Estimation for $\Upsilon(1S)$ production cross section at the NICA collider

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Generalized Parton Model (GPM)

Factorization schemes in different p_T-regions

The traditional Collinear Parton Model (CPM) is applicable in a region of high-p_T production

$$\mu \sim p_T \gg \Lambda_{QCD},$$

so we can neglect influence of small intrinsic $\mathbf{q_T}$ of initial partons $(\langle q_T^2 \rangle \simeq 1 \text{ GeV}^2)$.

But if we're interested in particle production in a region of $p_T \simeq \sqrt{\langle q_T^2 \rangle} \ll \mu$, we should take into account intrinsic q_T. It can be done within TMD approach, factorization for which has been proven in the limit $q_T \ll \mu$ [J. Collins, Camb. Monogr., Part. Phys. Nucl. Phys. Cosmol. 32, 1-624 (2011)]. In our case, the hard scale μ is given by bottomonium mass $m_\mathcal{C} = 9.5 \div 10.4$ GeV.
So we can use phenomenological TMD-ansatz, a so called Generalized Parton Model (GPM), initial partons in which are on-shell:

$$q_\mu = xP_\mu^+ + yP_\mu^- + q_{T\mu}, (q_\mu)^2 = 0, \tag{1}$$

and a factorized prescription for TMD parton distribution functions (PDFs) is used:

$$F_a(x, q_T, \mu_F) = f_a(x, \mu_F) G_a(q_T), \tag{2}$$

where $f_a(x, \mu_F)$ – corresponding CPM PDF, $G_a(q_T)$ – Gaussian distribution $G_a(q_T) = \exp(-q_T^2 / \langle q_T^2 \rangle_a)/(\pi \langle q_T^2 \rangle_a)$.

Factorization formula for the GPM

Within the GPM we can write the following expression for the differential cross-section of $2 \to 1$ hard subprocess $g(q_1) + g(q_2) \to \mathcal{C}(k)$:

$$d\sigma(pp \to \mathcal{C}X) = \int dx_1 \int d^2\mathbf{q_{1T}} \int dx_2 \int d^2\mathbf{q_{2T}} \times$$
$$\times F_g(x_1, q_{1T}, \mu_F) F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}), \quad (3)$$

where $\chi_{b\{0,2\}}(nP)$, and

$$d\hat{\sigma}(gg \to \mathcal{C}) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - k) \frac{\overline{|M(gg \to \mathcal{C})|^2}}{2x_1 x_2 s} \frac{d^4k}{(2\pi)^3} \delta_+(k^2 - m_{\mathcal{C}}^2). \quad (4)$$

In a case of $2 \to 2$ subprocess $g(q_1) + g(q_2) \to \mathcal{C}(k) + g(q_3)$, $\mathcal{C} = \Upsilon(nS)$ or $\chi_{b1}(nP)$ in formula (3) $d\hat{\sigma}(gg \to \mathcal{C})$ must be replaced by:

$$d\hat{\sigma}(gg \to \mathcal{C}g) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - k - q_3) \frac{\overline{|M(gg \to \mathcal{C}g)|^2}}{2x_1 x_2 s} \frac{d^3k}{(2\pi)^3 2k_0} \frac{d^4q_3}{(2\pi)^3} \delta_+(q_3^2). \quad (5)$$

Four-momenta of initial partons are on mass-shell ($q_1^2 = q_2^2 = 0$) and have longitudinal (along the Z-axis) and transverse parts:

$$q_1^\mu = \left(x_1 \frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{1T}^2}{2\sqrt{s}x_1}, \mathbf{q}_{1T}, x_1 \frac{\sqrt{s}}{2} - \frac{\mathbf{q}_{1T}^2}{2\sqrt{s}x_1} \right)^\mu, \quad (6)$$

$$q_2^\mu = \left(x_2 \frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{2T}^2}{2\sqrt{s}x_2}, \mathbf{q}_{2T}, -x_2 \frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{2T}^2}{2\sqrt{s}x_2} \right)^\mu. \quad (7)$$

NRQCD subprocesses

Basics of NRQCD factorization

The NRQCD framework [G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995)] describes heavy quarkonia in terms of Fock state decompositions. In case of orthoquarkonium state the wave function can be written as power series expansion in the velocity parameter $v \sim 1/\ln M_Q$:

$$|\mathcal{H}\rangle = \mathcal{O}(v^0)|Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v)|Q\bar{Q}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)|Q\bar{Q}[^1S_0^{(8)}]g\rangle \quad (8)$$

$$+\mathcal{O}(v^2)|Q\bar{Q}[^3S_1^{(1,8)}]gg\rangle + \ldots \quad (9)$$

In the NRQCD effects of short and long distances are separated, and then the cross-section of heavy-quarkonium production via a partonic subprocess $a + b \to \mathcal{H} + X$ can be presented in a factorized form:

$$d\hat{\sigma}(a + b \to \mathcal{H} + X) = \sum_n d\hat{\sigma}(a + b \to Q\bar{Q}[n] + X) \times \langle \mathcal{O}^{\mathcal{H}}[n] \rangle, \quad (10)$$

