MPAMCS 2014

Investigation of Mathematical Model of Nonlinear Optic Systems with 2D Feedback

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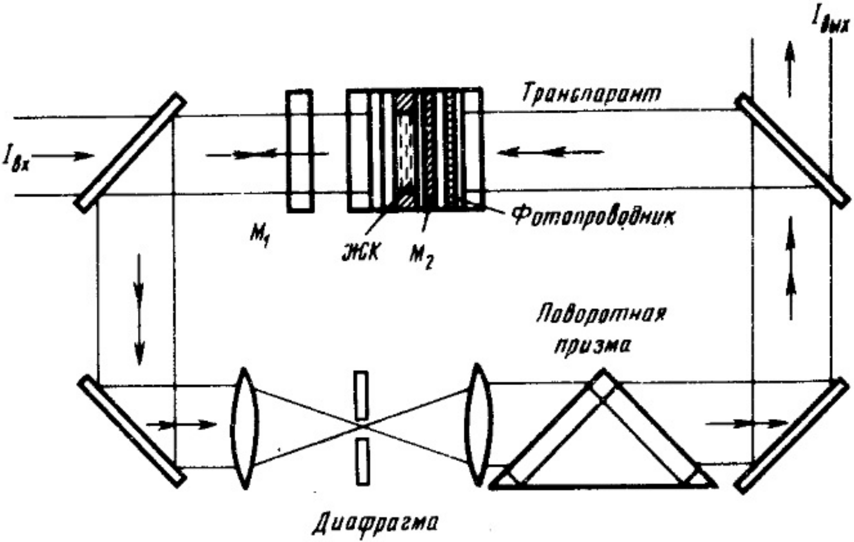
Optic System

S.Akhmanov, M. Vorontzov, V. Ivanov, JETP, 47, 1988.



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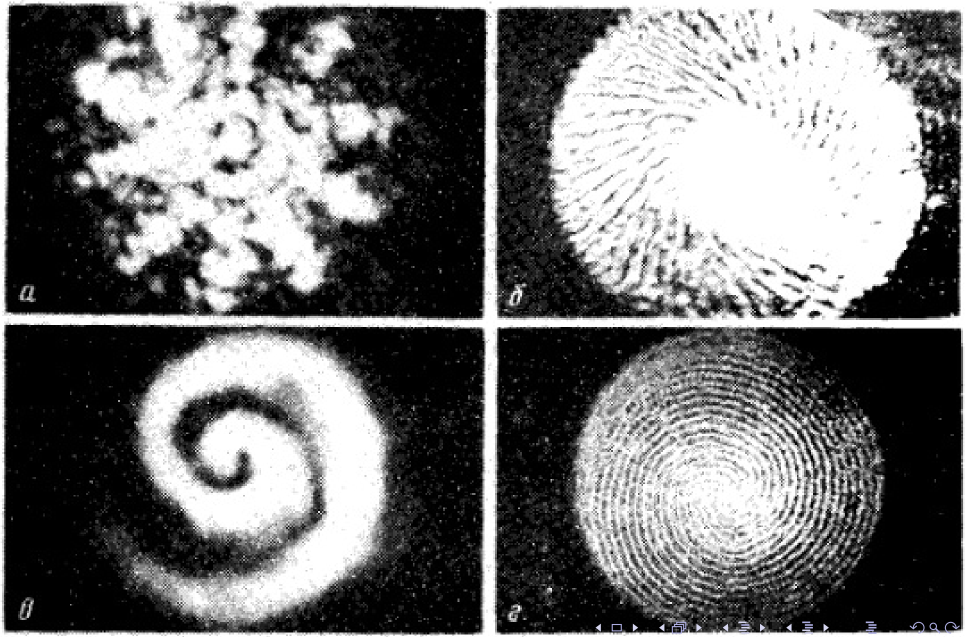

Optic System

$$u_t = D\Delta u - u + K(1 + \gamma\cos(u(x + \Delta x, t) + \phi_0))$$

$$\left.\frac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0,$$

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A. Skubachevskii, Nonl. Anal, 32, 1998.

Generalized Problem

Let $\Omega \subset \mathbb{R}^n$ be bounded Lischitz domain and unbounded operator $A\colon D(A) \to L_2(\Omega)$ is given by the "energy" form

$$a[u,\, v] = \sum_{i,j=1}^{n} (a_{ij} u_{x_i},\, v_{x_j}) + \sum_{i=1}^{n} (b_i u_{x_i},\, v) + c(u,\, v):$$

$$(Au,\, v)_{L_2(\Omega)} = a[u,\, v], \qquad \forall v \in V.$$

We consider the following problem:

$$u' + Au = f, \tag{1}$$

$$u\big|_{t=0} = \varphi. \tag{2}$$

Definition of Solution

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Definition

The function

$$u \in L_2\left(0, T; D(A)\right) \cap H^1\left(0, T; L_2(\Omega)\right)$$

is called the solution of problem (1)-(2), if it satisfies the equality (1) for almost all $t \in (0, T)$ *and initial condition (2).*

Conditions

Condition

Coeffitients b_i *and* c *are bounded operators in* $L_\infty(\Omega)$ *and* a_{ij} *are multipliers in* $H^s(\Omega)$ *for* $|s| < \varepsilon$.

The form $a[u, v]$ *is strongly coercitive, i.e.,*

$$\operatorname{Re} a[u, u] \geq C\|u\|^2_{H^1(Q)}.$$

Theorem

Assume that Condition holds. Then operator for all $f \in L_2(\Omega \times (0,T))$ and $\varphi \in H^1(\Omega)$ problem (1)-(2) has a unique strong solution given by

$$u(x,\, t) = G(t)\varphi(x) + \int\limits_0^t G(t-s)f(x,\, s)\, ds,$$

where $\{G(t)\}$ $(t \geq 0)$ is an analytic semigroup generated by operator $-A$.

Corollary

Assume that previouse Condition holds and $1 < p \leq 2$. Then for all $f \in L_p(0,T;L_2(\Omega))$ and $\varphi \in B_{2,p}^{2-\frac{2}{p}}(\Omega)$ problem (1)-(2) has a unique strong solution $u \in L_p(0,T;D(A)) \cap W^{1,p}(0,T;L_2(\Omega))$ given by

$$u(x,\, t) = G(t)\varphi(x) + \int\limits_0^t G(t-s)f(x,\, s)\, ds,$$

where $\{G(t)\}$ $(t \geq 0)$ is an analytic semigroup generated by operator $-A$.

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• strongly elliptic differential equations with bounded measurable coefficients in Lipschitz domains;
A. Axelsson, K. Stephen, and A. McIntosh,
J. London Math. Soc. 74(1) (2006) 113–130.

• operators, given by sesquilinear, satisfying our Condition,
M.S. Agranovich and A.M. Selitskii, Funktsional'nyi Analiz i Ego Prilozheniya 47(2) (2013) 2–17.