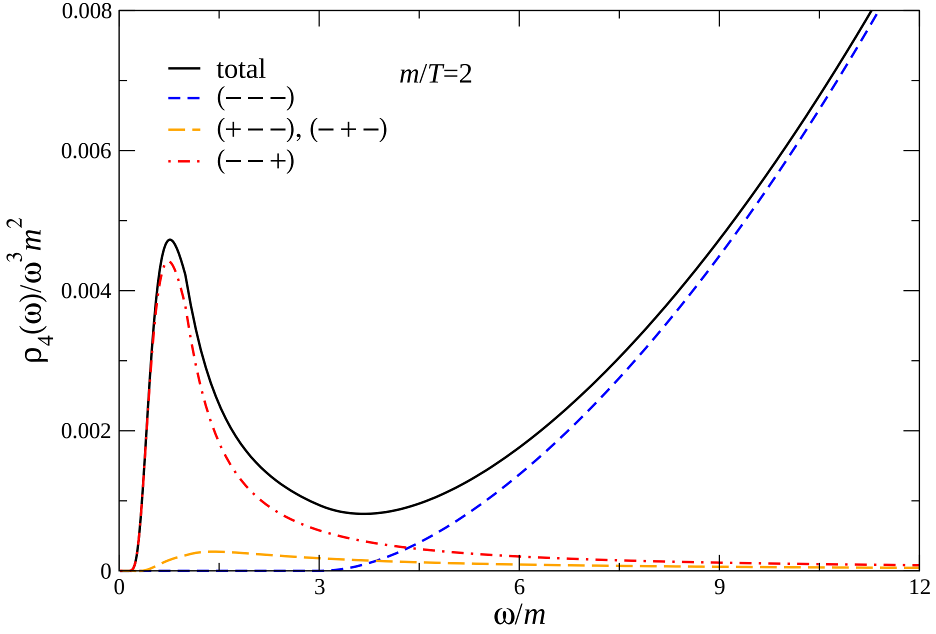
Free spectral functions



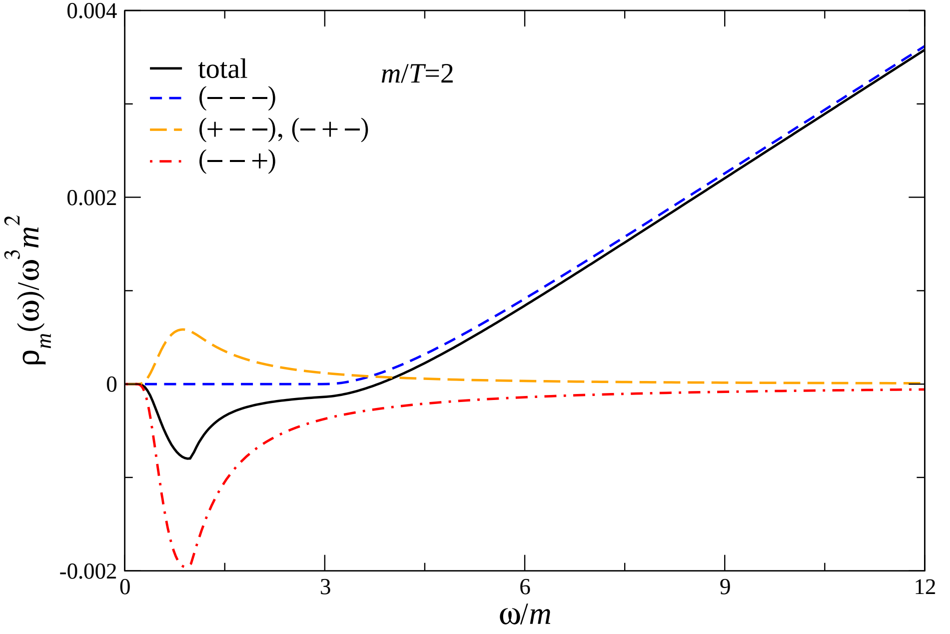
$\rho_4(\omega)$ $\rho_m(\omega)$

- decay: $\omega > 3m$ with m quark mass
- at $T > 0$ scattering contributions for all ω
- large ω: thermal contributions suppressed
- $\rho_m(\omega)$ not positive definite

