Moscow Institute of Physics and Technology
NRC "Kurchatov Institute" - ITEP

Kirill Gubarev

11D extension of 10D generalized supergravity

supervisor: Edvard Musaev

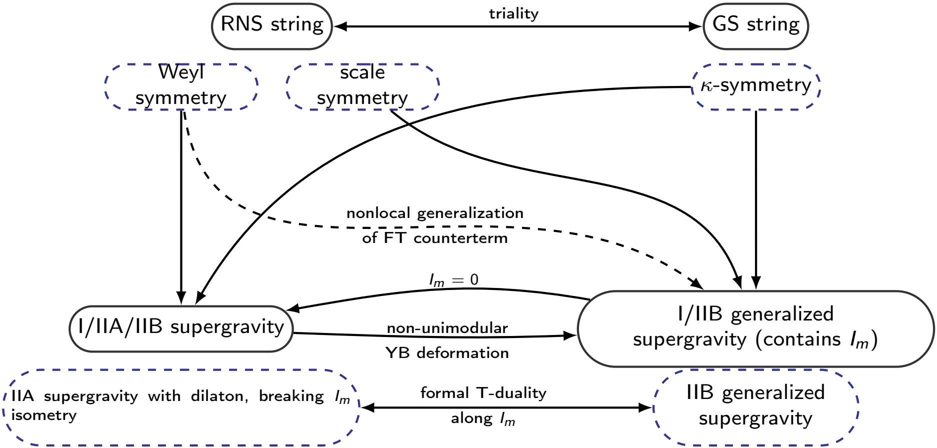


Рис.: Ordinary and generalized supergravities in 10D

Generalized 10D supergravity from bivector deformation in DFT

$$S_{DFT} = \int dx^D e^{-2d}\,\mathcal{R}, \qquad d = \phi - \frac{1}{4} log\, G. \tag{1}$$

DFT curvature

$$\begin{aligned}\mathcal{R} =& \mathcal{H}^{\mathcal{AB}}(2E^{\mathcal{M}}{}_{\mathcal{A}}\partial_{\mathcal{M}}\mathcal{F}_{\mathcal{B}} - \mathcal{F}_{\mathcal{A}}\mathcal{F}_{\mathcal{B}}) + \mathcal{F}_{\mathcal{ABC}}\mathcal{F}_{\mathcal{DEF}}(\frac{1}{4}\mathcal{H}^{\mathcal{AD}}\eta^{\mathcal{BE}}\eta^{\mathcal{CF}} - \frac{1}{12}\mathcal{H}^{\mathcal{AD}}\mathcal{H}^{\mathcal{BE}}\mathcal{H}^{\mathcal{CF}}) \\ & - 2\eta^{\mathcal{AC}}E^{\mathcal{M}}{}_{\mathcal{C}}\partial_{\mathcal{M}}\mathcal{F}_{\mathcal{A}} + \eta^{\mathcal{AB}}\mathcal{F}_{\mathcal{A}}\mathcal{F}_{\mathcal{B}} - \frac{1}{6}\mathcal{F}_{\mathcal{ABC}}\mathcal{F}_{\mathcal{DEF}}\eta^{\mathcal{AD}}\eta^{\mathcal{BE}}\eta^{\mathcal{CF}},\end{aligned} \tag{2}$$

$$\mathcal{H}_{\mathcal{AB}} = \begin{pmatrix} h_{ab} & 0 \\ 0 & h^{ab}\end{pmatrix}, \quad \eta_{\mathcal{AB}} = \begin{pmatrix} 0 & \delta_a^b \\ \delta_b^a & 0\end{pmatrix}, \quad \partial_{\mathcal{M}} = (\partial_m, \partial^m \equiv 0), \quad \partial_{\mathcal{A}} = E_{\mathcal{A}}^{\mathcal{M}}\partial_{\mathcal{M}}. \tag{3}$$

