

Topology, Thermal Simulations and Boundary Conditions

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in collaboration with

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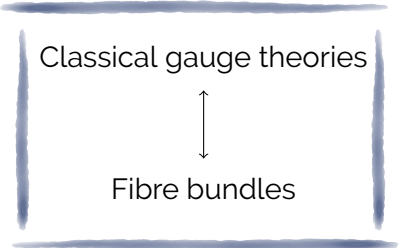


Topology and Freezing

Open-Boundary Conditions in Space

Switches

Some Topology



Classical gauge theories



Fibre bundles

Some Topology

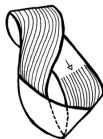
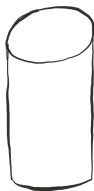
Classical gauge theories



Fibre bundles



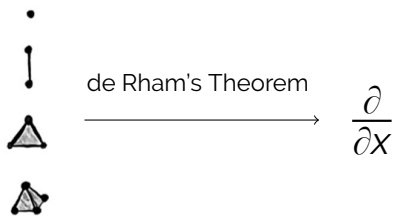
Can be 'twisted'!



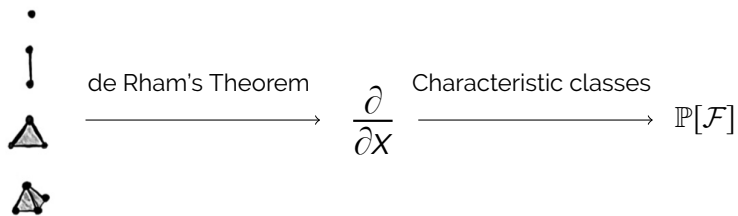
Cohomology and Classification



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$SU(N)$ bundles in $4D$ \longleftrightarrow 2^{nd} Chern class:

$$q = -\frac{1}{8\pi^2} \mathbf{Tr} (F \wedge F)$$

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Topological charge: $Q = -\frac{1}{16\pi^2} \int_M F_{\mu\nu} \tilde{F}^{\mu\nu}$

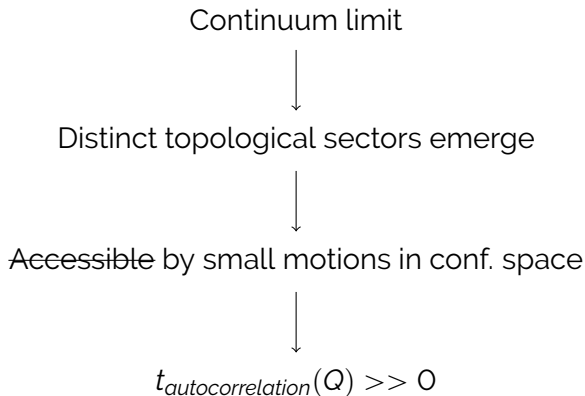


Remarks

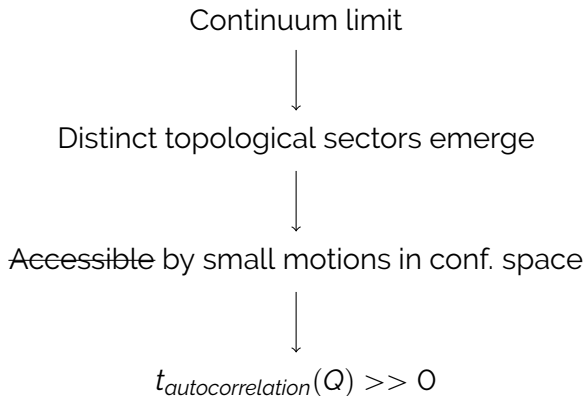
- For compact spaces $Q \in \mathbb{Z}$
- For non-compact spaces $Q \in \mathbb{R}$
- **Continuum** story

Refs: [Avis,Isham 1978] [DeWitt,Hart,Isham 1979]

Topological Freezing



Topological Freezing



A Solution: OBC

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T⁴

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T^4



Open-up the torus!