Free spectral functions



$\rho_4(\omega)$



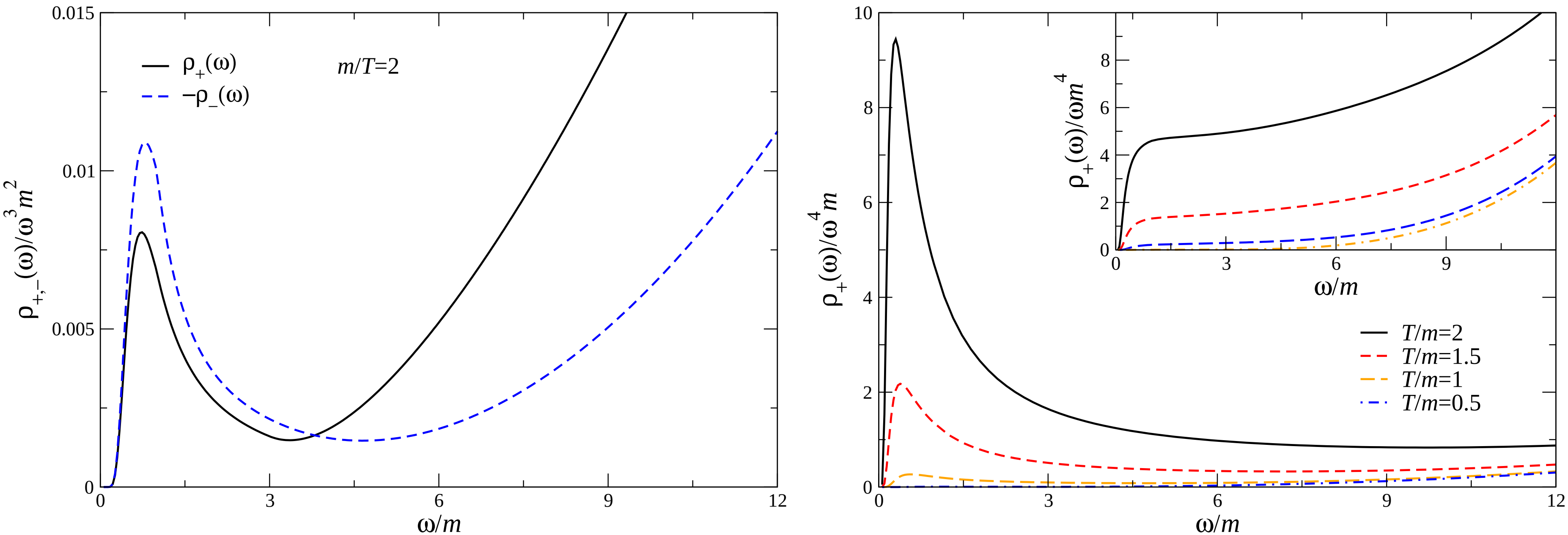
$\rho_m(\omega)$

$$\omega \gg T \gg m$$

$$\rho_4(\omega) = \frac{5\omega^5}{2048\pi^3}\left(1 + \frac{112\pi^4}{3}\frac{T^4}{\omega^4} + \dots\right)$$

$$\rho_m(\omega) = \frac{7m\omega^4}{512\pi^3}\left(1 - 4\pi^2\frac{T^2}{\omega^2} + \dots\right)$$

