At T=0 we recover the textbook potential which agrees very well with the Cornell potential. $V_{T=0}(r) = \lim_{\tau\to\infty} \frac{1}{\tau} \log\left[W_\square(r,\tau)\right] \in \mathbb{R}$ [Fig. 20]

At T=0 the Wilson loop has a well separated lowest lying δ-like state. Otherwise the above definition does NOT give the correct potential (How to compute $Q\bar{Q}$ spectrum 0911.1743, 1509.07366)

How to interpret the in-medium modification of $V_{QQ}(r) \in \mathbb{C}$? (see: 1506.08684)

At high temperatures $T > T_c$ the $Q\bar{Q}$ is surrounded by a QGP with free color charge carriers. First proposals were related to Debye screening.

Original idea: Use Gauss law for $V(r) = -\frac{\alpha}{r}$, i.e. $-\nabla^2 V = 4\pi\alpha\,\delta(r)$ and introduce background charge which is Boltzmann distributed

$$-\nabla^2 V = 4\pi\alpha\left(\delta + \langle \rho(r)\rangle\right) \qquad \rho(r) = \left(n_0 e^{-\beta V} - n_0\right) - \left(n_0 e^{+\beta V} - n_0\right) \overset{V/T \ll 1}{\approx} -2\beta n_0 V(r)$$

$$\Rightarrow \quad -\nabla^2 V + \underbrace{8\pi\alpha n_0 \beta}_{A^2} V = 4\pi\alpha\delta \quad \Rightarrow \quad V(r) = -\alpha\, e^{-Ar}/r$$

At this point no route for imaginary part, no confining potential @ T>0?

Modern approach: Use a generalized Gauss-law $\nabla\left(\frac{-\nabla V}{r^{a+1}}\right) = 4\pi q\,\delta(r)$

$a=-1,\ q=\alpha$ Coulomb, $a=1,\ q=\sigma$ string like potential.

Medium effects are introduced via in-medium permittivity of a weakly coupled gas of quarks and gluons. Disentangle non-perturbative T=0 physics from medium effects.

See what happens for Coulombic part of the potential: FT

$$p^2 V_c(p) = 4\pi\alpha_S \quad \Rightarrow \quad p^2 V_c(p) = \frac{4\pi\alpha_S}{\varepsilon(p)} \qquad \underbrace{\varepsilon^{-1}(p, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T \frac{p\, m_D^2}{(p^2+m_D^2)^2}}_{\text{from hard-thermal loop PT}}$$

$$\Rightarrow \quad -\nabla^2 V_c(r) + m_D^2 V_c(r) = \alpha_S\left(4\pi\delta(r) - i T m_D^2\, \phi(m_D r)\right)$$

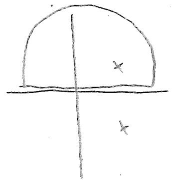
$$\phi(x) = 2\int_0^\infty dp\, \frac{\sin(px)}{px}\, \frac{p}{p^2+1}$$

Exercise: Show that this equation reproduces the results by Laine.

$$\mathrm{Re}\, V_c(r) = \frac{4\pi\alpha_s}{(2\pi)^3}\int_0^\infty dp \int_0^\pi d\theta \int_0^{2\pi} d\varphi\, \sin\theta\, \frac{p^2}{p^2+m_D^2}\, e^{ipr\cos\theta}$$

$$= \frac{2(2\pi)^2\alpha_s}{(2\pi)^3}\int_0^\infty dp \int_{-1}^1 dy\, e^{ipry}\, \frac{p^2}{p^2+m_D^2} = \frac{2\alpha_s}{(2\pi)}\int_0^\infty dp\left(\frac{e^{ipr}}{ipr}\frac{p^2}{p^2+m_D^2} - \frac{e^{-ipr}}{ipr}\frac{p^2}{p^2+m_D^2}\right)$$

$$= \frac{2\alpha_s}{(2\pi)} \int_{-\infty}^{\infty} \frac{e^{ipr}}{ir} \frac{p}{p^2+m_D^2} \qquad \frac{p}{(p+im_D)(p-im_D)} \to Res(p=im_D)\gamma = -\frac{im_D}{2im_D}$$



$$= -\alpha_s \frac{e^{-m_D r}}{r} \quad ✓$$

XV

$Im\, V_c$: $Im\, V_c(p) = 4\pi\alpha_s \pi T \frac{m_D^2}{p(p^2+m_D^2)^2}$

$$Im\, V_c(r) = 4\pi\alpha_s \pi T \int \frac{d^3p}{(2\pi)^3} e^{ipr\cos\vartheta} \frac{m_D^2}{p(p^2+m_D^2)^2} \quad \text{with } z = \frac{p}{m_D}$$

$$= \alpha_s T \int dp\, 2 \frac{\sin(z m_D r)}{z m_D r} \frac{z}{(z^2+1)^2} \quad ✓$$

Now a similar line of argument can be constructed for the in-medium potential contribution coming from the string-like vacuum part.

$$Re\, V_S(r) = -\frac{\Gamma(\frac{1}{4})}{2^{3/4}\sqrt{\pi}} \frac{\sigma}{\mu} D_{-1/2}(\sqrt{2}\mu r) + \frac{\Gamma(\frac{1}{4})}{2\Gamma(\frac{3}{4})} \frac{\sigma}{\mu} \qquad \mu^4 = m_D^2 \frac{2\cdot\alpha}{\sigma}$$

$$Im\, V_S(r) = -\frac{i\sigma m_D^2}{\mu^4} T\, \mathcal{Y}(\mu r)$$ $\mathcal{Y}(\mu r)$ integral expression arising from a Wronskian construction.

