Quantum-like model for unconscious-conscious interaction within theory of open quantum systems

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Part 1: Applications of quantum formalism to cognition and decision making: from cells to humans

Quantum(-like) operational representation of the process of decision making by cognitive systems

This talk is not about **quantum brain** in the spirit of R. Penrose (Nobel Prize 2020) and S. Hameroff. We do not try to reduce information processing by cognitive system to quantum physical effects.

In our modeling, the brain is a **black box** which information processing can be mathematically described within the formalism of quantum theory.

In particular, the standard modeling based on classical probability (Kolmogorov axioms 1933) seems to be inadequate to mental processes.

One can find a plenty of such nonclassical statistical data collected in cognitive psychology, game theory, decision making, social science, economics, finance, social and political sciences.

A. Khrennikov, Ubiquitous quantum structure: from psychology to fnances, Springer, Berlin-Heidelberg-New York, 2010.

Busemeyer, J.R., Bruza, P. D. Quantum models of cognition and decision, Cambridge University Press, Cambridge, 2012.

E. Haven and A. Khrennikov, Quantum Social Science, Cambridge Press, Cambridge, 2013.

In decision theory, such data was coupled to **probability fallacies** and **irrational behavior** of agents.

We propose to apply the most well developed non-classical theory of information and probability whichis based on the mathematical formalism of QM.

One may think that the appeal to quantum probability (and information) to model decision making by humans is too exotic.

However, we recall that as early as the 1970s, Tversky (one of the most cited psychologists of all time) and Kahneman (Nobel prize in economics in 2002, for prospect theory, which he co-developed with Tversky) have been demonstrating cases where **classical probability prescription and human behavior persistently diverge** (Tversky and Kahneman 1973, 1983).

Today, we are at the theoretical cross-roads, with huge divisions across conflicting, entrenched theoretical positions.

Should we continue relying on CP as the basis for descriptive and normative predictions in decision making (and perhaps ascribe inconsistencies to methodological idiosyncrasies)?

Should we abandon probability theory completely and instead pursue explanations based on heuristics, as Tversky and Kahneman proposed?

Quantum theory instead of heuristics of Tversky and Kahneman!

Classical versus quantum probability

Classical probability (CP) was mathematically formalized by Kolmogorov (1933). This is the calculus of probability measures, non-negative weight $p(A)$ is assigned to any event A.

The main property of CP is its additivity: if two events A_1, A_2 are disjoint, then the probability of disjunction of these events equals to the sum of probabilities:

$$P(A_1 \vee A_2) = P(A_1) + P(A_2).$$

Quantum probability (QP) is the calculus of complex amplitudes or in the abstract formalism complex vectors. Thus, instead of operations on probability measures one operates with vectors.

We can say that QP is a vector model of probabilistic reasoning.

Each complex amplitude ψ gives the probability by the Born's rule:

"Probability is the square of the absolute value of the complex amplitude"'.

$$p = |\psi|^2.$$

Interference of probabilities

By operating with complex probability amplitudes, instead of the direct operation with probabilities, one can violate the basic laws of CP, in particular, additivity of probability. One can get that, for disjoint events, the probability of disjunction is strictly smaller or larger than the sum of probabilities:

$$\boldsymbol{P(A_1 \vee A_2) < P(A_1) + P(A_2)}$$

or

$$\boldsymbol{P(A_1 \vee A_2) > P(A_1) + P(A_2),}$$

QP calculus leads to the formula

$$P(A_1 \vee A_2) = P(A_1) + P(A_2) + 2\cos\theta\sqrt{P(A_1)P(A_2)}.$$

The additional term is known as the interference term.

Interference is the basic feature of waves, so often one speaks about probability waves. Wave theory can be applied not only to waves propagating in the physical space-time, but also to waves propagating in the information space.

Applications to molecular biology: Quantum Information Biology

Quantum formalism was also applied to describe "decision making" in genetics and molecular biology - cells and proteins as decision makers:

Asano, M., Khrennikov, A., Ohya, M., Tanaka, Y., Yamato, I. *Quantum adaptivity in biology: from genetics to cognition*, (Springer, Heidelberg-Berlin-New York, 2015).

Generally quantum formalism gives the possibility to describe information processing by all biological systems from cells to human beings – on all scales of living matter. Surprisingly, the formal mathematical model is the same.

Cells are decision makers using by the same rules as people (or people use the same rules as cells). We can speak about quantum information biology. It is not quantum biophysics!

The following paradigm can be used to motivate the applications of quantum theory outside of physics.

Quantum-like paradigm (Khrennikov 1999):

The mathematical formalism of quantum information and probability theories can be used to model behavior not only of genuine quantum physical systems, but all context-sensitive systems, e.g., humans.

Contextual information processing cannot be based on complete resolution of ambiguity. It is meaningless to do this for the concrete context, if tomorrow context will be totally different.