Free spectral functions

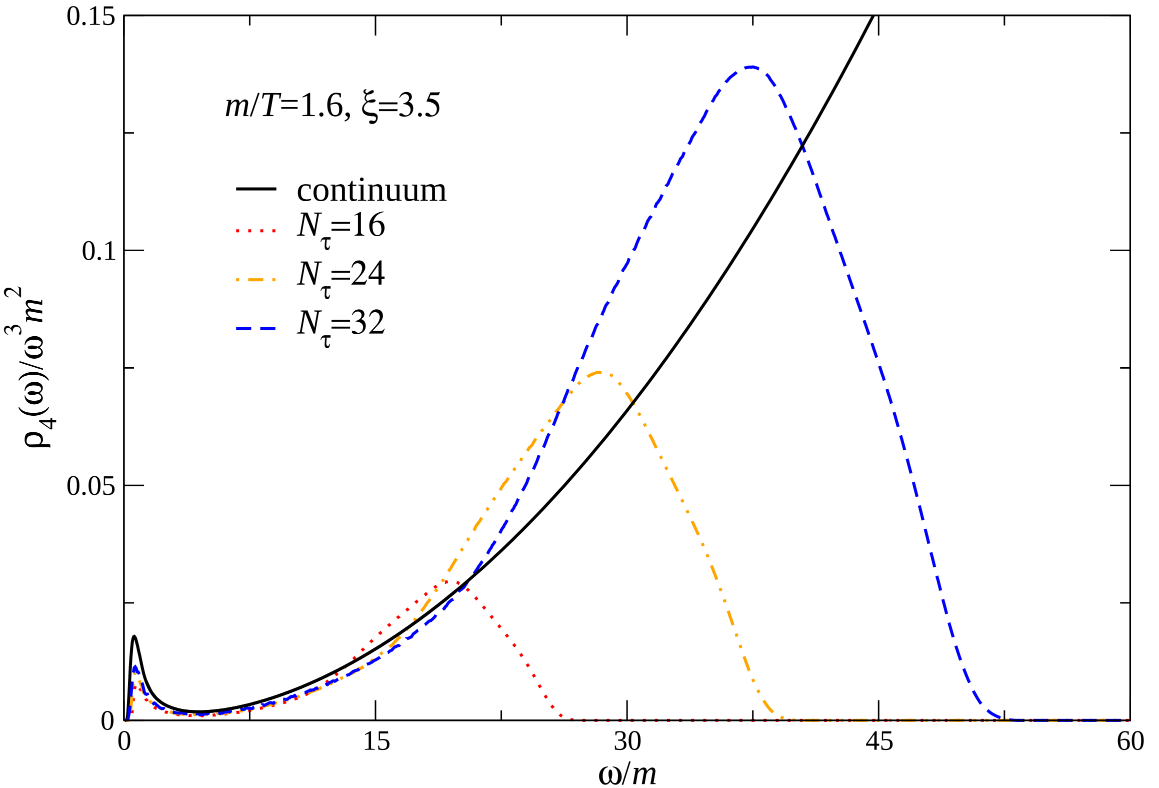


$$\rho_\pm(\omega) = \tfrac{1}{2}\left[\rho_m(\omega) \pm \rho_4(\omega)\right] \qquad\qquad \rho_+(\omega)$$

- thermal enhancement at $\omega \sim T \sim m$
- apparent peak depends on presentation/normalisation
- exponentially suppressed as $\omega \to 0$
- $\pm\rho_\pm(\omega) \geq 0 \qquad \rho_-(\omega) = -\rho_+(-\omega)$

Lattice free spectral functions

- lattice dispersion relation, sum over Brillouin zones
- maximal energy $\omega = 3\omega_{\mathbf{k},\max}$
- similar to mesons Karsch et al 03, GA & Martínez Resco 05
- no cusps due to two-loop Brillouin sum



