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ЯДРА

DESCRIPTION OF LOW-LYING STATE STRUCTURES WITH SKYRME INTERACTION

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We use the quasiparticle random phase approximation to study properties of the low-lying 2^+ states in the even–even nuclei around ^{132}Sn. Starting from a Skyrme interaction in the particle–hole channel and a density-dependent zero-range interaction in the particle–particle channel, the calculation within the finite-rank separable approximation for the residual interaction is performed.

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1. INTRODUCTION

New experiments [1–8] give spectroscopic observations in nuclei near ^{132}Sn and this is a good possibility to test theoretical approaches. An evolution of the low-energy spectrum in nuclei around ^{132}Sn is an increasingly important point of study in nuclear structure physics and nuclear astrophysics. To investigate this region, one can comprehend features of evolution of the shell closures in the neutron-rich nuclei since the shell structure is reflected in the low-energy spectrum behavior. Note also that there is a relation between the $N = 82$ shell closure and the $A \approx 130$ peak of the solar r-process abundance distribution, i.e., the structure peculiarities of the $N = 82$ isotones below ^{132}Sn are important for stellar nucleosynthesis.

By means of an effective nucleon–nucleon interaction which is taken whether as the nonrelativistic two-body force [9–11] or derived from the relativistic lagrangian [12], the quasiparticle random phase approximation (QRPA) with the self-consistent mean field is nowadays one of the successful tools to perform the nuclear structure calculations [13–17]. Such the QRPA calculations do not require to introduce new parameters since the residual interaction is derived from the same energy density functional as that determining the mean field. Among developments for nuclear structure studies, a finite-rank separable approximation for the residual interaction is particularly promising. The separable residual interaction can simplify the solution of the QRPA equations since there is no need to diagonalize matrices whose dimensions grow with the size of configuration space. Starting from a Skyrme interaction, the finite-rank separable approximation was proposed [18] for the particle–hole (p–h) residual interaction. This means that the self-consistent mean field can be calculated by the Hartree–Fock (HF) method with the original Skyrme interactions, whereas the RPA equations are solved with the finite-rank approximation for the p–h matrix elements. Alternative schemes to factorize the p–h interaction have also been considered in [19–21]. This approach was extended to include the pairing correlations within the BCS approach with the constant gap approximation [22]. Recently, we generalized our method to take into account the particle–particle (p–p) residual interaction [23].

A complexity of calculations taking into account a coupling between one-phonon and more complex states increases rapidly with the size of the configuration space. The separable form of the residual interaction is the practical advantage of the quasiparticle phonon model (QPM) [24] which allows one to perform structure calculations in very large configuration spaces. The QPM can do the detailed predictions for nuclei away from closed shells [25], but it is very difficult to extrapolate the phenomenological parameters of the model to new regions of nuclei. Recently, we have generalized our approach to take into account a coupling between the one- and two-phonon components of wave functions in [26], where we follow the basic QPM ideas. However, the single-quasiparticle spectrum and the parameters of the residual interaction are calculated with Skyrme forces.

In the present paper we describe our method for the one-phonon case [23]. As an application we present results of calculations for the low-lying

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quadrupole states in the $N = 80, 82, 84$ isotones and the $Z = 48, 50, 52$ isotopes around ^{132}Sn.

2. THE METHOD

This method has already been presented in detail [18, 22, 23]. Let us briefly describe this approach. The starting point of the method is the HF–BCS calculation [27] of the ground states, where spherical symmetry is imposed on the quasiparticle wave functions. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic-oscillator basis [28]. We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

$$a^+_{jm} = u_j \alpha^+_{jm} + (-1)^{j-m} v_j \alpha_{j-m}, \qquad (1)$$

where jm denote the quantum numbers $nljm$. The Hamiltonian includes the Skyrme interaction [29] in the p–h channel and the surface peaked density-dependent zero-range force

$$V_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left(1 - \frac{\rho(r_1)}{\rho_c}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2) \qquad (2)$$

in the p–p channel. The strength V_0 is a parameter fixed to reproduce the odd–even mass difference of nuclei in the studied region.

The residual interaction in the p–h channel V^{ph}_{res} and in the p–p channel V^{pp}_{res} can be obtained as the second derivative of the energy density functional with respect to the particle density ρ and the pair density $\tilde{\rho}$, respectively. Following our previous paper [18] we simplify V^{ph}_{res} by approximating it by its Landau–Migdal form. For Skyrme interactions all Landau parameters with $l > 1$ are zero. We keep only the $l = 0$ terms in V^{ph}_{res}. In this work we study only normal parity states and one can neglect the spin–spin terms since they play a minor role [22]. The Coulomb and spin–orbit residual interactions are also dropped. Therefore we can write the residual interaction in the following form:

$$V^a_{\text{res}}(\mathbf{r}_1, \mathbf{r}_2) = N_0^{-1}[F^a_0(r_1) + \qquad (3)$$
$$+ F'^a_0(r_1)(\tau_1 \cdot \tau_2)]\delta(\mathbf{r}_1 - \mathbf{r}_2),$$

where a is the channel index $a = \{ph, pp\}$; σ_i and τ_i are the spin and isospin operators, and $N_0 = 2k_{\text{F}} m^*/\pi^2\hbar^2$ with k_{F} and m^* standing for the Fermi momentum and nucleon effective mass. The expressions for F^{ph}_0, F'^{ph}_0 and F^{pp}_0, F'^{pp}_0 can be found in [30] and in [23], respectively.

