# The study of multiplicity distributions for prompt neutrons emitted in spontaneous fission of transfermium isotopes

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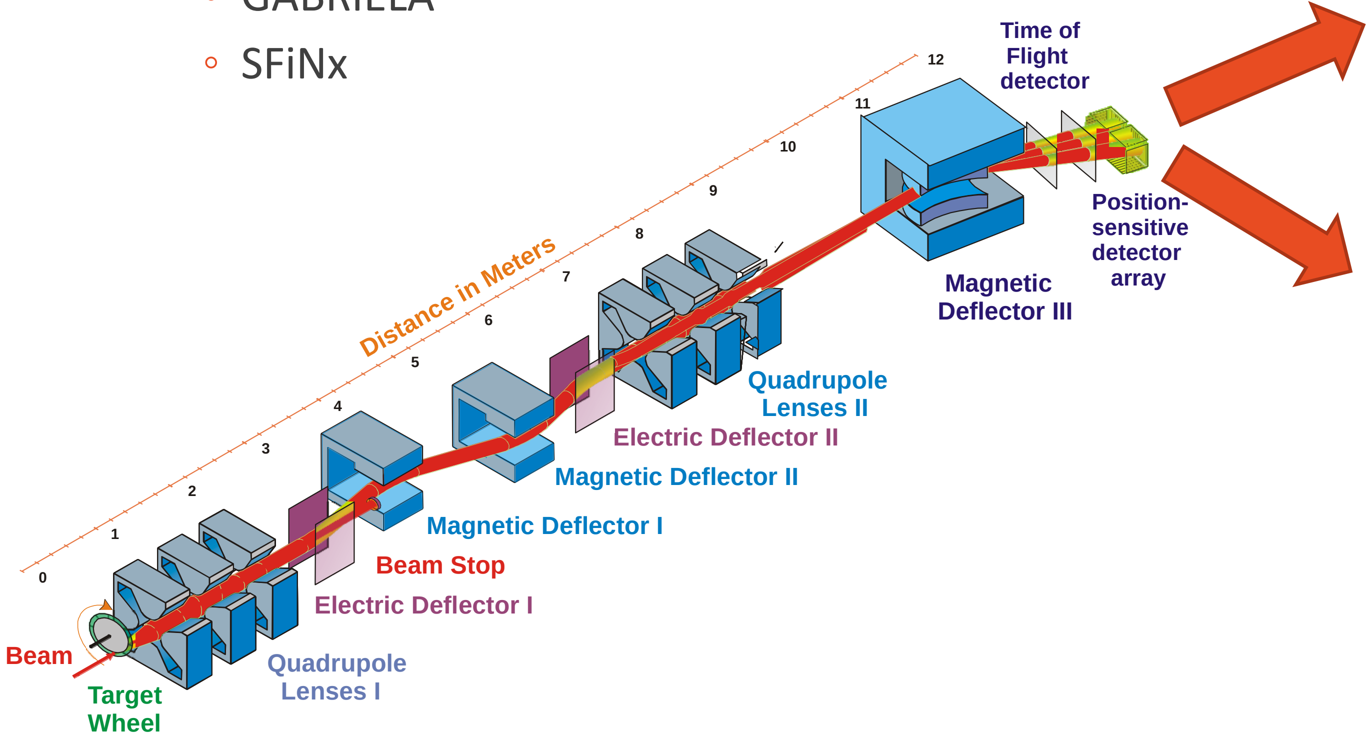
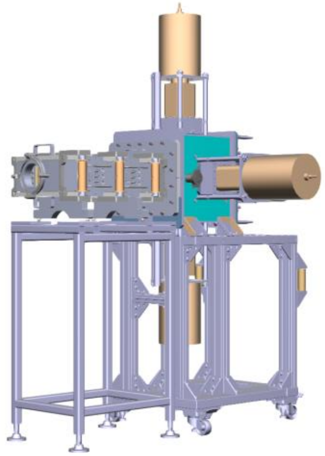
# Experimental setup

