Calculation of critical exponents and representative physical parameters of scaling behaviour of stochastic systems by quantum field theory methods.

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Theoretical understanding of turbulence is one the most important unsolved problems in classical physics. Of course, the concept of turbulence encompasses a wide class of physical phenomena of various nature, so that any exhaustive and completed "theory of turbulence" can hardly ever be constructed. However, there is a "canonical list" the of problems: existence and stability of solutions to the hydrodynamics equations, convective (thermally driven) turbulence, instability of laminar flows, decaying turbulence, etc., which have both practical and conceptual significance and have always been in the focus of theorists' attention. One of them is the problem of describing the fully developed (homogeneous, isotropic) hydrodynamic turbulence (in particular, its inertial-range behaviour).

Turbulent flows occurring in various liquids or gases at very high Reynolds numbers exhibit a number of general encouraging phenomena and features (cascades of energy and other conserved quantities, scaling behaviour with apparently universal "anomalous exponents," etc.), indicating that the developed turbulence can be described within the framework of a certain self-sufficient and internally consistent theory. The most notable feature of developed turbulence, which does not fit into the framework of the classical phenomenological Kolmogorov-Obukhov theory, is the "intermittency" phenomenon arising due to the strong energy flux fluctuations. In particular, it manifests itself in a singular dependence, presumably power-law, of various equal-time correlation and structure functions, on the distances and the integral scale, and is characterized by an infinite number of independent anomalous exponents (anomalous multi-scaling). Both experiments and numerical simulations show that the anomalous scaling is more strongly pronounced for the passive turbulent transfer of scalar and vector fields (temperature, impurity concentration, magnetic field) than that for the velocity field itself. Therefore, the problem of passive transfer is an essential integral part of the study of scaling in a turbulent medium.

A related class of problems is associated with the study of the role of turbulence in fluids near their critical points, at which the systems turn out to be extremely sensitive to external influences and hydrodynamic fluctuations, which ultimately lead to the emergence of new dynamic scaling universality classes both in classical and quantum systems, for example, in liquid helium. In the latter, vanishing of the viscosity coefficient immediately leads to arbitrarily large Reynolds numbers even at low flow rates.

Fluctuations of random velocity fields, including turbulent ones, affect many other stochastic processes in Nature. Among them, a special place is occupied by chemical reactions proceeding in random environments, models of non-equilibrium critical behaviour , describing the roughening of a randomly growing surface (like the Kardar-Parisi-Zhang equation), models of directed percolation, describing the propagation of fronts of forest-fires, epidemics, the growth of tumors and bacterial colonies, and others. It turns out that taking into account the turbulent motion of the environment significantly enriches the universality classes of such systems.

It should be emphasized that the dynamical physical quantities (velocity, impurity concentration, magnetic field, etc.) are random fields and their evolution is described by nonlinear stochastic differential equations. The main goal of theoretical research is to find various averaged statistical characteristics of those fields: correlation functions, response functions, structure functions and more complex objects. Suitable methods for achieving these goals are the methods of quantum field theory (renormalization group) and non-equilibrium statistical physics.

The authors have contributed significantly to the adaptation and improvement of these methods for solving turbulence problems. Original methods were developed to calculate the representative constants and parameters of turbulent systems using perturbation theory and the critical dimensions of the so-called "dangerous" composite operators forming a multi-fractal (intermittent) behaviour of statistical correlations of the investigated random fields.

Since the second half of the 90s of the last century, their use has led to several significant results in the theory of developed turbulence and the study of its influence on other stochastic processes in open systems. Below we briefly summarize the main results of the authors, published in leading international journals in 1995-2021, including four review articles [6, 14, 27, 35].

1. Within the framework of dimensional-analytical regularization [1], the Kolmogorov constants and the skewness factor have been calculated, which are consistent with the experimental values [2]–[5]. The kinetic energy spectrum in the energy-containing and inertial interval and the transfer function in the energy balance equation, expressed in terms of the triple correlation function of the velocity field, have been found [7]. The calculated Kolmogorov constant $\mathbf{C_k}$ and the kinetic energy spectrum are in a good agreement with experimental data, Fig. (1).

