Grodzins relation and some predictions for superheavy nuclei

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L.Grodzins, Phys. Lett. **2** (1962) 88

Grodzins Relation

$$E(2_1^+) \times B(E2; 0_1^+ \to 2_1^+) = \text{constant } Z^2 A^{-2/3}$$

- where $E(2_1^+)$ is given in keV and $B(E2; 0_1^+ \to 2_1^+)$ in $e^2 b^2$

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S.Raman *et al*., At. Data Nucl. Data Tables **78** (2001) 1

2.57(45)

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R.V.Jolos *et al*., Phys.Rev.C **71** (2005) 044305; J.B.Gupta, Phys.Rev.C **89** (2014) 034321; J.B.Gupta and V.Katoch, Int. J. Mod. Phys. E **28** (2019),1950055

L.Grodzins, Phys. Lett. **2** (1962) 88
R.V.Jolos, EAK, Phys. Lett. B 820 (2021) 136581

Grodzins Relation

The collective quadrupole Bohr Hamiltonian

$$H = -\frac{\hbar^2}{2}\sum_{\mu,\mu'}\frac{\partial}{\partial\alpha_{2\mu}}(B^{-1})_{\mu\mu'}\frac{\partial}{\partial\alpha_{2\mu'}} + V(\alpha_{2\mu}),$$

where the inverted inertia tensor $(B^{-1})_{\mu\mu'} = \sqrt{5}\sum_{LM} C^{LM}_{2\mu 2\mu'}(B^{-1})_{LM}$

using the ground state average of the double commutator

$$\left[[H, Q_{2\mu}], Q_{2\mu'}\right] = -\hbar^2 q^2 \sqrt{5}\sum_{LM} C^{LM}_{2\mu 2\mu'}(B^{-1})_{LM}$$

with $q = \frac{3}{4\pi} eZr_0^2 A^{2/3}$

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R.V.Jolos, EAK, Phys. Lett. B 820 (2021) 136581
R.V.Jolos, P.von Brentano, Phys. Rev. C **76** (2007) 024309

Grodzins Relation

The collective quadrupole Bohr Hamiltonian

$$E(2_1^+) \times B(E2; 0_1^+ \to 2_1^+) = \hbar^2 q^2 \langle 0_1^+ | (B^{-1})_{00} | 0_1^+ \rangle$$

$$(B^{-1})_{00}^{in} = \frac{2}{5}\frac{1}{B_{rot}} + \frac{2}{5}\frac{1}{B_\gamma} + \frac{1}{5}\frac{1}{B_\beta}$$

B_{rot} , B_γ , B_β - the inertia coefficients for rotational, γ and β motions.

$$B(E2; 0_1^+ \to 2_1^+) = \left(\frac{3}{4\pi} eZR_0^2\right)^2 \beta_2^2$$

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N.V.Shirikova *et al.*, Phys. Rev. C **105** (2022) 024309

Grodzins Relation

As a result

$$E(2_1^+) = \hbar^2 \frac{1}{\beta_2^2}\left(\frac{2}{5}\frac{1}{B_{rot}} + \frac{2}{5}\frac{1}{B_\gamma} + \frac{1}{5}\frac{1}{B_\beta}\right)$$

$$B_{rot} = \alpha \sum_{s,t} \frac{|\langle s|\frac{dV}{dr}\frac{1}{\sqrt{2}}(Y_{21}+Y_{2-1})|t\rangle|^2 \left(\varepsilon_s\varepsilon_t - (E_s-\lambda)(E_t-\lambda) - \Delta_s\Delta_t\right)}{2\varepsilon_s\varepsilon_t(\varepsilon_s+\varepsilon_t)^3},$$

$$B_\gamma = \alpha \sum_{s,t} \frac{|\langle s|\frac{dV}{dr}\frac{1}{\sqrt{2}}(Y_{22}+Y_{2-2})|t\rangle|^2 \left(\varepsilon_s\varepsilon_t - (E_s-\lambda)(E_t-\lambda) + \Delta_s\Delta_t\right)(\varepsilon_s+\varepsilon_t)}{2\varepsilon_s\varepsilon_t((\varepsilon_s+\varepsilon_t)^2-\omega_\gamma^2)^2}$$

$$B_\beta = \alpha \sum_{s,t} \frac{|\langle s|\frac{dV}{dr}Y_{20}|t\rangle|^2 \left(\varepsilon_s\varepsilon_t - (E_s-\lambda)(E_t-\lambda) + \Delta_s\Delta_t\right)(\varepsilon_s+\varepsilon_t)}{2\varepsilon_s\varepsilon_t((\varepsilon_s+\varepsilon_t)^2-\omega_\beta^2)^2},$$

where *s* and *t* are the quantum numbers of the single particle states, $E_{s(t)}$ - the single particle energy, $\varepsilon_{s(t)}$ - the single quasiparticle energy, λ - the Fermi level energy, $\Delta_{s(t)}$ - the pairing gap, ω_β (ω_γ) - the energy of the β (γ) phonon, $Y_{\lambda\mu}$ - the spherical function. $\alpha = 2\hbar^2 r_0^2 A^{2/3}$

