Quasiparticle random phase approximation with finite rank approximation for Skyrme interactions

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A finite rank separable approximation for particle-hole random phase approximation (RPA) calculations with Skyrme interactions is extended to take into account the pairing. As an illustration of the method energies and transition probabilities for the quadrupole and octupole excitations in some O, Ar, Sn, and Pb isotopes are calculated. The values obtained within our approach are very close to those that were calculated within quasiparticle RPA (QRPA) with the full Skyrme interaction. They are in reasonable agreement with experimental data.

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I. INTRODUCTION

Many properties of the nuclear states can be described within the random phase approximation (RPA) [1–4]. Among many microscopic nuclear models aiming at a description of the properties of nuclear excitations the most consistent model employs an effective interaction which can describe, throughout the Periodic Table, the ground states in the framework of the Hartree-Fock (HF) approximation and the excited states in the small amplitude limit of the time-dependent HF, or the RPA. Gogny's interaction [5] and the Skyrme-type interactions [6] are very popular now. Such models are quite successful not only for predicting accurately nuclear ground state properties [7,8] but also for calculating the main features of giant resonances in closed-shell nuclei [9,10] and single-particle strengths near closed shells [11]. Taking into account the pairing effects enables one to reproduce also many properties of collective states in open-shell nuclei [12–15].

It is well known that due to the anharmonicity of vibrations there is a coupling between one-phonon and more complex states [2,4]. The main difficulty is that the complexity of calculations beyond the standard RPA (e.g., for studying damping mechanisms of collective excitations) increases rapidly with the size of the configuration space and one has to work within limited spaces. From another point of view more phenomenological models that assume some simple separable form for the residual nucleon-nucleon interaction while the mean field is modelized by an empirical potential well allow one to calculate nuclear excitations in very large configuration spaces since there is no need to diagonalize matrices whose dimensions grow with the size of configuration space. The well-known quasiparticle-phonon model (QPM) of Soloviev [4] belongs to such a model. Very detailed predictions can be made by QPM for nuclei away from closed shells [16].

The possibility to solve easily the RPA problem in a large configuration space when the residual particle-hole (p-h) interaction is separable was the motivation for proposing in our previous work [17] a finite rank approximation for the p-h interaction resulting from Skyrme-type forces. Thus the self-consistent mean field can be calculated in the standard way with the original Skyrme interaction, whereas the RPA solutions would be obtained with the finite rank approximation to the p-h matrix elements. It was found that the finite rank approximation reproduces reasonably well the dipole and quadrupole strength distributions in Ar isotopes.

In the present work, we extend the finite rank RPA calculations to take into account pairing effects. As an application we present results of calculations for low-lying 2^+ and 3^- states in some O, Ar, Sn, and Pb isotopes. This paper is organized as follows: in Sec. II we sketch our method for constructing a finite rank interaction for the quasiparticle RPA (QRPA) case. In Sec. III we discuss details of calculations and show how this approach can be applied to treat different multipole states in wide excitation energy regions. Results of calculations for characteristics of the quadrupole and octupole states in some nuclei are given in Sec. IV. Conclusions are drawn in Sec. V.

II. HAMILTONIAN OF THE MODEL AND QRPA

We start from the effective Skyrme interaction [6] and use the notation of Ref. [18] containing explicit density dependence and all spin-exchange terms rather than the original form of Ref. [6] where density dependence at the HF level was introduced by a three-body contact force and where some spin-exchange terms were dropped. The exact p-h re-

sidual interaction $\tilde{V}_{res}$ corresponding to the Skyrme force and including both direct and exchange terms can be obtained as the second derivative of the energy density functional with respect to the density [19]. Following our previous paper [17] we simplify $\tilde{V}_{res}$ by approximating it by its Migdal-Landau form in the momentum space,

$$V_{res}(\mathbf{k}_1,\mathbf{k}_2)=N_0^{-1}\sum_{l=0}^{1}[F_l+G_l\sigma_1\sigma_2 + (F_l'+G_l'\sigma_1\sigma_2)\tau_1\tau_2]P_l\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_F^2}\right), \quad (1)$$

where $\mathbf{k}_i$, $\boldsymbol{\sigma}_i$, and τ_i are the nucleon momentum, spin and isospin operators, and $N_0=2k_Fm^*/\pi^2\hbar^2$ with k_F and m^* standing for the Fermi momentum and nucleon effective mass. For Skyrme interactions all Landau parameters with $l>1$ are zero. Here we keep only the $l=0$ terms in V_{res}, and in the coordinate representation one can write it in the following form:

$$V_{res}(\mathbf{r}_1,\mathbf{r}_2)=N_0^{-1}\{F_0(r_1)+G_0(r_1)\sigma_1\sigma_2 + [F_0'(r_1)+G_0'(r_1)\sigma_1\sigma_2]\tau_1\cdot\tau_2\}\delta(\mathbf{r}_1-\mathbf{r}_2). \quad (2)$$