Such systems process ambiguities, represented by superpositions of alternatives, superpositions of state vectors.

The use QP, instead of CP, can resolve some paradoxes of CP-based theory of decision making, economics, and game theory; e.g., the Allais (1953), Ellsberg (1961), and Machina (1987) paradoxes.

Typically a paradox (as a sign of irrational behavior) is probabilistically expressed as violation of the law of additivity of probability, i.e., that for disjoint events

$$P(A_1 \vee A_2) \neq P(A_1) + P(A_2).$$

The number of paradoxes generated by the classical decision making theory is really amazing. The authors of the recent review (Erev and Ert 2015) counted 35 basic paradoxes.

During many years DM-theory was developed through creation of paradoxes and resolving them through modifications of the theory, e.g., from expected utility theory to the prospect theory. But any modified theory suffered of new paradoxes.

The use of QP can resolve all such paradoxes, at least this is claimed in the recent paper:

M. Asano, I. Basieva, A. Khrennikov, M. Ohya, Y. Tanaka, A quantum- like model of selection behavior. J. Math. Psychology 78, 2-12 (2017).

QP was successfully applied to model a variety of psychological effects, e.g., the order, disjunction, and conjunction effects.

E. Haven and A. Khrennikov, Quantum mechanics and violation of the sure-thing principle: the use of probability interference and other concepts. J. Math. Psychology, 53, 378-388 (2009).

Quantum formalism: : states and observables

H is complex Hilbert space (finite dimensional). Pure states of a system S are normalized vectors of H.

Physical observable A is a Hermitian operator:

$$\hat{A} = \sum_x x \, E_x^A, \tag{1}$$

where E_x^A is the orthogonal projector onto the subspace of H corresponding to the eigenvalue x, i.e., $\mathrm{H}_x^A = E_x^A \mathrm{H}$.

The probability to get the answer $A = x$ for initial state ψ is given by the Born rule:

$$\Pr\{A = x \| \psi\} = \|E_x^A \psi\|^2 = \langle \psi | E_x^A \psi \rangle. \tag{2}$$

and according to the projection postulate (von Neumann) the post-measurement state is generated by the map:

$$\psi \to \mathrm{I}_x^A \psi = E_x^A \psi / \|E_x^A \psi\|. \tag{3}$$

This state transformation is by observation's feedback to the system which initially was in the state ψ, i.e., observations disturb systems' states.

The quantum state update is the basis of quantum generalization of classical Bayesian inference.

The projection update has properties which crucially differ from the classical probability update.

In particular, it generates violation of the law of total probability (playing the important role in Bayesian inference) and the order effect which is absent in the classical probability theory.

Non-Bayesian character of the quantum state and probability update is one of the distinguishing features of the quantum-like modeling of the brain's functioning. This is a consequence of the existence of incompatible quantum observables, those which values cannot be jointly assigned.

Indirect measurements

Bohr:, the results of quantum measurements are generated via interaction of a system S with a measurement apparatus M.

M consists of a complex physical device interacting with S and a pointer that shows the result of measurement, say spin up or spin down. An observer can see only outputs of the pointer and he associates these outputs with the values of the observable A for the system S.

So, the observer approaches only the pointer, not the system by itself.

Thus, the indirect measurement scheme involves:

- the states of the systems S and the apparatus M;
- the operator U representing the interaction-dynamics for the system $S + M$;
- the meter observable M_A giving outputs of the pointer of the apparatus M.

Formally, an *indirect measurement model*, as a "(general) measuring process", is a quadruple

$$(\boldsymbol{K}, \boldsymbol{\sigma}, \boldsymbol{U}, \boldsymbol{M_A})$$

consisting of a Hilbert space $\boldsymbol{K}$, a density operator $\boldsymbol{\sigma}$, a unitary operator $\boldsymbol{U}$ on the tensor product of the state spaces of $\boldsymbol{S}$ and $\boldsymbol{M}$, $\boldsymbol{U} : \boldsymbol{H} \otimes \boldsymbol{K} \to \boldsymbol{H} \otimes \boldsymbol{K}$, and a self-adjoint operator $\boldsymbol{M_A}$ on $\boldsymbol{K}$. By this measurement model, the Hilbert space $\boldsymbol{K}$ describes the states of the apparatus $\boldsymbol{M}$, the unitary operator $\boldsymbol{U}$ describes the time-evolution of the composite system $\boldsymbol{S} + \boldsymbol{M}$, the density operator $\boldsymbol{\sigma}$ describes the initial state of the apparatus $\boldsymbol{M}$, and the self-adjoint operator $\boldsymbol{M_A}$ describes the meter observable of the apparatus $\boldsymbol{M}$. Then, the output probability distribution in the system state $\boldsymbol{\rho}$ is given by

$$\text{(4)} \qquad \Pr\{\boldsymbol{A} = \boldsymbol{x} \| \boldsymbol{\rho}\} = \boldsymbol{Tr}[(\boldsymbol{I} \otimes \boldsymbol{E}^{\boldsymbol{M_A}}(\boldsymbol{x}))\boldsymbol{U}(\boldsymbol{\rho} \otimes \boldsymbol{\sigma})\boldsymbol{U}^\star],$$

where $E^{M_A}(x)$ is the spectral projection of M_A for the eigenvalue x.

