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Effects of phonon-phonon coupling on low-lying states in neutron-rich Sn isotopes

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Abstract. Starting from an effective Skyrme interaction we present a method to take into account the coupling between one- and two-phonon terms in the wave functions of excited states. The approach is a development of a finite rank separable approximation for the quasiparticle RPA calculations proposed in our previous work. The influence of the phonon-phonon coupling on energies and transition probabilities for the low-lying quadrupole and octupole states in the neutron-rich Sn isotopes is studied.

PACS. 21.60.Jz Hartree-Fock and random-phase approximations – 24.30.Cz Giant resonances – 27.30.+t $20 \leq A \leq 38$ – 27.40.+z $39 \leq A \leq 58$

1 Introduction

The experimental and theoretical studies of properties of the excited states in nuclei far from the β-stability line are presently the object of very intensive activity. The random phase approximation (RPA) [1–4] is a well-known and successful way to treat nuclear vibrational excitations. Using Gogny's [5] or Skyrme-type [6] effective nucleon-nucleon interactions the most consistent models can describe the ground states in the framework of the Hartree-Fock (HF) and Hartree-Fock-Bogoliubov (HFB) approximations and the excited states within the RPA and quasiparticle RPA (QRPA). Such models are quite successful not only to reproduce the nuclear ground-state properties [7,8], but also to describe the main features of nuclear excitations in closed-shell [9,10] and open-shell nuclei [11–14]. In the latter case the pairing correlations are very important.

Due to the anharmonicity of vibrations there is a coupling between one-phonon and more complex states [2,4] and the complexity of calculations beyond standard RPA or QRPA increases rapidly with the size of the configuration space, so one has to work within limited spaces. Making use of separable forces one can perform calculations of nuclear characteristics in very large configuration spaces since there is no need to diagonalize matrices whose dimensions grow with the size of configuration space. For example, the well-known quasiparticle-phonon model (QPM) [4] can do very detailed predictions for nuclei away from closed shells [15], but it is very difficult to extrapolate the phenomenalogical parameters of the nuclear Hamiltonian to new regions of nuclei.

That is why a finite rank approximation for the particle-hole (p-h) interaction resulting from the Skyrme forces has been suggested in our previous work [16]. Thus, the self-consistent mean field can be calculated with the original Skyrme interaction, whereas the RPA solutions would be obtained with the finite rank approximation to the p-h matrix elements. It was found that the finite rank approximation can reproduce reasonably well the dipole and quadrupole strength distributions in Ar isotopes. Alternative schemes to factorize the p-h interaction were considered in [17–19].

Recently, the finite rank approximation for p-h interactions of Skyrme type has been generalized to take into account the pairing correlations [20]. The QRPA was used to describe characteristics of the low-lying 2^+ and 3^- states and giant resonances in nuclei with very different mass numbers [20,21]. It was found that there is room for the phonon-phonon coupling effects in many cases. The first calculation to estimate this effect has been done for ^{112}Sn in [22].

In the present work, we extend our approach to take into account the coupling between the one- and two-phonon terms in the wave functions of excited states. As an application of the method we present results for low-lying 2^+ and 3^- states in neutron-rich Sn isotopes and compare them with recent experimental data [23] and other calculations [24–26].

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This paper is organized as follows: in sect. 2 we sketch our method allowing to consider effects of the phonon-phonon coupling. In sect. 3 we discuss details of calculations and show how this approach can be applied to treat the low-lying collective states. Results of calculations for properties of the quadrupole and octupole states in $^{124-134}$Sn isotopes are given in sect. 4. Conclusions are drawn in sect. 5.

2 Method of calculations

2.1 The model Hamiltonian and QRPA

We start from the effective Skyrme interaction [6] and use the notation of ref. [27] containing explicit density dependence and all spin-exchange terms. The single-particle spectrum is calculated within the HF method. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic-oscillator basis [28]. The p-h residual interaction $\tilde{V}_{\rm res}$ corresponding to the Skyrme force and including both direct and exchange terms can be obtained as the second derivative of the energy density functional with respect to the density [29]. Following our previous papers [16] we simplify $\tilde{V}_{\rm res}$ by approximating it by its Landau-Migdal form. For Skyrme interactions all Landau parameters $F_l, G_l, F_l^{'}, G_l^{'}$ with $l > 1$ are zero. Here, we keep only the $l = 0$ terms in $V_{\rm res}$ and in the coordinate representation one can write it in the following form:

