Polarization of QCD vacuum by the strong electromagnetic fields and deconfinement

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"Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics", July 10 - 14, 2017, Dubna

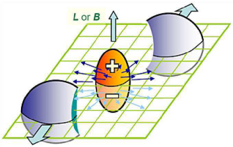
G.V. Efimov, Sh. Bilani, Ya. Burdanov, B. Galilo, A. Kalloniatis, L. von Smekal, S. Solunin, V. Tainov, V. Voronin

H. Pagels, and E. Tomboulis // Nucl. Phys. B143. 1978.
P. Minkowski, Nucl. Phys. B 177, 203 (1981);
H. Leutwyler, Phys. Lett. B**96**, 154 (1980); Nucl. Phys. B**179**, 129 (1981).
A. Eichhorn, H. Gies, J. M. Pawlowski, Phys. Rev. D83 (2011)

August 31, 2017

QCD IN EXTERNAL MAGNETIC BACKGROUNDS

Quarks are subject to electroweak interactions, which in general induce small corrections to strong interaction dynamics. Exceptions are expected in the presence of strong e.m. backgrounds, a situation relevant to many contexts:

- **Large magnetic fields are expected in a class of neutron stars known as magnetars ($B \sim 10^{10}$ Tesla on the surface) (Duncan-Thompson, 1992).**
- **Large magnetic fields ($B \sim 10^{16}$ Tesla, $\sqrt{|e|B} \sim 1.5$ GeV), may have been produced at the cosmological electroweak phase transition (Vachaspati, 1991).**
- 

  in non-central heavy ion collisions, largest magnetic fields ever created in a laboratory (B up to 10^{15} Tesla at LHC) with a possible rich associated phenomenology (e.g., chiral magnetic effect)

- QCD effective action and vacuum gluon configurations
- Gluon condensates and domain wall network as QCD vacuum

- Testing the domain model - static characteristics of QCD vacuum
- Bosonization – Effective meson action
- Meson properties: masses, decay constants, form factors
- Strong electromagnetic field as a trigger of deconfinement
- Overview of lattice QCD results – magnetic catalysis of deconfinement

- **Confinement of both static and dynamical quarks** $\longrightarrow$

$$W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu} \rangle$$
$$S(x,y) = \langle \psi(y)\bar{\psi}(x) \rangle$$

- **Dynamical Breaking of chiral** $SU_L(N_f) \times SU_R(N_f)$ **symmetry** $\longrightarrow \langle \bar{\psi}(x)\psi(x) \rangle$

- $U_A(1)$ **Problem** $\longrightarrow \eta'$ (χ, Axial Anomaly)

- **Strong CP Problem** $\longrightarrow Z(\theta)$

- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:
  hadron masses,
  form factors, scattering

Light mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**

QCD vacuum as a medium characterized by certain condensates,
quarks and gluons - elementary coloured excitations (confined),
mesons and baryons - collective colorless excitations

Deconfinement, chiral symmetry restoration under "extreme" conditions

Quantum effective action of QCD

QCD effective action and vacuum gluon configurations

In Euclidean functional integral for YM theory one has to allow the gluon condensates to be nonzero:

$$Z = N \int_{\mathcal{F}_B} DA \int_{\Psi} D\psi D\bar{\psi} \exp\{-S[A,\psi,\bar{\psi}]\}$$

$$\mathcal{F}_B = \left\{ A : \lim_{V\to\infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2 \right\}.$$

B.V. Galilo and S.N. , Phys. Rev. D84 (2011) 094017

L. D. Faddeev, [arXiv:0911.1013 [math-ph]]

H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$A^a_\mu = B^a_\mu + Q^a_\mu$, background gauge fixing condition $D(B)Q = 0$:

$$1 = \int_{\mathcal{B}} DB \Phi[A,B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega] \delta[D(B^\omega)Q^\omega]$$

Q^a_μ – local (perturbative) fluctuations of gluon field with zero gluon condensate: $Q \in \mathcal{Q}$;
B^a_μ are long range field configurations with nonzero condensate: $B \in \mathcal{B}$.

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \int_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)] \delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$

The character of background fields B has yet to be identified by the dynamics of fluctuations:

$$Z = N' \int_{\mathcal{B}} DB \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S_{\rm QCD}[B+Q,\psi,\bar{\psi}]\}$$
$$= \int_{\mathcal{B}} DB \exp\{-S_{\rm eff}[B]\}$$

Global minima of $S_{\rm eff}[B]$ – field configurations that are dominant in the limit $V \to \infty$. Homogeneous Abelian (anti-)self-dual fields are of particular interest.

$$B_\mu = -\frac{1}{2} n B_{\mu\nu} x_\nu, \ \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}$$
$$n = T^3 \ \cos\xi + T^8 \ \sin\xi.$$

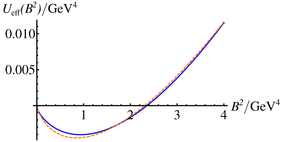
H. Pagels, and E. Tomboulis, Nucl. Phys. B 143 (1978) 485
P. Minkowski, Nucl. Phys. B177 (1981) 203
H. Leutwyler, Nucl. Phys. B 179 (1981) 129

$$G\left(z^2\right) \sim \frac{e^{-Bz^2}}{z^2}, \quad \tilde{G}\left(p^2\right) \sim \frac{1}{p^2}\left(1 - e^{-p^2/B}\right)$$

H. Leutwyler, Phys. Lett. B 96 (1980) 154

Gluon propagator ⇒ Regge trajectories

G.V. Efimov, and S.N. , Phys. Rev. D 51 (1995)



A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D 83, 045014 (2011)

Gluon condensates and domain wall network

Pure gluodynamics - Ginzburg-Landau approach:

$$\mathcal{L}_{\rm eff} = -\frac{1}{4\Lambda^2}\left(D^{ab}_{\nu}F^{b}_{\rho\mu}D^{ac}_{\nu}F^{c}_{\rho\mu} + D^{ab}_{\mu}F^{b}_{\mu\nu}D^{ac}_{\rho}F^{c}_{\rho\nu}\right) - U_{\rm eff}$$

$$U_{\rm eff} = \frac{\Lambda^4}{12}{\rm Tr}\left(C_1F^2 + \frac{4}{3}C_2F^4 - \frac{16}{9}C_3F^6\right),$$

B.V. Galilo, S.N. , Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

$$D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - iA^{ab}_{\mu} = \partial_{\mu} - iA^{c}_{\mu}(T^{c})^{ab},$$

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - if^{abc}A^{b}_{\mu}A^{c}_{\nu},$$

$$F_{\mu\nu} = F^{a}_{\mu\nu}T^{a}, \quad T^{a}_{bc} = -if^{abc}$$

$$C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

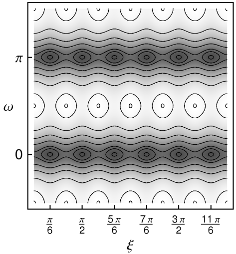
U_{eff} possesses degenerate discrete minima:

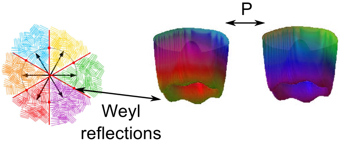
$$B_\mu = -\frac{1}{2} n_k B_{\mu\nu} x_\nu,\ \tilde{B}_{\mu\nu} = \pm B_{\mu\nu},$$

matrix n_k belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3\ \cos(\xi_k) + T^8\ \sin(\xi_k)\,, \quad \xi_k = \frac{2k+1}{6}\pi,\ k = 0, 1, \ldots, 5,$$

$$\vec{E}\vec{H} = B^2 \cos(\omega)$$





Domain wall network

ξ, $\langle g^2 F^2 \rangle \longrightarrow$ vacuum values

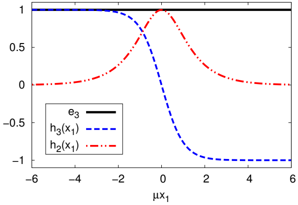
$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\Lambda^2 b_{\rm vac}^2 \partial_\mu\omega\partial_\mu\omega - b_{\rm vac}^4\Lambda^4\left(C_2 + 3C_3 b_{\rm vac}^2\right)\sin^2\omega,$$