The expressions for F_0,G_0,F_0',G_0' in terms of the Skyrme force parameters can be found in Ref. [18]. Because of the density dependence of the interaction the Landau parameters of Eq. (2) are functions of the coordinate $\mathbf{r}$. In what follows we use the second quantized representation and V_{res} can be written as

$$\hat{V}_{res}=\frac{1}{2}\sum_{1234}V_{1234}:a_1^+a_2^+a_4a_3:, \quad (3)$$

where a_1^+ (a_1) is the particle creation (annihilation) operator and 1 denotes the quantum numbers $(n_1l_1j_1m_1)$,

$$V_{1234}=\int\phi_1^*(\mathbf{r}_1)\phi_2^*(\mathbf{r}_2)V_{res}(\mathbf{r}_1,\mathbf{r}_2)\phi_3(\mathbf{r}_1)\phi_4(\mathbf{r}_2)\mathbf{dr}_1\mathbf{dr}_{2,}, \quad (4)$$

$$V_{1234}=\sum_{JM}\hat{J}^{-2}(-)^K\langle j_1m_1j_3-m_3|J-M\rangle \times\langle j_2m_2j_4-m_4|JM\rangle V_{1234}^J, \quad (5)$$

where $K=J+j_3+j_4-M-m_3-m_4$ and

$$V_{1234}^J=\langle j_1||Y_J||j_3\rangle\langle j_2||Y_J||j_4\rangle I_M(j_1j_2j_3j_4) - \sum_{L=J,J\pm1}\langle j_1||T_{JL}||j_3\rangle\langle j_2||T_{JL}||j_4\rangle I_S(j_1j_2j_3j_4). \quad (6)$$

In the above equation, $\langle j_1||Y_J||j_3\rangle$ is the reduced matrix element of the spherical harmonics $Y_{J\mu}$, $\hat{J}=\sqrt{2J+1}$, $T_{JL}^M(\hat{r},\sigma)=[Y_L\times\sigma]_J^M$ and $I_M(j_1j_2j_3j_4)$, and $I_S(j_1j_2j_3j_4)$ are the radial integrals:

$$I_M(j_1j_2j_3j_4)=N_0^{-1}\int_0^\infty(F_0(r) + F_0'(r)\tau_1\cdot\tau_2)u_{j_1}(r)u_{j_2}(r)u_{j_3}(r)u_{j_4}(r)\frac{dr}{r^2}, \quad (7)$$

$$I_S(j_1j_2j_3j_4)=N_0^{-1}\int_0^\infty(G_0(r) + G_0'(r)\tau_1\cdot\tau_2)u_{j_1}(r)u_{j_2}(r)u_{j_3}(r)u_{j_4}(r)\frac{dr}{r^2}, \quad (8)$$

where the radial wave functions $u(r)$ are related to the HF single-particle wave functions:

$$\phi_{i,m}(1)=\frac{u_i(r_1)}{r_1}\mathcal{Y}_{l_i,j_i}^m(\hat{r_1},\sigma_1). \quad (9)$$

As shown in Ref. [17], the radial integrals can be calculated accurately by choosing a large enough cutoff radius R and using an N-point integration Gauss formula with abscissas and weights r_k and w_k. Thus the residual interaction can be presented as a sum of N separable terms.

So we employ the hamiltonian including an average nuclear HF field, pairing interactions, the isoscalar and isovector particle-hole $(p\text{-}h)$ residual forces in a finite rank N:

$$H=\sum_\tau\left(\sum_{jm}^{\tau}(E_j-\lambda_\tau)a_{jm}^\dagger a_{jm}-\frac{1}{4}V_\tau^{(0)}:P_0^\dagger(\tau)P_0(\tau): -\frac{1}{2}\sum_{k=1}^{N}\sum_{q=\pm1}\sum_{\lambda\mu}\Big[(\kappa_0^{(M,k)} + q\kappa_1^{(M,k)}):M_{\lambda\mu}^{(k)+}(\tau)M_{\lambda\mu}^{(k)}(q\tau):+\sum_{L=\lambda,\lambda\pm1}(\kappa_0^{(S,k)} + q\kappa_1^{(S,k)}):S_{\lambda L\mu}^{(k)+}(\tau)S_{\lambda L\mu}^{(k)}(q\tau):\Big]\right), \quad (10)$$