The change of the state ρ of the system S caused by the measurement for the outcome $A = x$ is represented with the aid of the map $I_A(x)$ in the space of density operators defined as

$$I_A(x)\rho = Tr_K[(I \otimes E^{M_A}(x))U(\rho \otimes \sigma)U^\star], \tag{5}$$

where Tr_K is the partial trace over K.

System S = Unconscious

Measurment apparatus M = Consciousness

Part 2: Quantum-like modeling of brain's functioning: consciousness performing observations on unconsciousness

Once again, we do not consider quantum physical processes in the brain.

The quantum formalism is used as the most general measurement formalism.

Quantum mechanics has many interpretations. By the Copenhagen interpretation, it describes and predicts outcomes of measurements performed by observers on systems.

Separation system-observer places the crucial role in quantum methodology. The bg problems is establishing the boundary between a system and an observer.

We have to consider system-observer separation in information processing by the brain. How? It seems that the brain performs self-observations. Unconscious vs. conscious information processing in the brain

Unconsciousness

An essential part of information processing in the brain is performed unconsciously.

The information system for such processing (call it unconsciousness) is denoted by the symbol UC. The space of its states is denoted by $H \equiv H_{UC}$. In the quantum-like model, this is a complex Hilbert space.

We do not couple unconsciousness with James, Freud, and Jung

UC denotes a special information processors of the brain. It performs pre-observational processing of the mental state.

Consciousness

Perceptions and emotions are commonly treated as conscious entities. In our model the brain contains another information processing system generating conscious experiences; denote it by the symbol C.

Its functioning is modeled as performing measurements on the system UC.

Introduction of two systems UC and C matches the quantum measurement scheme, UC is the analog of a physical system exposed to measurements and C is the analog of a complex of measurement apparatuses.

The basic theories of consciousness

There are two basic competing theories of consciousness:

- the First Order Theory of Consciousness;
- the Higher Order Theory of Consciousness.

"First-order theorists ... argue that processing related to a stimulus is all that is needed for there to be phenomenal consciousness of that stimulus."

"In contrast, ... higher-order theorists argue that a first-order state resulting from stimulus-processing alone is not enough to make possible the conscious experience of a stimulus. ... consciousness exists by virtue of the relation between the first- and higher-order states."

The Higher Order Theory distinguishes between unconscious and conscious processing of mental information; what makes cognition conscious is a higher-order observation of the first-order processing.

Compare with Bohr:

"This crucial point ... implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear"

This viewpoint matches with the Higher Order Theory of Consciousness. A conscious experience is not simply introspection of the UC-state.

Perceptions and emotions

Perception representation of sensations

We follow to von Helmholtz theory of sensation-perception. Perceptions are not simply a copies of sensations, not "impressions like the imprint of a key on wax", but the results of complex signal processing including unconscious cognitive processing and conscious observation of unconscious states.

Context-representation via emotions

"Emotion schema are learned in childhood and used to categorize situations as one goes through life. As one becomes more emotionally experienced, the states become more differentiated: fright comes to be distinguished from startle, panic, dread, and anxiety."

In our terminology, each emotion-generation scheme is crystallized on of the basic life-contexts. Context-labeling is the basic function of emotions.

Contextualization of surrounding environment was one of the first cognitive tasks of biosystems and this ability was developed in parallel with establishing of sensation-perception system.

"Emotions represent adaptive reactions to environmental challenges; they are a result of human evolution; they provided optimal (from the viewpoint of computational resources) solutions to ancient and recurring problems that faced our ancestors."

We emphasize that in our model emotions are conscious.
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Unconscious and conscious counterparts of the processes of generation of perceptions and emotions

We shall be concentrated on functioning of two information processors transforming

- **sensations → perceptions,**
- **contexts → emotions.**

Both processors have conscious outputs.

Their functioning is strongly correlated; in the formalism quantum theory correlations are represented by entangled states.

We denote unconscious counterparts of these processors by the symbols UC_{per} and UC_{em}, respectively.

In modeling of the emotional coloring of perception (its contextualization), we shall consider the compound information system (UC_{per}, UC_{em}).

Basic and supplementary conscious experiences, application to decision making

Although we are mainly interested in emotional coloring of perceptions, the formalism under consideration can be applied to the very general class of compound information processing systems, $(\mathrm{UC}_{\mathrm{bas}}, \mathrm{UC}_{\mathrm{sup}})$.

