Calculation of neutrino oscillation probabilities in the Earth using Magnus expansion

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Motivation

- Atmospheric neutrinos are produced as decay products in hadronic showers from collisions of cosmic rays with nuclei in the Earth atmosphere.
- Atmospheric neutrinos propagate considerable distances in the Earth until they interact with the particles of detector. Propagation in matter has a noticeable effect on neutrino oscillations.
- The multipurpose experiment JUNO will detect and study neutrinos from various sources, including atmospheric neutrinos.
- Carrying out a statistical analysis with atmospheric neutrinos requires calculation of oscillation probabilities $P(\nu_\alpha \to \nu_\beta)$ with realistic matter density distribution of the Earth. In general case, the evolution equation of the neutrino flavor states in matter is not analytically solvable.

Evolution equation

For relativistic neutrinos, propagating in normal matter, the evolution equation for neutrino amplitudes in the mass basis has the following form

$$\mathrm{i}\frac{d}{d\xi}\Psi(\xi) = \Big[\underbrace{H_0}_{Vacuum} + \underbrace{v(\xi)\,W}_{Matter}\Big]\Psi(\xi),$$

where ξ is the distance travelled by the neutrinos. The vacuum part of effective Hamiltonian H depends on differences of squared neutrino masses and neutrino energy:

$$H_0 = \mathrm{diag}(0,\, \Delta m^2_{21},\, \Delta m^2_{31})/(2E_\nu),$$

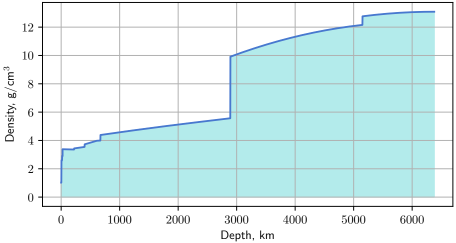
while the matter one depends on electron density along the neutrino path $v(\xi) \sim n_e(\xi)$ and oscillation parameters in mixing matrix U:

$$W = U^\dagger\,\mathrm{diag}(1,\, 0,\, 0)\,U.$$

For Dirac neutrinos mixing matrix is customarily expressed as $U = \mathcal{O}_{23}\Gamma\mathcal{O}_{13}\Gamma^\dagger$, where the orthogonal matrices O_{ij} represent rotations by mixing angles θ_{ij} in the respective planes, while $\Gamma = \mathrm{diag}(1, 1, \mathrm{e}^{\mathrm{i}\delta})$, with δ – CP-violating phase.

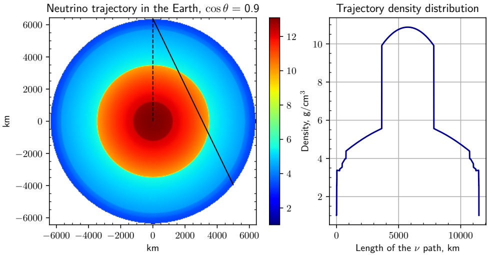
Model of the Earth

The Earth is assumed to be spherical. Mass density values are given by the Preliminary Reference Earth Model (PREM). The PREM values can be divided into 11 shells with characteristic density distributions. The values inside each shell are approximated by spline.

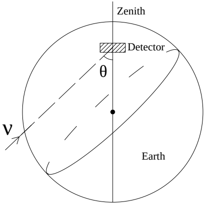


The density of electrons is calculated from mass density using electron fraction: $n_e(r) \sim Y_e(r)\,\rho(r)$, with the following values:

Region	Y_e
mantle + crust	0.4957
core	0.4656



Neutrinos propagate through different layers of matter inside the Earth. Each neutrino trajectory is parameterized by a zenith angle θ.



Magnus expansion method

- Numerical integration of the differential equations describing the matter neutrino oscillations with classical general purpose methods does not preserve the norm of the solution.
- An integrator based on the Magnus expansion preserves geometrical properties of the solution up to round-off error.
- This method combines computational efficiency with high accuracy.
- The Magnus approach consists in looking for an exponential representation of the solution:

$$\Psi(\xi) = \mathrm{e}^{\Omega(\xi,\xi_0)}\ \Psi(\xi_0),$$

where $\Omega(\xi,\, \xi_0) = \sum_{k=1}^{\infty} \Omega_k(\xi,\, \xi_0)$ is the Magnus expansion.