We sum over the proton (p) and neutron (n) indexes and the notation $\{\tau=(n,p)\}$ is used. A change $\tau\leftrightarrow-\tau$ means a change $p\leftrightarrow n$. The single-particle states are specified by the quantum numbers (jm), E_j are the single-particle energies and λ_τ the chemical potentials. $V_\tau^{(0)}$ is the interaction strength in the particle-particle channel; $\kappa^{(Mk)}$ $(\kappa^{(Sk)})$ are the multipole (spin-multipole) interaction strengths in the $p\text{-}h$ channel, and they can be expressed via the Landau parameters as

$$\begin{pmatrix} \kappa_0^{(M,k)} \\ \kappa_1^{(M,k)} \\ \kappa_0^{(S,k)} \\ \kappa_1^{(S,k)} \end{pmatrix} = -N_0^{-1} \frac{Rw_k}{2r_k^2} \begin{pmatrix} F_0(r_k) \\ F_0'(r_k) \\ G_0(r_k) \\ G_0'(r_k) \end{pmatrix}. \tag{11}$$

The monopole pair creation and the multipole and spin-multipole operators entering the normal products in Eq. (10) are defined as follows:

$$P_0^+(\tau) = \sum_{jm}^{\tau} (-1)^{j-m} a_{jm}^+ a_{j-m}^+, \tag{12}$$

$$M_{\lambda\mu}^{(k)+}(\tau) = \hat{\lambda}^{-1} \sum_{jj'mm'}^{\tau} (-1)^{j+m} \times \langle jmj'-m'|\lambda\mu\rangle f_{j'j}^{(\lambda k)}(\tau) a_{jm}^+ a_{j'm'}, \tag{13}$$

$$S_{\lambda L\mu}^{(k)+}(\tau) = \hat{\lambda}^{-1} \sum_{jj'mm'}^{\tau} (-1)^{j+m} \times \langle jmj'-m'|\lambda\mu\rangle f_{j'j}^{(\lambda Lk)}(\tau) a_{jm}^+ a_{j'm'}, \tag{14}$$

where $f_{j'j}$ are the single particle radial matrix elements of the multipole and spin-multipole operators:

$$f_{j_1 j_2}^{(\lambda k)} = u_{j_1}(r_k) u_{j_2}(r_k) i^\lambda \langle j_1 || Y_\lambda || j_2 \rangle, \tag{15}$$

$$f_{j_1 j_2}^{(\lambda Lk)} = u_{j_1}(r_k) u_{j_2}(r_k) i^L \langle j_1 || T_{\lambda L} || j_2 \rangle. \tag{16}$$

One can see that Hamiltonian (10) has the same form as the QPM Hamiltonian with N separable terms, but in contrast to the QPM all parameters of this Hamiltonian are expressed through parameters of the Skyrme forces.

In what follows we work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

$$a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m}. \tag{17}$$

Hamiltonian (10) can be represented in terms of bifermion quasiparticle operators and their conjugates [4]:

$$B(jj';\lambda\mu) = \sum_{mm'} (-1)^{j'+m'} \langle jmj'm'|\lambda\mu\rangle \alpha_{jm}^+ \alpha_{j'-m'}, \tag{18}$$

$$A^+(jj';\lambda\mu) = \sum_{mm'} \langle jmj'm'|\lambda\mu\rangle \alpha_{jm}^+ \alpha_{j'm'}^+. \tag{19}$$

We introduce the phonon creation operators

$$Q_{\lambda\mu i}^+ = \frac{1}{2} \sum_{jj'} [X_{jj'}^{\lambda i} A^+(jj';\lambda\mu) - (-1)^{\lambda-\mu} Y_{jj'}^{\lambda i} A(jj';\lambda-\mu)], \tag{20}$$

where the index λ denotes total angular momentum and μ is its z projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$, i.e., $Q_{\lambda\mu i}|0\rangle = 0$. We define the excited states for this approximation by $Q_{\lambda\mu i}^+|0\rangle$. For the QRPA the following relation is valid:

$$\langle 0|[Q_{\lambda\mu,i}, Q_{\lambda'\mu',i'}^+]|0\rangle = \delta_{\lambda\lambda'}\delta_{\mu\mu'} \frac{1}{2} \sum_{jj'} (X_{jj'}^{\lambda i} X_{jj'}^{\lambda i'} - Y_{jj'}^{\lambda i} Y_{jj'}^{\lambda i'}). \tag{21}$$

The quasiparticle energies (ε_j), the chemical potentials (λ_τ), the energy gap and the coefficients u and v of the Bogoliubov transformations [Eq. (17)] are determined from the BCS equations with the single-particle spectrum that is calculated within the HF method with the effective Skyrme interaction. Making use of the linearized equation-of-motion approach [1],

$$\langle 0|[\delta Q_{\lambda\mu i}, [H, Q_{\lambda\mu i}^+]]|0\rangle = \omega_{\lambda i} \langle 0|[\delta Q_{\lambda\mu i}, Q_{\lambda\mu i}^+]|0\rangle, \tag{22}$$

with the condition.