Fluxes

$$\mathcal{F}_{\mathcal{ABC}} = 3E_{\mathcal{N}[\mathcal{C}}\partial_{\mathcal{A}}E^{\mathcal{N}}{}_{\mathcal{B}]}, \qquad \mathcal{F}_{\mathcal{A}} = E_{\mathcal{N}\mathcal{A}}\partial^{\mathcal{B}}E^{\mathcal{N}}{}_{\mathcal{B}} + 2\partial_{\mathcal{A}}d. \tag{4}$$

Parametrization that corresponds to supergavity

$$E_{\mathcal{M}}{}^{\mathcal{A}} = \begin{pmatrix} e_m^a & 0 \\ -e_a^k B_{km} & e_a^m\end{pmatrix}. \tag{5}$$

Generalized 10D supergravity from bivector deformation in DFT

$$\tilde{E}_{\mathcal{M}}{}^{\mathcal{A}} = O[\beta]_{\mathcal{M}}{}^{\mathcal{N}} E_{\mathcal{N}}{}^{\mathcal{B}} = \begin{pmatrix} \tilde{e}^a_m & 0 \\ -\tilde{e}^k_a \tilde{B}_{km} & \tilde{e}_a^m \end{pmatrix}, \qquad O_{\mathcal{M}}{}^{\mathcal{N}} = \begin{pmatrix} \delta^n_m & -\beta^{nm} \\ 0 & \delta^m_n \end{pmatrix}. \tag{6}$$

For the deformation parameters, we use the polykilling ansatz

$$\beta^{mn} = r^{ab} k_a^m k_b^n, \quad I^m = \nabla_k \beta^{km} = \frac{1}{2} f_{ab}{}^c k_c{}^m r^{ab} \neq 0, \quad f_{de}{}^{[a} r^{b|d|} r^{c]e} = 0\,(\text{CYBE}) \tag{7}$$

under such deformations, the fluxes transform as

$$\delta \mathcal{F}_{\mathcal{ABC}} = 0, \quad \delta \mathcal{F}_a = 2 I^m b_{mn} e^n_a, \quad \delta \mathcal{F}^a = 2 I^a. \tag{8}$$

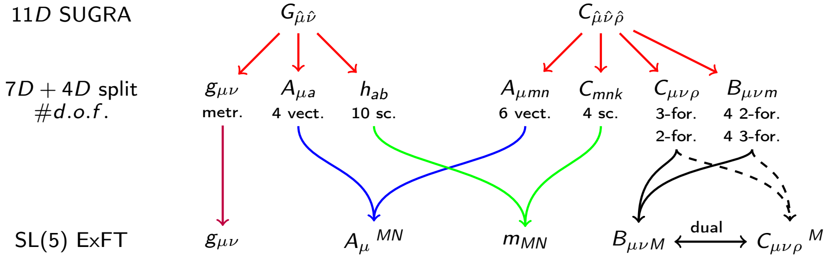
$$\begin{tikzcd}
E_{\mathcal{M}}{}^{\mathcal{A}} \arrow[r] \arrow[d, dashed, <->, "\mathcal{L}"] & \tilde{E}_{\mathcal{M}}{}^{\mathcal{A}} = O_{\mathcal{M}}{}^{\mathcal{N}} E_{\mathcal{N}}{}^{\mathcal{A}} \arrow[d, dashed, <->, "\mathcal{L}"] \\
\mathcal{F} \arrow[r] \arrow[d] & \tilde{\mathcal{F}} = \mathcal{F} + \delta\mathcal{F} \arrow[d] \\
\text{EoMs}(\mathcal{F})=0 \arrow[d, dash] & \text{EoMs}(\tilde{\mathcal{F}} - \delta\mathcal{F})=0 \arrow[d, dash] \\
\text{EoMs}_{SUGRA}(g, B, d) = 0 & \text{EoMs}_I(\tilde{g}, \tilde{B}, \tilde{d}) = 0
\end{tikzcd}$$

$$\mathcal{R} = R[h_4] + 4\,\nabla_m X^m - 4\,X_m X^m - \frac{1}{12} H^2 \tag{9}$$

$$X_m = I_m + Z_m, \quad Z_m = \partial_m \Phi - B_{mn} I^n\,. \tag{10}$$

Exceptional field theory

Exceptional field theory (ExFT) - this is a generalization of 11-dimensional supergravity in split form $d + (11 - d)$, explicitly covariant with respect to the U-duality group.