$$V_{\rm res}(\mathbf{r}_1,\mathbf{r}_2) = N_0^{-1}\Big[F_0(r_1) + G_0(r_1)\sigma_1\cdot\sigma_2 + (F_0^{'}(r_1) + G_0^{'}(r_1)\sigma_1\cdot\sigma_2)\tau_1\cdot\tau_2\Big]\delta(\mathbf{r}_1-\mathbf{r}_2)\,, \quad (1)$$

where σ_i and τ_i are the spin and isospin operators, and $N_0 = 2k_{\rm F}m^*/\pi^2\hbar^2$ with $k_{\rm F}$ and m^* standing for the Fermi momentum and nucleon effective mass. The expressions for $F_0, G_0, F_0^{'}, G_0^{'}$ in terms of the Skyrme force parameters can be found in ref. [27]. Because of the density dependence of the interaction the Landau parameters of eq. (1) are functions of the coordinate $\mathbf{r}$.

In what follows we use the second quantized representation and $V_{\rm res}$ can be written as

$$\hat{V}_{\rm res} = \frac{1}{2}\sum_{1234} V_{1234} : a_1^+ a_2^+ a_4 a_3 :\,, \quad (2)$$

where a_1^+ (a_1) is the particle creation (annihilation) operator and 1 denotes the quantum numbers $(n_1 l_1 j_1 m_1)$,

$$V_{1234} = \int \phi_1^*(\mathbf{r}_1)\phi_2^*(\mathbf{r}_2)V_{\rm res}(\mathbf{r}_1,\mathbf{r}_2)\phi_3(\mathbf{r}_1)\phi_4(\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2. \quad (3)$$

After integrating over the angular variables one needs to calculate the radial integrals. As is shown in [16,20] the radial integrals can be calculated accurately by choosing a large enough cut-off radius R and using a N-point integration Gauss formula with abscissas r_k and weights w_k. Thus, the two-body matrix element is a sum of N separable terms, *i.e.*, the residual interaction takes the form of a rank N separable interaction.

We employ a Hamiltonian including an average HF field, pairing interactions, the isoscalar and isovector particle-hole (p-h) residual forces in a finite rank separable form [20]:

$$H = \sum_\tau \Big(\sum_{jm}^{\tau}(E_j-\lambda_\tau)a_{jm}^\dagger a_{jm} - \frac{1}{4}V_\tau^{(0)} : P_0^\dagger(\tau)P_0(\tau) :\Big) + \hat{V}_{\rm res}\,, \quad (4)$$

where

$$P_0^+(\tau) = \sum_{jm}^{\tau}(-1)^{j-m}a_{jm}^+ a_{j-m}^+. \quad (5)$$

We sum over the proton (p) and neutron (n) indexes and the notation $\{\tau = (n,p)\}$ is used. A change $\tau \leftrightarrow -\tau$ means a change $p \leftrightarrow n$. The single-particle states are specified by the quantum numbers (jm), E_j are the single-particle energies, λ_τ the chemical potentials. $V_\tau^{(0)}$ is the interaction strength in the particle-particle channel. The Hamiltonian (4) has the same form as the QPM Hamiltonian with N separable terms [4,30], but the single-particle spectrum and parameters of the p-h residual interaction are calculated making use of the Skyrme forces.

In what follows we work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

$$a_{jm}^+ = u_j\alpha_{jm}^+ + (-1)^{j-m}v_j\alpha_{j-m}. \quad (6)$$

The Hamiltonian (4) can be represented in terms of bifermion quasiparticle operators and their conjugates [4]:

$$B(jj^{'};\lambda\mu) = \sum_{mm^{'}}(-1)^{j^{'}+m^{'}}\langle jmj^{'}m^{'} \mid \lambda\mu\rangle\alpha_{jm}^+\alpha_{j^{'}-m^{'}}^+, \quad (7)$$

$$A^+(jj^{'};\lambda\mu) = \sum_{mm^{'}}\langle jmj^{'}m^{'} \mid \lambda\mu\rangle\alpha_{jm}^+\alpha_{j^{'}m^{'}}^+. \quad (8)$$

