Bayesian Analysis of Hybrid EoS based on Astrophysical Observational Data

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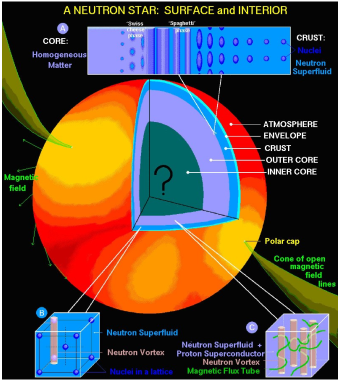
Qualification and Classification of EoS

- Estimation of different models of EoS from observational constraints
- Applying Bayesian Analysis for the estimation
- Finding suggestions for observation which could be most selective for the models of EoS

Neutron Star Structure

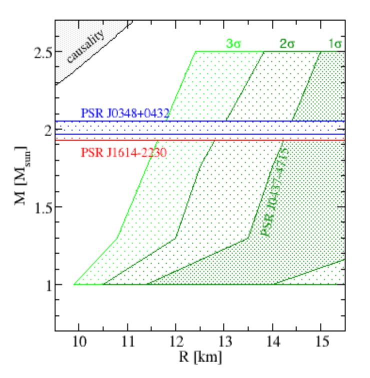


Credit: Dany Page

Observational Constraints

Mass and Radius Constraints

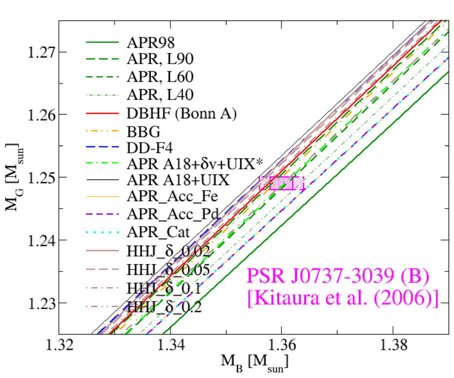
Radius and maximum mass constraints are given from PSR J0437-4715 [1] and PSR J0348+0432 [2] correspondingly.



Observational Constraints

Gravitational Binding Energy Constraint

A constraint on the gravitational binding energy is taken from the neutron star B in the binary system J0737-3039 (B) [3].



Observational Constraints

Three Statistically Independent Constraints

- A radius constraint from the nearest millisecond pulsar PSR J0437-4715 [1].
- A maximum mass constraint from PSR J0348+0432 [2].
- A constraint on the gravitational binding energy from the neutron star *B* in the binary system PSR J0737-3039 (B) [3].

Tolman–Oppenheimer–Volkoff equations

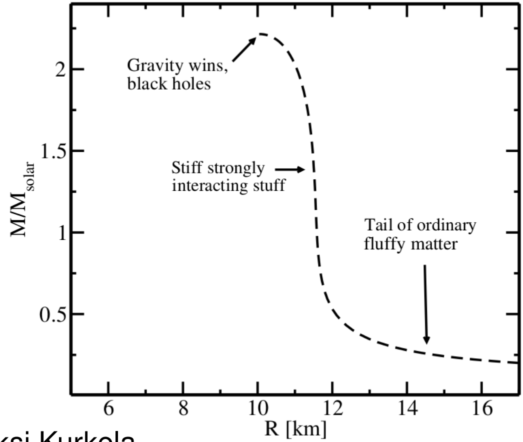
TOV equations

$$\begin{cases} \dfrac{dm(r)}{dr} = C_1 \epsilon r^2 \\ \dfrac{dm_B(r)}{dr} = C_1 n_B m_N \dfrac{r^2}{(1-2C_2 m/r)} \\ \dfrac{dp(\epsilon, r)}{dr} = -C_2 \dfrac{(\epsilon + p)(m + C_1 p r^3)}{r(r - 2C_2 m)} \end{cases} \quad (1)$$

Constants

$$C_1 = 1.11269 \cdot 10^{-5} \frac{\mathrm{M}_\odot}{\mathrm{km}^3} \frac{\mathrm{fm}^3}{\mathrm{MeV}} \qquad C_2 = 1.4766 \frac{\mathrm{km}}{\mathrm{M}_\odot}$$