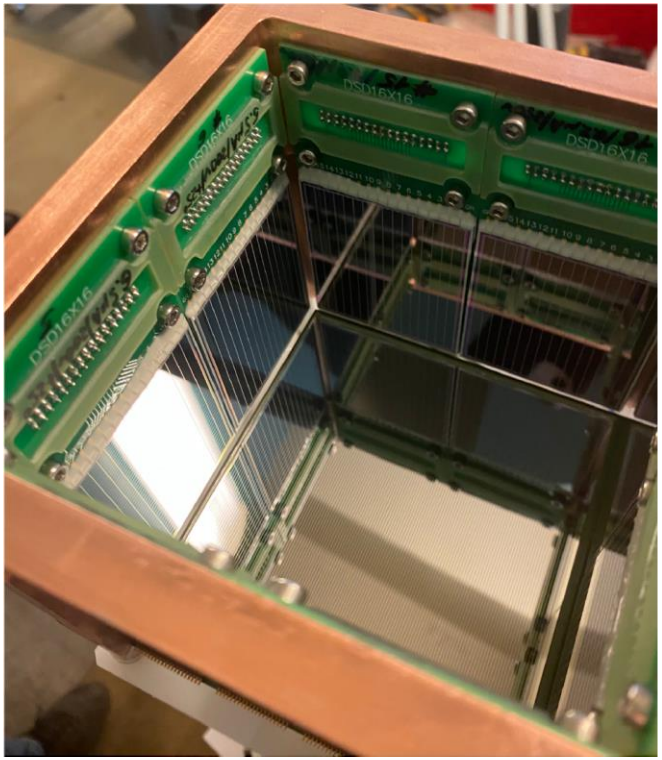
- Velocity filter SHELS
- Detecting systems:
  - GABRIELA
  - SFiNx



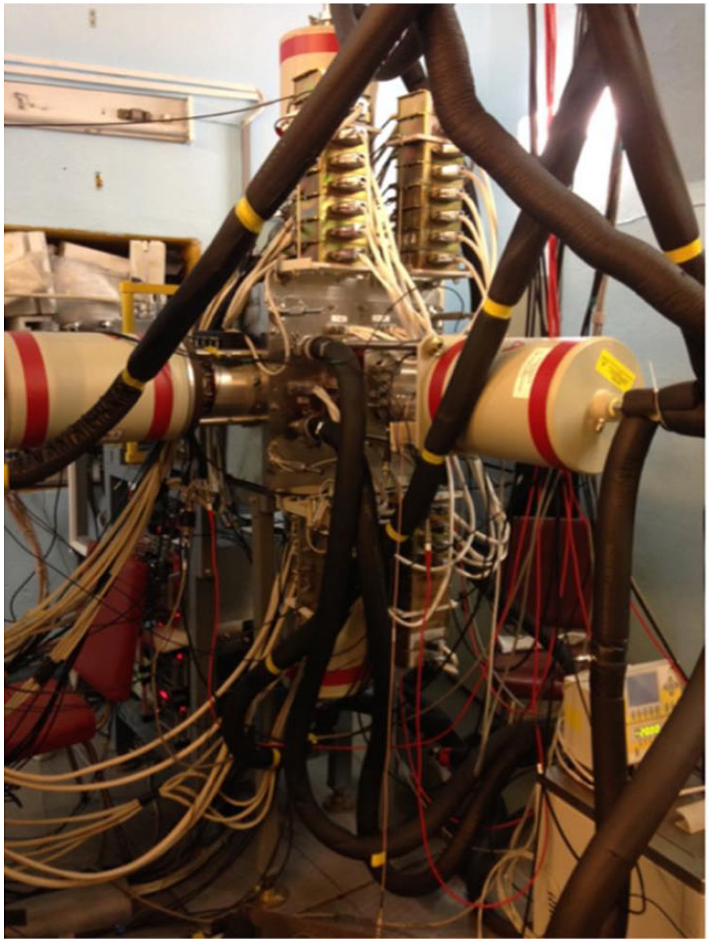
# DSSD particle detector

- To detect particles emitted in decays or reactions of unstable nuclei:
  - Evaporation residue in complete fusion reactions
  - Alphas
  - Betas and conversion electrons
  - Protons
  - Fission fragments
- What is measured:
  - Energy
  - Emission time
  - Emission coordinates