Spectral functions

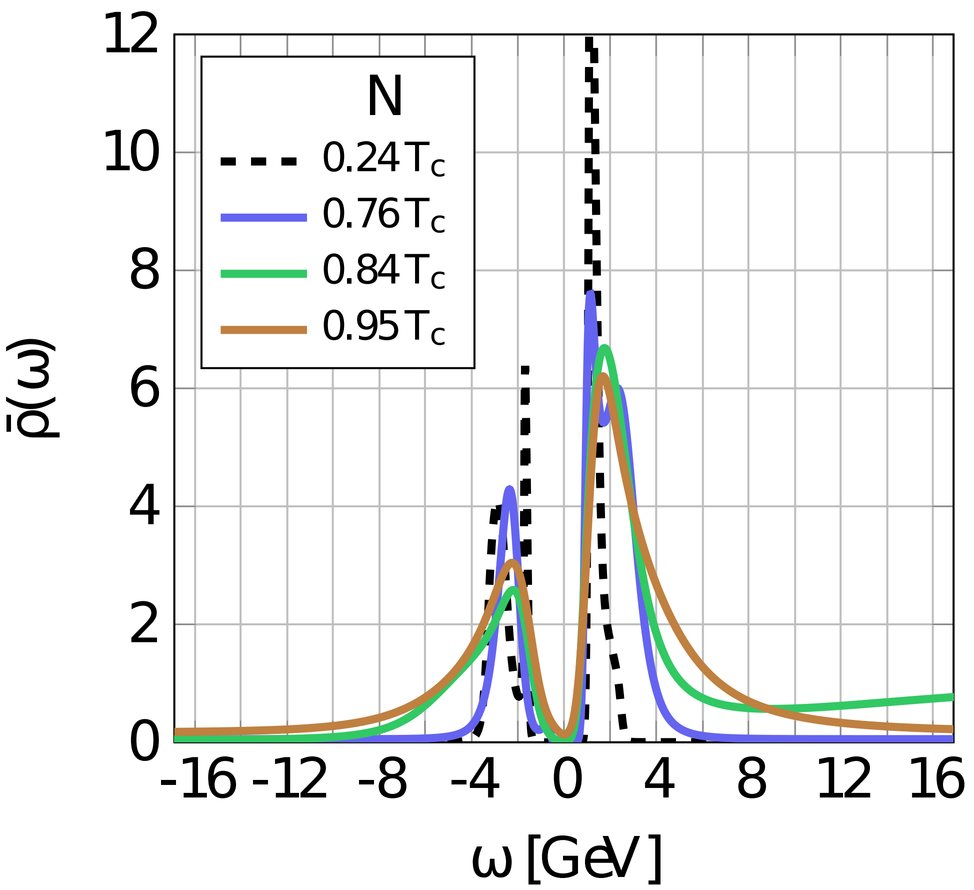
extract same information from spectral functions

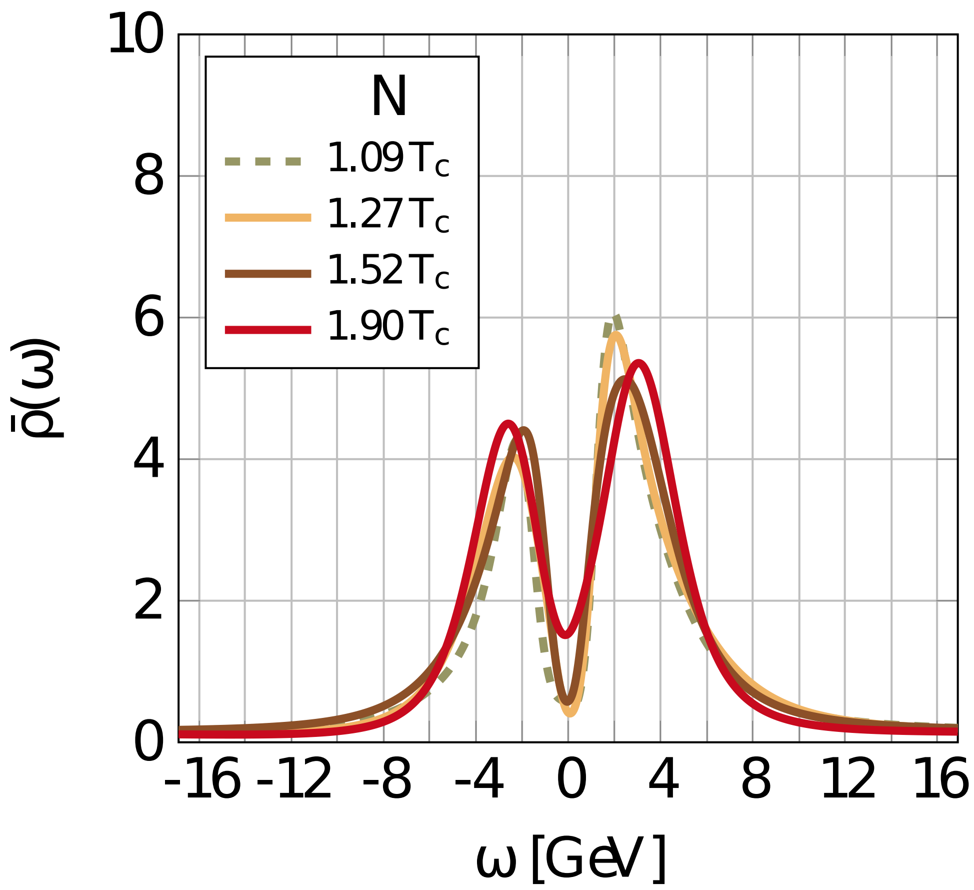
$$G_\pm(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\pm(\omega) \qquad K(\tau,\omega) = \frac{e^{-\omega\tau}}{1+e^{-\omega/T}}$$