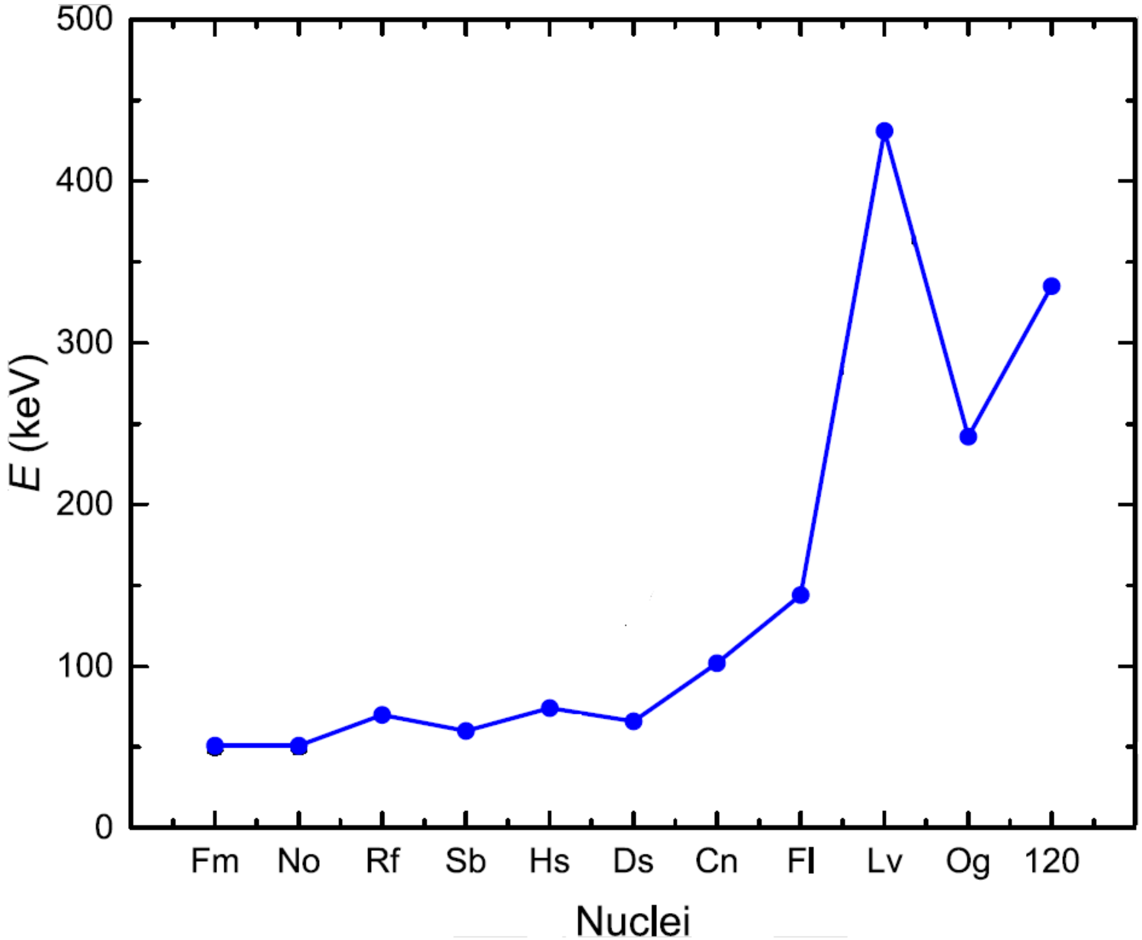
N.V.Shirikova *et al.*, Phys. Rev. C **105** (2022) 024309

Results

Nucleus	β_2	$E(2_1^+)$ (keV)
^{258}Fm	0.274	51
^{262}No	0.256	51
^{266}Rf	0.235	70
^{270}Sb	0.242	60
^{274}Hs	0.237	74
^{278}Ds	0.197	66
^{282}Cn	0.160	102
^{286}Fl	-0.154	144
^{290}Lv	0.078	431
^{294}Og	-0.105	242
^{298}120	-0.092	335

N.V.Shirikova *et al.*, Phys. Rev. C **105** (2022) 024309

Results



R.V.Jolos, P. von Brentano,
Phys. Rev. C **76** (2007) 024309;
Phys. Rev. C **77** (2008) 064317;
Phys. Rev. C **78** (2008) 064309

Model

- B_{rot}: the single particle level scheme and the monopole pairing
- B_{β} and B_{γ}: the residual forces => energies of the γ- and β vibrations, more sensitive to the choice of the theoretical model

Deformed nuclei: $\frac{B_{\gamma}}{B_{rot}} = 4$ and $\frac{B_{\beta}}{B_{rot}} = 12 \rightarrow E(2_1^+) = \frac{0.52\hbar^2}{\beta_2^2} B_{rot}$