where n denotes the set of quantum numbers of the $Q\bar{Q}$ pair, and its nonperturbative transitions into $\mathcal{H}$ is described by the NMEs $\langle \mathcal{O}^{\mathcal{H}}[n] \rangle$.
In the general case, the partonic cross-section of quarkonium production from the $Q\bar{Q}$ Fock state $n =^{2S+1} L_J^{(1,8)}$ has the form:

$$d\hat{\sigma}(a + b \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}] \to \mathcal{H}) = d\hat{\sigma}(a + b \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]) \times \frac{\langle \mathcal{O}^{\mathcal{H}}[^{2S+1}L_J^{(1,8)}] \rangle}{N_{col} N_{pol}},$$

where $N_{col} = 2N_c$ for color-singlet state, $N_{col} = N_c^2 - 1$ for color-octet state, and $N_{pol} = 2J + 1$.

Amplitude of specified state

The definition of partonic cross-section of $Q\bar{Q}[^{2S+1}L_J^{(1,8)}]$ production is following:

$$d\hat{\sigma}(a+b \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]) = \frac{1}{2x_1x_2S}\overline{|\mathcal{A}(a+b \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}])|^2}d\Phi. \qquad (11)$$

The projectors on spin-zero and spin-one states:

$$\Pi_0 = \frac{1}{8m^3}\left(\frac{\hat{p}}{2} - \hat{q} - m\right)\gamma^5\left(\frac{\hat{p}}{2} + \hat{q} + m\right), \Pi_1^\alpha = \frac{1}{8m^3}\left(\frac{\hat{p}}{2} - \hat{q} - m\right)\gamma^\alpha\left(\frac{\hat{p}}{2} + \hat{q} + m\right).$$

The projectors on color-singlet and color-octet states:

$$C_1 = \frac{\delta_{ij}}{\sqrt{N_c}},\ C_8 = \sqrt{2}T^a_{ij},$$

respectively, where T^a with $a = 1, \cdots, N_c^2 - 1$ are the generators of the color gauge group $SU(N_c)$.

For example, the amplitude of $Q\bar{Q}$ production in state $^3S_1^{(1,8)}$ is:

$$\mathcal{A}(a+b \to Q\bar{Q}[^3S_1^{(1,8)}]) = Tr[C_{1,8}\Pi_1^\alpha \times \mathcal{A}(a+b \to Q\bar{Q})\varepsilon_\alpha(p)]_{q=0},$$

where $\varepsilon_\alpha(p)$ is the polarization 4-vector of a spin-one particle with momentum p^μ and mass $M = p^2$. And the polarization sum then is:

$$\sum_{J_z}\varepsilon_\alpha(p)\varepsilon^*_{\alpha'}(p) = \mathcal{P}_{\alpha\alpha'}(p) = -g_{\alpha\alpha'} + \frac{p_\alpha p'_\alpha}{M^2}.$$

Long- and short-distance matrix elements

We adopt the following values of color-singlet LDMEs [E.J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995)]: $\langle \mathcal{O}^{\Upsilon(1S)}[^3S_1^{(1)}]\rangle = 9.28$ GeV^3, $\langle \mathcal{O}^{\Upsilon(2S)}[^3S_1^{(1)}]\rangle = 4.63$ GeV^3, $\langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(1)}]\rangle = 3.54$ GeV^3, $\langle \mathcal{O}^{\chi_{bJ}(1P)}[^3P_J^{(1)}]\rangle = (2J+1)\times 2.03$ GeV^5 and $\langle \mathcal{O}^{\chi_{bJ}(2P)}[^3P_J^{(1)}]\rangle = (2J+1)\times 2.36$ GeV^5.
Squared LO in α_S amplitudes for $2\to 1$ subprocesses in CPM are well-known [P.L. Cho, A.K. Leibovich (1996)]:

$$\overline{|\mathcal{A}(g+g\to \mathcal{C}[^3P_0^{(1)}]|^2} = \frac{8}{3}\pi^2\alpha_s^2\frac{\langle \mathcal{O}^{\mathcal{C}}[^3P_0^{(1)}]\rangle}{M^3}, \quad (12)$$

$$\overline{|\mathcal{A}(g+g\to \mathcal{C}[^3P_1^{(1)}]|^2} = 0, \quad (13)$$

$$\overline{|\mathcal{A}(g+g\to \mathcal{C}[^3P_2^{(1)}]|^2} = \frac{32}{45}\pi^2\alpha_s^2\frac{\langle \mathcal{O}^{\mathcal{C}}[^3P_2^{(1)}]\rangle}{M^3}, \quad (14)$$

Long- and short-distance matrix elements

There is only one relevant LO in α_S $2 \to 2$ partonic subprocess, which describes direct production of $\Upsilon(nS)$ via color-singlet intermediate state $[^3S_1^{(1)}]$ as it is in the Color-Singlet Model.