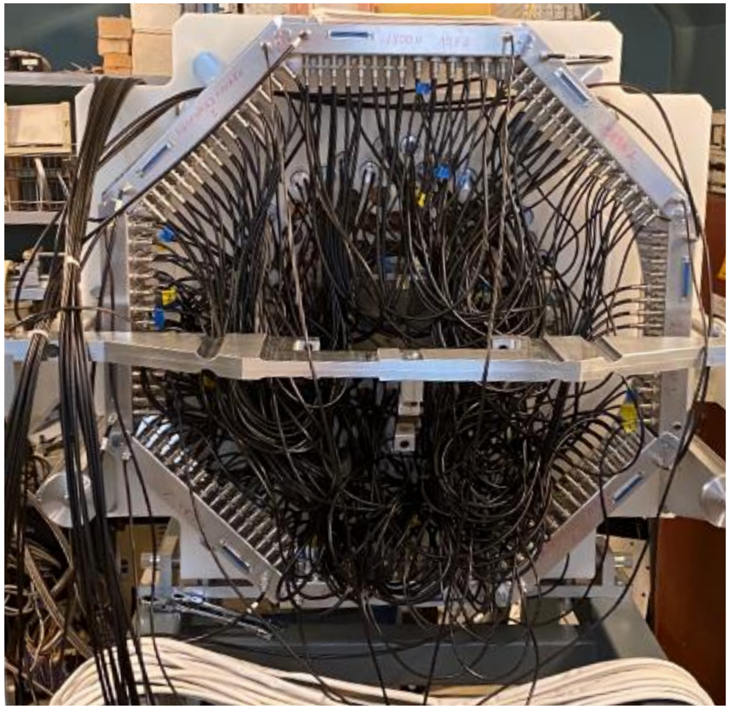
# High-pure Ge detectors array

- To detect:
  - Gamma-rays emitted at decays
- What is measured:
  - Energy with good resolution
  - Emission time
- Anti-Compton protection:
  - BGO detector around each Germanium



# Neutrons $^3$He detector array

- To detect:
  - Neutrons
  - Prompt gamma-ray of spontaneous fission
- What is measured
  - Neutrons multiplicity
  - Two coordinates of counter that registered the neutron
- Spontaneous fission identification:
  - BGO detector to detect prompt gamma rays of spontaneous fission



# Neutrons multiplicity as an information about fission process

- Average number of emitted neutrons helps to restore energy balance equation of the spontaneous fission process
- Variance of the emitted neutrons multiplicity distribution can be used to estimate variance of fragments excitation energy
- Neutron multiplicity distribution shape is able to point at possible exotic modes of spontaneous fission.

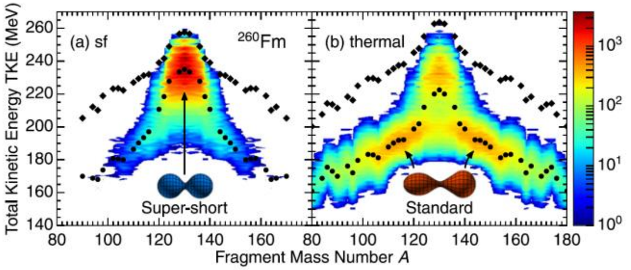



FIG. 2. Contour plots (on a logarithmic scale) of the number of fission events for $^{260}$Fm in the $A$-TKE plane based on $10^5$ spontaneous or thermal fission events. Also shown are $Q^*_{LH}$ (filled diamonds) and $\overline{\mathrm{TKE}}(A)$ (filled circles). Typical scission shapes are shown for the compact, symmetric super-short and the elongated, asymmetric standard modes.

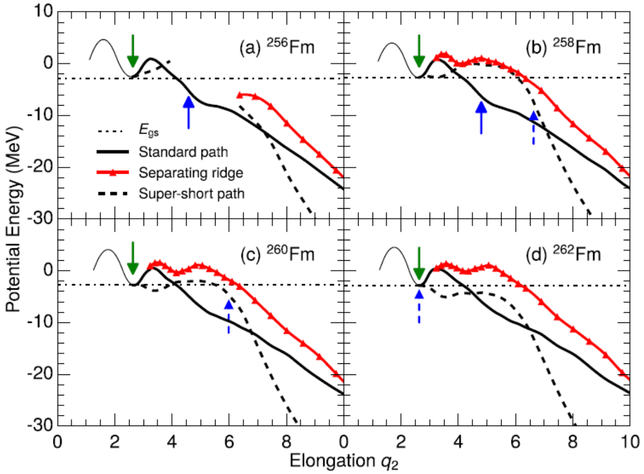



FIG. 1. The potential energy along the paths for the standard (solid) and the super-short (dashed) fission modes, as function of the overall quadrupole moment $q_2$ for $^{256}$Fm (a), $^{258}$Fm (b), $^{260}$Fm (c), and $^{262}$Fm (d); the two paths are separated by a ridge (red line). The ground-state energy is shown by the horizontal line. The shape evolutions start at the isomeric minima (down-pointing arrows). For spontaneous fission, the extra energy $\Delta E$ is reset typically at the up-pointing arrows.