- *ill-posed* inversion problem
- use Maximum Entropy Method (MEM)
- featureless default model
- construct $\rho_+(\omega) \geq 0$ for all ω
- $\rho_-(\omega) = -\rho_+(-\omega)$

Baryon spectral functions

- nucleon

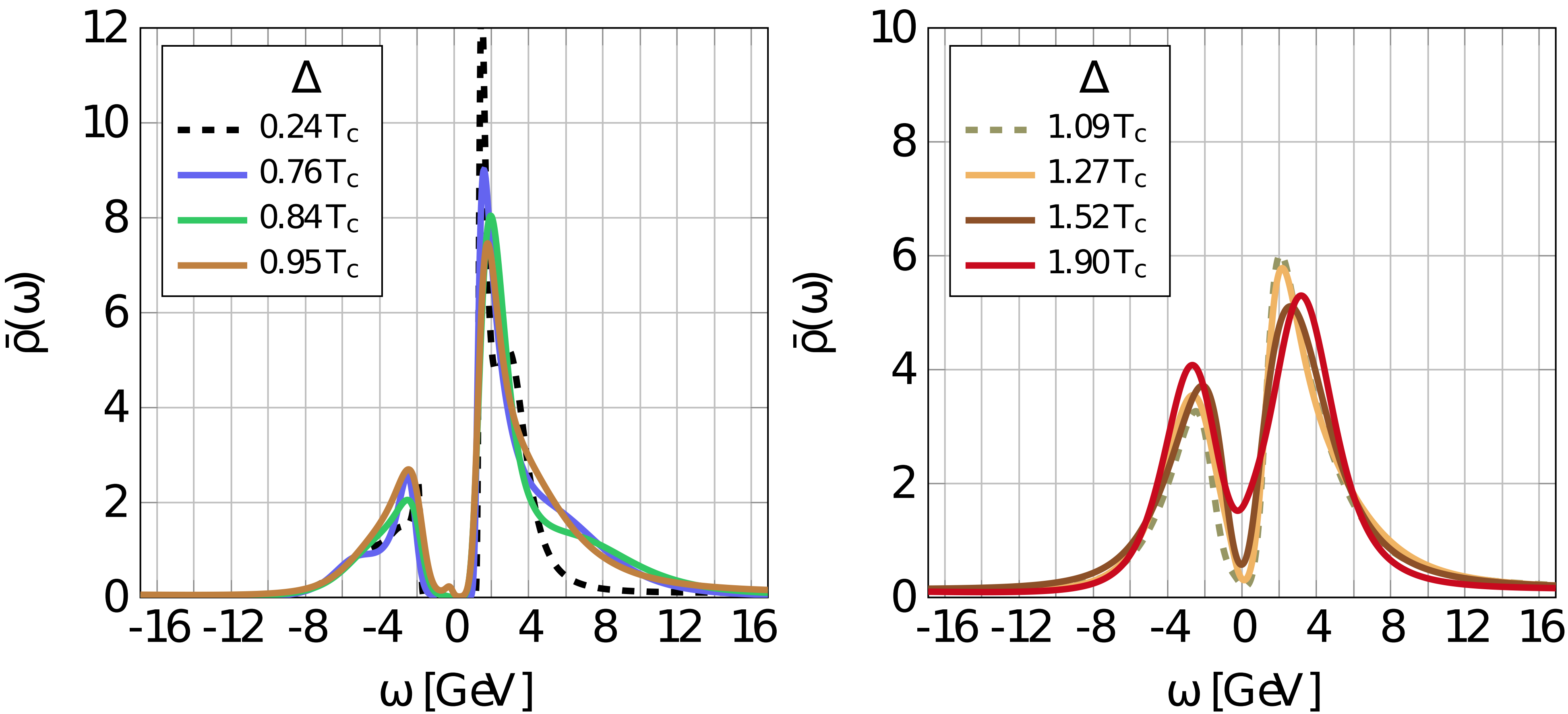




- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

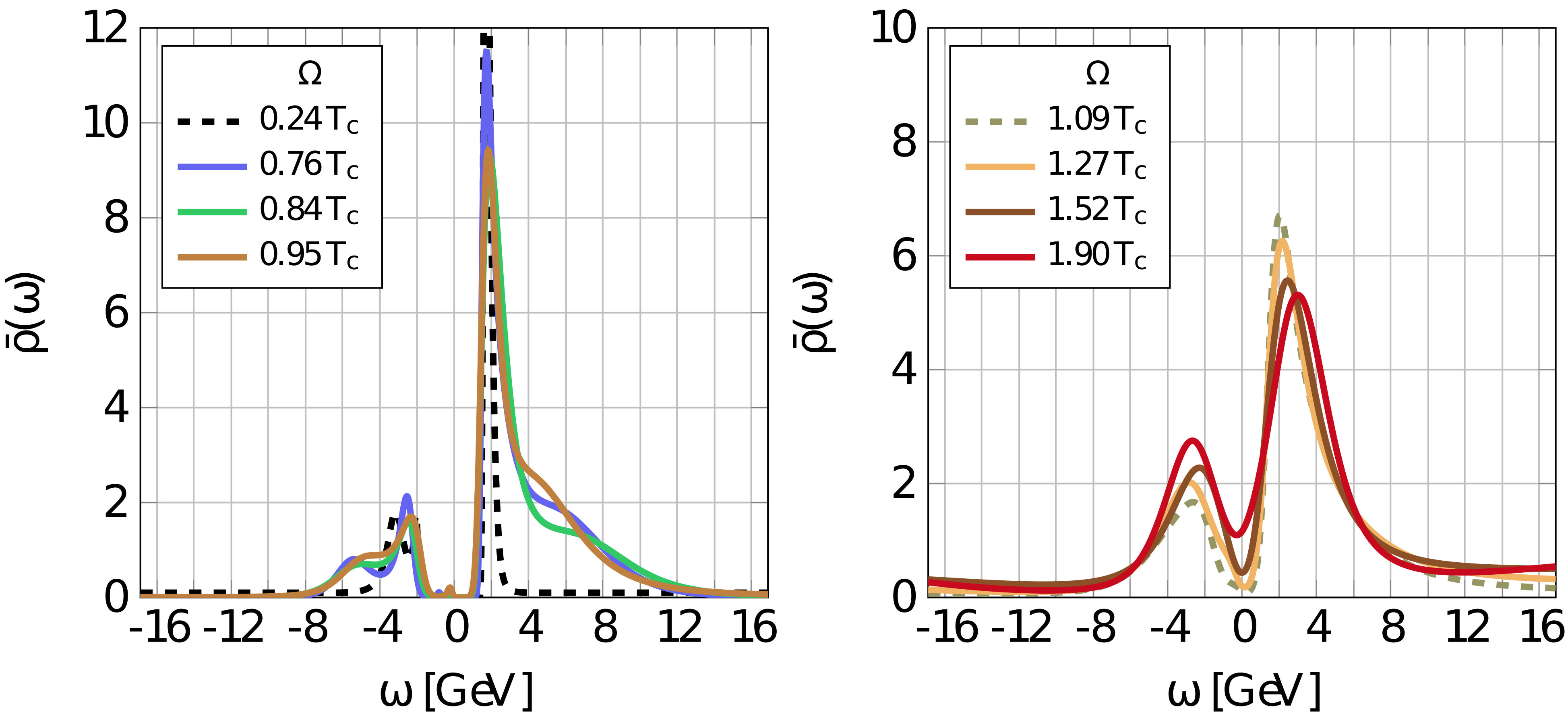
- Δ



- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

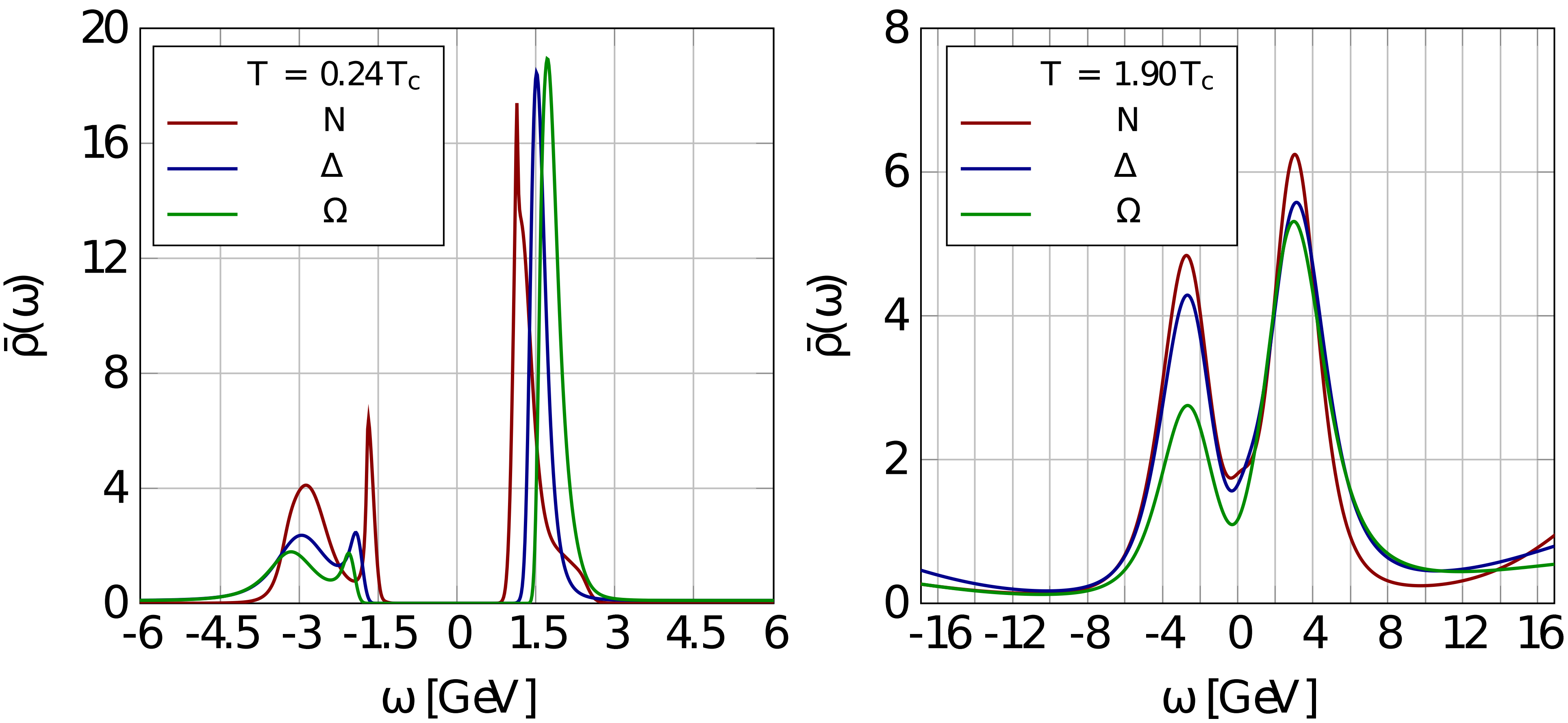
- Ω



- groundstates below T_c
- degeneracy emerging above T_c, finite m_s

Baryon spectral functions

- all channels: low and high temperature



- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

- results consistent with correlator analysis
- latter is on firmer ground, due to inversion uncertainties
- effect of heavier s quark visible

Summary: baryons in medium

in hadronic phase

- pos-parity groundstates mostly T independent
- stronger T dependence in neg-parity groundstates reduction in mass, near degeneracy close to T_c
- relevant for heavy-ion phenomenology?

in quark-gluon plasma

- pos/neg parity channels degenerate: parity doubling
- linked to deconfinement transition and chiral symmetry restoration
- correlator and spectral function analysis consistent
- effect of heavier s quark noticeable

Outlook: baryons in medium

lattice

- Wilson fermions: no chiral symmetry at short distances
- manifestly chiral fermions?

physics

- strangeness dependence
- physical light quarks
- phenomenology

understanding

- models?
- holography?