

# Axial vortical effect and hyperon polarization

Helmholtz - DIAS International Summer School "Hadron  
Structure, Hadronic Matter and Lattice", JINR, Dubna

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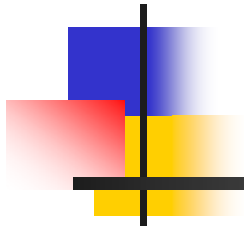
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# To appear in Nature

## Global $\Lambda$ hyperon polarization in nuclear collisions: evidence for the most vortical fluid

The extreme temperatures and energy densities generated by ultra-relativistic collisions between heavy nuclei produce a state of matter with surprising fluid properties<sup>1</sup>. Non-central collisions have angular momentum on the order of  $1000\hbar$ , and the resulting fluid may have a strong vortical structure<sup>2-4</sup> that must be understood to properly describe the fluid. It is also of particular interest because the restoration of fundamental symmetries of quantum chromodynamics is expected to produce novel physical effects in the presence of strong vorticity<sup>15</sup>. However, no experimental indications of fluid vorticity in heavy ion collisions have so far been found. Here we present the first measurement of an alignment between the angular momentum of a non-central collision and the spin of emitted particles, revealing that the fluid produced in heavy ion collisions is by far the most vortical system ever observed. We find that  $\Lambda$  and  $\bar{\Lambda}$  hyperons show a positive polarization of the order of a few percent, consistent with some hydrodynamic predictions<sup>5</sup>. A previous measurement<sup>6</sup> that reported a null result at higher collision energies is seen to be consistent with the trend of our new observations, though with larger statistical uncertainties. These data provide the first experimental access to the vortical structure of the “perfect fluid”<sup>7</sup> created in a heavy ion collision. They should prove valuable in the development of hydrodynamic models that quantitatively connect observations to the theory of the Strong Force. Our results extend the recent discovery<sup>8</sup> of **hydrodynamic spin alignment to the subatomic realm.**

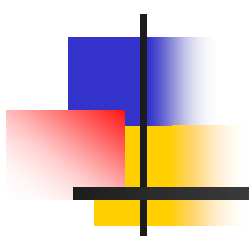
arXiv:1701.06657v1 [nucl-ex] 23 Jan 2017



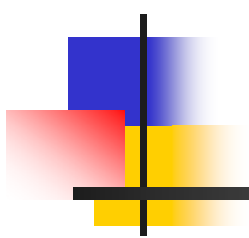
*Polarization data has often been the graveyard of fashionable theories.  
If theorists had their way, they might just ban such measurements altogether out of self-protection.*

*J.D. Bjorken  
St. Croix, 1987*

# Outline

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- QCD factorization and hadronic polarization (Lecture 1)
  - Axial anomaly and transport in hadronic media (Lecture 2)
  - Vorticity and hyperon polarization (Lecture 3)

# Outline of Lecture 1

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- QCD and factorization
  - Hadronic structure ~ hadronic matrix elements of quark and gluon operators
  - Density matrix and axial current
  - Single spin asymmetries and QCD

# QCD

## Why QCD?

Major scientific problem - mass of the Universe

~ 70% - Dark Energy

~ 25% - Dark Matter

~ 5% - Visible Matter

almost all of which is due to QCD!

1. Almost all of visible matter = protons.

Binding energy of nuclei and electrons in atoms - negligible.

Binding energy of nucleons in nuclei - dominant (current quark mass/proton mass ~ 1% )

(Current=fundamental) quarks are very light - chiral symmetry.

2. Fundamental theory of strong interaction - responsible for nuclear phenomena:

3. However - currently directly applicable only at large energy/momenta transfer - "hard" processes. Also very important - background for any search of new physics at hadronic colliders.

# QCD like QED

## What is QCD?

### 1. Local gauge theory (like QED)

Global phase transformation of Dirac electron field

$$\Psi(x) \rightarrow e^{i\alpha}\Psi(x) \quad (1)$$

Invariance -(Charge) conservation law

Local phase transformation

$$\Psi(x) \rightarrow e^{i\alpha(x)}\Psi(x) \quad (2)$$

Invariance - (Minimal)interaction with photon field.

$$\bar{\psi}\hat{A}\psi; \quad A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu\alpha(x) \quad (3)$$

# QCD unlike QED

## 2. Non-abelian (unlike QED)

Dirac quark field - intrinsic degree of freedom (colour) First evidence - from baryon spectroscopy:  $\Delta^{++}$  - 3 quarks of different colours.