The p–h matrix elements and the antisymmetrized p–p matrix elements can be written as the separable form in the angular coordinates [18, 22, 23]. After integrating over the angular variables one needs to calculate the radial integrals

$$I^a(j_1 j_2 j_3 j_4) = \qquad (4)$$
$$= N_0^{-1} \int_0^\infty \left(F^a_0(r) + F'^a_0(r)\tau_1 \cdot \tau_2\right) \times$$
$$\times u_{j_1}(r) u_{j_2}(r) u_{j_3}(r) u_{j_4}(r) \frac{dr}{r^2},$$

where $u_j(r)$ is the radial part of the single-particle wave function. The radial integrals (4) can be calculated accurately by choosing a large enough cutoff radius R and using a N-point integration Gauss formula with abscissas r_k and weights w_k:

$$I^a(j_1 j_2 j_3 j_4) \simeq \qquad (5)$$
$$\simeq N_0^{-1} \frac{R}{2} \sum_{k=1}^{N} \frac{w_k}{r_k^2} \left(F^a_0(r_k) + F'^a_0(r_k)\tau_1 \cdot \tau_2\right) \times$$
$$\times u_{j_1}(r_k) u_{j_2}(r_k) u_{j_3}(r_k) u_{j_4}(r_k).$$

Thus, the residual interaction can be expressed as a sum of N separable terms. The Hamiltonian of our method has the same form as the Hamiltonian of the well-known quasiparticle-phonon model [24], but the single-quasiparticle spectrum and the parameters of the residual interaction are calculated by the Skyrme forces.

We introduce the phonon creation operators

$$Q^+_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} (X^{\lambda i}_{jj'} A^+(jj'; \lambda\mu) - \qquad (6)$$
$$- (-1)^{\lambda-\mu} Y^{\lambda i}_{jj'} A(jj'; \lambda - \mu)),$$

where the index λ denotes total angular momentum and μ is its z projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$. We define the excited states as $Q^+_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|[Q_{\lambda\mu i}, Q^+_{\lambda\mu i'}]|0\rangle = \delta_{ii'}. \qquad (7)$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations [27]

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}. \qquad (8)$$

In QRPA problems there appear two types of interaction matrix elements, the $\mathcal{A}^{(\lambda)}_{(j_1 j'_1)(j_2 j'_2)}$ matrix related to forward-going graphs and the $\mathcal{B}^{(\lambda)}_{(j_1 j'_1)(j_2 j'_2)}$ matrix related to backward-going graphs. Solutions of this set of linear equations yield the eigen energies and the amplitudes X, Y of the excited states. The

Table 1. Energies, $B(E2)$ values for up transitions to the first 2^+ states

Nucleus	Energy, MeV		$B(E2\uparrow)$, e^2 fm^4	
	Exp.	Theory	Exp.	Theory
^{124}Cd	0.61	1.28		2240
^{126}Cd	0.65	1.35		1790
^{128}Cd	0.65	1.45		1310
^{130}Cd	1.33	1.58		810
^{132}Cd		1.46		1040
^{126}Sn	1.14	2.74	1000 ± 300	1570
^{128}Sn	1.17	2.83	730 ± 60	1240
^{130}Sn	1.22	2.97	230 ± 50	790
^{132}Sn	4.04	4.46	1100 ± 300	1360
^{134}Sn	0.73	2.05	290 ± 50	190
^{128}Te	0.74	1.09	3830 ± 60	4660
^{130}Te	0.84	1.27	2950 ± 70	3560
^{132}Te	0.97	1.49	1720 ± 170	2440
^{134}Te	1.28	1.72	960 ± 120	1280
^{136}Te	0.61	1.46	1030 ± 150	1830

dimension of the matrices $\mathcal{A}$, $\mathcal{B}$ is the space size of the two-quasiparticle configurations. One can find a prescription how the finite-rank approximation can simplify the solution of the QRPA equations in [23]. The QRPA equations (8) can be reduced to the secular equation and the matrix dimensions never exceed $6N \times 6N$ independently of the configuration space size. If we omit terms of the residual interaction in the p–p channel, then the matrix dimension is reduced by a factor 3 [18, 22]. So this approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform structure calculations in very large configuration spaces.