Mass–Radius plot



Credit: Aleksi Kurkela

Laboratory of Information Technologies

Tolman–Oppenheimer–Volkoff equations

TOV equations

$$\begin{cases} \dfrac{dp}{d\epsilon} = F(m, dp/d\epsilon, r, \epsilon) \\ \dfrac{dm}{d\epsilon} = C_1 \epsilon r^2 F(m, dp/d\epsilon, r, \epsilon) \\ \dfrac{dm_B}{d\epsilon} = C_1 n_B m_N \dfrac{r^2}{(1 - 2C_2 m/r)} F(m, dp/d\epsilon, r, \epsilon) \end{cases} \quad (2)$$

$$F(m, dp/d\epsilon, r, \epsilon) = \frac{dp/d\epsilon}{-C_2 \dfrac{(\epsilon + p)(m + C_1 p r^3)}{r(r - 2C_2 m)}} \quad (3)$$

EoS Parametrization

Hybrid EoS

$$p(\epsilon) = p^{I}(\epsilon)\,\Theta(\epsilon_c - \epsilon) + p^{I}(\epsilon_c)\,\Theta(\epsilon - \epsilon_c)\,\Theta(\epsilon_c - \epsilon + \Delta\epsilon) + p^{II}(\epsilon)\,\Theta(\epsilon - \epsilon_c - \Delta\epsilon),$$

where $p^{I}(\epsilon)$ is given by a pure hadronic EoS (here well known model of APR), and $p^{II}(\epsilon)$ represents the high density nuclear matter [4] used here as quark matter given in the bag-like form.

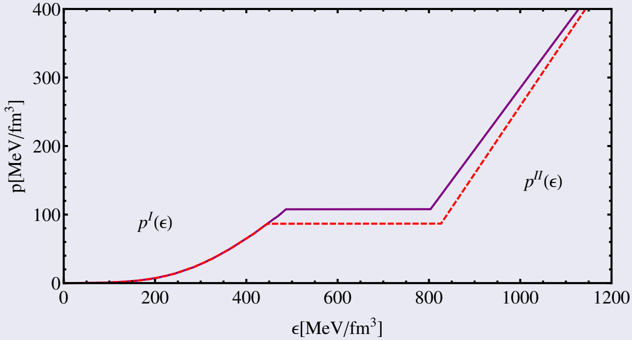
Bag-Like Form of QM EoS

$$p^{II}(\epsilon) = c_{QM}^2 \epsilon - B,$$

where c_{QM}^2 is the squared speed of sound in quark matter and B is the bag constant.

EoS Parametrization

Hybrid EoS



EoS Parametrization

Hybrid EoS Pareameters

$$400 \leq \epsilon_c \left[MeV/fm^3\right] \leq 1000 \quad : \quad \epsilon_c(k) \qquad k = 1 \ldots N_1 = 10$$

$$0 \leq \gamma = \frac{\Delta\epsilon}{\epsilon_c} \leq 1 \quad : \quad \gamma(l) \qquad l = 1 \ldots N_2 = 10$$

$$0.3 \leq c^2_{QM} \leq 1 \quad : \quad c^2_{QM}(m) \qquad m = 1 \ldots N_3 = 10$$

Vector of Parameters

For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values:

$$\pi_i = \vec{\pi}\left(\epsilon_c(k), \gamma(l), c^2_{\mathrm{QM}}(m)\right),$$

$i = 1 \ldots N$ (here $N = \prod_{q=1}^{3} N_q$) and $i = N_1 \times N_2 \times k + N_2 \times l + m$

Qualification of EoS Set from Observation

Goal

To find the set π_i corresponding to an EoS and thus a sequence of configurations which contains the most probable one based on the given constraints using BA (calculate of *a posteriori* probabilities of π_i).

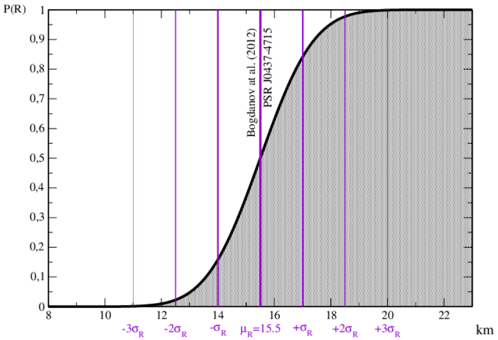
Unification of *a priori* probabilities

$P(\pi_i) = 1$ for $\forall i$.

Calculation of Probabilities

Probability of Corresponding to Radius Constraint for π_i

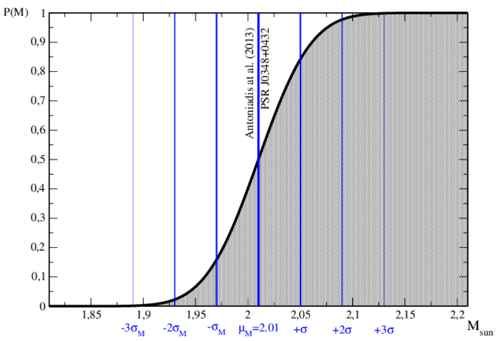
$P\left(E_B\,|\pi_i\right) = \Phi(R_i, \mu_B, \sigma_B)$, here R_i is max radius given by π_i.
$\mu_B = 15.5\ \mathrm{km}$ and $\sigma_B = 1.5\ \mathrm{km}$ [1].