We introduce the phonon creation operators

$$Q_{\lambda\mu i}^+ = \frac{1}{2}\sum_{jj^{'}}\Big(X_{jj^{'}}^{\lambda i}\,A^+(jj^{'};\lambda\mu) - (-1)^{\lambda-\mu}Y_{jj^{'}}^{\lambda i}\,A(jj^{'};\lambda-\mu)\Big), \quad (9)$$

where the index λ denotes the total angular momentum and μ is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $\mid 0\rangle$, *i.e.* $Q_{\lambda\mu i}\mid 0\rangle = 0$. We define the excited states for this approximation by $Q_{\lambda\mu i}^+\mid 0\rangle$. The quasiparticle energies (ε_j), the chemical potentials (λ_τ), the energy gap and the coefficients u,v of the Bogoliubov transformations (6)

are determined from the BCS equations with the single-particle spectrum that is calculated within the HF method with the effective Skyrme interaction. Making use of the linearized equation-of-motion approach [1]

$$\langle 0|\left[\delta Q_{\lambda\mu i},\left[H,Q^{+}_{\lambda\mu i}\right]\right]\mid 0\rangle=\omega_{\lambda i}\langle 0|\left[\delta Q_{\lambda\mu i},Q^{+}_{\lambda\mu i}\right]\mid 0\rangle, \tag{10}$$

with the normalization condition

$$\langle 0\mid[Q_{\lambda\mu i},Q^{+}_{\lambda\mu i'}]\mid 0\rangle=\delta_{ii'}, \tag{11}$$

one can get the QRPA equations [3,4]:

$$\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ -\mathcal{B} & -\mathcal{A}\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}=w\begin{pmatrix}X\\ Y\end{pmatrix}. \tag{12}$$

In QRPA problems there appear two types of interaction matrix elements, the $A^{(\lambda)}_{(j_1j_1')(j_2j_2')}$ matrix related to forward-going graphs and the $B^{(\lambda)}_{(j_1j_1')_\tau(j_2j_2')_{q\tau}}$ matrix related to backward-going graphs. Solutions of this set of linear equations yield the eigenenergies and the amplitudes X,Y of the excited states. The dimension of the matrices $\mathcal{A},\mathcal{B}$ is the space size of the two-quasiparticle configurations. For our case expressions for $\mathcal{A},\mathcal{B}$ and X,Y are given in [20]. Using the finite rank approximation we need to invert a matrix of dimension $4N\times 4N$ independently of the configuration space size [16,20]. Therefore, this approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform structure calculations in very large configuration spaces.

2.2 Phonon-phonon coupling

Our calculations [20] show that, for the normal-parity states one can neglect the spin-multipole terms of the p-h residual interaction (1). Using the completeness and orthogonality conditions for the phonon operators one can express the bifermion operators $A^{+}(jj';\lambda\mu)$ and $A(jj';\lambda\mu)$ through the phonon ones and the initial Hamiltonian (4) can be rewritten in terms of quasiparticle and phonon operators in the following form:

$$H=h_0+h_{QQ}+h_{QB}\,, \tag{13}$$

$$h_0=\sum_{jm}\varepsilon_j\,\alpha^{+}_{jm}\,\alpha_{jm}\,, \tag{14}$$

$$h_{QQ}=-\frac{1}{4}\sum_{\lambda\mu ii'\tau}W^{\lambda ii'}(\tau)\,Q^{+}_{\lambda\mu i}Q_{\lambda\mu i'}\,, \tag{15}$$

$$h_{QB}=-\frac{1}{2}\sum_{\lambda\mu i\tau}\sum_{jj'}{}^{\tau}\,\Gamma^{\lambda i}_{jj'}(\tau)\left((-)^{\lambda-\mu}Q^{+}_{\lambda\mu i}+Q_{\lambda-\mu i}\right)\times B(jj';\lambda-\mu)+\text{h.c.}\,. \tag{16}$$

The coefficients W,Γ of the Hamiltonian (13) are sums of N combinations of phonon amplitudes, the Landau parameters, the reduced matrix elements of the spherical harmonics and radial parts of the HF single-particle wave function (see appendix A). It is worth pointing out that the term h_{QB} is responsible for the mixing of the configurations and, therefore, for the description of many characteristics of the excited states of even-even nuclei [4].

To take into account the mixing of the configurations in the simplest case one can write the wave functions of excited states as

$$\Psi_\nu(\lambda\mu)=\Big\{\sum_i R_i(\lambda\nu)Q^{+}_{\lambda\mu i}+\sum_{\lambda_1 i_1\lambda_2 i_2}P^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda\nu)\left[Q^{+}_{\lambda_1\mu_1 i_1}Q^{+}_{\lambda_2\mu_2 i_2}\right]_{\lambda\mu}\Big\}|0\rangle \tag{17}$$

with the normalization condition

$$\sum_i R_i^2(J\nu)+2\sum_{\lambda_1 i_1\lambda_2 i_2}(P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2=1\,. \tag{18}$$