3. RESULTS

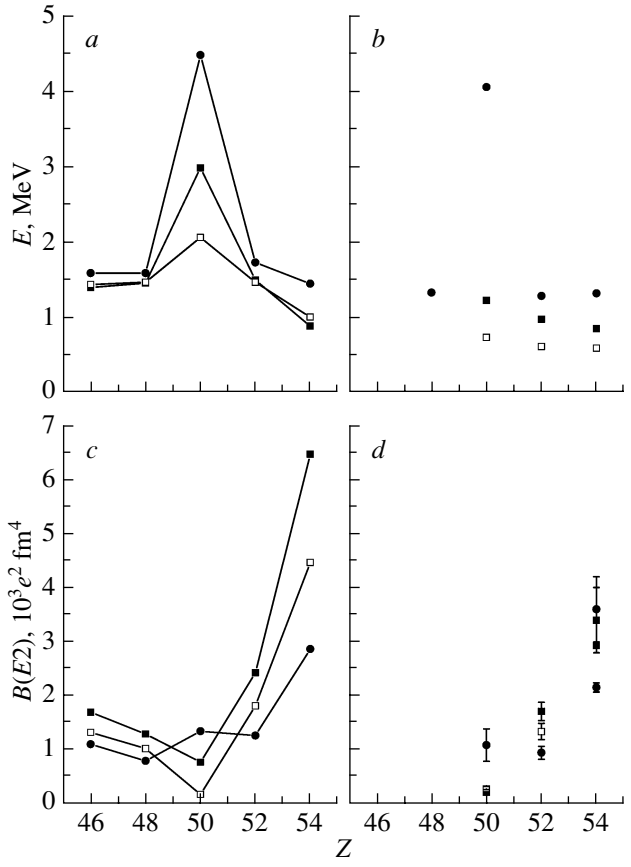
We apply our approach to study characteristics of the low-lying 2^+ states in the even–even nuclei around ^{132}Sn. We use the Skyrme interaction SLy4 in the p–h channel [29] together with the isospin-invariant surface-peaked pairing force (2). The pairing strength is equal to -940 MeV fm^3. Besides that we use the soft cutoff at 10 MeV above the Fermi energies as introduced in [23]. Note also that the Landau parameters F_0^{ph}, $F_0^{'ph}$ expressed in terms of the Skyrme force parameters [30] depend on k_{F}. As it is pointed out in our previous works [18, 22], one needs to adopt some effective value for k_{F} to give an accurate representation of the original p–h Skyrme interaction. For the present calculations we use the nuclear matter value for k_{F}.

Table 2. Structure of the $2^+_{1,4}$ states in ^{130}Te (the largest components are given)

State	$\{n_1 l_1 j_1, n_2 l_2 j_2\}_\tau$	X	Y	Structure, %
2^+_1	$\{2d_{5/2}, 2d_{5/2}\}_p$	0.77	0.18	28
	$\{1g_{7/2}, 2d_{5/2}\}_p$	0.28	0.07	7
	$\{1g_{7/2}, 1g_{7/2}\}_p$	0.76	0.21	27
	$\{1h_{11/2}, 1h_{11/2}\}_n$	0.69	0.31	19
2^+_4	$\{2d_{5/2}, 2d_{5/2}\}_p$	-0.40	0.07	8
	$\{1g_{7/2}, 2d_{5/2}\}_p$	-0.18	0.03	3
	$\{1g_{7/2}, 1g_{7/2}\}_p$	-0.74	0.09	27
	$\{1h_{11/2}, 1h_{11/2}\}_n$	0.99	0.05	48

We study the 2^+_1-state energies and transition probabilities in $^{126-130}$Pd, $^{124-132}$Cd, $^{124-134}$Sn, $^{128-136}$Te, $^{134-138}$Xe. Results of our calculations for energies and the $B(E2)$ values and the available experimental data [1, 2, 4, 6, 31] are shown in Table 1 and the figure. One can see that there is the correct description of the isotopic and isotonic dependences of the properties of the first quadrupole states. The 2^+_1 energies have a maximal value at $N = 82$ and at $Z = 50$. Such a behavior corresponds to a standard evolution of the energies near closed shells. On the other hand, the structure peculiarities are reflected in the $B(E2)$ evolutions. The $B(E2)$ value at $N = 82$ ($Z = 50$) is either a maximal value in the Sn isotopes (the $N = 82$ isotones), or a minimal value in the Pd, Cd, Te, Xe isotopes (the $N = 80$, 84 isotones).

In $^{124-132}$Cd the proton phonon amplitudes are dominant ones and the contribution of the main proton configuration $\{1g_{9/2}, 1g_{9/2}\}$ increases from 79% in ^{124}Cd to 89% in ^{128}Cd, while the main neutron configuration $\{1h_{11/2},\ 1h_{11/2}\}$ exhausts about 13, 11, and 7% of the wave function normalization in ^{124}Cd, ^{126}Cd, and ^{128}Cd, respectively. The closure of the neutron subshell $1h_{11/2}$ in ^{130}Cd leads to the vanishing the neutron pairing and as a result the energy of the first neutron two-quasiparticle pole $\{2f_{7/2}, 1h_{11/2}\}$ in ^{130}Cd is larger than energies of the first neutron poles in ^{128,132}Cd. This yields that in ^{130}Cd the leading contribution (about 97%) comes from proton configuration $\{1g_{9/2}, 1g_{9/2}\}$ and the $B(E2)$ value is reduced.