- The term Ω_k is built up from multiple integrals of nested commutators of k matrices $A = -iH$ evaluated at k different times.
- To maintain accuracy, the variable step is used, which automatically adjusts during the integration process.

Segment calculation

The neutrino trajectory crosses different density layers and can be divided into segments. Each segment in neutrino trajectory corresponds to one density layer. The algorithm calculates solution along trajectory until the end of a segment and in a point of discontinuity takes the calculated final state as an initial state for the next segment.

$$\text{The first segment:}\quad \underbrace{\Psi_0}_{Initial} \to \underbrace{\Psi_1}_{Final}$$

$$\vdots$$

$$\text{The last segment:}\quad \underbrace{\Psi_{N-1}}_{Initial} \to \underbrace{\Psi_N}_{Final}$$

Software

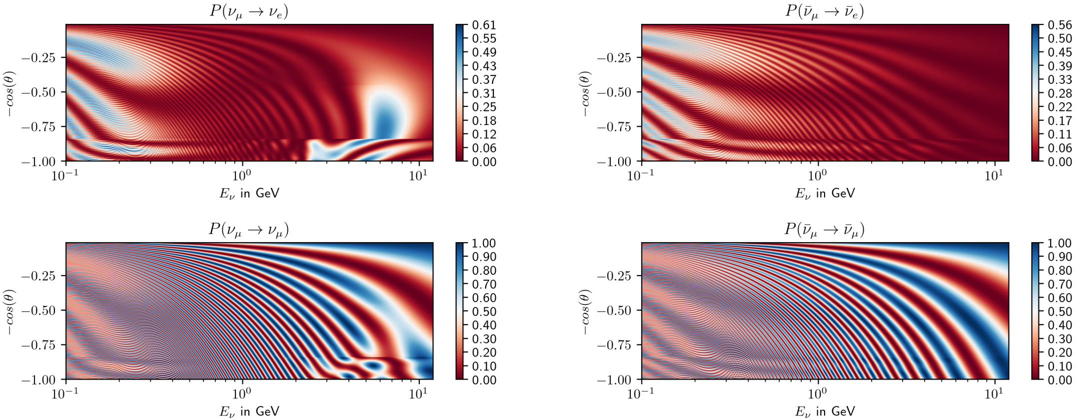
The program was made within the Global Neutrino Analysis framework (GNA) with the following structure:

- The user interface is implemented in Python.
- Algorithm are implemented in C++.
- The interface is linked with C++ programs via PyROOT.

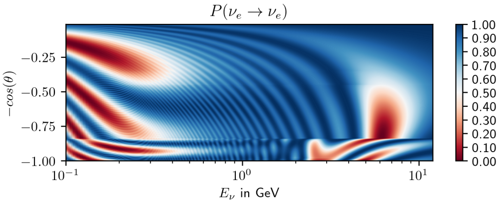
Results: Oscillation probabilities

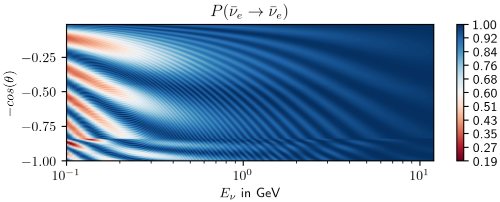
The following values of oscillation parameters were used to present the results:

Oscillation parameter	δ	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	Δm^2_{21}, eV^2	Δm^2_{32}, eV^2	Mass ordering
Value	0	0.307	0.0218	0.545	7.53×10^{5}	2.453×10^{-3}	Normal



Results: Oscillation probabilities





Computational performance

Energy range (GeV)	Calculation time
0.1 – 1	$\approx 160\mu$s
1 – 10	$\approx 70\mu$s
10 – 100	$\approx 60\mu$s

About $\sim 10^4$ values are required to carry out a single step of statistical analysis of oscillation experiments with neutrino energy $E_\nu = 1 - 10$ GeV. It takes calculation time about fractions of seconds – seconds.

Summary

- The program to calculate neutrino oscillation probabilities in the Earth within the GNA framework was made.
- Computational performance is acceptable for usage this program in statistical analysis of atmospheric neutrinos.

Thank you for your attention!