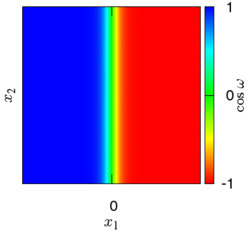
leads to sine-Gordon equation

$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\rm vac}^2\Lambda^2\left(C_2 + 3C_3 b_{\rm vac}^2\right),$$

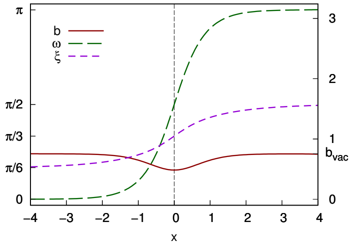
and the standard kink solution

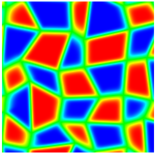
$$\omega(x_\nu) = 2\ \mathrm{arctg}\left(\exp(\mu x_\nu)\right)$$

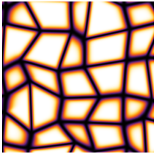




"Domain wall involving the topological charge density (in units of $\langle g^2F^2\rangle$), $su(3)$ angle ξ and the background action density simultaneously"







The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta^i_\nu x_\nu - q^i)).$$

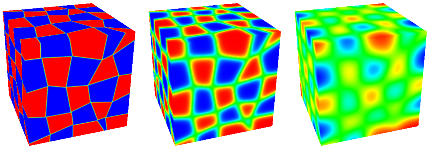
A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i).$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta^{ij}_\nu x_\nu - q^{ij})$$

S.N., V.E. Voronin, Eur.Phys.J. A51 (2015) 4



$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle = B^2$

$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle \ll B^2$

"Phase transitions and heterophase fluctuations" V. I. Yukalov, Phys. Rep. 208, 396 (1991)

What could stabilize a finite mean size of the domains?

Lower dimensional defects?

Quark (quasi-)zero modes?

Domain bulk - harmonic confinement

Elementary color charged excitations - fluctuations, eigenmodes decay in all four directions.

Eigenvalue problem for scalar field in $\mathbb{R}^4$: *H. Leutwyler, Nucl. Phys. B 179 (1981);*

$$B_\mu = B_{\mu\nu}x_\nu, \widetilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu}.$$

$$-\left(\partial_\mu - iB_\mu\right)^2 G = \delta \quad \longrightarrow \quad G\left(x-y\right) \sim \frac{e^{-B(x-y)^2/4}}{(x-y)^2}$$

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = \lambda\Phi \quad \longrightarrow \quad \left[\beta^+_\pm\beta_\pm + \gamma^+_+\gamma_+ + 1\right]\Phi = \frac{\lambda}{4B}\Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}}x_\mu + \partial_\mu,$$

$$\beta^+_\pm = \frac{1}{2}(\alpha^+_1 \pm i\alpha^+_2), \quad \gamma^+_\pm = \frac{1}{2}(\alpha^+_3 \pm i\alpha^+_4), \quad \alpha^+_\mu = \frac{1}{\sqrt{B}}x_\mu - \partial_\mu.$$

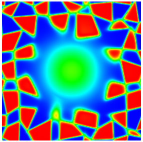
The eigenfunctions and eigenvalues - 4-dim. harmonic oscillator

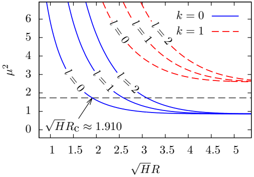
$$\Phi_{nmkl}(x) = \frac{1}{\pi^2\sqrt{n!m!k!l!}}\left(\beta^+_+\right)^k\left(\beta^+_-\right)^l\left(\gamma^+_+\right)^n\left(\gamma^+_-\right)^m\Phi_{0000}, \ \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$$\lambda_r = 4B(r+1), \ r = k+n \text{ self-dual field}, \ r = l+n \text{ anti-self-dual field}$$

Domain wall junctions - deconfinement

S.N. , V.E. Voronin, Eur.Phys.J. A51 (2015) 4





The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = 0, \quad \Phi(x) = 0, \quad x \in \partial\mathcal{T}, \quad \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2, \ (x_3, x_4) \in \mathrm{R}^2\right\}$$

The solutions are quasi-particle excitations

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3)e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3)e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta}\phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3)e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3)e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta}\phi_{alk}(r),$$

$$p_0^2 = p_3^2 + \mu^2_{akl}, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu^2_{akl}},$$

$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z},$$

In general near the boundaries

$$\mathrm{div}\vec{H} \neq 0, \quad \mathrm{div}\vec{E} \neq 0$$

The description of the domain walls as well as separation of the Abelian part in the general network in terms of the vector potential requires application of the gauge field parametrization suggested by L.D. Faddeev, A. J. Niemi (2007); K.-I. Kondo, T. Shinohara, T. Murakami(2008); Y.M. Cho (1980, 1981); L.Prokhorov, S.V. Shabanov (1989,1999)
The Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

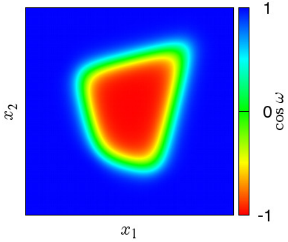
$$\begin{aligned}
\hat{A}_\mu(x) &= \hat{V}_\mu(x) + \hat{X}_\mu(x), \, \hat{V}_\mu(x) = \hat{B}_\mu(x) + \hat{C}_\mu(x), \\
\hat{B}_\mu(x) &= [n^a A^a_\mu(x)]\hat{n}(x) = B_\mu(x)\hat{n}(x), \\
\hat{C}_\mu(x) &= g^{-1}\partial_\mu \hat{n}(x) \times \hat{n}(x), \\
\hat{X}_\mu(x) &= g^{-1}\hat{n}(x) \times \left(\partial_\mu \hat{n}(x) + g\hat{A}_\mu(x) \times \hat{n}(x)\right),
\end{aligned}$$

where $\hat{A}_\mu(x) = A^a_\mu(x)t^a$, $\hat{n}(x) = n_a(x)t^a$, $n^a n^a = 1$, and

$$\partial_\mu \hat{n} \times \hat{n} = if^{abc}\partial_\mu n^a n^b t^c, \; [t^a, t^b] = if^{abc}t^c.$$

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

Both the color and space orientation of the field can become frustrated at the junction location and, thus, develop the singularities in the vector potential. The potential singularities cover the whole range of defects – vortex-like, dyon-like and zero-dimensional instanton-like defects.

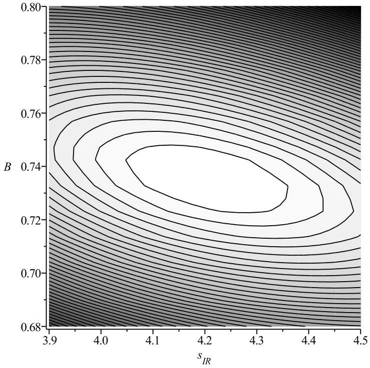


Rough estimation!

To mimic finite size of the region of homogeneity of the background field, let us introduce infra-red cutoff s_{IR} both to the quark and glue potentials,

$$U_{\rm eff}(B, s_{IR}) = \Lambda^4 \left\{ B^2 a \ln(bB^2 + s_{IR}^{-2}) + \frac{N_f}{8\pi^2} \int\limits_0^{s_{IR}} \frac{ds}{s^3} {\rm Tr}_n \left[s^2 \coth^2(s\hat{n}B) - 1 - \frac{2}{3} s^2 \hat{n}^2 B^2 \right] \right\}.$$

with $a = .00528$ and $b = .433$.



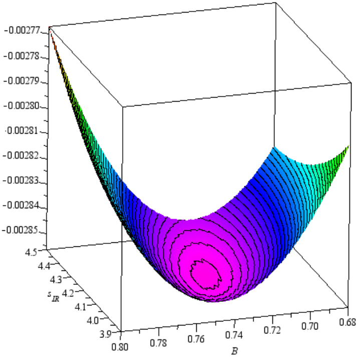
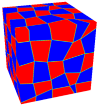


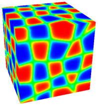
Figure : Effective potential (in units of Λ^4) as a function of angles B (in units of Λ^2) and s_{IR} (in units Λ^{-2}) for $N_f = 3$. One can see that the minimum of the potential is achieved at $B \approx .74$, $s_{IR} \approx 4.2$.