(a, b) The 2^+_1-state energies and (c, d) $B(E2; 0^+_{g.s} \to 2^+_1)$ values in the $N = 80$ (■), 82 (●), 84 (□) isotones.

The structure of the 2^+_1 in ^{126,128,130}Pd is similar to that in ^{128,130,132}Cd. We get the noncollective structure with the domination of the proton configuration $\{1g_{9/2},\ 1g_{9/2}\}$. In ^{128}Pd, as is discussed for ^{130}Cd, the contribution of the proton $\{1g_{9/2},\ 1g_{9/2}\}$ increases to 96%. This results in the $B(E2)$-value reduction.

Such the $B(E2)$ behavior in Sn isotopes was explained in more detail in [23, 26]. The reason is related with the proportion between the phonon amplitudes for neutrons and protons. The neutron amplitudes are dominant and the lowest two-quasiparticle poles are neutron ones. In ^{130}Sn the main neutron (proton) configuration is $\{1h_{11/2},\ 1h_{11/2}\}$ ($\{2d_{5/2},\ 1g_{9/2}\}$) and it exhaust 86% (7%) of the wave function normalization. Due to the closure of the neutron subshell $1h_{11/2}$ in ^{132}Sn, the energy of the first neutron two-quasiparticle pole $\{2f_{7/2},\ 1h_{11/2}\}$ in ^{132}Sn is greater than energies of the first poles in ^{130,134}Sn and the contribution of the $\{2f_{7/2},\ 1h_{11/2}\}$ configuration in the doubly magic ^{132}Sn is about 61%. Since the first pole in ^{132}Sn is closer to the proton poles, the contribution of the proton two-quasiparticle configurations is greater than those in the neighboring isotopes. In ^{134}Sn we get again the noncollective structure of the 2^+_1 and the contribution of the main neutron configuration $\{2f_{7/2},\ 2f_{7/2}\}$ is about 96%.

Let us discuss the effect of the closure of the neutron subshell $1h_{11/2}$ in the Te and Xe isotopes. There is a rather detailed discussion of the properties of the 2^+_1 states in $^{128-136}$Te in [23]. In the present analysis, one can compare the tendencies in the Te and Xe isotopes. In general, the behavior of the 2^+_1 energies and the $B(E2)$ values of ^{134,136,138}Xe is similar to that of ^{132,134,136}Te and such a behavior reflects the shell structure in this region. The lowest two-quasiparticle poles are proton ones. For the 2^+_1 states, the proton phonon amplitudes are dominant. All neutron configurations exhaust about 17% (20%) and 28% (26%) of the wave function normalization in ^{132}Te (^{134}Xe) and ^{136}Te (^{138}Xe), respectively. In ^{134}Te and ^{136}Xe, the contribution of the neutron configurations is less than 5%, i.e., the contribution of the proton configurations is greater than those in the neighboring isotopes. However, the contribution of the dominant proton configuration $\{1g_{7/2},\ 1g_{7/2}\}$ (65% of ^{134}Te and 49% of ^{136}Xe) is more than in one and a half time than in the neighboring isotopes and as a result the $B(E2)$ value is reduced. It is worth to mention that the first prediction of the anomalous behavior of 2^+ excitations around ^{132}Sn based on the QRPA calculations with a separable quadrupole-plus-pairing hamiltonian has been done in [32].

The dominance of the neutron–proton attraction is one of the main characteristics of the effective nucleon–nucleon interaction. On the other hand, the collective-quadrupole isovector valence-shell excitations, so-called mixed-symmetry states [8, 33], are sensitive to the proton–neutron interaction. That is why it is interesting to study the characteristics of the lowest isovector-collective states in nuclei discussed above. As an example we consider ^{130}Te. Our calculation shows that the 2^+_1 is the collective state, which has a typical isoscalar character. Table 2 shows the dominant phonon amplitudes X, Y of the 2^+_1 state, the neutron and proton amplitudes are in phase. The next fairly collective state is the 2^+_4 state. The 2^+_4 energy is equal to 3.1 MeV, $B(E2; 0^+_{g.s} \to \to 2^+_4) = 340\ e^2$ fm^4. One can see that the dominant neutron and proton amplitudes of the 2^+_4 state are in the phase opposition and it corresponds to an isovector excitation. Let us remark that the 2^+_4 state is only the best candidate for the mixed-symmetry state in ^{130}Te. To make a final conclusion about the mixed-symmetry state one needs to calculate the

$B(M1; 2_4^+ \to 2_1^+)$ value and such a calculation is in progress.