$$\langle 0|[Q_{\lambda\mu i}, Q_{\lambda\mu i'}^+]|0\rangle = \delta_{ii'}, \tag{23}$$

one can derive the QRPA equations [3,4]

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = w \begin{pmatrix} X \\ Y \end{pmatrix}. \tag{24}$$

In QRPA problems there appear two types of interaction matrix elements, the $A_{(j_1 j_1')_\tau (j_2 j_2')_{q\tau}}^{(\lambda)}$ matrix related to forward-going graphs, and the $B_{(j_1 j_1')_\tau (j_2 j_2')_{q\tau}}^{(\lambda)}$ matrix related to backward-going graphs. For our case we get obtain the following expressions:

$$\begin{aligned} A_{(j_1 \geqslant j_1')_\tau (j_2 \geqslant j_2')_{q\tau}}^{(\lambda)} &= \varepsilon_{j_1 j_1'} \delta_{j_2 j_1} \delta_{j_2' j_1'} \delta_{q1} - \hat{\lambda}^{-2} (1+\delta_{j_2 j_2'})^{-1} \\ &\times \sum_{k=1}^{N} [(\kappa_0^{(M,k)} + q\kappa_1^{(M,k)}) u_{j_1 j_1'}^{(+)} f_{j_1 j_1'}^{(\lambda k)}(\tau) u_{j_2 j_2'}^{(+)} f_{j_2 j_2'}^{(\lambda k)}(q\tau) \\ &+ (\kappa_0^{(S,k)} + q\kappa_1^{(S,k)}) u_{j_1 j_1'}^{(-)} f_{j_1 j_1'}^{(\lambda\lambda k)}(\tau) u_{j_2 j_2'}^{(-)} f_{j_2 j_2'}^{(\lambda\lambda k)}(q\tau)], \end{aligned} \tag{25}$$

$$\begin{aligned} B_{(j_1 \geqslant j_1')_\tau (j_2 \geqslant j_2')_{q\tau}}^{(\lambda)} &= -\hat{\lambda}^{-2} (1+\delta_{j_2 j_2'})^{-1} \sum_{k=1}^{N} [(\kappa_0^{(M,k)} \\ &+ q\kappa_1^{(M,k)}) u_{j_1 j_1'}^{(+)} f_{j_1 j_1'}^{(\lambda k)}(\tau) u_{j_2 j_2'}^{(+)} f_{j_2 j_2'}^{(\lambda k)}(q\tau) \\ &- (\kappa_0^{(S,k)} + q\kappa_1^{(S,k)}) \\ &\times u_{j_1 j_1'}^{(-)} f_{j_1 j_1'}^{(\lambda\lambda k)}(\tau) u_{j_2 j_2'}^{(-)} f_{j_2 j_2'}^{(\lambda\lambda k)}(q\tau)], \end{aligned} \tag{26}$$

where $\varepsilon_{jj'} = \varepsilon_j + \varepsilon_{j'}$ and $u_{jj'}^{(\pm)} = u_j v_{j'} \pm v_j u_{j'}$.

One can find a prescription how to solve this system and to find the eigen-energies and phonon amplitudes in the Appendix (also also Ref. [17]). The matrix dimensions never exceed $4N\times 4N$ independently of the configuration space size. The derived equations have the same form as the QRPA equations in the QPM [4,20], but the single-particle spectrum and parameters of the p-h residual interaction are calculated making use of the Skyrme forces.