A Solution: OBC

[Luescher, Schaeffer 2011]: $F_{0\mu}(x)|_{x_0=0} = F_{0\mu}(x)|_{x_0=l_0} = 0$

$$\begin{array}{c} \downarrow \\ T \neq 0 \end{array}$$

Spatial OBC: $F_{i\mu}(x)|_{x_i=0} = F_{i\mu}(x)|_{x_i=l_i} = 0$

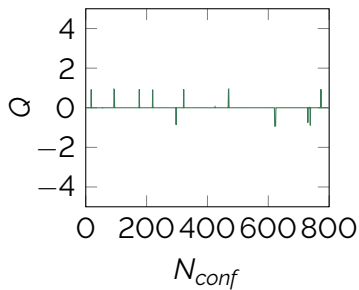
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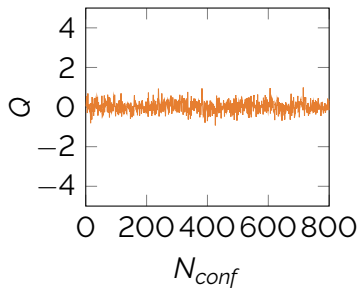
$T \neq 0$

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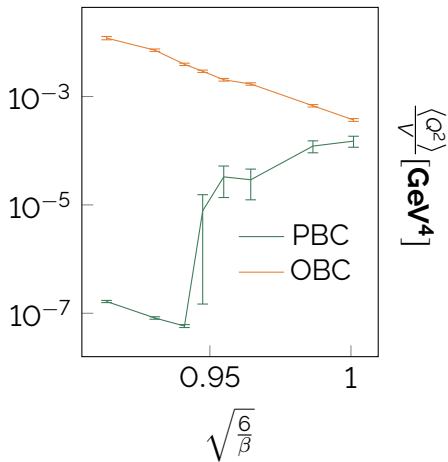
PBC



