# Electron Wigner Distributions from Light Front Wavefunctions

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## Introduction

- One of the most challenging problem in QCD is to understand the internal structure of hadrons. To aim this, generalized parton distributions (GPDs) and transverse momentum distributions (TMDs) are proved to be an excellent tools. The ultimate understanding of the structure can be obtained by joint position and momentum distributions named as Wigner distributions. These distributions contain the most general one-body information of partons, corresponding to the full one-body density matrix in both momentum and position space, and reduce in certain limits to TMDs and GPDs.
- We study the Wigner distributions for a physical electron, which reveal the multidimensional images of the electron. The physical electron is considered as a composite system of a bare electron and photon. The Wigner distributions for unpolarized, longitudinally polarized and transversely polarized electron are presented in transverse momentum plane as well as in impact parameter plane. We also evaluate all the leading twist generalized transverse momentum distributions (GTMDs) for electron.

## Light front QED Model

We evaluate the results for the Wigner distribution of the physical electron by considering it as a two particle state (electron and photon). The two particle Fock state for an electron with $J^z=\frac{1}{2}$ has four possible combinations[1]

$$\Psi^{\uparrow}(P^+,\mathbf{P}_\perp=\mathbf{0}_\perp)=\int\frac{dx\;d^2\mathbf{p}_\perp}{\sqrt{x(1-x)}16\pi^3}\Big[\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{p}_\perp)\,|+\tfrac{1}{2}+1;x\;P^+,\mathbf{p}_\perp\rangle+\psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{p}_\perp)|+\tfrac{1}{2}-1;x\;P^+,\mathbf{p}_\perp\rangle+\psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{p}_\perp)|-\tfrac{1}{2}+1;x\;P^+,\mathbf{p}_\perp\rangle+\psi^{\uparrow}_{-\frac{1}{2}-1}(x,\mathbf{p}_\perp)\,|-\tfrac{1}{2}-1;x\;P^+,\mathbf{p}_\perp\rangle\Big].$$

Similarly, two particle Fock state for an electron with $J^z=-\frac{1}{2}$ also has four possible combinations:

$$\Psi^{\downarrow}(P^+,\mathbf{P}_\perp=\mathbf{0}_\perp)=\int\frac{dx\;d^2\mathbf{p}_\perp}{\sqrt{x(1-x)}16\pi^3}\Big[\psi^{\downarrow}_{+\frac{1}{2}+1}(x,\mathbf{p}_\perp)\,|+\tfrac{1}{2}+1;x\;P^+,\mathbf{p}_\perp\rangle+\psi^{\downarrow}_{+\frac{1}{2}-1}(x,\mathbf{p}_\perp)|+\tfrac{1}{2}-1;x\;P^+,\mathbf{p}_\perp\rangle+\psi^{\downarrow}_{-\frac{1}{2}+1}(x,\mathbf{p}_\perp)|-\tfrac{1}{2}+1;x\;P^+,\mathbf{p}_\perp\rangle+\psi^{\downarrow}_{-\frac{1}{2}-1}(x,\mathbf{p}_\perp)\,|-\tfrac{1}{2}-1;x\;P^+,\mathbf{p}_\perp\rangle\Big],$$

where

$$\varphi(x,\mathbf{p}_\perp)=\frac{e}{\sqrt{1-x}}\frac{1}{M^2-\frac{\mathbf{p}_\perp^2+m^2}{x}+\frac{\mathbf{p}_\perp^2+\lambda^2}{1-x}}.$$

## Unpolarized Wigner distributions

- Wigner distribution in the light-front framework is defined as[2,3]

$$\rho^{[\Gamma]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;S)=\int\frac{d^2\Delta_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}W^{[\Gamma]}(\Delta_\perp,\mathbf{p}_\perp,x;S),$$

where

$$W^{[\Gamma]}(\Delta_\perp,\mathbf{p}_\perp,x;S)=\int\frac{dz^-d^2z_\perp}{2(2\pi)^3}e^{ip\cdot z}\langle P'';S|\bar{\psi}(-z/2)\Gamma\mathcal{W}_{[-\frac{z}{2},\frac{z}{2}]}\psi(z/2)|P';S\rangle,$$

with $\Gamma$, for example $\gamma^+,\gamma^+\gamma^5,i\sigma^{j+}\gamma_5$ and $S$ is the spin of the composite system. We have defined the $P'=(P^+,P'^-,\frac{\Delta_\perp}{2})$ and $P''=(P^+,P''^-,-\frac{\Delta_\perp}{2})$ are the initial and final momentum of the composite system. We define the unpolarized Wigner distribution

$$\begin{aligned}\rho_{UU}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)&=\frac{1}{2}\Big[\rho^{[\gamma^+]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;+\hat{e}_z)+\rho^{[\gamma^+]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;-\hat{e}_z)\Big],\\&=\frac{4e^2}{2(2\pi)^2 16\pi^3}\int d\Delta_x d\Delta_y\int dx\cos(\Delta_x b_x+\Delta_y b_y)\\&\times\Big[\frac{1+x^2}{x^2(1-x)^2}\Big(\mathbf{p}_\perp^2-\frac{(1-x)^2}{4}\mathbf{\Delta}_\perp^2\Big)+\Big(M-\frac{m}{x}\Big)^2\Big]\varphi^\dagger(\mathbf{p}''_\perp)\varphi(\mathbf{p}'_\perp).\end{aligned}$$