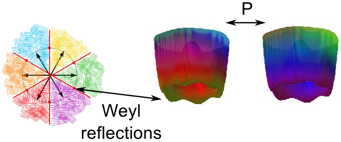
An ensemble of almost everywhere (in R^4) homogeneous Abelian (anti-)self-dual gluon fields

$$\langle : g^2 F^2 : \rangle \neq 0, \qquad \chi = \int d^4x \langle Q(x) Q(0) \rangle \neq 0, \qquad \langle Q(x) \rangle = 0$$

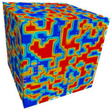
Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$







Domain wall network (S.N., V.E. Voronin, EPJA (2015); A. Kalloniatis, S.N., PRD (2001))



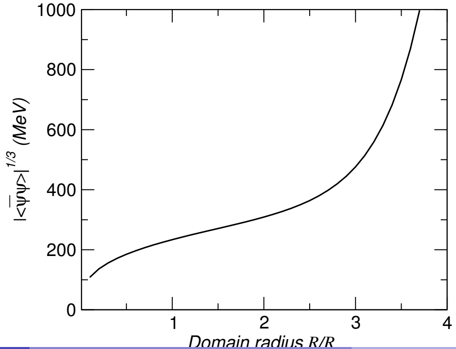
Lattice confining configuration (P.J. Moran, D.B. Leinweber, arXiv:0805.4246v1 [hep-lat])

Testing the model: characteristics of the domain wall network ensemble

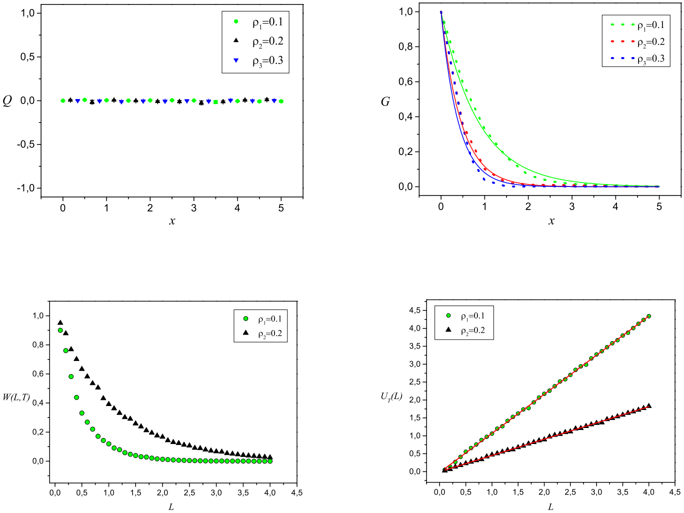
Spherical domains

A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006), Eur.Phys.J. A51 (2015), arXiv:1603.01447 [hep-ph] (2016)

Area law
Spontaneous chiral symmetry breaking
$U_A(1)$ is broken by anomaly
There is no strong CP violation



PRELIMINARY!: pure glue, domains - tubes with two finite dimensions (mean topological charge, two-point correlator of top. charge density, Wilson loop and static potential)



P. Olesen,"Confinement and random fluxes", Nucl. Phys. B, Volume 200 (1982) 381-390.

Hadronization

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)
A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006)

$$\mathcal{Z} = \int dB \int_\Psi \mathcal{D}\psi \mathcal{D}\bar\psi \int_\mathcal{Q} \mathcal{D}Q \delta[D(B)Q] \Delta_{\rm FP}[B,Q] e^{-S^{\rm QCD}[Q+B,\psi,\bar\psi]} =$$

$$\int dB \int_\Psi \mathcal{D}\psi \mathcal{D}\bar\psi \exp\left\{ \int dx \bar\psi \left(i \not\partial + g \not B - m \right) \psi \right\} W[j]$$

$$W[j] = \exp\left\{ \sum_n \frac{g^n}{n!} \int dx_1 \dots \int dx_n j^{a_1}_{\mu_1}(x_1) \dots j^{a_n}_{\mu_n}(x_n) G^{a_1 \dots a_n}_{\mu_1 \dots \mu_n}(x_1, \dots, x_n | B) \right\}$$

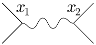
$$j^a_\mu = \bar\psi \gamma_\mu t^a \psi,$$

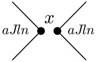
Next step: $W[j]$ is truncated up to the term including two-point gluon correlation function.

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar\psi \exp\left\{ \int dx \bar\psi \left(i\not\partial + g\not B - m \right)\psi \right.$$
$$\left. + \frac{g^2}{2}\int dx_1 dx_2 G^{a_1 a_2}_{\mu_1\mu_2}(x_1, x_2|B) j^{a_1}_{\mu_1}(x_1) j^{a_2}_{\mu_2}(x_2) \right\}$$

Fierz transform, center of mass coordinates $\longrightarrow \int dz dx G(z|B) J^{aJ}(x,z) J^{aJ}(x,z)$

α_s  $= \alpha_s(0)$  $\left[1 + \Pi^{\mathrm{R}}(p^2)\right]$; $\Pi^{\mathrm{R}}(0) = 0$

0  $z \;\to \dfrac{e^{-\frac{1}{4}Bz^2}}{4\pi^2 z^2}$ $\qquad \int dx_1\, dx_2$ 

$= \int dx \sum_{aJln}$ 

$\to \alpha_s(p) \frac{1-\exp\left(-p^2/B\right)}{p^2}$

$$J^{aJ}(x,z) = \sum_{nl} \left(z^2\right)^{l/2} f^{nl}_{\mu_1\ldots\mu_l}(z) J^{aJln}_{\mu_1\ldots\mu_l}(x), \quad J^{aJln}_{\mu_1\ldots\mu_l}(x) = \bar q(x) V^{aJln}_{\mu_1\ldots\mu_l}\left(\frac{\overleftrightarrow{D}(x)}{B}\right) q(x),$$

$$f^{nl}_{\mu_1\ldots\mu_l} = L_{nl}\left(z^2\right) T^{(l)}_{\mu_1\ldots\mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}}.$$

$T^{(l)}_{\mu_1\ldots\mu_l}$ are irreducible tensors of four-dimensional rotational group

$$\int_0^\infty du \rho_l(u) L_{nl}(u) L_{n'l}(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u} \leftrightarrow \frac{e^{-Bz^2}}{z^2} \quad \text{gluon propagator}$$

Effective meson action for composite colorless fields:

$$Z = \mathcal{N} \lim_{V\to\infty} \int D\Phi_{\mathcal{Q}} \exp\left\{ -\frac{B}{2}\frac{h^2_{\mathcal{Q}}}{g^2 C_{\mathcal{Q}}} \int dx \Phi^2_{\mathcal{Q}}(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad \mathcal{Q} = (aJln)$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(-M^2_{\mathcal{Q}}|B), \quad h^{-2}_{\mathcal{Q}} = \frac{d}{dp^2}\tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(p^2)|_{p^2=-M^2_{\mathcal{Q}}}.$$

$$W_k[\Phi] = \sum_{\mathcal{Q}_1\dots\mathcal{Q}_k} h_{\mathcal{Q}_1}\dots h_{\mathcal{Q}_k} \int dx_1 \dots \int dx_k \Phi_{\mathcal{Q}_1}(x_1)\dots\Phi_{\mathcal{Q}_k}(x_k)\Gamma^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k|B)$$

$$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} = \overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)} - \Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}G^{(1)}_{\mathcal{Q}_2}},$$

$$\Gamma^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3} = \overline{G^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3}(x_1,x_2,x_3)} - \frac{3}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)}$$
$$+\frac{1}{2}\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)},$$

$$\Gamma^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4} = \overline{G^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_1,x_2,x_3,x_4)} - \frac{4}{3}\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(3)}_{\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_2,x_3,x_4)}$$
$$-\frac{1}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$
$$+\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$
$$-\frac{1}{6}\Xi_4(x_1,x_2,x_3,x_4)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)G^{(1)}_{\mathcal{Q}_4}(x_4)}.$$

$$\overline{G^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k)} = \int dB_j \mathrm{Tr} V_{\mathcal{Q}_1}\left(x_1|B^{(j)}\right) S\left(x_1,x_2|B^{(j)}\right)\dots$$
$$\dots V_{\mathcal{Q}_k}\left(x_k|B^{(j)}\right) S\left(x_k,x_1|B^{(j)}\right)$$

$$\overline{G^{(l)}_{\mathcal{Q}_1\dots\mathcal{Q}_l}(x_1,\dots,x_l) G^{(k)}_{\mathcal{Q}_{l+1}\dots\mathcal{Q}_k}(x_{l+1},\dots,x_k)} =$$
$$\int dB_j \mathrm{Tr}\left\{V_{\mathcal{Q}_1}\left(x_1|B^{(j)}\right) S\left(x_1,x_2|B^{(j)}\right)\dots V_{\mathcal{Q}_k}\left(x_l|B^{(j)}\right) S\left(x_l,x_1|B^{(j)}\right)\right\}$$
$$\times \mathrm{Tr}\left\{V_{\mathcal{Q}_{l+1}}\left(x_{l+1}|B^{(j)}\right) S\left(x_{l+1},x_{l+2}|B^{(j)}\right)\dots V_{\mathcal{Q}_k}\left(x_k|B^{(j)}\right) S\left(x_k,x_{l+1}|B^{(j)}\right)\right\},$$

Bar denotes integration over all configurations of the background field with measure dB_j.