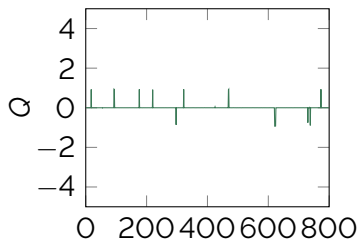
OBC



$T \approx 1.28T_c$

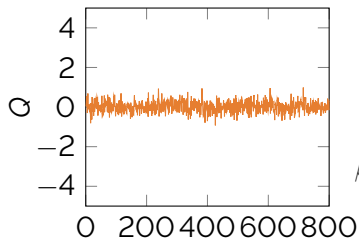


PBC



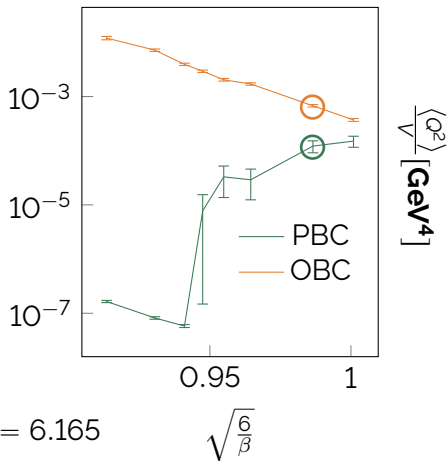
N_{conf}

OBC

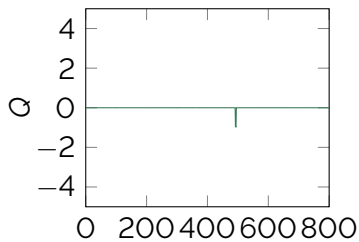


N_{conf}

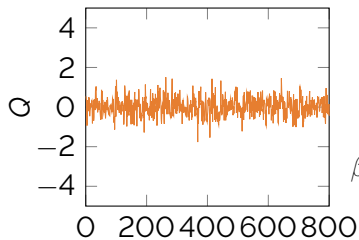
$T \approx 1.28T_c$



PBC

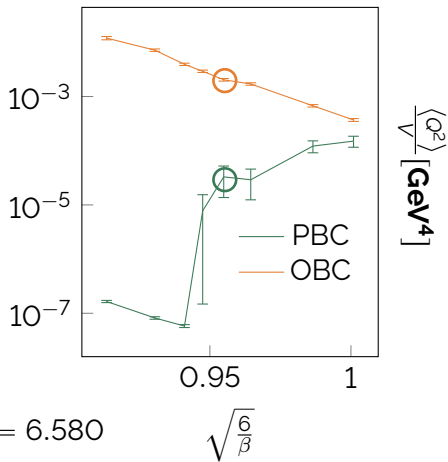


N_{conf}
OBC

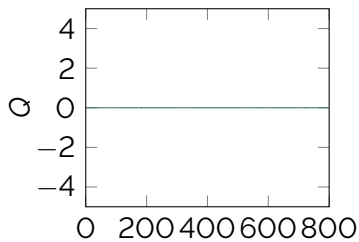


N_{conf}

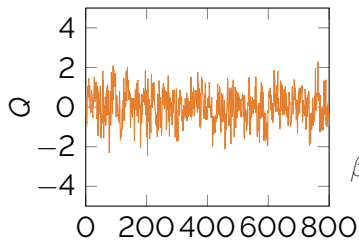
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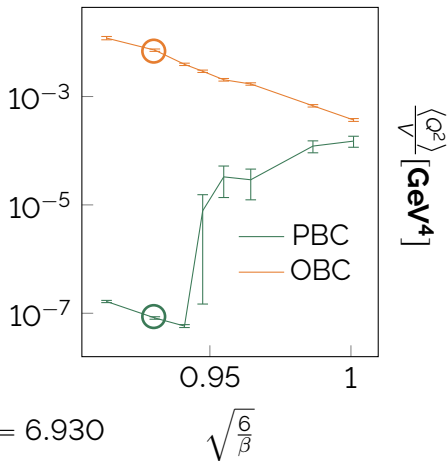


N_{conf}
OBC

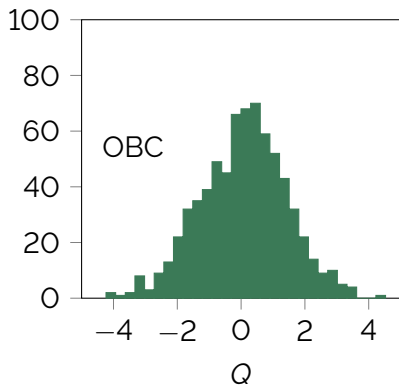
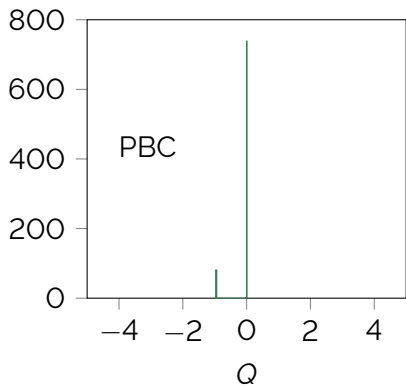


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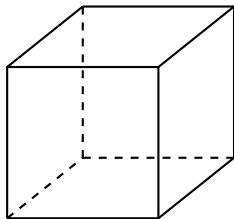
$$32^4, \beta = 6.872, T \approx 0$$



Finite Size Effects

$$V_s = L_s^3$$

t

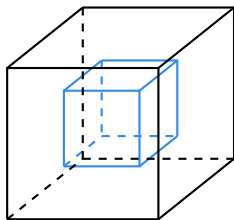


Finite Size Effects

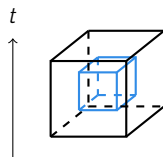
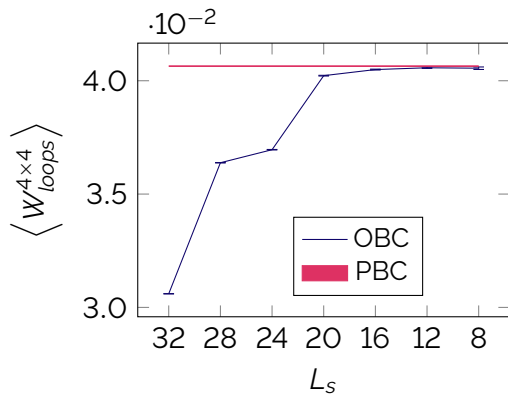
$$V_S = L_S^3$$

$$V'_S = L'_S{}^3$$

t



Finite Size Effects



Outlooks: Switching between BCs

Do n_{PBC} sweeps with PBC

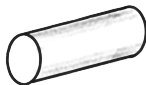


Outlooks: Switching between BCs

Do n_{PBC} sweeps with PBC



Switch PBC to OBC
in some direction(s)