III. DETAILS OF CALCULATIONS

In this work we use generally the standard parametrization SIII [21] of the Skyrme force. Some examples of calculations with other parameter sets are presented in Sec. IV. Spherical symmetry is assumed for the HF ground states. The pairing constants V_τ^0 are fixed to reproduce the odd-even mass difference of neighboring nuclei. As a result constant pairing gaps have values that are very close to $\Delta = 12.0A^{-1/2}$ besides a case of semimagic nuclei. It is well known [12,14] that the constant gap approximation leads to an overestimation of occupation probabilities for subshells that are far from the Fermi level and it is necessary to introduce a cutoff in the single-particle space. Above this cutoff subshells do not participate in the pairing effect. In our calculations we choose the BCS subspace to include all subshells lying below 5 MeV. In order to perform RPA calculations, the single-particle continuum is discretized [22] by diagonalizing the HF Hamiltonian on a basis of 12 harmonic-oscillator shells and cutting off the single-particle spectra at the energy of 190 MeV. This is sufficient to exhaust practically all the energy-weighted sum rule. As shown in our previous calculations [17] we have adopted the value $N=24$ for the finite rank approximation for the dipole and quadrupole excitations in Ar isotopes. Increasing the mass number and the multipolarity of excitations demands an increase of the rank to keep the calculations accurate. Our investigations enable us to conclude that $N=45$ is enough for multipolarities $\lambda\leqslant 3$ in nuclei with $A\leqslant 208$. Increasing N, for example, up to $N=60$ in ^{208}Pb changes results for energies and transition probabilities not more than by 1%, so all calculations in what follows have been done with $N=45$. Our calculations show that, for normal parity states, one can neglect the spin-multipole interactions as a rule and this reduces by a factor of 2 the total matrix dimension. For example, for the octupole excitations in ^{206}Pb we need to invert a matrix having a dimension $2N=90$ instead of diagonalizing a 1376×1376 matrix as it would be without the finite rank approximation. For light nuclei the reduction of matrix dimensions due to the finite rank approximation is 3 or 4. So, for heavy nuclei our approach gives a large gain in comparison with an exact diagonalization.

The Landau parameters F_0, G_0, F_0', and G_0' expressed in terms of the Skyrme force parameters [18] depend on k_F. As it is pointed out in our previous work [17] one needs to adopt some effective value for k_F to give an accurate representation of the original p-h Skyrme interaction. To fix the effective values of k_F for the Landau parameters we use the self-consistency relation [23]. From the set of equations,

$$\frac{1}{2}\sum_\tau \frac{\delta U_\tau^{self}}{\delta\rho_\tau} = N_0^{-1}(F_0+F_0'), \tag{27}$$

$$\frac{1}{2}\sum_\tau \frac{\delta U_\tau^{self}}{\delta\rho_{-\tau}} = N_0^{-1}(F_0-F_0'), \tag{28}$$

one can get obtain the average field potential corresponding to such a choice of the residual interaction:

$$\begin{aligned} U_\tau^{self} = {} & \rho\left[t_0\left(1+\frac{x_0}{2}\right)+\frac{k_F^2}{4}[t_1(2+x_1)+t_2(2+x_2)]\right] \\ & -\rho_\tau\left[t_0\left(\frac{1}{2}+x_0\right)-\frac{k_F^2}{4}[t_2(1+2x_2)-t_1(1+2x_1)]\right] \\ & +\frac{1}{24}t_3\{\rho^{\alpha+1}(2+\alpha)(2+x_3)-(1+2x_3) \\ & \times[2\rho^\alpha\rho_\tau+\alpha\rho^{\alpha-1}(\rho_n^2+\rho_p^2)]\}. \end{aligned} \tag{29}$$

This potential can be compared to $(m^*/m)U^{HF}(r)$ which is the leader term of a local equivalent potential in the Skyrme-HF approach. It is possible to evaluate the effective value k_F for every nucleus. One can show that this value is larger than the nuclear matter value in order to compensate for the effects of the neglected terms F_1 and G_1. To calculate the dipole strength distributions we choose k_F so that the spurious isoscalar dipole mode appears at zero excitation energy. The strongest renormalization of the k_F values in comparison with the nuclear matter value takes place in light nuclei. For ^{208}Pb the effective value k_F becomes rather close to the nuclear matter one.

IV. RESULTS OF CALCULATIONS

As a first example we examine the multipole strength distributions in ^{36}Ar. The calculated strength distributions are displayed in Fig. 1. For the giant dipole resonance (GDR) and giant quadrupole resonance (GQR) QRPA gives results that are very similar to our previous calculations with the particle-hole RPA [17], because the influence of pairing on the giant resonance properties is weak. This is not the case for the first 2^+ and 3^- states that will be discussed later. For the GDR energy centroid we obtain $E_c=19.9$ MeV, and this value is rather close to the empirical systematics [24] $E_c=(31.2A^{-1/3}+20.6A^{-1/6})$ MeV. The isoscalar GQR energy centroid is equal to $E_c=18.8$ MeV, that can be compared with the empirical value $E_c=63A^{-1/3}=19.1$ MeV. For the isovector GQR our calculation gives $E_c=30.5$ MeV, that is about 10% less than predicted by the empirical systematics. It is worth mentioning that experimental data for the giant resonances in light nuclei are very scarce.