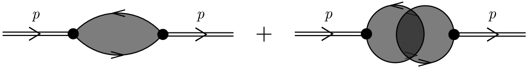
$$\langle \exp(iB_{\mu\nu}J_{\mu\nu})\rangle = \frac{\sin W}{W}$$

$$W = \sqrt{2B^2\left(J_{\mu\nu}J_{\mu\nu} \pm J_{\mu\nu}\widetilde{J}_{\mu\nu}\right)}$$

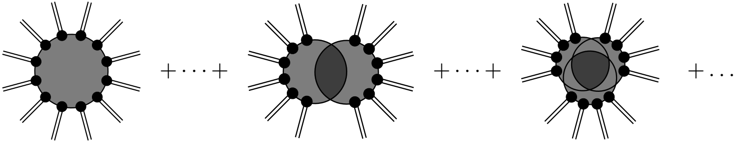
$J_{\mu\nu}$ is a tensor, composed of the momenta $p_{1\mu_1}\dots p_{n\mu_n}$ - arguments of the meson vertex

$$\tilde{\Gamma}^{(n)}(p_{1\mu_1}\dots p_{n\mu_n})$$

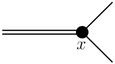
$$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} =$$



$$\Gamma^{(n)}_{\mathcal{Q}_1\mathcal{Q}_2\ldots\mathcal{Q}_n} =$$



Meson-quark vertex operators $\Leftarrow J^{aJln}_{\mu_1\ldots\mu_l} = \bar{q}(x) V^{aJln}_{\mu_1\ldots\mu_l} q(x)$



$$V^{aJln}_{\mu_1\ldots\mu_l}(x) = M^a \Gamma^J \left\{ \left\{ F_{nl}\left(\frac{\overleftrightarrow{D}^2(x)}{B^2}\right) T^{(l)}_{\mu_1\ldots\mu_l}\left(\frac{1}{i}\frac{\overleftrightarrow{D}(x)}{B}\right)\right\}\right\},$$

$$F_{nl}(s) = s^n \int_0^1 dt t^{n+l} \exp(st) = \int_0^1 dt t^{n+l} \frac{\partial^n}{\partial t^n} \exp(st),$$

$$\overleftrightarrow{D} = \overleftarrow{D} \xi_{f'} - \overrightarrow{D} \xi_f, \xi_f = \frac{m_f}{m_f + m_{f'}}$$

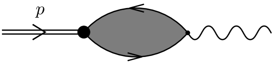
Quark propagator in homogeneous Abelian (anti-)self-dual field

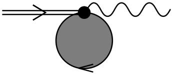
 $= \dfrac{\longrightarrow}{m(0)}\left[1 + \Sigma^{\mathrm{R}}(p^2)\right];\ \Sigma^{\mathrm{R}}(0) = 0 \qquad S(x,y) = \exp\left(-\frac{i}{2} x_\mu B_{\mu\nu} y_\nu\right) H(x-y),$

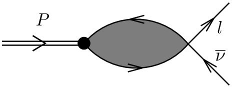
$$\widetilde{H}_f(p|B) = \frac{1}{\upsilon B^2}\int_0^1 ds e^{(-p^2/\upsilon B^2)s}\left(\frac{1-s}{1+s}\right)^{m_f^2/2\upsilon B^2}\left[p_\alpha\gamma_\alpha \pm is\gamma_5\gamma_\alpha \frac{B_{\alpha\beta}}{\upsilon B^2} p_\beta + \right.$$
$$\left. + m_f\left(P_\pm + P_\mp \frac{1+s^2}{1-s^2} - \frac{i}{2}\gamma_\alpha \frac{B_{\alpha\beta}}{\upsilon B^2}\gamma_\beta \frac{s}{1-s^2}\right)\right]$$

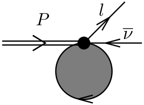
$$\tilde{H}_f(p|B) = \frac{m}{2\upsilon\Lambda^2}\mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2\upsilon\Lambda^2}\mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2\upsilon\Lambda^2}\mathcal{H}_V(p^2) \pm i\gamma_5\gamma_\alpha \frac{f_{\alpha\beta}p_\beta}{2\upsilon\Lambda^2}\mathcal{H}_A(p^2) \qquad (2)$$
$$+\sigma_{\alpha\beta}\frac{m f_{\alpha\beta}}{4\upsilon\Lambda^2}\mathcal{H}_T(p^2).$$

Weak and electromagnetic interactions









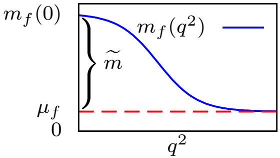
Masses of radially excited mesons

The parameters of the model are

$$\alpha_s(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R$$

$$\langle \alpha_s F^2 \rangle = \frac{B^2}{\pi} \quad \chi_{\rm YM} = \frac{B^4 R^4}{128\pi^2}$$

Dynamical chiral symmetry breaking:



$$\tilde{m} = 136 \text{ MeV}$$

$$\mu_{u/d} = m_{u/d} - \tilde{m}$$

$$\mu_s = m_s - \tilde{m}$$

$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$

$$\Lambda^2 \Phi^{(0)}_{\mathcal{Q}_1} = \sum_{k=1}^{\infty} \frac{g^k}{k} \sum_{\mathcal{Q}_1 \dots \mathcal{Q}_k} \Phi^{(0)}_{\mathcal{Q}_2} \dots \Phi^{(0)}_{\mathcal{Q}_k} \Gamma^{(k)}_{\mathcal{Q}_1 \dots \mathcal{Q}_k},$$

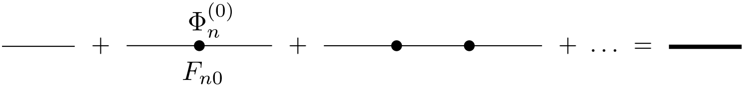


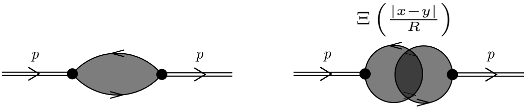
Figure : Mass corrections to the quark propagator due to the constant scalar condensates $\Phi_n^{(0)}$ coupled to nonlocal form factor F_{n0}. Summation over the radial number n is assumed.

Asymptotic Regge spectrum :

$$M_n^2 \sim Bn, \ n \gg 1$$
$$M_l^2 \sim Bl, \ l \gg 1$$

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995)

η and η'!