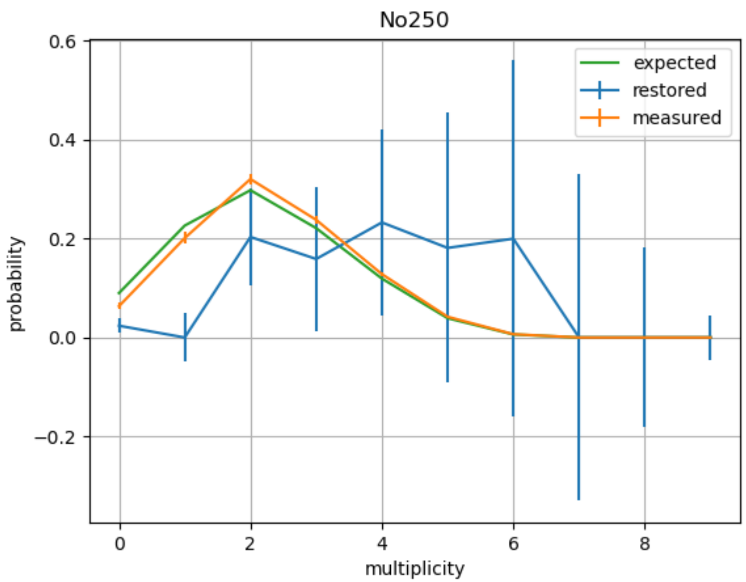

1. M. ALBERTSSON et al., PHYSICAL REVIEW C 104, 064616 (2021)

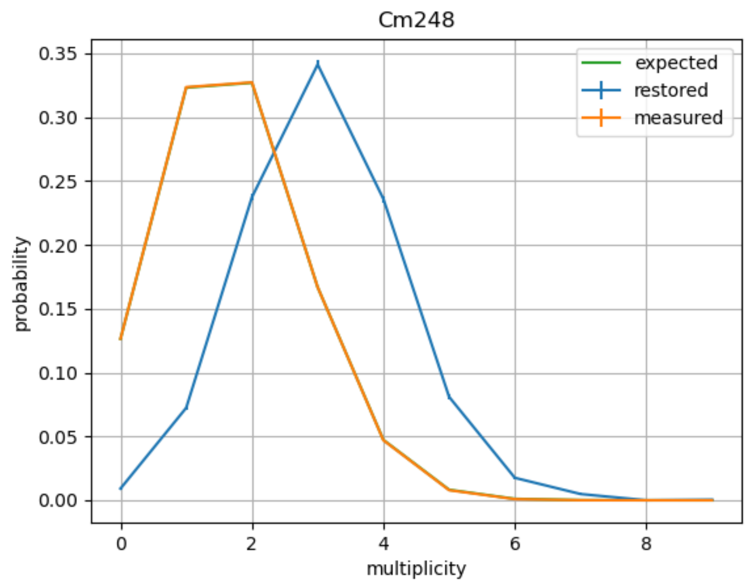
# Restoring technique

- Due to detectors efficiency is far from 100% the measured distribution is heavily distorted.