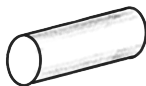
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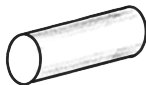


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Switch back



Outlooks: Switching between BCs

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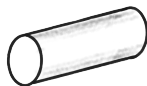
Switch PBC to OBC
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Switch back

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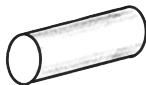
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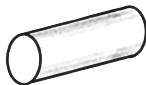


Switch back

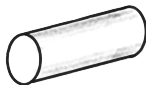
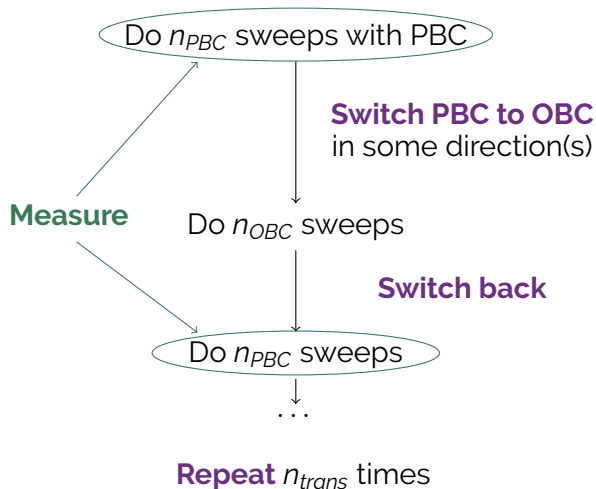
Do n_{PBC} sweeps



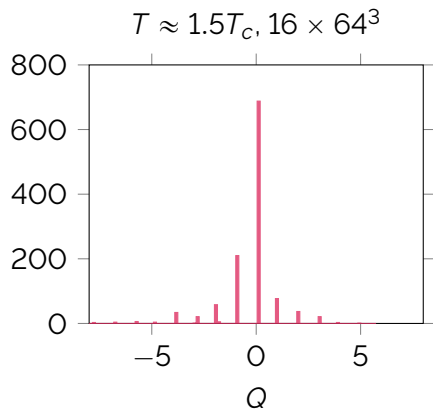
Repeat n_{trans} times



Outlooks: Switching between BCs

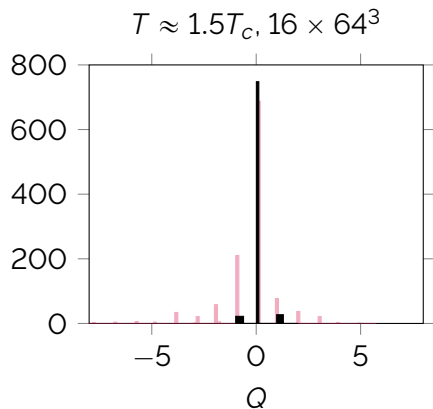


First Results



Generates higher Q 's

First Results



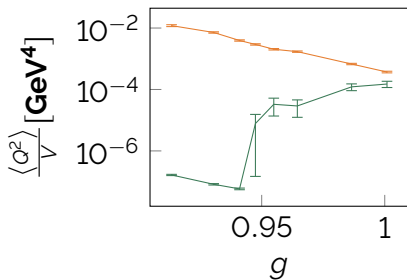
Generates higher Q's

but

oversamples

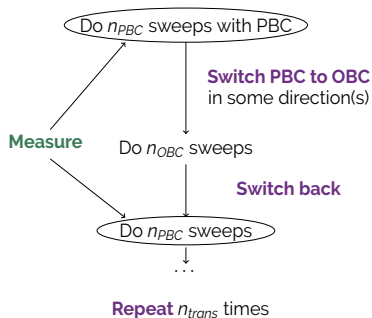
To Sum Up

- ▶ **OBC also solve topological freezing at finite temperature**
- ▶ May use them algorithmically



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- ▶ **May use them algorithmically**



Thank you!

(... and <https://www.pinterest.com/pin/571746115169525297/> for the Moebius strip)