Calculation of Probabilities

Probability of Corresponding to Mass Constraint for π_i

$P\left(E_A \,|\pi_i\right) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i.
$\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$ [2].



Calculation of Probabilities

Probability of Corresponding to $M - M_B$ Constraint for π_i

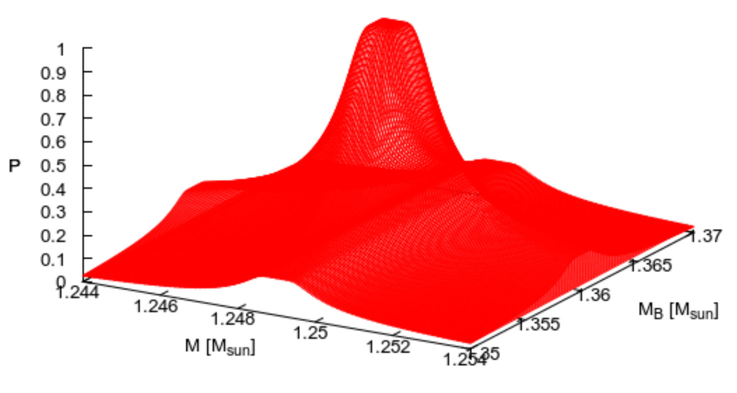
We need to estimate the probability for the closeness of a theoretical point $M_i = (M_i, M_{Bi})$ to the observed point $\mu_K = (\mu_G, \mu_B)$. The required probability can be calculated using the following formula

$$P(E_K | \pi_i) = [\Phi(\xi_G) - \Phi(-\xi_G)] \cdot [\Phi(\xi_B) - \Phi(-\xi_B)],$$

where $\Phi(x) = \Phi(x, 0, 1)$, $\xi_G = \sigma_{M_G}/d_{M_G}$ and $\xi_B = \sigma_{M_B}/d_{M_B}$, with d_{M_G} and d_{M_B} being the absolute values of components of the vector $\mathbf{d}_\mathrm{i} = \mu - \mathbf{M}_i$, where $\mu_\mathbf{B} = (\mu_G, \mu_B)^T$ is given in

Calculation of Probabilities

Probability of $M - M_B$ for π_i



Calculation of Probabilities

Probability of All Constraints for π_i

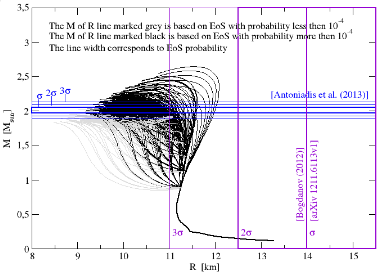
Taking to the account assumption that these measurements are independent on each other we can calculate complete conditional probability:

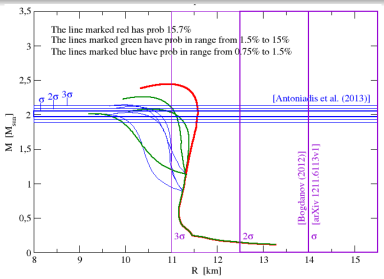
$$P(E|\pi_i) = P(E_A|\pi_i) \times P(E_B|\pi_i) \times P(E_K|\pi_i)$$

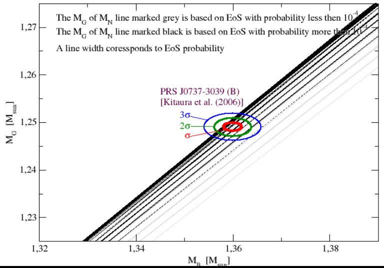
Calculation of *a posteriori* Probabilities of π_i

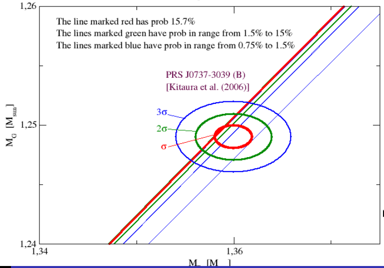
Now, we can calculate probability of π_i using Bayes' theorem:

$$P(\pi_i|E) = \frac{P(E|\pi_i)\,P(\pi_i)}{\sum\limits_{j=0}^{N-1} P(E|\pi_j)\,P(\pi_j)}$$