- To restore the “real” distribution we use Tikhonov regularization method.
  - $n$ – maximum multiplicity
  - $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{1},\boldsymbol{n}}$ – real prompt neutrons multiplicity distribution vector
  - $K \in \mathbb{R}^{\boldsymbol{n}\,\boldsymbol{n}}$ – detector response matrix
    - $\varepsilon$ – single neutron registration efficiency
    - $K_{i,j} = \frac{j!}{i!(j-i)!}\varepsilon^{i}(1-\varepsilon)^{j-i}$
  - $\boldsymbol{y} \in \mathbb{R}^{\boldsymbol{1},\boldsymbol{n}}$ – measured prompt neutrons multiplicity distribution vector
    - $\boldsymbol{y} = K\boldsymbol{x}$
  - $\boldsymbol{x_d} = (K^T K)^{-1} K\boldsymbol{y}$ – direct solution most often does not make sense

# Direct solution examples




1357 s.f. events

845224 s.f. events

Single neutron registration efficiency ≈ 56%

# Regularization method

- The essence of the method is to use a priori information to find an approximate solution. [2]

- Using a prior information:
  - Distribution is smooth
  - Distribution is flat at tails

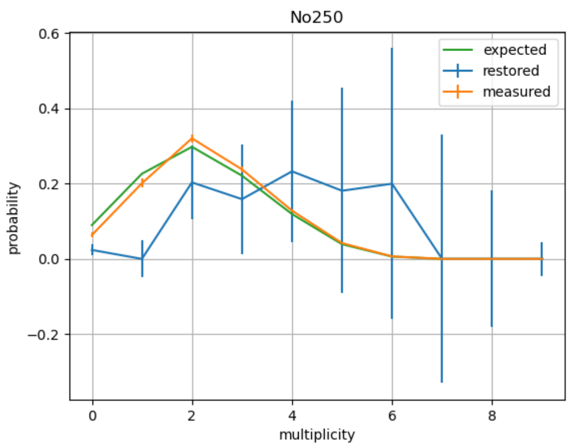
- Regularization matrix:

$$R = \begin{pmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & -3 & 2 \end{pmatrix} \in \mathbb{R}^{(n-1)\times n}$$

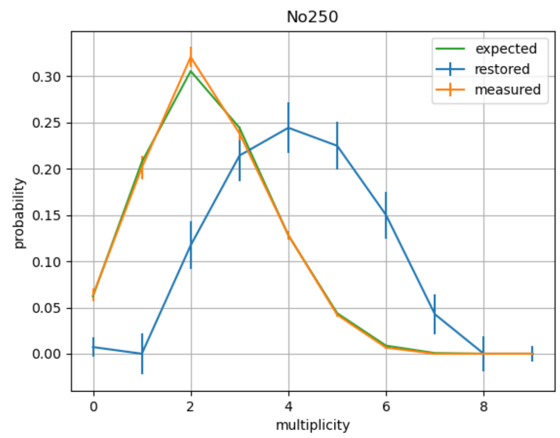
- Regularized solution: $x_r = (L^T L + \alpha s^2 R^T R)^{-1} L^T g$
  - $L \sim K; g \sim y$
  - $s = \sqrt[n]{\prod_0^n s_i}$ ; where $s_i$ are uncertainties of the measured distribution
  - $\alpha$ – regularization parameter

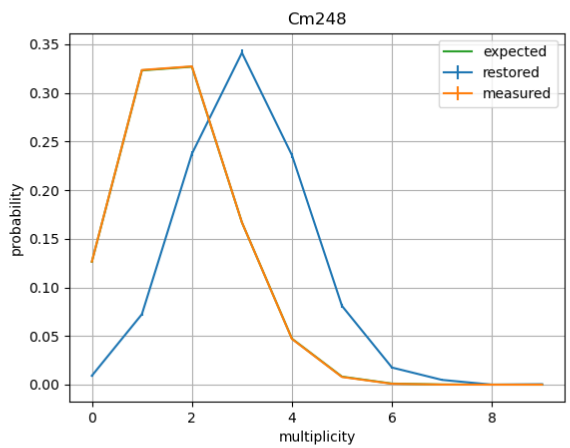
2. R. Mukhin, et al., Physics of Particles and Nuclei Letters, 2021, V 18, 4, p 439

# Restoring examples

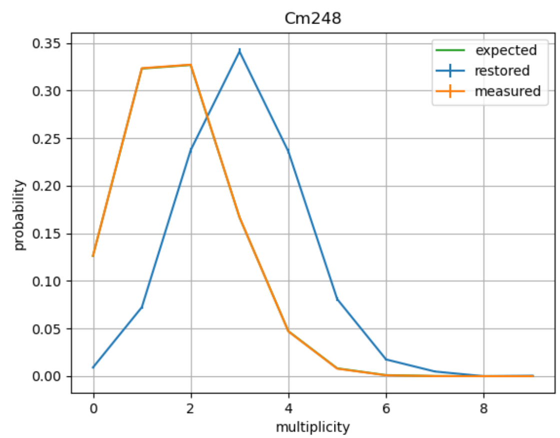



$\alpha = 89$




$\alpha = 89$



# Models used

## ISP [3]

- Improved scission point model
- Pros:
  - No adjustable or fitted parameters are used in the calculation of PES
- Cons:
  - Assumption of statistical equilibrium at the scission point