Polarization operator

Polarization operation for $l = 0$:

$$\Pi_J^{nn'}\left(-M^2; m_f, m_{f'}; B\right) =$$

$$\frac{B}{4\pi^2}\mathrm{Tr}_v \int_0^1 dt_1 \int_0^1 dt_2 \int_0^1 ds_1 \int_0^1 ds_2 \left(\frac{1-s_1}{1+s_1}\right)^{m_f^2/4vB} \left(\frac{1-s_2}{1+s_2}\right)^{m_{f'}^2/4vB} \times$$

$$\times\, t_1^n t_2^{n'} \frac{\partial^n}{\partial t_1^n} \frac{\partial^{n'}}{\partial t_2^{n'}} \frac{1}{\Phi_2^2} \left[\frac{M^2}{B}\frac{F_1^{(J)}}{\Phi_2^2} + \frac{m_f m_{f'}}{B}\frac{F_2^{(J)}}{(1-s_1^2)(1-s_2^2)} + \frac{F_3^{(J)}}{\Phi_2}\right] \exp\left\{\frac{M^2}{2vB}\frac{\Phi_1}{\Phi_2}\right\}.$$

$$\Phi_1 = s_1 s_2 + 2\left(\xi_1^2 s_1 + \xi_2^2 s_2\right)(t_1+t_2)v,$$

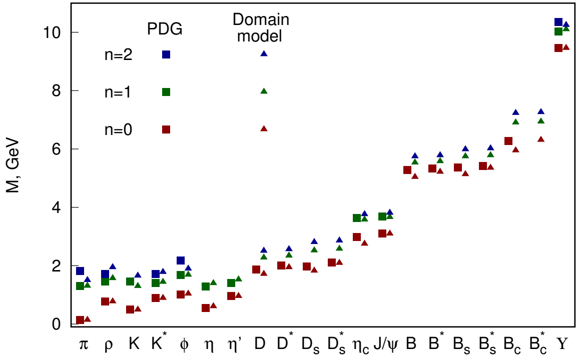
$$\Phi_2 = s_1 + s_2 + 2(1+s_1 s_2)(t_1+t_2)v + 16(\xi_1^2 s_1 + \xi_2^2 s_2) t_1 t_2 v^2,$$

$$F_1^{(P)} = (1+s_1 s_2)\left[2(\xi_1 s_1 + \xi_2 s_2)(t_1+t_2)v + 4\xi_1\xi_2(1+s_1 s_2)(t_1+t_2)^2 v^2 + s_1 s_2(1-16\xi_1\xi_2 t_1 t_2 v^2)\right],$$

$$F_1^{(V)} = \left(1 - \frac{1}{3}s_1 s_2\right)\left[s_1 s_2 + 16\xi_1\xi_2 t_1 t_2 v^2 + 2(\xi_1 s_1 + \xi_2 s_2)(t_1+t_2)v\right] + 4\xi_1\xi_2(1-s_1^2 s_2^2)(t_1-t_2)^2 v^2,$$

$$F_2^{(P)} = (1+s_1 s_2)^2, \quad F_2^{(V)} = (1-s_1^2 s_2^2),$$

$$F_3^{(P)} = 4v(1+s_1 s_2)(1-16\xi_1\xi_2 t_1 t_2 v^2), \; F_3^{(V)} = 2v(1-s_1 s_2)(1-16\xi_1\xi_2 t_1 t_2 v^2).$$



PDG
Domain model
n=2
n=1
n=0
M, GeV
0
2
4
6
8
10
π ρ K K* ϕ η η' D D* Ds Ds* ηc J/ψ B B* Bs Bs* Bc Bc* Υ

Table : Model parameters fitted to the masses of $\pi, \rho, K, K^*, \eta', J/\psi, \Upsilon$ and used in calculation of all other quantities.

$m_{u/d}$, MeV	m_s, MeV	m_c, MeV	m_b, MeV	Λ, MeV	α_s	R, fm
145	376	1566	4879	416	3.45	1.12

Table : Masses of light mesons. $\tilde{M}$ denotes the value in the chiral limit.

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	$\widetilde{M}$ (MeV)	Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	$\widetilde{M}$ (MeV)
π	0	140	140	0	ρ	0	775	775	769
$\pi(1300)$	1	1300	1310	1301	$\rho(1450)$	1	1450	1571	1576
$\pi(1800)$	1	1812	1503	1466	ρ	2	1720	1946	2098
K	0	494	494	0	K^*	0	892	892	769
$K(1460)$	1	1460	1302	1301	$K^*(1410)$	1	1410	1443	1576
K	2		1655	1466	$K^*(1717)$	1	1717	1781	2098
η	0	548	621	0	ω	0	775	775	769
η'	0	958	958	872	ϕ	0	1019	1039	769
$\eta(1295)$	1	1294	1138	1361	$\phi(1680)$	1	1680	1686	1576
$\eta(1475)$	1	1476	1297	1516	ϕ	2	2175	1897	2098

Table : Masses of heavy-light mesons and their lowest radial excitations .

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)
D	0	1864	1715	D^*	0	2010	1944
D	1		2274	D^*	1		2341
D	2		2508	D^*	2		2564
D_s	0	1968	1827	D_s^*	0	2112	2092
D_s	1		2521	D_s^*	1		2578
D_s	2		2808	D_s^*	2		2859
B	0	5279	5041	B^*	0	5325	5215
B	1		5535	B^*	1		5578
B	2		5746	B^*	2		5781
B_s	0	5366	5135	B_s^*	0	5415	5355
B_s	1		5746	B_s^*	1		5783
B_s	2		5988	B_s^*	2		6021
B_c	0	6277	5952	B_c^*	0		6310
B_c	1		6904	B_c^*	1		6938
B_c	2		7233	B_c^*	2		7260

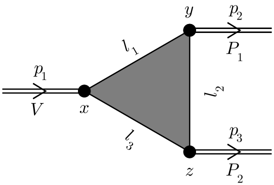
Table : Masses of heavy quarkonia.

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)
$\eta_c(1S)$	0	2981	2751
$\eta_c(2S)$	1	3639	3620
η_c	2		3882
$J/\psi(1S)$	0	3097	3097
$\psi(2S)$	1	3686	3665
$\psi(3770)$	2	3773	3810
$\Upsilon(1S)$	0	9460	9460
$\Upsilon(2S)$	1	10023	10102
$\Upsilon(3S)$	2	10355	10249

Table : Decay and transition constants of various mesons

Meson	n	f_P^{exp} (MeV)	f_P (MeV)	Meson	n	$g_{V\gamma}^{\mathrm{exp}}$	$g_{V\gamma}$
π	0	130	140	ρ	0	0.2	0.2
$\pi(1300)$	1	—	29	ρ	1		0.034
K	0	156	175	ω	0	0.059	0.067
$K(1460)$	1	—	27	ω	1		0.011
D	0	205	212	ϕ	0	0.074	0.069
D	1	—	51	ϕ	1		0.025
D_s	0	258	274	J/ψ	0	0.09	0.057
D_s	1	—	57	J/ψ	1		0.024
B	0	191	187	Υ	0	0.025	0.011
B	1	—	55	Υ	1		0.0039
B_s	0	253	248				
B_s	1	—	68				
B_c	0	489	434				
B_c	1		135				

Stong decays: g_{VPP}



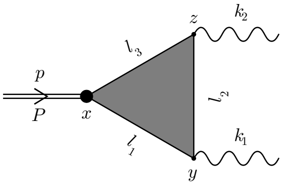
Decay	g_{VPP} [*]	g_{VPP}
$\rho^0 \to \pi^+\pi^-$	5.95	7.58
$\omega \to \pi^+\pi^-$	0.17	0
$K^{*\pm} \to K^\pm\pi^0$	3.23	3.54
$K^{*\pm} \to K^0\pi^\pm$	4.57	5.01
$\varphi \to K^+K^-$	4.47	5.02
$D^{*\pm} \to D^0\pi^\pm$	8.41	7.9
$D^{*\pm} \to D^\pm\pi^0$	5.66	5.59

local color gauge invariance

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Pion transition form factor

$$T_a^{\mu\nu}(x,y,z) = h_P \sum_n u_n^a \int d\sigma_B \mathrm{Tr}\; t_a e_f^2 V^n(x)\gamma_5 S(x,y|B)\gamma_\mu S(y,z|B)\gamma_\nu S(z,x|B),$$