Global transformation

$$\Psi_{\rho}(x) \rightarrow e^{it_{\rho\theta\alpha}} \Psi_{\rho}(x) \quad (4)$$

( $t$ -Gell Mann matrices) - Colour charge conservation. Moreover, all observed hadrons are colour singlet.

Local transformation invariance - (minimal interaction with gluon field)

$$\bar{\psi}_{\alpha} \hat{A}^{a\alpha} t_{\alpha\beta}^a \psi_{\beta}; \quad (5)$$

$N = 3$  quark colours -  $(N^2 - 1)/2 = 8$  gluon colours.

New ingredient - self interaction of gluons. Dramatic effect for charge renormalization. RG - invariant ( $\mu$ ) - running ( $Q^2$ ) coupling.

QED - screening - growing with  $Q^2$ , or in back direction - zero charge.

QCD - decreasing with  $Q^2$  (asymptotic freedom) - growing in back direction - confinement.

Many reasons (but no rigorous proof - worth  $\$10^6$ ) that it is absolute. Explains the non existence of free coloured particles. Nuclear forces - remnant of strong colour forces like van der Waals forces. Unlike to them - short distance rather than long distance - mass gap - crucial ingredient of confinement.

# Applying Asymptotic Freedom

How to explore the asymptotic freedom?

Processes typically contain hadrons on-shell.

The main tool -QCD factorization

Separate perturbatively calculable "hard" subprocesses and non-perturbative "soft" distribution/fragmentation functions.

Due to confinement problem - uncalculable BUT

1. Good objects for Non-perturbative methods (Lattice) and models
2. Universal = process independent.

"Zoology" of various non-perturbative inputs - like zoology in pre-Darwinian era.

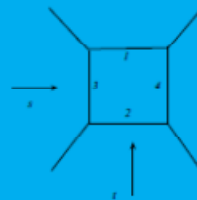
Factorization - based on the analysis of Feynman diagrams asymptotics

Useful tool  $\alpha$ -representation

$$\frac{1}{k^2 - m^2 + i\varepsilon} = i \int_0^\infty d\alpha e^{\alpha(i(k^2 - m^2) - \varepsilon)} \quad (6)$$

Large momenta - small  $\alpha$ .

Integral over momenta - Gaussian - easily performed. Remaining integrals over  $\alpha$ s - determined by the diagram topology. Elastic scattering of scalar massless particles) - box diagram



$$\sim \int_0^\infty \frac{\prod d\alpha}{(\sum \alpha)^2} e^{i(s\alpha_1\alpha_2 + t\alpha_3\alpha_4)/\sum \alpha} \quad (7)$$



# Appearance of subprocess

Asymptotic  $s \rightarrow \infty$  - small unless  $\alpha_1\alpha_2$  is small - rapidly oscillating function.

At least one of  $\alpha$ s whose removal splits diagram to two (connected) components in which momentum with large square enters ("kills" the dependence of process on the respective large variable) MUST be small: this is just the reason for subprocess appearance.

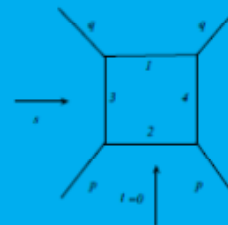
Electric circuits analogy : momentum  $\rightarrow$  current.

Large current due to its conservation should flow at least at one of the (afterwards) removed conductors.

The most known hard subprocess - Deep Inelastic Scattering  $\gamma^*(q)N(p) \rightarrow X$ .

Optical theorem: Total cross section - imaginary part of forward scattering amplitude.

Simplest model - again box diagram

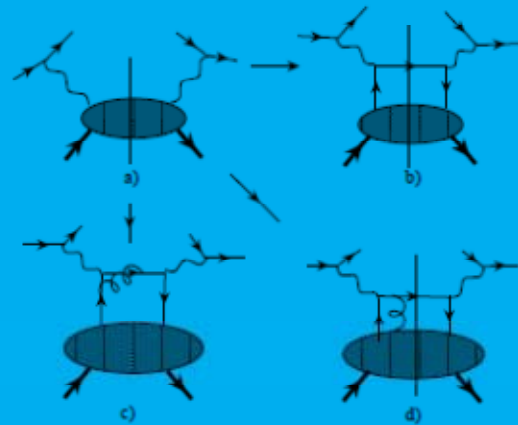


$$\sim \int_0^\infty \frac{\prod d\alpha}{(\sum \alpha)^2} e^{i(s\alpha_1\alpha_2 + q^2\alpha_1(\alpha_3 + \alpha_4))/\sum \alpha} \quad (8)$$

Large variables  $Q^2 = -q^2$ ,  $s = (p + q)^2$  -  $\alpha_1 \rightarrow 0$  - HANDBAG subprocess.