Spherical nuclei: $\frac{B_{\gamma}}{B_{rot}} = \frac{B_{\beta}}{B_{rot}} = 1 \rightarrow E(2_1^+) = \frac{\hbar^2}{\beta_2^2} B_{rot}$

$0.16 < \beta_2 < 0.20$ $\qquad$ $E(2_1^+) = \frac{0.76\hbar^2}{\beta_2^2} B_{rot}$

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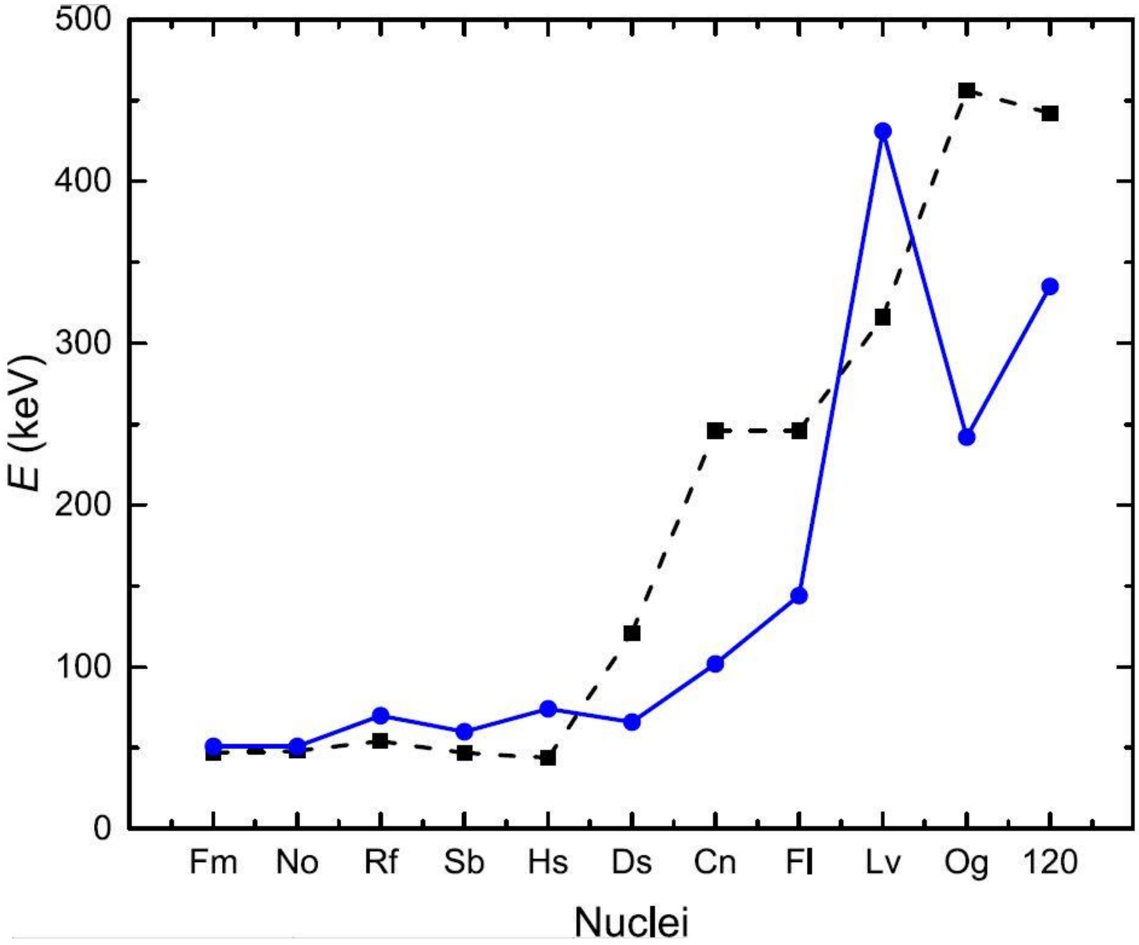
Model

Results

Nucleus	β_2	$E(2_1^+)$ (keV)	$E(2_1^+)$ (keV)
^{258}Fm	0.274	51	47
^{262}No	0.256	51	48
^{266}Rf	0.235	70	54
^{270}Sb	0.242	60	47
^{274}Hs	0.237	74	44
^{278}Ds	0.197	66	121
^{282}Cn	0.160	102	246
^{286}Fl	-0.154	144	246
^{290}Lv	0.078	431	316
^{294}Og	-0.105	242	456
^{298}120	-0.092	335	442

Results



Conclusion

- *The excitation energies of the first 2^+ states of the chain of even-even superheavy nuclei with Z from 100 to120 are predicted*
- *At the beginning of the studied region of nuclei where quadrupole deformation is large the energies of the 2_1^+ states do not exceed 80 keV i.e. correspond to rotational states.*
- *With decrease of deformation $E(2_1^+)$ rises sharply and reaches a maximum value in ^{294}Og or in ^{290}Lv*

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Thank you!