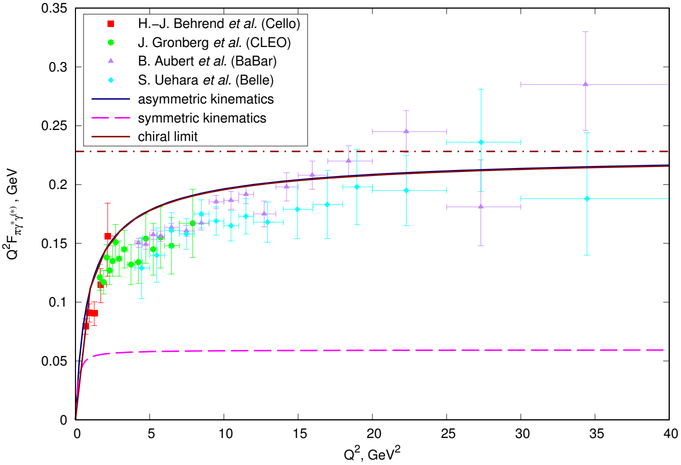
In momentum representation, the diagram has the following structure:

$$T_a^{\mu\nu}(p^2,k_1^2,k_2^2) = ie^2\delta^{(4)}(p-k_1-k_2)\varepsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}T_a(p^2,k_1^2,k_2^2).$$

$$F_{P\gamma}\left(Q^2\right) = T\left(-M_P^2, Q^2, 0\right).$$

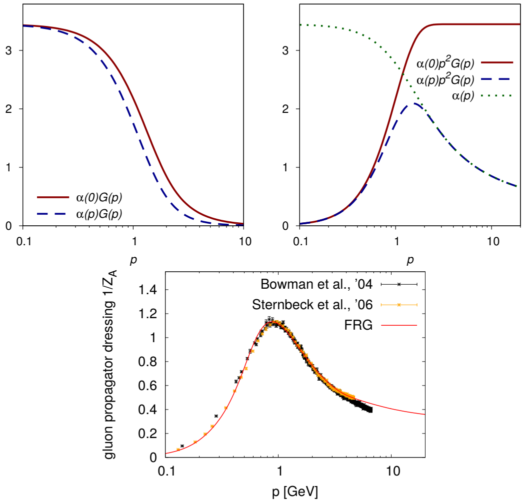
$$\Gamma(P\to\gamma\gamma) = \frac{\pi}{4}\alpha^2 M_P^3 g_{P\gamma\gamma}^2$$

$$g_{P\gamma\gamma} = T(-M_P^2,0,0) = F_{P\gamma}(0).$$

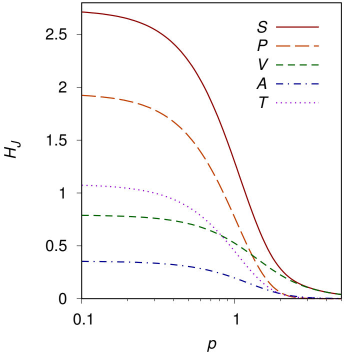


$$g_{\pi\gamma\gamma} = 0.272\text{GeV}^{-1} \qquad (g^{\text{exp}}_{\pi\gamma\gamma} = 0.274\text{GeV}^{-1}).$$

$$F_{\pi\gamma^*\gamma^*}\left(Q^2\right) = T\left(-M_P^2, Q^2, Q^2\right).$$



Functional RG, DSE, Lattice QCD



$$\tilde{H}(p) = \frac{m}{2v\Lambda^2}\mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2}\mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2}\mathcal{H}_V(p^2) \pm i\gamma_5\gamma_\alpha \frac{f_{\alpha\beta}p_\beta}{2v\Lambda^2}\mathcal{H}_A(p^2) + \sigma_{\alpha\beta}\frac{m f_{\alpha\beta}}{4v\Lambda^2}\mathcal{H}_T(p^2). \quad (3)$$

Estimate!

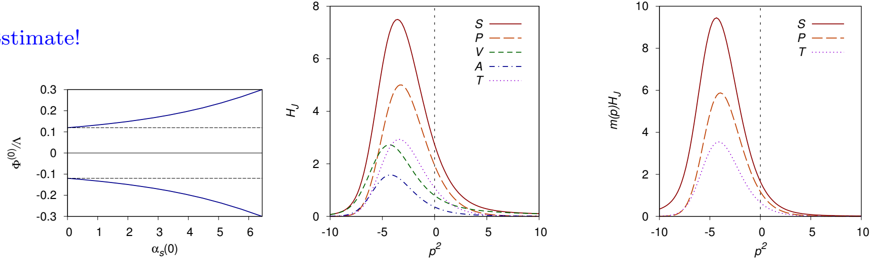


Figure : Scalar quark condensate (LHS). Momentum dependence of the scalar (solid line), pseudoscalar (long dash), vector (dash), axial (dash dot) and tensor (dot) form factors (central plot) in the quark propagator (5), and scalar, pseudoscalar and tensor form factors (RHS plot) multiplied by the quark mass.

$$\Lambda^2 \Phi^{(0)}_{\mathcal{Q}_1} = \sum_{k=1}^{\infty} \frac{g^k}{k} \sum_{\mathcal{Q}_1 \dots \mathcal{Q}_k} \Phi^{(0)}_{\mathcal{Q}_2} \dots \Phi^{(0)}_{\mathcal{Q}_k} \Gamma^{(k)}_{\mathcal{Q}_1 \dots \mathcal{Q}_k},$$

$$m(p) = \bar{m}(0) F_{00}(p^2), \quad F_{00}(p) = \left[1 - \exp\left(-\frac{p^2}{\Lambda^2}\right)\right] \frac{\Lambda^2}{p^2}, \quad \bar{m}(0) = \frac{1}{3} g \Phi^{(0)}, \tag{4}$$

$$\tilde{H}(p) = \frac{m}{2v\Lambda^2} \mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2} \mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2} \mathcal{H}_V(p^2) \pm i\gamma_5 \gamma_\alpha \frac{f_{\alpha\beta} p_\beta}{2v\Lambda^2} \mathcal{H}_A(p^2) + \sigma_{\alpha\beta} \frac{m f_{\alpha\beta}}{4v\Lambda^2} \mathcal{H}_T(p^2). \tag{5}$$

Bethe-Salpeter approach

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **51**, no. 1, 10 (2015) [arXiv:1409.5076 [hep-ph]]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **50**, 126 (2014) [arXiv:1406.4370 [hep-ph]].

S. M. Dorkin, L. P. Kaptari and B. Kampfer, arXiv:1412.3345 [hep-ph]
S. M. Dorkin, L. P. Kaptari, T. Hilger and B. Kampfer, Phys. Rev. C **89**, no. 3, 034005 (2014) [arXiv:1312.2721 [hep-ph]]

$$S^{-1}(p) = Z_2 S_0^{-1}(p) + 4\pi Z_2^2 C_F \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k+p) \gamma^\nu (\delta_{\mu\nu} - k_\mu k_\nu / k^2) \, \frac{\alpha_{\rm eff}(k^2)}{k^2},$$

$$\alpha_{\rm eff}(q^2) = \pi \eta^7 x^2 e^{-\eta^2 x} \; + \; \frac{2\pi \gamma_m \left(1 - e^{-y}\right)}{\ln\left[e^2 - 1 + (1+z)^2\right]}, \; x = q^2/\Lambda^2, \; y = q^2/\Lambda_t^2, \; z = q^2/\Lambda_{\rm QCD}^2$$

Harmonic confinement - 4-dim. oscillator

R. P. Feynman, M, Kislinger, and F. Ravndal, Phys. Rev. D **3** (1971) 2706.

H. Leutwyler and J. Stern, "Harmonic Confinement: A Fully Relativistic Approximation to the Meson Spectrum," Phys. Lett. B **73** (1978) 75;

H. Leutwyler and J. Stern, "Relativistic Dynamics on a Null Plane," Annals Phys. **112** (1978) 94.

Laguerre polynomials

$$\mathcal{S}_2 = -\frac{1}{2}\int d^4x \int d^4z D(z)\Phi^2_{Jc}(x,z)$$

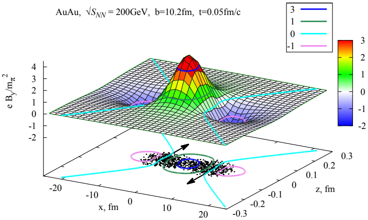
$$-2g^2 \int d^4x d^4x' d^4z d^4z' D(z) D(z') \Phi_{Jc}(x,z) \Pi_{Jc,J'c'}(x,x';z,z') \Phi_{J'c'}(x',z'),$$

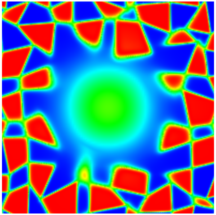
$$\Phi^{aJ}(x,z) = \sum_{nl} \left(z^2\right)^{l/2} \varphi^{nl}(z) \Phi^{aJln}(x).$$