Using the variational principle in the form

$$\delta\left(\langle\Psi_\nu(\lambda\mu)\,|H\mid\Psi_\nu(\lambda\mu)\rangle-E_\nu(\langle\Psi_\nu(\lambda\mu)\,|\Psi_\nu(\lambda\mu)\rangle-1)\right)=0\,, \tag{19}$$

one obtains a set of linear equations for the unknown amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$:

$$(\omega_{Ji}-E_\nu)R_i(J\nu)+\sum_{\lambda_1 i_1\lambda_2 i_2}U^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)=0\,, \tag{20}$$

$$\sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)R_i(J\nu)+2(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-E_\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)=0\,. \tag{21}$$

$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is the matrix element coupling one- and two-phonon configurations [4,31]:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)=\langle 0|Q_{Ji}h_{QB}\left[Q^{+}_{\lambda_1 i_1}Q^{+}_{\lambda_2 i_2}\right]_J|0\rangle. \tag{22}$$

The expression of $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is given in appendix B. The number of linear equations (20), (21) equals the number of one- and two-phonon configurations included in the wave function (17).

The energies of the excited states E_ν are solutions of the secular equation

$$F(E_\nu)\equiv\det\Big|(\omega_{\lambda i}-E_\nu)\delta_{ii'}-\frac{1}{2}\sum_{\lambda_1 i_1,\lambda_2 i_2}\frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i)U^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-E_\nu}\Big|=0\,, \tag{23}$$

where the rank of the determinant equals the number of the one-phonon configurations. Using eqs. (20), (21) and the normalization condition (18), one can find the amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$.

It is necessary to point out that the equations derived above have the same form as the basic QPM equations [4,31], but the single-particle spectrum and the p-h residual interaction are determined making use of the Skyrme interactions.

Table 1. Energies and $B(E2)$ values for up-transitions to the first 2^+ states.

Nucleus	Energy (MeV)			$B(E2\uparrow)$ (e^2b^2)		
	Exp.	Theory		Exp.	Theory	
		QRPA	2PH		QRPA	2PH
^{124}Sn	1.13	1.92	1.03	0.1660±0.0040	0.177	0.151
^{126}Sn	1.14	1.96	1.30	0.10±0.03	0.149	0.133
^{128}Sn	1.17	2.08	1.48	0.073±0.006	0.111	0.100
^{130}Sn	1.22	2.37	1.73	0.023±0.005	0.064	0.058
^{132}Sn	4.04	4.47	4.03	0.14±0.06	0.136	0.129
^{134}Sn	0.73	1.65	1.34	0.029±0.006	0.016	0.015

3 Details of calculations

We apply the present approach to study characteristics of the low-lying vibrational states in the neutron-rich Sn isotopes. In this paper we use the parametrization SLy4 [32] of the Skyrme interaction. This parametrization was proposed to describe isotopic properties of nuclei from the β-stability line to the drip lines. Spherical symmetry is assumed for the HF ground states.

The pairing constants V_τ^0 are fixed to reproduce the odd-even mass difference of neighbouring nuclei. It is well known [11,12] that the constant gap approximation leads to an overestimate of occupation probabilities for subshells that are far from the Fermi level and it is necessary to introduce a cut-off in the single-particle space. Above this cut-off, subshells do not participate in the pairing effect. In our calculations we choose the BCS subspace to include all subshells lying below 5 MeV.

In order to perform QRPA calculations, the single-particle continuum is discretized [28] by diagonalizing the HF Hamiltonian on a basis of twelve harmonic-oscillator shells and cutting off the single-particle spectra at the energy of 100 MeV. This is sufficient to exhaust practically all the energy-weighted sum rule.

The Landau parameters F_0, G_0, $F_0^{'}$, $G_0^{'}$ expressed in terms of the Skyrme force parameters [27] depend on $k_{\rm F}$. As is pointed out in our previous works [16,20] one needs to adopt some effective value for $k_{\rm F}$ to give an accurate representation of the original p-h Skyrme interaction. For the present calculations we use the nuclear matter value for $k_{\rm F}$.

Our previous investigations [20] enable us to conclude that $N = 45$ for the rank of our separable approximation is enough for multipolarities $\lambda \leq 3$ in nuclei with $A \leq 208$. Increasing N, for example, up to $N = 60$ in ^{208}Pb changes results for energies and transition probabilities not more than by 1%. Our calculations show that, for the natural-parity states one can neglect the spin-multipole interactions and this reduces by a factor 2 the total matrix dimension, *i.e.*, the matrix dimensions never exceed $2N\times 2N$ independently of the configuration space size [16,20].