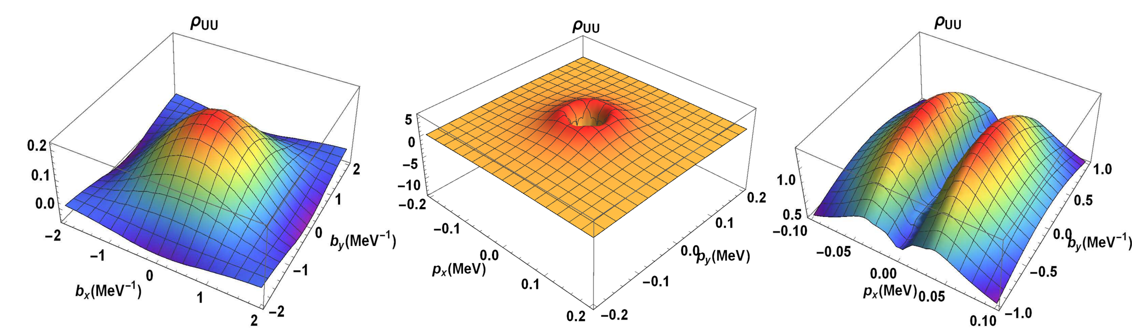

Figure 1: Plots of Wigner distribution $\rho_{UU}(\mathbf{b}_\perp,\mathbf{p}_\perp)$ for physical electron in impact-parameter space with fixed transverse momentum $\mathbf{p}_\perp=0.4\;MeV\;\hat{e}_x$ (left panel), in momentum space with fixed impact-parameter $\mathbf{b}_\perp=0.4\;MeV^{-1}\;\hat{e}_x$ (middle panel) and in mixed space (right panel).

## Longitudinal-unpolarized polarized Wigner distributions

- The longitudinally-unpolarized Wigner distribution is defined as

$$\begin{aligned}\rho_{LU}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)&=\frac{1}{2}\Big[\rho^{[\gamma^+]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;+\hat{e}_z)-\rho^{[\gamma^+]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;-\hat{e}_z)\Big],\\&=\frac{4\;e^2}{2(2\pi)^2 16\pi^3}\int d\Delta_x d\Delta_y\int dx\sin(\Delta_x b_x+\Delta_y b_y)\\&\quad\frac{(\Delta_x p_y-\Delta_y p_x)}{x^2(1-x)}(x^2-1)\varphi^\dagger(\mathbf{p}''_\perp)\varphi(\mathbf{p}'_\perp).\end{aligned}$$

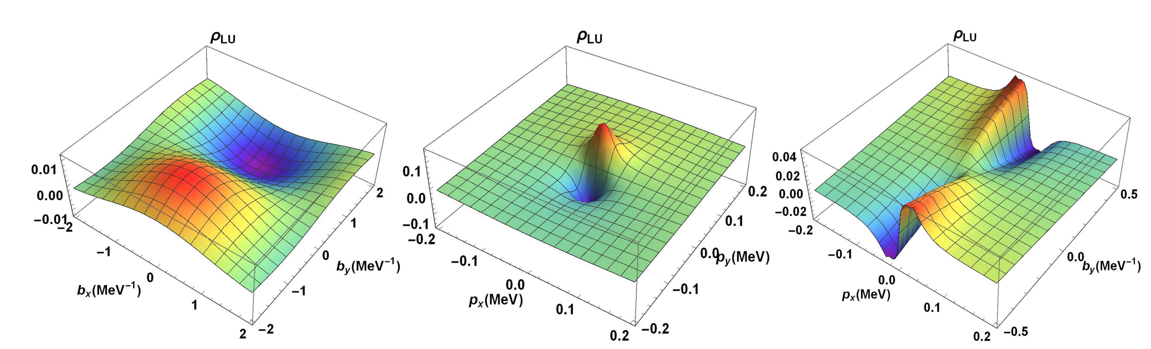

Figure 2: Plots of Wigner distribution $\rho_{LU}(\mathbf{b}_\perp,\mathbf{p}_\perp)$ for physical electron in impact-parameter space with fixed transverse momentum $\mathbf{p}_\perp=0.4\;MeV\;\hat{e}_x$ (left panel), in momentum space with fixed impact-parameter $\mathbf{b}_\perp=0.4\;MeV^{-1}\;\hat{e}_x$ (middle panel) and in mixed space (right panel).