"Polarization of QCD vacuum by the strong electromagnetic fields"

- **Relativistic heavy ion collisions - strong electromagnetic fields**

V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A **24** *(2009) 5925*
V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

One-loop quark contribution to the effective potential in the presence of arbitrary homogenous Abelian fields

$$U_{\rm eff}(G) = -\frac{1}{V}\ln\frac{\det(i\not{D}-m)}{\det(i\not{\partial}-m)} = \frac{1}{V}\int\limits_V d^4x\texttt{Tr}\int\limits_m^\infty dm'\left[S(x,x|m') - S_0(x,x|m')\right]|$$

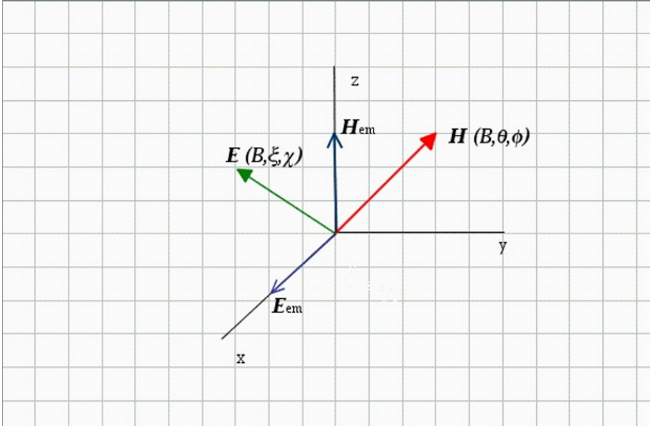
$$U_{\rm eff}^{\rm ren}(G) = \frac{B^2}{8\pi^2}\int\limits_0^\infty\frac{ds}{s^3}\texttt{Tr}_n\left[s\varkappa_+\coth(s\varkappa_+)s\varkappa_-\coth(s\varkappa_-) - \mathbf{1} - \frac{s^2}{3}(\varkappa_+^2+\varkappa_-^2)\right]e^{-\frac{m^2}{B}s},$$

$$\varkappa_\pm = \frac{1}{2B}\sqrt{\mathcal{Q}\sigma_\pm} = \frac{1}{2B}\left(\sqrt{2(\mathcal{R}+\mathcal{Q})}\pm\sqrt{2(\mathcal{R}-\mathcal{Q})}\right),$$

$$\mathcal{R} = (H^2-E^2)/2 + \hat{n}^2B^2 + \hat{n}B(H\cos(\theta) + iE\cos(\chi)\sin(\xi))$$

$$\mathcal{Q} = \hat{n}BH\cos(\xi) + i\hat{n}BE\sin(\theta)\cos(\phi) + \hat{n}^2B^2(\sin(\theta)\sin(\xi)cos(\phi-\chi) + cos(\theta)cos(\xi))$$

Y. M. Cho and D. G. Pak, Phys.Rev. Lett., 6 (2001) 1047

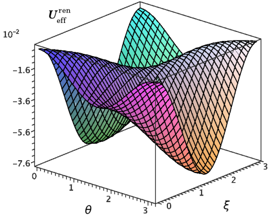


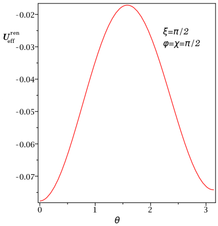
$$H_i = H\delta_{i3}, \quad E_j = E\delta_{j1}, \quad H^{\rm c} = \{B, \theta, \phi\}, \quad E^{\rm c} = \{B, \xi, \chi\}$$

$H \neq 0$, $E \neq 0$ and arbitrary gluon field

$$\Im(U_{\text{eff}}) = 0 \Longrightarrow \cos(\chi)\sin(\xi) = 0, \ \sin(\theta)\cos(\phi) = 0$$

Effective potential (in units of $B^2/8\pi^2$) for the electric $E = .5B$ and the magnetic $H = .9B$ fields as functions of angles θ and ξ ($\phi = \chi = \pi/2$)

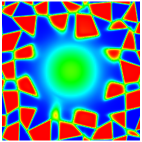


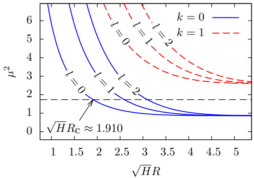


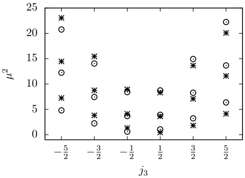
Minimum is at $\theta = \pi$ and $\xi = \pi/2$:
orthogonal to each other chromomagnetic and chromoelectric fields: $\mathcal{Q} = 0$.
Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies?!

B.V. Galilo and S.N., Phys. Rev. D84 (2011) 094017.
M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)
G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

S.N., V.E. Voronin, Eur.Phys.J. A51 (2015) 4







The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2\Phi = 0, \quad \Phi(x) = 0, \quad x \in \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2, \ (x_3, x_4) \in \mathrm{R}^2\right\}$$

The solutions are quasi-particle excitations

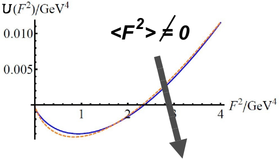
$$\phi^a(x) = \sum_{lk} \int\limits_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int\limits_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta} \phi_{alk}(r),$$

$$p_0^2 = p_3^2 + \mu^2_{akl}, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu^2_{akl}},$$

$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z},$$

A. Eichorn, et al,
Phys.Rev.D83, 2011

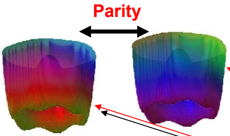


Weyl group, CP and the kink-like field configurations in the effective SU(3) gauge theory

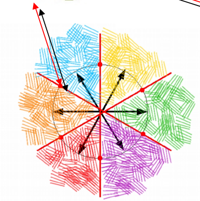
«Tendency» to dynamical breaking of CP and colour gauge symmetry:

SU(3) ⟷ **Weyl reflections ?**

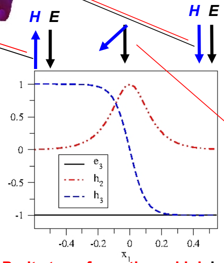
Topological defects(domain walls and lower dim. deffects at their intersections) bring disorder into ensemble of vacuum gluon configurations — on average SU(3) and CP <u>are not broken!</u>



Ensemble of domain structured vacuum fields:
dynamical quark confinement, chiral symmetry breaking.
In bulk of domain color is confined, color charged quasi-particles are localized at the boundaries.

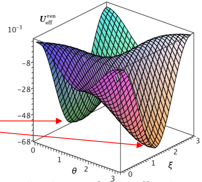


Weyl reflections in the root space of *color su(3)* — kink between boundaries of Weyl chambers



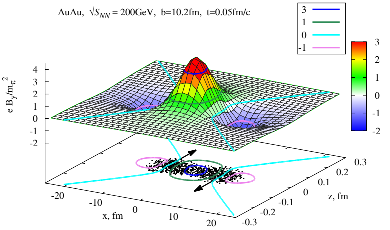
<u>Parity transformation</u> - kink interpolates Between self- and antiself-dual Abelian gluon configurations

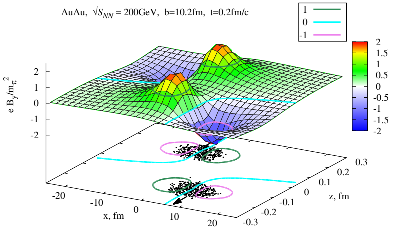
Strong crossed electromagnetic field creates relatively stable domain wall defect and thus triggers deconfinement of color charged particles in the space-time region of the relativistic heavy ion collision



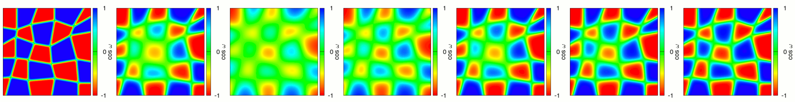
Quark contribution to QCD effective potential for Abelian gluon field in the presence of the strong crossed electromagnetic field

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





Magnetic field $eB \gtrsim m_\pi^2$ in the region $5fm \times 5fm \times .2fm \times .2fm/c$



Green region (**"Spaghetti vacuum"**) and the color charged quasi-particles

LQCD Results - based on the talk:

Confinining properties of QCD in strong magnetic backgrounds

Massimo D'Elia
University of Pisa & INFN

Based on arXiv:1607.08160, in collaboration with
C. Bonati, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo

XII Quark Confinement and the Hadron Spectrum - Thessaloniki, 2 September 2016

"Confinining properties of QCD in strong magnetic backgrounds" M. D'Elia, C. Bonati, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo
arXiv:1607.08160

Focus of this talk:

Effects of the magnetic field on the static quark potential

- A previous study has shown that the quark-antiquark potential becomes anisotropic, with a string tension smaller (larger) in the direction parallel to $\vec{B}$
  (C. Bonati et al., arXiv:1403.6094)
- The issue is interesting both by itself and for possible phenomenological consequences, e.g. for heavy quark bound states.