The two-phonon configurations of the wave function (17) are constructed from natural-parity phonons with multipolarities $\lambda = 2,3,4,5$. All one-phonon configurations with energies below 8 MeV for $^{124-130,134}$Sn and 10 MeV for ^{132}Sn are included in the the wave function (17). The cut-off in the space of the two-phonon configurations is 21 MeV. An extension of the space for one- and two-phonon configurations does not change results for energies and transition probabilities practically.

4 Results of calculations

As an application of the method we investigate effects of the phonon-phonon coupling on energies and transition probabilities to 2_1^+ and 3_1^- states in $^{124-134}$Sn.

Results of our calculations for the 2_1^+ energies and transition probabilities $B(E2)$ are compared with experimental data [23,33] in table 1. Columns "QRPA" and "2PH" give values calculated within the QRPA and taking into account the phonon-phonon coupling, respectively.

As is seen from table 1, there is a remarkable increase of the 2_1^+ energy and $B(E2\uparrow)$ in ^{132}Sn in comparison with those in ^{130,134}Sn. Such a behaviour of $B(E2\uparrow)$ is related with the proportion between the QRPA amplitudes for neutrons and protons in Sn isotopes. The neutron amplitudes are dominant in all Sn isotopes and the contribution of the main neutron configuration $\{1h_{11/2}, 1h_{11/2}\}$ increases from 81.2% in ^{124}Sn to 92.8% in ^{130}Sn when neutrons fill the subshell $1h_{11/2}$. At the same time the contribution of the main proton configuration $\{2d_{5/2}, 1g_{9/2}\}$ is decreasing from 9.3% in ^{124}Sn to 3.9% in ^{130}Sn. The closure of the neutron subshell $1h_{11/2}$ in ^{132}Sn leads to the vanishing of the neutron paring. The energy of the first neutron two-quasiparticle pole $\{2f_{7/2}, 1h_{11/2}\}$ in ^{132}Sn is greater than energies of the first poles in ^{130,134}Sn and the contribution of the $\{2f_{7/2}, 1h_{11/2}\}$ configuration in the doubly magic ^{132}Sn is about 61%. Furthermore, the first pole in ^{132}Sn is closer to the proton poles. This means that the contribution of the proton two-quasiparticle configurations is greater than those in the neighbouring isotopes and, as a result, the main proton configuration $\{2d_{5/2}, 1g_{9/2}\}$ in ^{132}Sn exhausts about 33%. In ^{134}Sn the leading contribution (about 99%) comes from the neutron configuration $\{2f_{7/2}, 2f_{7/2}\}$ and, as a result, the $B(E2)$ value is reduced. Such a behaviour of the 2_1^+ energies and

Table 2. Energies and $B(E3)$ values for up-transitions to the first 3^- states.

Nucleus	Energy (MeV)			$B(E3\uparrow)$ (e^2b^3)		
	Exp.	Theory		Exp.	Theory	
		QRPA	2PH		QRPA	2PH
^{124}Sn	2.60	3.64	3.25	0.073±0.010	0.208	0.196
^{126}Sn	2.72	4.16	3.76		0.191	0.176
^{128}Sn		4.66	4.22		0.181	0.161
^{130}Sn		5.17	4.75		0.183	0.159
^{132}Sn	4.35	5.66	5.36		0.202	0.191
^{134}Sn		5.01	4.51		0.128	0.111

$B(E2)$ values in the neutron-rich Sn isotopes reflects the shell structure in this region. It is worth mentioning that the first prediction of the anomalous behaviour of 2^+ excitations around ^{132}Sn based on the QRPA calculations with a separable quadrupole-plus-pairing Hamiltonian has been done in [24]. In comparison with other QRPA calculations of Sn isotopes done with the Gogny force [26], and especially with Skyrme forces [25], the present QRPA results for 2^+_1 energies are in agreement but our $B(E2)$ values are somewhat larger. One possible cause for this discrepancy may lie in the fact that different prescriptions for the residual interaction in the p-p channel are adopted in ref. [25] and here.

One can see from table 1 that the inclusion of the two-phonon terms results in a decrease of the energies and a reduction of transition probabilities. Note that the effect of the two-phonon configurations is important for the energies and this effect becomes weak in ^{132}Sn. There is some overestimate of the energies for the QRPA calculations and taking into account the two-phonon terms improves the description of the 2^+_1 energies. The reduction of the $B(E2)$ values is small in most cases due to the crucial contribution of the one-phonon configuration in the wave function structure.