The M of R line marked grey is based on EoS with probability less then 10^-4
The M of R line marked black is based on EoS with probability more then 10^-4
The line width corresponds to EoS probability
σ 2σ 3σ
[Antoniadis et al. (2013)]
[Bogdanov (2012)]
[arXiv 1211.6113v1]
3σ
2σ
σ
M [M_sun]
R [km]


The line marked red has prob 15.7%
The lines marked green have prob in range from 1.5% to 15%
The lines marked blue have prob in range from 0.75% to 1.5%
σ 2σ 3σ
[Antoniadis et al. (2013)]
[Bogdanov (2012)]
[arXiv 1211.6113v1]
3σ
2σ
σ
M [M_sun]
R [km]


The M_G of M_N line marked grey is based on EoS with probability less then 10^-4
The M_G of M_N line marked black is based on EoS with probability more then 10^-4
A line width coresspods to EoS probability
PRS J0737-3039 (B)
[Kitaura et al. (2006)]
3σ
2σ
σ
M_G [M_sun]
M_B [M_sun]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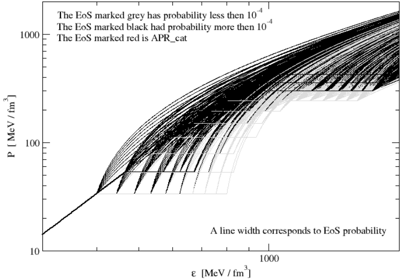
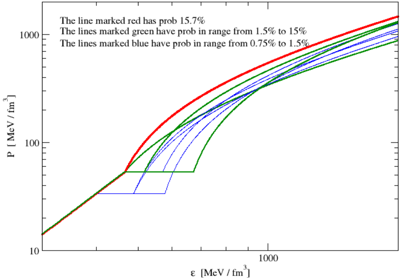
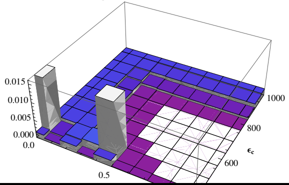
[Kitaura et al. (2006)]
3σ
2σ
σ
M_G [M_sun]

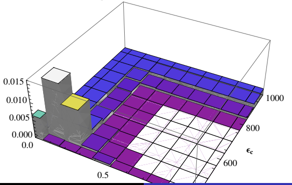




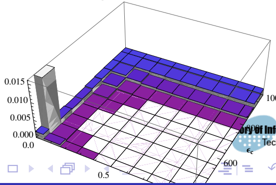
c^2_{QM}=0.922222



c^2_{QM}=0.844444



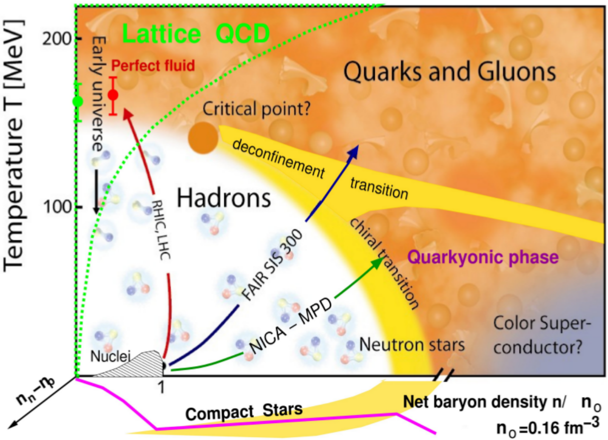
c^2_{QM}=0.533333



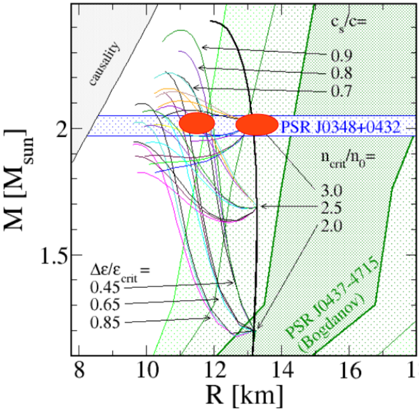
Conclusions

- The most probable set of parameters resulting from the Bayesian Analysis point out to a quite stiff EoS with a smooth phase transition.
- Less probable configurations have jump in phase transition. Most of these EoS are pretty much stiff as well.
- The 7 most probable EoS do not allow a "third family".

Phase Diagram



Fake measurements



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In the end, there can be only one.
– Duncan MacLeod

Thanks for your attention!