In this talk I discuss results reported in arXiv:1607.08160 (C. Bonati et al.), which try to achieve the following goals:

- A complete determination of the angular dependence of the potential
- An extrapolation to the continuum limit
- An extension to finite temperature

LATTICE SETUP

$$Z(B) = \int \mathcal{D}U \, e^{-\mathcal{S}_{YM}} \prod_{f=u,d,s} \det \left(D^f_{\mathsf{st}}[B]\right)^{1/4} .$$

- **pure gauge: Symanzik tree level improved gauge action**

- **fermion sector: 2-level stout improved rooted staggered fermions**

- **physical quark masses**

- **explored lattice spacings and sizes:**

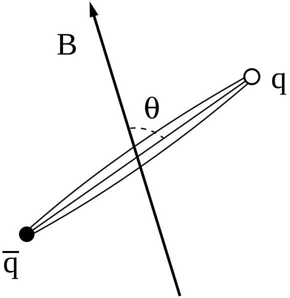
$a = 0.2173,\ 0.1535,\ 0.1249,\ 0.0989$ **fm** $\quad L_s a \sim 5$ **fm in all cases**

- **numerical simulations on FERMI (BG/Q at CINECA) thanks to PRACE allocation**

For non-zero background field $\vec{B}$, we want to study the potential not just for parallel or orthogonal directions, but for generic orientations.

In principle, one can either rotate the spatial side of the Wilson loop, or rotate $\vec{B}$ and perform new simulations.

Rotating the loop on the lattice introduces new cusps and renormalization effects, so we chose the second solution



Each component of the field gets quantized in the presence of spatial periodic b.c.

$$eB_x = 6\pi b_x/(a^2 N_z N_y)\,; \quad b_x \in \mathbb{Z}$$
$$eB_y = 6\pi b_y/(a^2 N_x N_z)\,; \quad b_y \in \mathbb{Z}$$
$$eB_z = 6\pi b_z/(a^2 N_x N_y)\,; \quad b_z \in \mathbb{Z}$$

we performed different simulations at fixed $B_x^2 + B_y^2 + B_z^2$ and different $\vec{B}$ orientations

Expected symmetries and ansatz for $V(r, \theta, \phi)$

- by residual rotational symmetry around $\vec{B}$: $V(r, \theta, \phi) = V(r, \theta)$

- by symmetry under $\vec{B} \to -\vec{B}$: $V(r, \pi - \theta) = V(r, \theta)$

- We make the assumption the potential is Cornell like along each direction

$$V(r, \theta) = -\frac{\alpha(\theta, B)}{r} + \sigma(\theta, B) r + V_0(\theta, B)$$

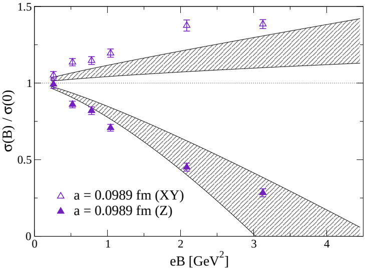
and write a Fourier expansion in θ for each term:

$$\begin{aligned} V(r, \theta) = & -\frac{\bar{\alpha}(B)}{r}\left(1 - \sum_{n=1} c_{2n}^{\alpha}(B)\cos(2n\theta)\right) \\ & + \bar{\sigma}(B) r \left(1 - \sum_{n=1} c_{2n}^{\sigma}(B)\cos(2n\theta)\right) \\ & + \bar{V}_0(B)\left(1 - \sum_{n=1} c_{2n}^{V_0}(B)\cos(2n\theta)\right). \end{aligned}$$

The continuum extrapolated results for σ predict a vanishing longitudinal string tension for $eB \sim 4$ GeV^2

This is outside the range explored for the continuum extrapolation, $eB \lesssim 1$ GeV^2.

Can we trust the prediction?



Cut-off effects are large for $eB \gtrsim 1/a^2$. We could extend to larger B just on the finest lattice spacing.

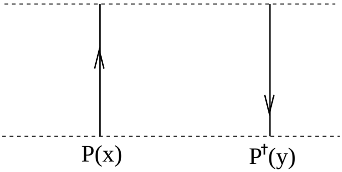
The decrease of $\sigma_{||}$ is steady, even if it somewhat undershoots the continuum band extrapolated to large B.

Simulations at finer lattice spacings should clarify the issue in the future.

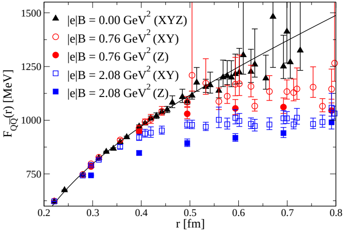
Finite T results

At finite T, the quark-antiquark potential is measured from Polyakov loop correlators

$$\langle \mathrm{Tr} P(\vec{x})\, \mathrm{Tr} P^{\dagger}(\vec{y}) \rangle \sim \exp\left(- \frac{F_{\bar{q}q}(r,T)}{T} \right)$$



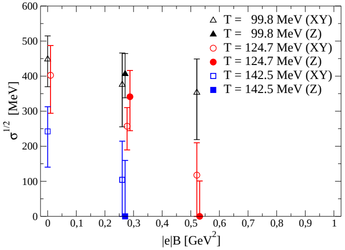
Results at $T \sim 100$ MeV on a $N_t = 20$ lattice



Although a small anisotropy is still visible, the main effect of B seems to suppress the potential in all directions

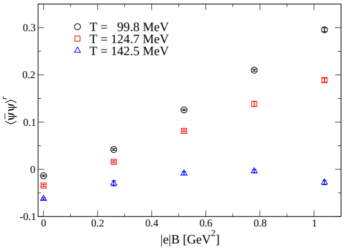
The string tension tends to disappear

A fit to the Cornell potential works in a limited range of distances and permits to obtain a determination of σ, which shows a steady decrease in all directions.



We can call this effect deconfinement catalysis

It is interesting to notice that this happens before (in temperature) inverse magnetic catalysis is visible in the chiral condensate



Is the decrease of T_c as a function of B related to a change in the confining properties?

CONCLUSIONS

- The magnetic field leads to a quadrupole-like deformation of the static quark-antiquark potential

- Most of the effect seems related to a modification of the string tension

- We have hints that $\sigma_{\|}$ could vanish in the vacuum for eB of the order of 10 GeV. Future simulations on finer lattice spacings could confirm this possibility.

- At finite T, the main effect is a general suppression of the potential leading to a precocious loss of confining properties: deconfinement catalysis.
  That could be important for heavy ion physics in the thermal medium, think for instance of J/ψ suppression and related issues.

Summary

$\langle g^2 F^2 \rangle \neq 0 \longrightarrow$ domain wall network, almost everywhere abelian (anti-)self-dual gluon fields.

An ensemble of almost everywhere Abelian homogeneous (anti-)self-dual gluon fields represented by the domain wall networks looks like a suitable framework for studying mechanisms of confinement, chiral symmetry realisation and hadronization.

Background of domain wall networks - harmonic confinement.

(Anti-)self-duality - quark zero mode driven realization of chiral symmetry.

Quark and gluon propagators - qualitative agreement with FRG and DSE.

Meson effective action - quantitatively correct phenomenology both with respect to confinement and chiral symmetry.

Polarization effects in QCD vacuum due to the strong electromagnetic fields, deconfinement, chiral symmetry restoration.

Electromagnetic fields as trigger of deconfinemen.