4. CONCLUSIONS

The QRPA calculations with the finite rank separable approximation for the Skyrme-type interactions in the particle–hole and particle–particle channels are presented. The suggested approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform structure calculations in very large configuration spaces. Using the same set of parameters we have investigated the evolution of the 2_1^+ state energies and the $B(E2)$ values in $^{126-130}$Pd, $^{124-132}$Cd, $^{124-134}$Sn, $^{128-136}$Te, $^{134-138}$Xe. The results of our calculations for the energies and the $B(E2)$ values describe correctly the isotopic and isotonic dependences. We give predictions for the structure of the 2_1^+ state in $^{126-130}$Pd, $^{124-132}$Cd. As an illustration of the method we study the lowest isovector collective quadrupole state in ^{130}Te. It is found that the 2_4^+ state is the best candidate for the mixed-symmetry state, but one needs to calculate the $M1$ transition probability between the 2_4^+ and 2_1^+ states.

The present analysis have been made within the one-phonon approximation. A systematic study taking into account the coupling between the one- and two-phonon terms is still underway.

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ОПИСАНИЕ СТРУКТУРЫ НИЗКОЛЕЖАЩИХ СОСТОЯНИЙ С ПОМОЩЬЮ ВЗАИМОДЕЙСТВИЯ СКИРМА

А. П. Северюхин, В. В. Воронов, Нгуен Ван Джай

Свойства низколежащих состояний 2^+ в четно-четных ядрах вблизи ^{132}Sn изучены в приближении случайных фаз. Расчеты выполнены с силами Скирма в канале частица–дырка и зависящим от плотности дельта-функциональным взаимодействием в канале частица–частица на основе сепарабельного приближения для остаточного взаимодействия.

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Description of Low-Lying State Structures with Skyrme Interaction*

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1. INTRODUCTION

New experiments [1–8] give spectroscopic observations in nuclei near ^{132}Sn and this is a good possibility to test theoretical approaches. An evolution of the low-energy spectrum in nuclei around ^{132}Sn is an increasingly important point of study in nuclear structure physics and nuclear astrophysics. To investigate this region, one can comprehend features of evolution of the shell closures in the neutron-rich nuclei since the shell structure is reflected in the low-energy spectrum behavior. Note also that there is a relation between the $N = 82$ shell closure and the $A \approx 130$ peak of the solar r-process abundance distribution, i.e., the structure peculiarities of the $N = 82$ isotones below ^{132}Sn are important for stellar nucleosynthesis.

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quadrupole states in the $N = 80, 82, 84$ isotones and the $Z = 48, 50, 52$ isotopes around ^{132}Sn.

2. THE METHOD

This method has already been presented in detail [18, 22, 23]. Let us briefly describe this approach. The starting point of the method is the HF–BCS calculation [27] of the ground states, where spherical symmetry is imposed on the quasiparticle wave functions. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic-oscillator basis [28]. We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

$$a^+_{jm} = u_j \alpha^+_{jm} + (-1)^{j-m} v_j \alpha_{j-m}, \qquad (1)$$

where jm denote the quantum numbers $nljm$. The Hamiltonian includes the Skyrme interaction [29] in the p–h channel and the surface peaked density-dependent zero-range force

$$V_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left(1 - \frac{\rho(r_1)}{\rho_c}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2) \qquad (2)$$

in the p–p channel. The strength V_0 is a parameter fixed to reproduce the odd–even mass difference of nuclei in the studied region.

The residual interaction in the p–h channel V^{ph}_{res} and in the p–p channel V^{pp}_{res} can be obtained as the second derivative of the energy density functional with respect to the particle density ρ and the pair density $\tilde{\rho}$, respectively. Following our previous paper [18] we simplify V^{ph}_{res} by approximating it by its Landau–Migdal form. For Skyrme interactions all Landau parameters with $l > 1$ are zero. We keep only the $l = 0$ terms in V^{ph}_{res}. In this work we study only normal parity states and one can neglect the spin–spin terms since they play a minor role [22]. The Coulomb and spin–orbit residual interactions are also dropped. Therefore we can write the residual interaction in the following form:

$$V^a_{\text{res}}(\mathbf{r}_1, \mathbf{r}_2) = N_0^{-1}[F^a_0(r_1) + F^{'a}_0(r_1)(\tau_1 \cdot \tau_2)]\delta(\mathbf{r}_1 - \mathbf{r}_2), \qquad (3)$$

where a is the channel index $a = \{ph, pp\}$; σ_i and τ_i are the spin and isospin operators, and $N_0 = 2k_{\text{F}}m^*/\pi^2\hbar^2$ with k_{F} and m^* standing for the Fermi momentum and nucleon effective mass. The expressions for F^{ph}_0, $F^{'ph}_0$ and F^{pp}_0, $F^{'pp}_0$ can be found in [30] and in [23], respectively.