Results of our calculations for the 3^-_1 energies and the transition probabilities $B(E3)$ compared to experimental data [34] are shown in table 2. As for the quadrupole excitations the influence of coupling between one- and two-phonon terms in the wave functions of the 3^-_1 states leads to the decrease of the energies and the reduction of transition probabilities. In spite of the fact that the 3^-_1 states have strong collectivity and many two-quasiparticle configurations give a contribution in the QRPA wave functions in Sn isotopes, the phonon-phonon coupling is not very strong in this case. Our calculation shows that the main reason is the smallness of the matrix elements coupling the one-phonon configuration $\{3^-_1\}$ and the two-phonon configuration $\{2^+_1;3^-_1\}$ ($U^{2^+_1}_{3^-_1}(3^-_1)$). As a result, the decrease of the 3^-_1 energies is about 10%. In the present paper we neglect the p-p channel that can be important for collective phonons and can reduce the collectivity of states [4,35]. This can give an additional lowering of energies and transition probabilities , but this is not the case for ^{132}Sn. Comparing with the QRPA results of ref. [25] for 3^-_1 energies and transition probabilities we find that the energies are in general agreement, whereas our calculated $B(E3)$ are larger than those of ref. [25]. Again, the reason may be in the different treatments of the residual interaction in the p-p channel. It is worth mentioning that experimental data for 3^-_1 states in the neutron-rich Sn isotopes are very scarce.

Table 3. $(M_n/M_p)/(N/Z)$ ratios for the first 2^+, 3^- states.

State	^{124}Sn	^{126}Sn	^{128}Sn	^{130}Sn	^{132}Sn	^{134}Sn
2^+_1	0.99	0.99	0.98	0.97	0.81	1.44
3^-_1	0.94	0.92	0.89	0.86	0.83	0.91

An additional information about the structure of the first $2^+, 3^-$ states can be extracted by looking at the ratio of the multipole transition matrix elements M_n/M_p that depends on the relative contributions of the proton and neutron configurations. In the framework of the collective model for isoscalar excitations this ratio is equal to $M_n/M_p = N/Z$ and any deviation from this value can indicate an isovector character of the state. The M_n/M_p ratio can be determined experimentally by using different external probes [36–38]. Our calculated values for the M_n/M_p ratios for the 2^+_1 and 3^-_1 states are shown in table 3. The calculated M_n/M_p ratios are rather close to N/Z except 2^+_1 in ^{134}Sn. It is worth noting that the deviation of the ratio for the 2^+_1 state in ^{132}Sn correlates with the increase of the contribution of the proton two-quasiparticle configurations.

5 Conclusions

The finite rank separable approximation for the QRPA calculations with Skyrme interactions that was proposed in our previous work is extended to take into account the coupling between one- and two-phonon terms in the wave functions of excited states. The suggested approach enables one to reduce considerably the dimensions of the matrices that must be diagonalized to perform structure calculations in very large configuration spaces. As an application of the method we have studied the behaviour of

$$\mathcal{Y}_{\tau}^{\lambda ki} = \frac{2\left(2\lambda+1\right)^{2}}{\left(D_{M}^{\lambda ik}\left(\tau\right)\left(\kappa_{0}^{(M,k)}+\kappa_{1}^{(M,k)}\right)+D_{M}^{\lambda ik}\left(-\tau\right)\left(\kappa_{0}^{(M,k)}-\kappa_{1}^{(M,k)}\right)\right)^{2}} \tag{A.3}$$

the energies and transition probabilities of the 2_1^+ and 3_1^- states in $^{124-134}$Sn. The tendency of our QRPA results is to overestimate the energies. The inclusion of the two-phonon configurations results in a decrease of the energies and a reduction of transition probabilities. However, we find that the effects modify significantly only the energies of quadrupole states. The maximum energy correction is about 46% and it occurs for the 2_1^+ state in ^{124}Sn, whereas the effect becomes weaker in ^{132}Sn. We find also that, for the energies of 3_1^- states, the effect is minor and changes the QRPA energies by about 10%. In this case, the main discrepancies between measured and calculated energies are too large to be overcome by the inclusion of the two-phonon configurations and one should seek for improvements in the effective interaction used. Our calculations show that the influence of the phonon coupling is small for $B(E2)$ and $B(E3)$ values. We emphasize also that the inclusion of the two-phonon terms does not change the effect of a remarkable increase of the QRPA value of $B(E2;0^+ \to 2_1^+)$ in the doubly closed shell nucleus ^{132}Sn in comparison with its neighbours. A systematical study of the influence of the two-phonon terms taking into account the p-p channel on properties of the low-lying states is now in progress.