$$m_{MN} = \mathcal{E}_M{}^A \mathcal{E}_N{}^B m_{AB} \tag{11}$$

Parametrization that corresponds to supergavity

$$m_{MN} = \mathcal{E}_M{}^A \mathcal{E}_N{}^B m_{AB}, \quad \mathcal{E}_M{}^A = e^{\frac{1}{10}} \begin{bmatrix} e^{-\frac{1}{2}} e_m{}^a & e^{\frac{1}{2}} V^a \\ 0 & e^{\frac{1}{2}} \end{bmatrix}, \quad \partial_{5m} = \partial_m, \quad \partial_{mn} = 0, \tag{12}$$

$e = \det(e_m^a)$, $V^m = \frac{1}{3!}\varepsilon^{mnkl} C_{nkl}$

Ansatz for fields

We consider theory on the background $M_{11} = M_7 \times M_4$

$$\begin{aligned} g_{\mu\nu} &= g_{\mu\nu}(y^\mu, x^m), & m_{MN} &= m_{MN}(x^m), \\ A_\mu{}^{MN} &= 0, & B_{\mu\nu M} &= 0. \end{aligned} \tag{13}$$

$$g_{\mu\nu}(y^\mu, x^m) = e^{-2\phi(x^m)} e^{\frac{2}{5}} \bar{g}_{\mu\nu}(y^\mu), \quad m_{MN} = e^{-\phi} e^{\frac{1}{5}} M_{MN}. \tag{14}$$

Lagrangian

$$\begin{aligned} m\, \mathcal{L}' = {} & Y_{AB} Y_{CD} m^{AC} m^{BD} - \frac{1}{2} Y_{AB} Y_{CD} m^{AB} m^{CD} + 32 Z^{ABC} Z^{DEF} m_{AD} m_{BE} m_{CF} \\ & + 32 Z^{ABC} Z^{DEF} m_{AC} m_{BD} m_{EF} - \frac{7}{3} \theta_{AB} \theta_{CD} m^{AC} m^{BD} + \bar{e} \mathcal{R}[\bar{g}_{(7)}], \end{aligned} \tag{15}$$

Fluxes

$$\mathcal{F}_{ABC}{}^D = \frac{3}{2} Z_{ABC}{}^D - \frac{1}{2} \theta_{[AB} \delta_{C]}{}^D + \delta_{[A}{}^D Y_{B]C}, \tag{16}$$

$$\begin{aligned} \theta_{AB} &= E^{-1} E^{MN}{}_{AB} \partial_{MN} E - E^M{}_{[A} \partial_{MN} E^N{}_{B]}, \\ Y_{AB} &= -E^M{}_{(A} \partial_{MN} E^N{}_{B)}, \\ Z^{FGE} &= -\frac{1}{24} \epsilon^{FGABC} E^E_M \partial_{AB} E^M_C - \frac{1}{48} \epsilon^{GABCE} E^F_M \partial_{AB} E^M_C + \frac{1}{48} \epsilon^{FABCE} E^G_M \partial_{AB} E^M_C. \end{aligned} \tag{17}$$

Generalized Yang-Baxter deformation

$$\tilde{E}_M{}^A = O[\Omega]_M{}^N E_N{}^B = e^{-\frac{\tilde{\phi}}{2}} \begin{bmatrix} \tilde{e}^{1/2}\tilde{e}^m{}_a & 0 \\ -\tilde{e}^{1/2}\tilde{V}^m & \tilde{e}^{-1/2} \end{bmatrix}, \qquad O[\Omega] = \begin{bmatrix} \delta_m{}^n & 0 \\ \frac{1}{3!}\epsilon_{mpqr}\Omega^{pqr} & 1 \end{bmatrix}. \quad (18)$$