# Quarks in hadrons

It appears at any order of perturbation theory.



- a) Blob - hadronic matrix elements of quark fields - basic animal of our Zoo.
- b) Radiative corrections
- c) Higher twists

$$W \sim \int d^4z \langle P | \varphi(0) \varphi(z) | P \rangle H(z) \quad (9)$$

Expand matrix element to the power series: Factorization (b) ensures that all singular in  $z$  terms appear only in  $H$ .

$$\langle P | \varphi(0) \varphi(z) | P \rangle = \sum \frac{1}{n!} z^{\nu_1} \dots z^{\nu_n} \langle P | \varphi(0) \partial^{\nu_1} \dots \partial^{\nu_n} \varphi(0) | P \rangle \quad (10)$$

For small  $z$  - only first term contribute BUT in pseudo-Euclidian space only  $z^2$  is small, while  $(zP)$  is large.

# Twist

Leading twist all indices (number = spin) are carried by large vector  $P$ . Higher twists (c+...) dimension is not compensated by spin suppressed as  $M$ .

$$\langle P | \varphi(0) \partial^{\nu_1} \dots \partial^{\nu_n} \varphi(0) | P \rangle = i^n a_n P^{\nu_1} \dots P^{\nu_n} \quad (11)$$

$$W \sim \int d^4 z H(z) \sum \frac{1}{n!} a_n (i P z)^n \quad (12)$$

Last (but not least) step: moments

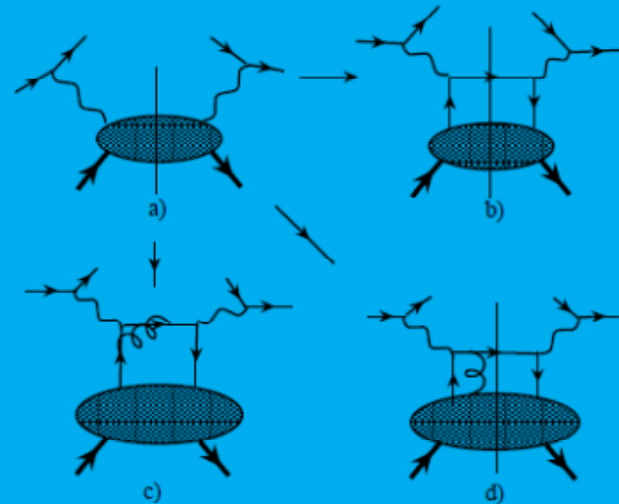
$$a_n = \int_0^1 dx f(x) x^n \quad (13)$$

$$W \sim \int_0^1 dx f(x) \int d^4 z H(z) \sum \frac{1}{n!} (i x P z)^n = \int_0^1 dx f(x) H(x P) \quad (14)$$

Parton "model" is derived. Radiative corrections (b)  
 $Q^2$ -dependence (Lecture 4).

# Spin 1/2 quarks

Back to DIS



Factorization for toy model of scalar quarks - hadronic matrix elements of quark fields. Realistic case - both quarks and hadrons (nucleons) are spin 1/2 particles

$$\langle P | \varphi(0) \varphi(z) | P \rangle \rightarrow \langle P, S | \psi_\alpha(0) E(0, z) \bar{\psi}_\beta | P, S \rangle \quad (1)$$

$E(0, z)$  - gluonic string providing gauge invariance of non-local operator (sum of the longitudinal gluons at Fig. (d)).

Quark spin - "contained" in indices  $\alpha, \beta$ . Nucleon spin - covariant polarization  $S$ : Scalar quarks distributions - probabilities to find quarks in nucleon. Dirac quarks - spin density matrix inside nucleon.