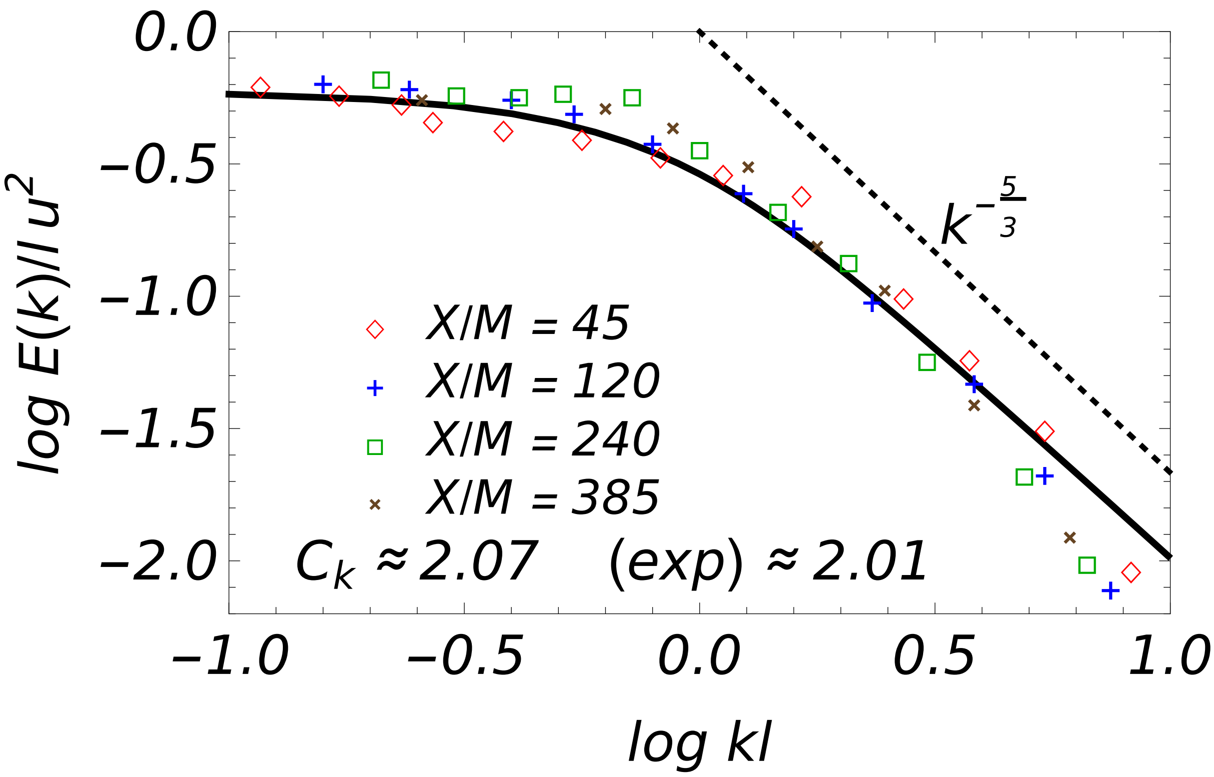

Figure 1: Longitudinal spectrum of energy as a wave number function (see [7] for details).

The leading correction to the Kolmogorov spectrum due to compressibility (second order in the Mach number) and anisotropic corrections to the Kolmogorov constant have been calculated. The validity of the Second Kolmogorov hypothesis for the energy spectrum has been justified, based on the assumption of isotropisation of the flow in the inertial interval [8]–[12]. The values of the Prandtl number for a passive scalar impurity and the magnetic Prandtl number in the models of developed turbulence with broken mirror – spatial parity – symmetry have been calculated [13, 14].

2. The advection of scalar and vector impurities by a turbulent medium has been investigated. Up to the three-loop approximation, the critical dimensions of the "dangerous" composite operators have been calculated, which determine the anomalous (intermittent) behaviour of the concentration and magnetic field in the inertial range [15]–[20]. It has been found that anisotropy, mirror symmetry breaking, and compressibility of the medium do not destroy the stability of critical regimes, but contribute to the anomalous dimensions of the composite operators [21]–[30].

3. A quantum-field model has been formulated that describes the kinetics of the autocatalytic annihilation reaction in the presence of fluctuations of the density and velocity field. The RG methods have been used to study the asymptotic behaviour of the particle density at large times near the critical dimension two, and it has been shown that arbitrarily small fluctuations of the velocity field dominate the density fluctuations, accelerating the process of annihilation of particles. Calculations have been carried out up to the two-loop order [31, 32, 33]. The influence of sources and sinks simulating the interaction of active particles with a chemically active medium (chemical radicals) has been studied, and the corresponding actions have been constructed [34]. An integro-differential nonlinear equation for the mean particle density has been obtained, which is a generalization of the well-known kinetic equation in the mean-field approximation [35, 36]. The influence of hydrodynamic fluctuations on the processes of directed percolation has been investigated. New scaling regimes have been established; the critical exponents, that determine the temporal asymptotics of the average number of active agents, the survival probability of active clusters, and the effective radius, have been calculated [37]–[43].

4. In [44]–[51] systematical investigation has been carried out of the influence of the motion of a medium (including turbulent one) on the scaling behaviour of a number of models of random growth of surfaces (interfaces between media, smoke and flame fronts, landscapes – the Kardar-Parisi-Zang models [44, 47] and Pastor-Satorras-Rotman [45, 46] and their modifications) and stochastic models of self-organized criticality (the Hwa-Kardar model [49, 50] and its variants). In all cases, possible types of scaling behaviour ("universality classes") and regions of their stability have been found. The corresponding critical dimensions have been calculated in the leading orders of the renormalization group expansions, and in some cases, exactly.

5. The influence of turbulent fluctuations on the phase transition to the superfluid state of Helium described in the framework of the models of critical dynamics has been analyzed. A new critical regime has been established, and it has been shown that the developed turbulent fluctuations that appear due to the vanishing viscosity suppress the phase transition to a superfluid state. They affect the stability of previously known fixed points, at one of which, as is commonly believed, the phase transition takes place [52]–[56]. Analysis, which takes into account compressible infrared-relevant hydrodynamic modes, supports the generalized "model A" – proposed on the basis of quantum kinetic theory – in which the fixed point is unique [57, 58].

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