Show that the analytic formula with a single T-dependent parameter excellently reproduces the lattice ReV and ImV. All in-medium effects on the scales investigated here are summarized in $m_D(T)$. Around T_c the values of ImV from the lattice and Gauss-law deviate more than at high T.

Note: Need to be careful with the use of the word potential, since $V_{Q\bar{Q}} \in \mathbb{C}$ is NOT the usual potential for the wave function of $Q\bar{Q}$ but it governs the time evolution of the quarkonium propagator

$$D^>(r,t) = \langle \psi(r,t)\, \psi^*(r,0) \rangle$$ (see e.g. 1110.1203)

Solving the Schrödinger equation for $D^>$ gives access to quarkonium spectral functions $\rho_{Q\bar{Q}}(\omega) = \lim_{r\to 0} \int dt\, e^{-i\omega t} D^>(r,t)$. Show Fig 24

Fig 25

⇒ Plus: These spectra are much more precise than those obtained from direct NRQCD reconstructions.

Minus: Since only the static potential was used to evolve $D^>$, possibly systematic errors present. (question of accuracy)

Nevertheless we can use these spectra to estimate the ratio of J/ψ to ψ' at the phase boundary @ T=155 MeV and find good agreement with experiment. |Fig 26| (XVI)

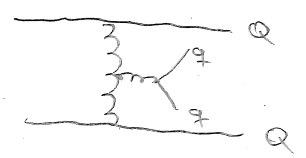
Last step: Towards a real time evolution of the $Q\bar{Q}$ wave function

Issue: Potential derived in the $M\to\infty$ limit, where heavy quarks cannot annihilate and singlet to octet transition weak. Using $V_{Q\bar{Q}}\in\mathbb{C}$ for the wave function leads to $|\psi_{QQ}|\propto e^{-|\mathrm{Im}\,V|t}$ heavy quarks are lost.

Resolution: Damping of the correlator on the other hand has clear physical meaning. $\langle \psi(r,t)\,\psi^*(r,0)\rangle \propto e^{-|\mathrm{Im}\,V|t}$ signals loss of memory of initial conditions, i.e. the system decoheres over time as it approaches thermal equilibrium.

Current research interest: How to realize $Q\bar{Q}$ evolution that incorporates decoherence.

One intuitive proposal: Imaginary part is related to kicks by medium partons on the gluon holding the $Q\bar{Q}$ together. (Landau damping) Implement as stochastic perturbation of a real-valued potential.



$$\psi(t)=\exp\Big[i\int dt\Big(-\frac{\nabla^2}{2M}+\underset{\mathbb{R}}{V(r)}+\eta\Big)\Big]\psi(0)\qquad \langle\eta\rangle=0\quad \langle\eta\eta'\rangle=2|\mathrm{Im}\,V|\,\delta(t-t')$$

discretized $=\dfrac{2|\mathrm{Im}\,V|}{\Delta t}\delta_{tt'}$

Expand evolution for infinitesimal time step:

$$\psi(t+\Delta t)\approx\Big[1+i\Delta t\Big(-\frac{\nabla^2}{2M}+V(r)+\eta\Big)-\underbrace{\frac{\Delta t^2}{2}\eta^2}_{\mathcal{O}(\Delta t)\,!}\Big]\psi(t)$$

This stochastic Schrödinger equation implements unitary time evolution and reproduces the correct correlator

$$i\partial_t\langle\psi(r,t)\,\psi^*(r,0)\rangle=\Big(-\frac{\nabla^2}{2M}+V(r)-i\,|\mathrm{Im}\,V|\Big)\langle\psi(t)\,\psi^*(0)\rangle$$

Ongoing work to understand limitations of the stochastic Schrödinger equation and how to generalize it in the context of so called Open Quantum Systems (key words: Master equation, Lindblad equation)

Summary:

① Heavy quarkonium is a unique system that allows us to test our understanding of microscopic QCD with well established experimental data both in vacuum (masses; lifetimes) as well as at high temperatures in heavy-ion collision (production yields)

② Effective field theories allow us to efficiently compute quarkonium correlators in thermal equilibrium using standard lattice QCD simulations as input. From these correlators in-medium spectral properties can be extracted using Bayesian inference of spectral functions.

③. Understanding of $Q\bar{Q}$ real-time dynamics is emerging based on the concept of an in-medium potential whose T-dependence as found in lattice QCD can be understood from a phenomenological picture of a sharply coupled $Q\bar{Q}$ being immersed in a bath of weakly coupled quarks and gluons