# Density matrix of quarks inside hadrons

Recall first the free quark (or electron) density matrix

$$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$$

At large energies mass is suppressed and longitudinal polarization is enhanced  $S \rightarrow \xi p/m$ ,  $\xi$  is the degree of longitudinal polarization.  $\rho \rightarrow \frac{1}{2}\hat{p}(1 + \xi\gamma_5)$

Consider longitudinally polarized nucleon; expansion over full set of Dirac matrices and making use of Lorentz invariance:

$$\langle P, \xi | \psi_\alpha(0) \hat{E}(0, z) \bar{\psi}_\beta(z) | P, \xi \rangle = \int dx e^{i(Pz)x} [q(x) \hat{P} + \Delta q \hat{P} \gamma_5 \xi] + O(M) \quad (2)$$

The density matrix of massless quarks is reproduced except spin-dependent and spin-independent terms enter with separate probabilistic weights: spin-dependent and spin independent distributions.

# Flavours and gluons

Distributions may be defined for each quark (and antiquark!) flavour and also for gluons:

$$\langle P, \xi | A^\mu(0) \tilde{E}(0, z) A^\nu(z) | P, \xi \rangle = \int dx e^{i(Pz)x} [G(x) g_\perp^{\mu\nu} + i \Delta G(x) \xi \varepsilon^{\mu\nu\rho\sigma} P_\rho n_\sigma] \quad (3)$$

Physical light-cone gauge  $n^2 = (An) = 0$ .  $g_\perp$ -in the plane transverse to  $P, n$ . Density matrix of circular polarized gluon.

Generally speaking, spin-averaged and spin-dependent distributions are unrelated, but  $|\Delta q(x)| \leq q(x)$ ,  $|\Delta G(x)| \leq G(x)$  (otherwise, in principle, one may get negative cross sections, as  $q(x) \pm \Delta q(x)$ ,  $G(x) \pm \Delta G(x)$  enter to the scattering on the nucleons of definite helicity) QCD corrections - Lecture 4.

What are the other constraints for the distributions?

# Constraining lowest moments

## Sum rules

The moments of parton distributions - local operators. Lowest moments  $\int dx \dots, \int dx$  - conserved operator - fixed by the respective conservation law. Physically: although details of parton distributions are defined by non-perturbative dynamics, averaged characteristics are constrained

- $\int dx$  - local vector current - matrix elements are fixed by charge conservation (which can be electric, baryonic, hypercharge)

So for  $u$ - quarks in the proton

$$\int_0^1 dx [u(x) - \bar{u}(x)] = 2 \quad (4)$$

for  $d$

$$\int_0^1 dx [d(x) - \bar{d}(x)] = 1 \quad (5)$$

and for  $s$  (and any other)

$$\int_0^1 dx [s(x) - \bar{s}(x)] = 0 \quad (6)$$

Therefore  $q(x) - \bar{q}(x)$  carry quantum numbers - "valence" (but not constituent) quarks  $q(x) + \bar{q}(x)$  - "sea" quarks.

# Momentum and spin – Axial Anomaly appears

$\int dx x \dots$  - current operator with derivative - energy momentum tensor; physically - weighting with momentum fraction

$$\int_0^1 dx x (\sum [q(x) + \bar{q}(x)] + G(x)) = 1 \quad (7)$$

Experimentally quark contribution  $\sim 0.5$  - historically the first evidence for gluon existence.

What about spin-dependent distributions?  $\int dx$ -axial current. Some matrix elements are known from  $\beta$ -decay.  $\langle p | J_5^\mu | n \rangle$  - due to isospin invariance  $\rightarrow \langle p | J_5^\mu | p \rangle = \langle n | J_5^\mu | n \rangle$  - Bjorken sum rule.

$$\int_0^1 dx (\Delta u(x) + \Delta \bar{u}(x) - \Delta d(x) - \Delta \bar{d}(x)) = \frac{1}{6} g_A \quad (8)$$

Is there any sum rule similar to momentum sum rule (polarized partons should carry total nucleon spin. like spin-averaged partons carry its momentum)

Feynman: Is there any constraint...?

Total angular momentum conservation

$$\int_0^1 dx (\sum (\Delta q(x) + \Delta \bar{q}(x)) + \Delta G(x) + L_q(x) + L_G(x)) = \frac{1}{2} \quad (9)$$

However: Orbital angular momenta are nonlocal Do not appear in inclusive processes cross-sections. Require non-forward matrix elements for its measurement (Lecture 3)

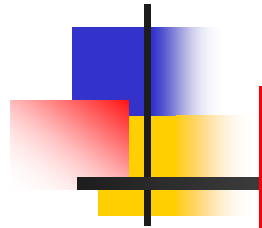
Another conserved operator - quark-gluon current (due to axial anomaly)

$$\int_0^1 dx (\sum (\Delta q(x) + \Delta \bar{q}(x)) + N_f \frac{\alpha_S}{2\pi} \Delta G(x)) = const \quad (10)$$

Two faces of nucleon spin structure



# Single and double spin asymmetries



Spin asymmetries: single vs double.