The squared amplitude for this partonic subprocess reads [R. Gastmans, W. Troost and T. T. Wu, Phys. Lett. B **184**, 257-260 (1987)]:

$$\overline{|\mathcal{A}(g+g \to \mathcal{C}[^3S_1^{(1)}]+g|^2} = \pi^3\alpha_s^3 \frac{\langle \mathcal{O}^{\mathcal{C}}[^3S_1^{(1)}]\rangle}{M^3} \frac{320M^4}{81(M^2-\hat{t})^2(M^2-\hat{u})^2(\hat{t}+\hat{u})^2} \times$$
$$\times (M^4\hat{t}^2 - 2M^2\hat{t}^3 + \hat{t}^4 + M^4\hat{t}\hat{u} - 3M^2\hat{t}^2\hat{u} + 2\hat{t}^3\hat{u} + M^4\hat{u}^2 -$$
$$- 3M^2\hat{t}\hat{u}^2 + 3\hat{t}^2\hat{u}^2 - 2M^2\hat{u}^3 + 2\hat{t}\hat{u}^3 + \hat{u}^4). \quad (15)$$

Estimation of bottomonium production cross section at NICA

Predictions for J/ψ (short reminder), $|y| \leq 3$, $\sqrt{S} = 24$ GeV.

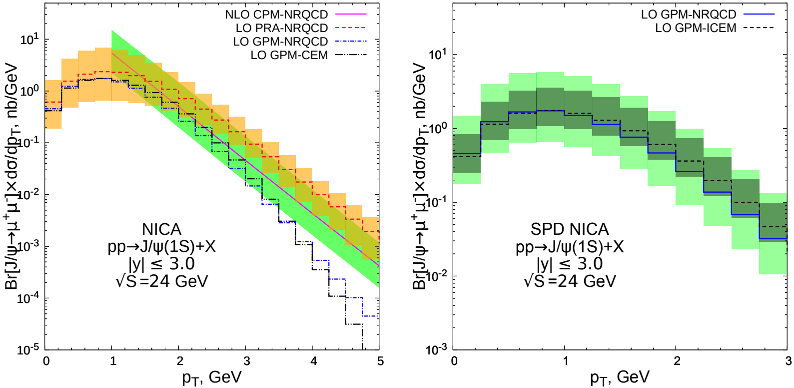

Figure 1 : Prompt J/ψ transverse momentum distribution at $\sqrt{s} = 24$ GeV, $|y| \leq 3$. Left panel: GPM results with $\langle q_T^2 \rangle = 1$ GeV^2 are shown by dash-dotted (NRQCD) and dash-double-dotted (ICEM) histograms. Solid and dashed histograms with uncertainty bands are PRA [A.V. Karpishkov, M.A. Nefedov and V.A. Saleev, Phys. Rev. D **104**, 016008 (2021)] and NLO CPM [M. Butenschön and B.A. Kniehl, private communication] predictions respectively. Right panel: GPM predictions in NRQCD (solid histogram with light green uncertainty band) and ICEM (dashed histogram with dark-green uncertainty band) approaches with their uncertainty bands shown.

Predictions for $\Upsilon(1S)$, $|y| \leq 3$, $\sqrt{S} = 27$ GeV.

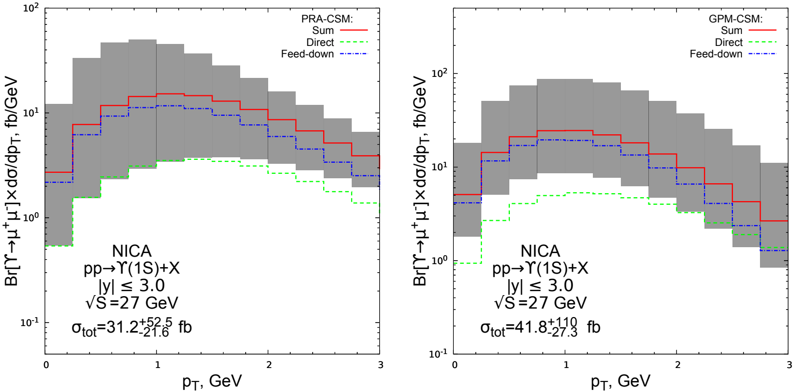

Figure 2 : Prompt $\Upsilon(1S)$ transverse momentum distributions at $\sqrt{s} = 27$ GeV, $|y| \leq 3$. Left panel: PRA results. Right panel: GPM predictions with $\langle q_T^2 \rangle = 1$ GeV^2.

Predictions for $\Upsilon(1S)$, $|y| \leq 3$, $\sqrt{S} = 27$ GeV.

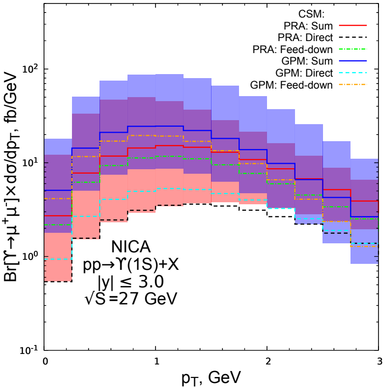

Figure 3 : A comparison between transverse momentum distributions for prompt $\Upsilon(1S)$ production within PRA and GPM at $\sqrt{s} = 27$ GeV, $|y| \leq 3$.

Thank you for your attention!