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Appendix A.

The coefficients of the Hamiltonian (13) are given by the following expressions:

$$W^{\lambda ii'}\left(\tau\right) = \sum_{k=1}^{N}\left(\frac{D_{M}^{\lambda ik}\left(\tau\right)}{\sqrt{2\mathcal{Y}_{\tau}^{\lambda ki'}}}+\frac{D_{M}^{\lambda i'k}\left(\tau\right)}{\sqrt{2\mathcal{Y}_{\tau}^{\lambda ki}}}\right), \tag{A.1}$$

$$\Gamma_{jj'}^{\lambda i}\left(\tau\right) = \sum_{k=1}^{N}\frac{f_{jj'}^{(\lambda k)}v_{jj'}^{(-)}}{\sqrt{2\mathcal{Y}_{\tau}^{\lambda ki}}}, \tag{A.2}$$

where

$$D_{M}^{\lambda ik}\left(\tau\right) = \sum_{jj'}^{\tau} f_{jj'}^{(\lambda k)}u_{jj'}^{(+)}\left(X_{jj'}^{\lambda i}+Y_{jj'}^{\lambda i}\right),$$

see eq. (A.3) *above,*

$$v_{jj'}^{(-)} = u_j u_{j'} - v_j v_{j'}\; u_{jj'}^{(+)} = u_j v_{j'} + v_j u_{j'}.$$

In the above expressions $f_{jj'}^{(\lambda k)}$ denotes the single-particle radial matrix elements [20]:

$$f_{j_1 j_2}^{(\lambda k)} = u_{j_1}(r_k)u_{j_2}(r_k)i^{\lambda}\langle j_1||Y_{\lambda}||j_2\rangle,$$

where $u_{j_1}(r_k)$ is the radial part of the HF single-particle wave function at the abscissas of the N-point integration Gauss formula r_k. $\kappa_0^{(M,k)}$ and $\kappa_1^{(M,k)}$ are defined by the Landau parameters as

$$\begin{pmatrix}\kappa_0^{(M,k)}\\ \kappa_1^{(M,k)}\end{pmatrix} = -N_0^{-1}\frac{Rw_k}{2r_k^2}\begin{pmatrix}F_0(r_k)\\ F_0'(r_k)\end{pmatrix}.$$

Appendix B.

The matrix elements $U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji)$ have the following form:

$$\begin{aligned} U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i) = (-1)^{\lambda_1+\lambda_2+\lambda}\sqrt{(2\lambda_1+1)(2\lambda_2+1)}\sum_{\tau}\sum_{j_1 j_2 j_3}^{\tau} \\ \times\left(\Gamma_{j_1 j_2}^{\lambda i}\left(\tau\right)\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ j_2 & j_1 & j_3\end{Bmatrix}\left(X_{j_2 j_3}^{\lambda_2 i_2}Y_{j_3 j_1}^{\lambda_1 i_1}+X_{j_3 j_1}^{\lambda_1 i_1}Y_{j_2 j_3}^{\lambda_2 i_2}\right)\right. \\ +\Gamma_{j_1 j_2}^{\lambda_1 i_1}\left(\tau\right)\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ j_3 & j_2 & j_1\end{Bmatrix}\left(Y_{j_3 j_1}^{\lambda_2 i_2}Y_{j_2 j_3}^{\lambda i}+X_{j_2 j_3}^{\lambda i}X_{j_3 j_1}^{\lambda_2 i_2}\right) \\ \left.+\Gamma_{j_1 j_2}^{\lambda_2 i_2}\left(\tau\right)\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ j_1 & j_3 & j_2\end{Bmatrix}\left(Y_{j_2 j_3}^{\lambda_1 i_1}Y_{j_3 j_1}^{\lambda i}+X_{j_3 j_1}^{\lambda i}X_{j_2 j_3}^{\lambda_1 i_1}\right)\right). \end{aligned} \tag{B.1}$$