The p–h matrix elements and the antisymmetrized p–p matrix elements can be written as the separable form in the angular coordinates [18, 22, 23]. After integrating over the angular variables one needs to calculate the radial integrals

$$I^a(j_1 j_2 j_3 j_4) = N_0^{-1} \int_0^\infty \left(F^a_0(r) + F^{'a}_0(r)\tau_1 \cdot \tau_2\right) \times u_{j_1}(r) u_{j_2}(r) u_{j_3}(r) u_{j_4}(r) \frac{dr}{r^2}, \qquad (4)$$

where $u_j(r)$ is the radial part of the single-particle wave function. The radial integrals (4) can be calculated accurately by choosing a large enough cut-off radius R and using a N-point integration Gauss formula with abscissas r_k and weights w_k:

$$I^a(j_1 j_2 j_3 j_4) \simeq N_0^{-1} \frac{R}{2} \sum_{k=1}^{N} \frac{w_k}{r_k^2} \left(F^a_0(r_k) + F^{'a}_0(r_k)\tau_1 \cdot \tau_2\right) \times u_{j_1}(r_k) u_{j_2}(r_k) u_{j_3}(r_k) u_{j_4}(r_k). \qquad (5)$$

Thus, the residual interaction can be expressed as a sum of N separable terms. The Hamiltonian of our method has the same form as the Hamiltonian of the well-known quasiparticle-phonon model [24], but the single-quasiparticle spectrum and the parameters of the residual interaction are calculated by the Skyrme forces.

We introduce the phonon creation operators

$$Q^+_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} (X^{\lambda i}_{jj'} A^+(jj'; \lambda\mu) - (-1)^{\lambda-\mu} Y^{\lambda i}_{jj'} A(jj'; \lambda - \mu)), \qquad (6)$$

where the index λ denotes total angular momentum and μ is its z projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$. We define the excited states as $Q^+_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|[Q_{\lambda\mu i}, Q^+_{\lambda\mu i'}]|0\rangle = \delta_{ii'}. \qquad (7)$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations [27]

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}. \qquad (8)$$

In QRPA problems there appear two types of interaction matrix elements, the $\mathcal{A}^{(\lambda)}_{(j_1 j'_1)(j_2 j'_2)}$ matrix related to forward-going graphs and the $\mathcal{B}^{(\lambda)}_{(j_1 j'_1)(j_2 j'_2)}$ matrix related to backward-going graphs. Solutions of this set of linear equations yield the eigen energies and

Table 1. Energies, $B(E2)$ values for up transitions to the first 2^+ states

Nucleus	Energy, MeV		$B(E2\uparrow)$, e^2 fm^4	
	Expt.	Theory	Expt.	Theory
^{124}Cd	0.61	1.28		2240
^{126}Cd	0.65	1.35		1790
^{128}Cd	0.65	1.45		1310
^{130}Cd	1.33	1.58		810
^{132}Cd		1.46		1040
^{126}Sn	1.14	2.74	1000 ± 300	1570
^{128}Sn	1.17	2.83	730 ± 60	1240
^{130}Sn	1.22	2.97	230 ± 50	790
^{132}Sn	4.04	4.46	1100 ± 300	1360
^{134}Sn	0.73	2.05	290 ± 50	190
^{128}Te	0.74	1.09	3830 ± 60	4660
^{130}Te	0.84	1.27	2950 ± 70	3560
^{132}Te	0.97	1.49	1720 ± 170	2440
^{134}Te	1.28	1.72	960 ± 120	1280
^{136}Te	0.61	1.46	1030 ± 150	1830

the amplitudes X, Y of the excited states. The dimension of the matrices $\mathcal{A}$, $\mathcal{B}$ is the space size of the two-quasiparticle configurations. One can find a prescription how the finite-rank approximation can simplify the solution of the QRPA equations in [23]. The QRPA equations (8) can be reduced to the secular equation and the matrix dimensions never exceed $6N \times 6N$ independently of the configuration space size. If we omit terms of the residual interaction in the p–p channel, then the matrix dimension is reduced by a factor 3 [18, 22]. So this approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform structure calculations in very large configuration spaces.

3. RESULTS

We apply our approach to study characteristics of the low-lying 2^+ states in the even–even nuclei around ^{132}Sn. We use the Skyrme interaction SLy4 in the p–h channel [29] together with the isospin-invariant surface-peaked pairing force (2). The pairing strength is equal to -940 MeV fm^3. Besides that we use the soft cutoff at 10 MeV above the Fermi energies as introduced in [23]. Note also that the Landau parameters F_0^{ph}, $F_0^{'ph}$ expressed in terms of the Skyrme force parameters [30] depend on k_{F}. As it is pointed out in our previous works [18, 22], one needs to adopt some effective value for k_{F} to give an accurate representation of the original p–h Skyrme interaction. For the present calculations we use the nuclear matter value for k_{F}.