Results of our calculations and experimental data [25] for the 2_1^+ state energies and transition probabilities $B(E2)$ in several nuclei are shown in Table I. One can see that there is a satisfactory agreement with experimental data. Results of our calculations for O and Ar isotopes are close to those of the QRPA with Skyrme forces [12,26] and all calculations fail to reproduce the $B(E2)$ value in ^{18}O. Making use of the

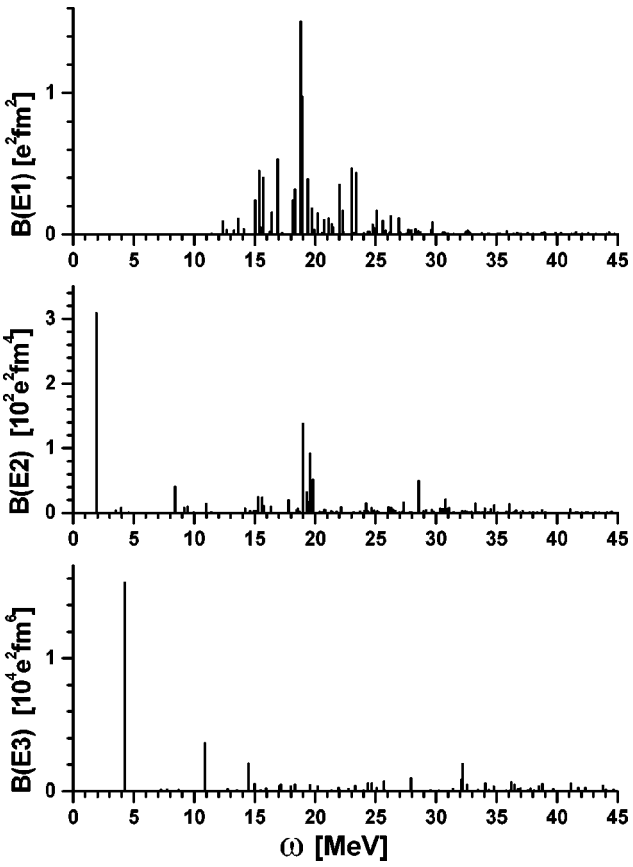

FIG. 1. The multipole strength distributions in ^{36}Ar.

SGII interaction [18] improves the description for the O isotopes and gives practically the same results for the Ar isotopes, but for Sn and Pb isotopes the results become much worse. Calculations with the SkI4 force [27] do not change the above conclusions. The evolution of the $B(E2)$ values in the Ar isotopes demonstrates clearly the pairing effects. The experimental and calculated $B(E2)$ values in ^{38}Ar are three times less than those in ^{36,40}Ar. The neutron shell closure leads to the vanishing of the neutron pairing and a reduction of the proton gap. As a result there is a remarkable reduction of the $E2$ transition probability in ^{38}Ar. Some overestimate of the energies indicates that there is room for two-phonon effects. Indeed, it was found in calculations performed within the QPM for ^{208}Pb [28] that the two-phonon configurations can shift down the 2_1^+ energy by more than 1 MeV. The $B(E2)$ value reduction is about 10% in this case. The study of the influence of two-phonon configurations on properties of the low-lying states within our approach is in progress now.

TABLE I. Energies and $B(E2)$ values for up-transitions to the first 2^+ states.

Nucleus	Energy (MeV)		$B(E2\uparrow)$ (e^2fm^4)	
	Expt.	Theory	Expt.	Theory
^{18}O	1.98	4.75	45±2	14
^{20}O	1.67	4.17	28±2	20
^{36}Ar	1.97	1.91	300±30	310
^{38}Ar	2.17	2.51	130±10	110
^{40}Ar	1.46	2.17	330±40	290
^{112}Sn	1.26	1.49	2400±140	2600
^{114}Sn	1.30	1.51	2400±500	2100
^{206}Pb	0.80	0.96	1000±20	1700
^{208}Pb	4.09	5.36	3000±300	2000

TABLE II. Energies and $B(E3)$ values for up transitions to the first 3^- states.

Nucleus	Energy (MeV)		$B(E3\uparrow)$ (e^2fm^6)	
	Expt.	Theory	Expt.	Theory
^{18}O	5.10	6.15	1250±50	1200
^{20}O	5.61	7.28	530±180	1000
^{36}Ar	4.18	4.26	11 100±1100	16 000
^{38}Ar	3.81	3.21	10 000±3000	15 000
^{40}Ar	3.68	4.85	8700±1000	12 000
^{112}Sn	2.36	2.73	87 000±12 000	97 000
^{114}Sn	2.28	2.31	100 000±12 000	97 000
^{206}Pb	2.65	2.92	650 000±40 000	750 000
^{208}Pb	2.62	2.66	611 000±9000	860 000

Results of our calculations for the 3_1^- energies and the transition probabilities $B(E3)$ are compared with experimental data [29] in Table II. Generally there is a better agreement between theory and experiment than in the case of quadrupole excitations. Other choices of the Skyrme forces do not improve the agreement obtained with SIII.