For the deformation parameters, we use the polykilling ansatz

$$\Omega^{mnk} = \frac{1}{6}\rho^{\alpha\beta\gamma} k_\alpha{}^m k_\beta{}^n k_\gamma{}^k, \quad (19)$$

satisfying the condition (a generalization of the Yang-Baxter equation)

$$6\rho^{[i2|i7j1}\rho^{|i3i4|j2} f_{j1j2}{}^{|i5]} + \rho^{j1j2[i2}\rho^{i3i4i5]} f_{j1j2}{}^{i7} = 0. \quad (20)$$

Under such deformations, the fluxes transform as

$$\delta\mathcal{F}_{ABC}{}^D = \frac{1}{4} E^m{}_C E^n{}_A E^k{}_B E_l{}^E J^{lp}\epsilon_{kmnp}, \quad (21)$$

$$J^{mn} = k_{i_1}{}^m k_{i_4}{}^n \rho^{i_1 i_2 i_3} f_{i_2 i_3}{}^{i_4}, \quad (22)$$

that under reduction gives

$$J^{mn} = \rho^{i_1 i_2 i_3} f_{i_2 i_3}{}^{i_4} k_{i_1}{}^m k_{i_4}{}^n \underset{i=(*,\alpha),\ m=(*,\bar{m})}{\longrightarrow} I^{\bar{m}} \equiv J^{*\bar{m}} = \rho^{*\alpha\beta} f_{\alpha\beta}{}^\gamma k_\gamma{}^{\bar{m}}. \quad (23)$$

Construction of generalized supergravity equations

$$
\begin{tikzcd}
E_M{}^A \arrow[r] \arrow[d, leftrightarrow, dashed, "\mathcal{L}"] & \tilde{E}_M{}^A = O_M{}^N E_N{}^A \arrow[d, leftrightarrow, dashed, "\mathcal{L}"] \\
\mathcal{F} \arrow[r] \arrow[d] & \tilde{\mathcal{F}} = \mathcal{F} + \delta\mathcal{F} \arrow[d] \\
\text{EoMs}(\mathcal{F})=0 \arrow[d, dash] & \text{EoMs}(\tilde{\mathcal{F}} - \delta\mathcal{F})=0 \arrow[d, dash] \\
\text{EoMs}_{SUGRA}(g, C) = 0 & \text{EoMs}_J(\tilde{g}, \tilde{C}) = 0
\end{tikzcd}
$$

Generalized equations (ϕ and C_{mnk})

$$
\begin{aligned}
0 &= \frac{1}{7} e^{2\phi} \mathcal{R}[\bar{g}_{(7)}] + \frac{1}{6} (\nabla V)^2 + \tilde{\nabla}^m X_m - 6 X_m X^m - 2 J^{mn} J_{mn} + \frac{4}{3} J_{mn} J^{nm}, \\
0 &= \tilde{\nabla}^m F_{mnkl} - 6\Big(X^m F_{mnkl} + 2 J^{pm} C_{m[nk} J_{l]p} - J^{pm} J_{p[n} C_{kl]m} \Big),
\end{aligned} \tag{24}
$$

where

$$
\begin{aligned}
F_{mnkl} &= 4 \partial_{[m} C_{nkl]}, \qquad Z_m = \epsilon_{mnkl} J^{nk} V^l, \\
X_m &= \partial_m \phi - \frac{2}{3} Z_m, \qquad \tilde{\nabla}_m = \nabla_m - \partial_m \phi.
\end{aligned} \tag{25}
$$

Thank you for your attention!

Рис.: "Deformations open the way to the world of new knowledge"