Table 2. Structure of the $2^+_{1,4}$ states in ^{130}Te (the largest components are given)

State	$\{n_1 l_1 j_1, n_2 l_2 j_2\}_\tau$	X	Y	Structure, %
2^+_1	$\{2d_{5/2}, 2d_{5/2}\}_p$	0.77	0.18	28
	$\{1g_{7/2}, 2d_{5/2}\}_p$	0.28	0.07	7
	$\{1g_{7/2}, 1g_{7/2}\}_p$	0.76	0.21	27
	$\{1h_{11/2}, 1h_{11/2}\}_n$	0.69	0.31	19
2^+_4	$\{2d_{5/2}, 2d_{5/2}\}_p$	-0.40	0.07	8
	$\{1g_{7/2}, 2d_{5/2}\}_p$	-0.18	0.03	3
	$\{1g_{7/2}, 1g_{7/2}\}_p$	-0.74	0.09	27
	$\{1h_{11/2}, 1h_{11/2}\}_n$	0.99	0.05	48

We study the 2^+_1-state energies and transition probabilities in $^{126-130}$Pd, $^{124-132}$Cd, $^{124-134}$Sn, $^{128-136}$Te, $^{134-138}$Xe. Results of our calculations for energies and the $B(E2)$ values and the available experimental data [1, 2, 4, 6, 31] are shown in Table 1 and Fig. 1. One can see that there is the correct description of the isotopic and isotonic dependences of the properties of the first quadrupole states. The 2^+_1 energies have a maximal value at $N = 82$ and at $Z = 50$. Such a behavior corresponds to a standard evolution of the energies near closed shells. On the other hand, the structure peculiarities are reflected in the $B(E2)$ evolutions. The $B(E2)$ value at $N = 82$ ($Z = 50$) is either a maximal value in the Sn isotopes (the $N = 82$ isotones), or a minimal value in the Pd, Cd, Te, Xe isotopes (the $N = 80$, 84 isotones).

In $^{124-132}$Cd the proton phonon amplitudes are dominant ones and the contribution of the main proton configuration $\{1g_{9/2}, 1g_{9/2}\}$ increases from 79% in ^{124}Cd to 89% in ^{128}Cd, while the main neutron configuration $\{1h_{11/2},\ 1h_{11/2}\}$ exhausts about 13, 11, and 7% of the wave function normalization in ^{124}Cd, ^{126}Cd, and ^{128}Cd, respectively. The closure of the neutron subshell $1h_{11/2}$ in ^{130}Cd leads to the vanishing the neutron pairing and as a result the energy of the first neutron two-quasiparticle pole $\{2f_{7/2}, 1h_{11/2}\}$ in ^{130}Cd is larger than energies of the first neutron poles in ^{128,132}Cd. This yields that in ^{130}Cd the leading contribution (about 97%) comes from proton configuration $\{1g_{9/2}, 1g_{9/2}\}$ and the $B(E2)$ value is reduced.

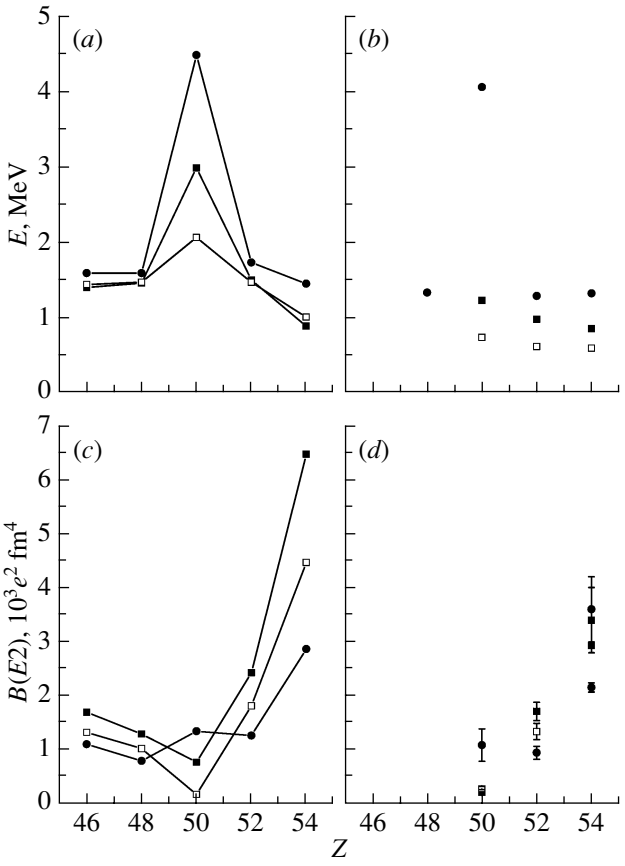

Fig. 1. (a, b) The 2_1^+-state energies and (c, d) $B(E2; 0_{g.s}^+ \to 2_1^+)$ values in the N = (■) 80, (●) 82, and (□) 84 isotones.

The structure of the 2_1^+ in ^{126,128,130}Pd is similar to that in ^{128,130,132}Cd. We get the noncollective structure with the domination of the proton configuration $\{1g_{9/2},\ 1g_{9/2}\}$. In ^{128}Pd, as is discussed for ^{130}Cd, the contribution of the proton $\{1g_{9/2},\ 1g_{9/2}\}$ increases to 96%. This results in the $B(E2)$ value reduction.