An additional information about the structure of the first 2^+ and 3^- states can be extracted by looking at the ratio of the multipole transition matrix elements M_n/M_p that depend on the relative contributions of the proton and neutron configurations. In the framework of the collective model for isoscalar excitations this ratio is equal to $M_n/M_p=N/Z$ and any deviation from this value can indicate an isovector character of the state. The M_n/M_p ratio can be determined experimentally by using different external probes [30–32]. Recently [13,26], QRPA calculations of the M_n/M_p ratios for the 2_1^+ states in some O and Ar isotopes have been done. The predicted results are in good agreement with experimental data [26]. Our calculated values of the M_n/M_p ratios for the 2_1^+ and 3_1^- states are shown in Tables III and IV, respec-

TABLE III. $(M_n/M_p)/(N/Z)$ ratios for the first 2^+ states.

Nucleus	^{18}O	^{20}O	^{36}Ar	^{38}Ar	^{40}Ar
Theory	2.1	2.1	0.9	0.5	0.9
Expt.	0.88±0.19[a]	2.17±0.53[a]	1.41±0.50[b]	–	0.68±0.21[b]
	1.62±0.02[c]	1.9±0.3[c]			

[a]Reference [13].
[b]Reference [26].
[c]Reference [32].

TABLE IV. $(M_n/M_p)/(N/Z)$ ratios for the first 3^- states.

Nucleus	^{18}O	^{20}O	^{36}Ar	^{38}Ar	^{40}Ar
Theory	0.8	0.8	0.9	1.0	0.9

tively. Our results support the conclusions of Refs. [13,26] about the isovector character of the 2_1^+ states in ^{18,20}O and ^{38}Ar. As one can see from Table IV, our calculations predict that the M_n/M_p ratios for the 3_1^- states are rather close to N/Z, thus indicating their isoscalar character. This conclusion remains valid for the SGII and SkI4 parameter sets.

V. CONCLUSION

A finite rank separable approximation for the particle-hole RPA calculations with Skyrme interactions that was proposed in our previous work is extended to take into account the pairing correlations. The QRPA equations are derived for this case. These equations are used to study the evolution of quadrupole and octupole excitations in nuclei away from stability. It is shown that the suggested approach enables one to reduce remarkably the dimensions of the matrices that must be inverted to perform structure calculations in very large configuration spaces.

As an illustration of the method we have used the finite rank *p-h* interaction derived from the Skyrme force SIII to calculate the energies and transition probabilities of the 2_1^+ and 3_1^- states in some O, Ar, Sn, and Pb isotopes. The values calculated within our approach are very close to those that were calculated in the QRPA with the full Skyrme interactions. They are generally in a reasonable agreement with experimental data. A further development will be to take into account the coupling between the one- and two-phonon terms, and such investigations are in progress now.

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APPENDIX

For the sake of completeness we show how the finite rank form of the residual forces (10) can simplify the solution of the RPA equations (24). In the $4N$-dimensional space we introduce a vector $\binom{\mathcal{D}_S(\tau)}{\mathcal{D}_M(\tau)}$ by its components

$$\boldsymbol{\mathcal{D}}^{\mathbf{k}}_{\beta}(\tau)=\begin{pmatrix} D^k_\beta(\tau) \\ D^k_\beta(-\tau)\end{pmatrix}, \quad \beta=\{M,S\}, \tag{A1}$$

where

$$D^{\lambda ik}_M(\tau)=\sum_{jj'}^{\tau} f^{(\lambda k)}_{jj'}u^{(+)}_{jj'}(X^{\lambda i}_{jj'}+Y^{\lambda i}_{jj'}),$$

$$D^{\lambda ik}_S(\tau)=\sum_{jj'}^{\tau} f^{(\lambda\lambda k)}_{jj'}u^{(-)}_{jj'}(X^{\lambda i}_{jj'}-Y^{\lambda i}_{jj'}).$$

The index k run over the N-dimensional space (k $=1,2,\ldots,N$). Solving the system of Eqs. (24) one can obtain the following expressions for the phonon amplitudes:

$$X^{\lambda i}_{jj'}(\tau)=\frac{1}{(\varepsilon_{jj'}-\omega_{\lambda i})}\sum_{k=1}^{N}\frac{1}{\sqrt{2\mathcal{Y}^{\lambda ki}_\tau}} \times[u^{(+)}_{jj'}f^{(\lambda k)}_{jj'}+u^{(-)}_{jj'}f^{(\lambda\lambda k)}_{jj'}z^{\lambda ik}(\tau)], \tag{A2}$$