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## Transverse Wigner distributions

- The transverse Wigner distributions is defined as

$$\begin{aligned}\rho_{TT}(\mathbf{b}_\perp,\mathbf{p}_\perp,x)&=\frac{1}{2}\delta_{ij}\Big[\rho^{[i\sigma^{+j}\gamma_5]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;+\hat{e}_i)-\rho^{[i\sigma^{+j}\gamma_5]}(\mathbf{b}_\perp,\mathbf{p}_\perp,x;-\hat{e}_i)\Big],\\&=\frac{4e^2}{16\pi^3(2\pi)^2}\int d\Delta_x\,d\Delta_y\int dx\cos(\Delta_x b_x+\Delta_y b_y)\\&\times\frac{1}{x(1-x)^2}\Big(\mathbf{p}_\perp^2-\frac{(1-x)^2}{4}\mathbf{\Delta}_\perp^2\Big)\varphi^\dagger(\mathbf{p}''_\perp)\varphi(\mathbf{p}'_\perp).\end{aligned}$$

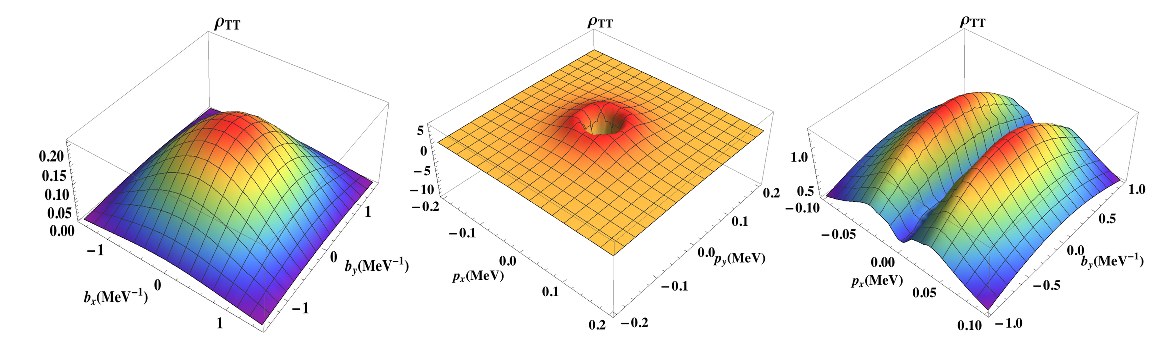


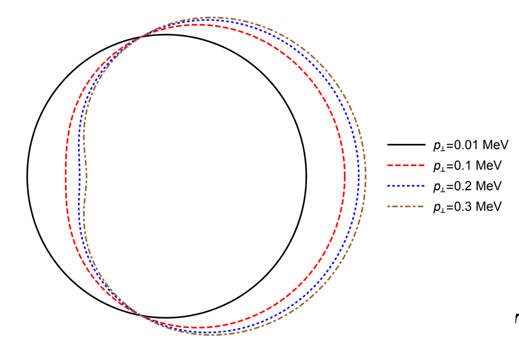
Figure 3: Plots of Wigner distribution $\rho_{TT}(\mathbf{b}_\perp,\mathbf{p}_\perp)$ for physical electron in impact-parameter space with fixed transverse momentum $\mathbf{p}_\perp=0.4\;MeV\;\hat{e}_x$ (left panel), in momentum space with fixed impact-parameter $\mathbf{b}_\perp=0.4\;MeV^{-1}\;\hat{e}_x$ (middle panel) and in mixed space (right panel).

## Transverse shape of an electron

- The shapes of an electron can be obtained by using the following relation which was first introduced in [4],

$$\frac{\hat{\rho}_{RELT}(\mathbf{p}_\perp,)/M}{\tilde{f}_1(\mathbf{p}_\perp{}^2)}=1+\frac{\tilde{h}_1(\mathbf{p}_\perp^2)}{\tilde{f}_1(\mathbf{p}_\perp^2)}\cos\phi_n+\frac{\mathbf{p}_\perp^2}{2M^2}\cos(2\phi-\phi_n)\frac{\tilde{h}^\perp_{1T}(\mathbf{p}_\perp^2)}{\tilde{f}_1(\mathbf{p}_\perp^2)},$$

where $\phi$ is the angle between $\mathbf{p}_\perp$ and $\mathbf{S}$ and $\phi_n$ is the angle between $\mathbf{n}$ and $\mathbf{S}$. $\mathbf{n}$ is the unit vector which describes the arbitrary spin of the particle in a fixed direction. $\mathbf{S}$ is the physical electron polarization in the transverse direction. Further, $f_1$, $h_1$ and $h^\perp_{1T}$ are the unpolarized electron distribution, transversity, and pretzelous distributions respectively and $\tilde{f}(\mathbf{p}_\perp^2)=\int dx f(x,\mathbf{p}_\perp^2)$.



Transverse shape of electron for different values of $\mathbf{p}_\perp$. The shapes are denoted with different lines.

Since the pretzelous distribution $h^\perp_{1T}$ is zero in this model, thus the shape of an electron explicitly depends upon $f_1$ and $h_1$. We show that how the shape of an electron emerges when we take the different angles between $\mathbf{n}$ and $\mathbf{S}$.

## References

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