The latter is used for determining stable repeatable and evolutionary fixed contexts for the former.

Simplest generalization of the perception-emotion scheme is emotional contextualization of decision making which modeling is based the compound system $(\mathrm{UC}_{\mathrm{dm}}, \mathrm{UC}_{\mathrm{em}})$.

Generation of conscious experiences (basic and supplementary) is modeled quantum observables; denote the corresponding classes by the symbols O_{bas} and O_{sup}. In particular, we shall consider the pairs $(O_{\mathrm{per}}, O_{\mathrm{em}})$ and $(O_{\mathrm{dm}}, O_{\mathrm{em}})$.

Incompatible conscious observables

bservables C_1 and C_2 are called compatible if they can be jointly measurable and the joint probability distribution (JPD) $p_\psi(C_1 = x_1, C_2 = x_2)$ is well defined; observables which cannot be jointly measurable and, hence, their JPD cannot be defined are called incompatible.

In the mathematical formalism, compatibility and incompatibility are formalized through commutativity and noncommutativity, respectively. If observables are described as Hermitian operators C_1, C_2, compatibility is encoded as $[C_1, C_2] = 0$. Incompatibility is encoded as $[C_1, C_2] \neq 0$.

For compatible observables, JPD is given by the following extension of the Born's rule:

(6)

$$\boldsymbol{p_\psi(C_1 = x_1, C_2 = x_2) = |\langle E_{x_1}^{C_1} E_{x_2}^{C_2}\psi, \psi\rangle|^2 = |\langle E_{x_2}^{C_2} E_{x_1}^{C_1}\psi, \psi\rangle|^2.}$$

We stress that the space of observables O_{per} can contain incompatible perceptions as well as O_{em} can contain incompatible emotions.

In the mental framework, incompatibility can be interpreted very naturally: there exist say emotions which can be experienced simultaneously; say happiness and sadness, pride and shame.

There exist incompatible, i.e., jointly unobservable perceptions and other conscious experiences.

The necessity to operate with various incompatible entities is the main roots of the use of the quantum(-like) information representation.

In the absence of incompatibility, i.e., if, for the same mental state, the brain were able to construct the consistent probabilistic representation (in the form of JPD) of all possible combinations of say emotions, the quantum state formalism would be unnecessary.

In quantum measurement theory, selection of observables co-measurable with A is considered as specification of measurement context of A-measurements; the A-value in the B-context can differ from the A-value in the C-context, for the same premeasurment state ψ.

This is the essence of contextuality playing so important role in quantum information theory [?]:

Definition 1. *If $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ are three quantum observables, such that $\boldsymbol{A}$ is compatible with $\boldsymbol{B}$ and $\boldsymbol{C}$, a measurement of $\boldsymbol{A}$ might give different result depending upon whether $\boldsymbol{A}$ is measured with $\boldsymbol{B}$ or with $\boldsymbol{C}$.*

We note that contextual behavior corresponds to the case of incompatible quantum observables B and C, i.e.,

If all observables are pairwise commute, it is possible to construct the noncontextual model of measurement based

on the joint probability distribution for triple outcomes

$$p_{ABC}(x_k, b_m, c_n|\psi) = \|E_k^A E_m^B E_n^C \psi\|^2 = ... = \|E_k^A E_n^C E_m^B E_n^C \psi\|^2. \tag{7}$$

If B and C are incompatible, such a model is impossible. This is the contextuality scenario.

Bell type inequalities and experimental testing of emotional contextuality

In quantum physics, experimental testing of the Bell type inequalities is the hot topic. In psychology and decision making, they have been tested by a few authors.

This talk can stimulate such experimenting in consciousness studies with joint measurements of the pairs $(A, B) =$ (perception, emotion) or (decision making, emotion).

As in physics and the previous psychological experiments, it is natural to test the CHSH inequality.

Consider two incompatible perceptions A and A' and two incompatible emotions B and B', and form cyclically their correlations.

The CHSH correlation function is given by the following combination of correlations:

$$C_{\text{CHSH}} = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle \tag{8}$$

and, for dichotomous observables yielding values ± 1 and in the absence of contextuality the following inequality holds:

$$|C_{\mathrm{CHSH}}| \leq 2. \tag{9}$$

Violation of this inequality gives the measure of contextuality. In quantum physics, the maximal value of C_{CHSH} is $2\sqrt{2}$, Tsirelson bound.

Surprisingly in decision making we got the same bound.

Irina Basieva, V. H. Cervantes, Ehtibar N. Dzhafarov, Andrei Khrennikov, True Contextuality Beats Direct Influences in Human Decision Making. Journal of Experimental Psychology: General 148, 1925-1937, 2020.