$$Y^{\lambda i}_{jj'}(\tau)=\frac{1}{(\varepsilon_{jj'}+\omega_{\lambda i})}\sum_{k=1}^{N}\frac{1}{\sqrt{2\mathcal{Y}^{\lambda ki}_\tau}} \times[u^{(+)}_{jj'}f^{(\lambda k)}_{jj'}-u^{(-)}_{jj'}f^{(\lambda\lambda k)}_{jj'}z^{\lambda ik}(\tau)], \tag{A3}$$

where

$$\mathcal{Y}^{\lambda ki}_\tau=\frac{2(2\lambda+1)^2}{[D^{\lambda ik}_M(\tau)(\kappa^{(M,k)}_0+\kappa^{(M,k)}_1)+D^{\lambda ik}_M(-\tau)(\kappa^{(M,k)}_0-\kappa^{(M,k)}_1)]^2},$$

$$z^{\lambda ik}(\tau)=\frac{D^{\lambda ik}_S(\tau)(\kappa^{(S,k)}_0+\kappa^{(S,k)}_1)+D^{\lambda ik}_S(-\tau)(\kappa^{(S,k)}_0-\kappa^{(S,k)}_1)}{D^{\lambda ik}_M(\tau)(\kappa^{(M,k)}_0+\kappa^{(M,k)}_1)+D^{\lambda ik}_M(-\tau)(\kappa^{(M,k)}_0-\kappa^{(M,k)}_1)}.$$

Using Eqs. (A1) and Eqs. (A2) and (A3) the RPA equations (24) can be reduced to the following system of equations:

$$\begin{pmatrix}\mathcal{M}_{MM}(\tau)-1 & \mathcal{M}_{MS}(\tau)\\ \mathcal{M}_{SM}(\tau) & \mathcal{M}_{SS}(\tau)-1\end{pmatrix}\begin{pmatrix}\boldsymbol{\mathcal{D}}_M(\tau)\\ \boldsymbol{\mathcal{D}}_S(\tau)\end{pmatrix}=0, \tag{A4}$$

where $\mathcal{M}$ is the $2N\times 2N$ matrix:

$$\mathcal{M}^{kk'}_{\beta\beta'}(\tau)=\begin{pmatrix} (\kappa_0^{(\beta',k')}+\kappa_1^{(\beta',k')})T^{kk'}_{\beta\beta'}(\tau) & (\kappa_0^{(\beta',k')}-\kappa_1^{(\beta',k')})T^{kk'}_{\beta\beta'}(\tau) \\ (\kappa_0^{(\beta',k')}-\kappa_1^{(\beta',k')})T^{kk'}_{\beta\beta'}(-\tau) & (\kappa_0^{(\beta',k')}+\kappa_1^{(\beta',k')})T^{kk'}_{\beta\beta'}(-\tau) \end{pmatrix}, \quad 1\leqslant k,k'\leqslant N. \tag{A5}$$

The matrix elements $T^{kk'}$ have the following forms:

$$T^{kk'}_{MM}(\tau)=\sum_{jj'}^{\tau}\frac{f^{(\lambda k)}_{jj'}f^{(\lambda k')}_{jj'}(u^{(+)}_{jj'})^2\varepsilon_{jj'}}{\hat{\lambda}^2(\varepsilon^2_{jj'}-\omega^2_{\lambda i})},$$

$$T^{kk'}_{MS}(\tau)=\sum_{jj'}^{\tau}\frac{f^{(\lambda k)}_{jj'}f^{(\lambda\lambda k')}_{jj'}u^{(+)}_{jj'}u^{(-)}_{jj'}\omega_{\lambda i}}{\hat{\lambda}^2(\varepsilon^2_{jj'}-\omega^2_{\lambda i})},$$

$$T^{kk'}_{SM}(\tau)=T^{k'k}_{MS}(\tau),$$

$$T^{kk'}_{SS}(\tau)=\sum_{jj'}^{\tau}\frac{f^{(\lambda\lambda k)}_{jj'}f^{(\lambda\lambda k')}_{jj'}(u^{(-)}_{jj'})^2\varepsilon_{jj'}}{\hat{\lambda}^2(\varepsilon^2_{jj'}-\omega^2_{\lambda i})}.$$

Thus the RPA eigenvalues $\omega_{\lambda i}$ are the roots of the secular equation

$$\det\begin{pmatrix} \mathcal{M}_{MM}(\tau)-1 & \mathcal{M}_{MS}(\tau) \\ \mathcal{M}_{SM}(\tau) & \mathcal{M}_{SS}(\tau)-1 \end{pmatrix}=0. \tag{A6}$$

The phonon amplitudes corresponding to the RPA eigenvalue $\omega_{\lambda i}$ are determined by Eqs. (A2) and (A3), taking into account the normalization condition (23).

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