References

1. D.J. Rowe, *Nuclear Collective Motion, Models and Theory* (Barnes and Noble, 1970).
2. A. Bohr, B. Mottelson, *Nuclear Structure*, Vol. **2** (Benjamin, New York, 1975).
3. P. Ring, P. Schuck, *The Nuclear Many Body Problem* (Springer, Berlin, 1980).
4. V.G. Soloviev, *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Institute of Physics, Bristol and Philadelphia, 1992).
5. D. Gogny, in *Nuclear Self-consistent Fields*, edited by G. Ripka, M. Porneuf (North-Holland, Amsterdam, 1975).
6. D. Vautherin, D.M. Brink, Phys. Rev. C **5**, 626 (1972).
7. H. Flocard, P. Quentin, Annu. Rev. Nucl. Part. Sci. **28**, 523 (1978).
8. J. Dobaczewski, W. Nazarewicz, T.R. Werner, J.F. Berger, C.R. Chinn, J. Dechargé, Phys. Rev. C **53**, 2809 (1996).
9. G. Colò, P.F. Bortignon, N. Van Giai, A. Bracco, R.A. Broglia, Phys. Lett. B **276**, 279 (1992).
10. G. Colò, N. Van Giai, P.F. Bortignon, R.A. Broglia, Phys. Rev. C **50**, 1496 (1994).

11. E. Khan, Nguyen Van Giai, Phys. Lett. B **472**, 253 (2000).
12. G. Colò, N. Van Giai, P.F. Bortignon, M.R. Quaglia, Phys. Lett. B **485**, 362 (2000).
13. G. Colò, P.F. Bortignon, Nucl. Phys. A **696**, 427 (2001).
14. E. Khan, N. Sandulescu, M. Grasso, Nguyen Van Giai, Phys. Rev. C **66**,024309 (2002).
15. S. Galès, Ch. Stoyanov, A.I. Vdovin, Phys. Rep. **166**, 127 (1988).
16. Nguyen Van Giai, Ch. Stoyanov, V.V. Voronov, Phys. Rev. C **57**, 1204 (1998).
17. T. Suzuki, H. Sagawa, Prog. Theor. Phys. **65**, 565 (1981).
18. P. Sarriguren, E. Moya de Guerra, A. Escuderos, Nucl. Phys. A **658**, 13 (1999).
19. V.O. Nesterenko, J. Kvasil, P.-G. Reinhard, Phys. Rev. C **66**, 044307 (2002).
20. A.P. Severyukhin, Ch. Stoyanov, V.V. Voronov, Nguyen Van Giai, Phys. Rev. C **66**, 034304 (2002).
21. A.P. Severyukhin, V.V. Voronov, Ch. Stoyanov, Nguyen Van Giai, Phys. At. Nucl. **66**, 1434 (2003).
22. A.P. Severyukhin, V.V. Voronov, Ch. Stoyanov, Nguyen Van Giai, Nucl. Phys. A **722**, 123c, (2003).
23. D.C. Radford *et al.*, Phys. Rev. Lett. **88**, 22501 (2002); Eur. Phys. J. A **15**, 171 (2002); `http:// www.phy.ornl.gov`, HRIBF Newsletter, July 2003.
24. J. Terasaki, J. Engel, W. Nazarewicz, M. Stoitsov, Phys. Rev. C **66**, 054313 (2002).
25. G. Colò, P.F. Bortignon, D. Sarchi, D.T. Khoa, E. Khan, Nguyen Van Giai, Nucl. Phys. A **722**, 111c (2003).
26. G. Giambrone *et al.*, Nucl. Phys. A **726**, 3 (2003).
27. Nguyen Van Giai, H. Sagawa, Phys. Lett. B **106**, 379 (1981).
28. J.P. Blaizot, D. Gogny, Nucl. Phys. A **284**, 429 (1977).
29. G.F. Bertsch, S.F. Tsai, Phys. Rep. C **18**, 126 (1975).
30. V.G. Soloviev, Yad. Fiz. **50**, 40 (1989).
31. V.V. Voronov, V.G. Soloviev, Phys. Part. Nucl. **14**, 583 (1983).
32. E. Chabanat , P. Bonche, P. Haensel, J. Meyer, R. Schaeffer, Nucl. Phys. A **635**, 231 (1998).
33. S. Raman, C.W. Nestor jr., P. Tikkanen, At. Data Nucl. Data Tables **78**, 1 (2001).
34. T. Kibedi, R.H. Spear, At. Data Nucl. Data Tables **80**, 35 (2002).
35. T. Kubo, H. Sakamoto, T. Kammuri, T. Kishimoto, Phys. Rev. C **54**, 2331 (1996).
36. A.M. Bernstein, V.R. Brown, V.A. Madsen, Comments Nucl. Part. Phys. **11**, 203 (1983).
37. M.A. Kennedy, P.D. Cottle, K.W. Kemper, Phys. Rev. C **46**, 1811 (1992).
38. J.K. Jewell *et al.*, Phys. Lett. B **454**, 181 (1999).