Such the $B(E2)$ behavior in Sn isotopes was explained in more detail in [23, 26]. The reason is related with the proportion between the phonon amplitudes for neutrons and protons. The neutron amplitudes are dominant and the lowest two-quasiparticle poles are neutron ones. In ^{130}Sn the main neutron (proton) configuration is $\{1h_{11/2},\ 1h_{11/2}\}$ ($\{2d_{5/2},\ 1g_{9/2}\}$) and it exhaust 86% (7%) of the wave function normalization. Due to the closure of the neutron subshell $1h_{11/2}$ in ^{132}Sn, the energy of the first neutron two-quasiparticle pole $\{2f_{7/2},\ 1h_{11/2}\}$ in ^{132}Sn is greater than energies of the first poles in ^{130,134}Sn and the contribution of the $\{2f_{7/2},\ 1h_{11/2}\}$ configuration in the doubly magic ^{132}Sn is about 61%. Since the first pole in ^{132}Sn is closer to the proton poles, the contribution of the proton two-quasiparticle configurations is greater than those in the neighboring isotopes. In ^{134}Sn we get again the noncollective structure of the 2_1^+ and the contribution of the main neutron configuration $\{2f_{7/2},\ 2f_{7/2}\}$ is about 96%.

Let us discuss the effect of the closure of the neutron subshell $1h_{11/2}$ in the Te and Xe isotopes. There is a rather detailed discussion of the properties of the 2_1^+ states in $^{128-136}$Te in [23]. In the present analysis, one can compare the tendencies in the Te and Xe isotopes. In general, the behavior of the 2_1^+ energies and the $B(E2)$ values of ^{134,136,138}Xe is similar to that of ^{132,134,136}Te and such a behavior reflects the shell structure in this region. The lowest two-quasiparticle poles are proton ones. For the 2_1^+ states, the proton phonon amplitudes are dominant. All neutron configurations exhaust about 17% (20%) and 28% (26%) of the wave function normalization in ^{132}Te (^{134}Xe) and ^{136}Te (^{138}Xe), respectively. In ^{134}Te and ^{136}Xe, the contribution of the neutron configurations is less than 5%, i.e., the contribution of the proton configurations is greater than those in the neighboring isotopes. However, the contribution of the dominant proton configuration $\{1g_{7/2},\ 1g_{7/2}\}$ (65% of ^{134}Te and 49% of ^{136}Xe) is more than in one and a half time than in the neighboring isotopes and as a result the $B(E2)$ value is reduced. It is worth to mention that the first prediction of the anomalous behavior of 2^+ excitations around ^{132}Sn based on the QRPA calculations with a separable quadrupole-plus-pairing hamiltonian has been done in [32].

The dominance of the neutron–proton attraction is one of the main characteristics of the effective nucleon–nucleon interaction. On the other hand, the collective-quadrupole isovector valence-shell excitations, so-called mixed-symmetry states [8, 33], are sensitive to the proton–neutron interaction. That is why it is interesting to study the characteristics of the lowest isovector-collective states in nuclei discussed above. As an example we consider ^{130}Te. Our calculation shows that the 2_1^+ is the collective state, which has a typical isoscalar character. Table 2 shows the dominant phonon amplitudes X, Y of the 2_1^+ state, the neutron and proton amplitudes are in phase. The next fairly collective state is the 2_4^+ state. The 2_4^+ energy is equal to 3.1 MeV, $B(E2; 0_{\rm g.s}^+ \to 2_4^+) = 340\ e^2$ fm^4. One can see that the dominant neutron and proton amplitudes of the 2_4^+ state are in the phase opposition and it corresponds to an isovector excitation. Let us remark that the 2_4^+ state is only the best candidate for the mixed-symmetry state in ^{130}Te. To make a final conclusion about the mixed-symmetry state one needs to calculate the

$B(M1; 2_4^+ \to 2_1^+)$ value and such a calculation is in progress.

4. CONCLUSIONS

The QRPA calculations with the finite rank separable approximation for the Skyrme-type interactions in the particle–hole and particle–particle channels are presented. The suggested approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform structure calculations in very large configuration spaces. Using the same set of parameters we have investigated the evolution of the 2_1^+ state energies and the $B(E2)$-values in $^{126-130}$Pd, $^{124-132}$Cd, $^{124-134}$Sn, $^{128-136}$Te, $^{134-138}$Xe. The results of our calculations for the energies and the $B(E2)$ values describe correctly the isotopic and isotonic dependences. We give predictions for the structure of the 2_1^+ state in $^{126-130}$Pd, $^{124-132}$Cd. As an illustration of the method we study the lowest isovector collective quadrupole state in ^{130}Te. It is found that the 2_4^+ state is the best candidate for the mixed-symmetry state, but one needs to calculate the $M1$ transition probability between the 2_4^+ and 2_1^+ states.

The present analysis have been made within the one-phonon approximation. A systematic study taking into account the coupling between the one- and two-phonon terms is still underway.

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