DIS structure function  $F_1, F_2$  - averaged over spin.

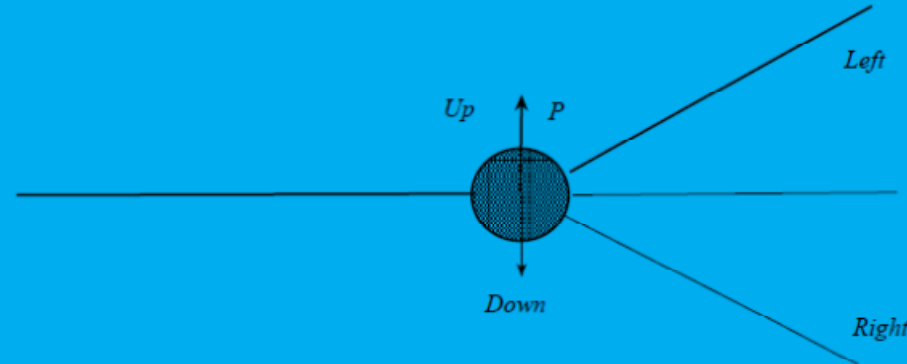
$G_1, G_2$  - for polarised leptons AND nucleons - double spin asymmetries

What about Single Spin Asymmetries (only one particle is polarized)?

Simple experiment - Complicated Theory

# Simple example

Simplest example - (non-relativistic) elastic pion-nucleon scattering  $\pi \vec{N} \rightarrow \pi N$




$M = a + ib(\vec{\sigma}\vec{n})$   $\vec{n}$  is the normal to the scattering plane.

Density matrix:  $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$ ,

Differential cross-section:  $d\sigma \sim 1 + A(\vec{P}\vec{n})$ ,  $A = \frac{2\text{Im}(ab^*)}{|a|^2 + |b|^2}$

# Single Spin Asymmetries and Spin-Orbital Interactions

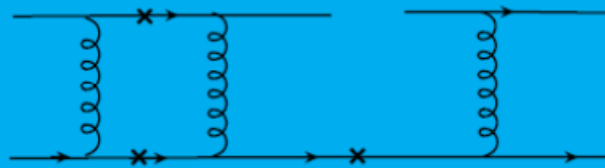


The same for the case of initial or final state polarization.  
Various possibilities to measure the effects: change sign of  $\vec{n}$  or  $\vec{P}$ : left-right or up-down asymmetry.  
Qualitative features of the asymmetry  
Transverse momentum required (to have  $\vec{n}$ )  
Transverse polarization (to maximize  $(\vec{P}\vec{n})$ )  
Interference of amplitudes  
IMAGINARY phase between amplitudes - absent in Born approximation

# Single Spin Asymmetries in pQCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like  $q - e$  scattering in DIS):

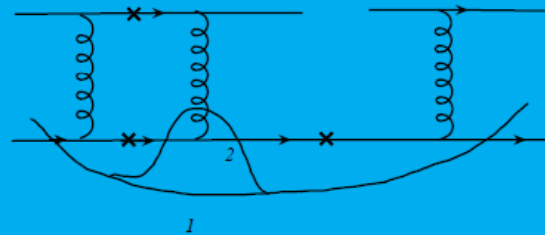


$$A \sim \frac{\alpha_s m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

# NPQCD: twist 3 and T-odd distributions

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop  $\rightarrow$  Born diagram

At Large distances - quark distribution  $\rightarrow$  quark-gluon correlator.

Physically - process proceeds in the external gluon field of the hadron.

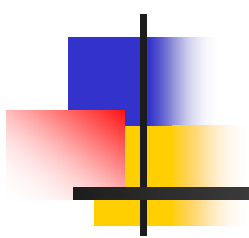
Leads to the shift of  $\alpha_S$  to non-perturbative domain AND

"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.

# Summary of Lecture 1

- 
- QCD factorization – “Zoo” of parton distributions (correlations)
  - Hadron structure encoded in hadronic matrix elements of quark/gluon fields – natural objects of Lattice/NP QCD
  - Spin – related to axial current and **Anomaly** (Lecture 2)
  - Single Spin Asymmetries – **Spin Orbital Interactions** (Lecture 3)