3. A.V. Andreev, et al., EPJa, 2006, V. 30, 3, pp 579-589

## GEF [4]

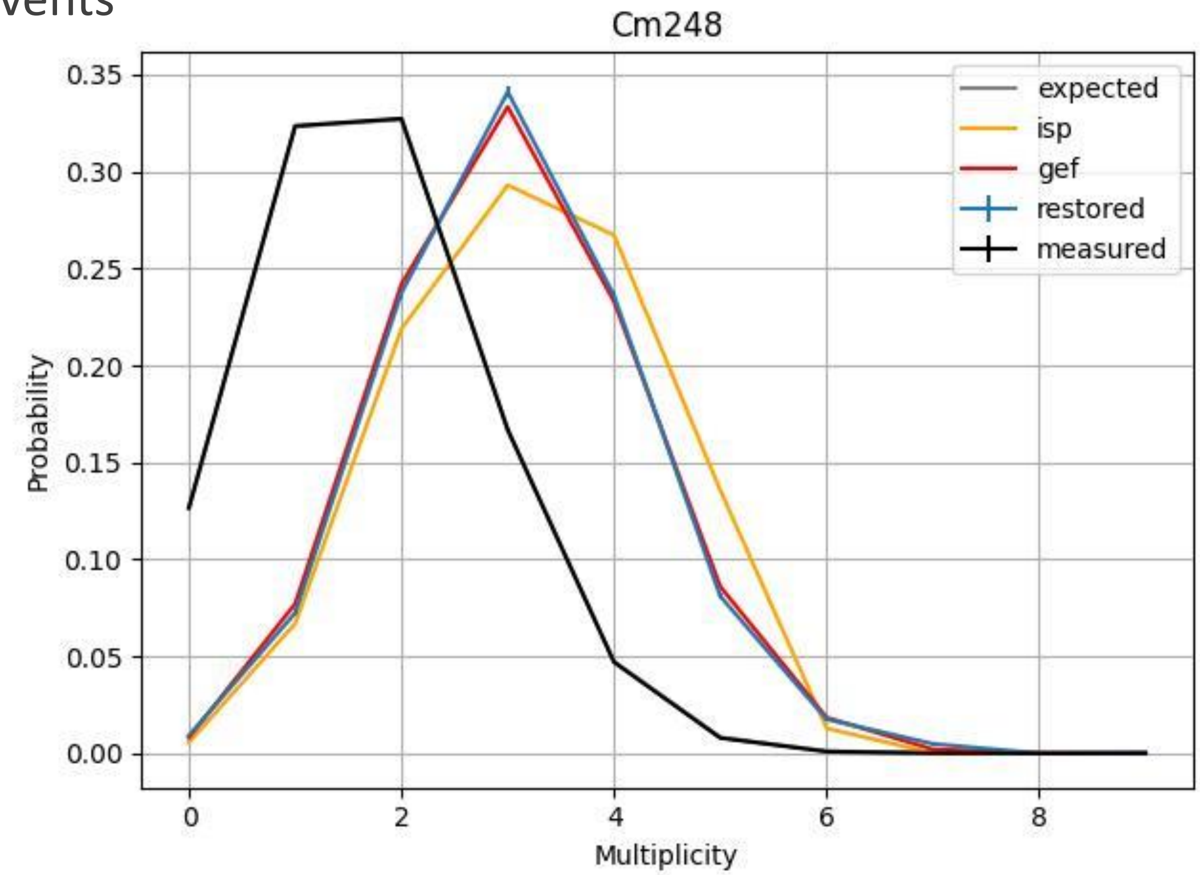
- The General fission model
- Pros:
  - Dynamics of motion along the PES is taken into account explicitly
- Cons:
  - PES is adjusted to fit experimental data

4. K.-H. Schmidt, et al., General description of fission, GEF model, © OECD 2014

*PES - potential energy surface

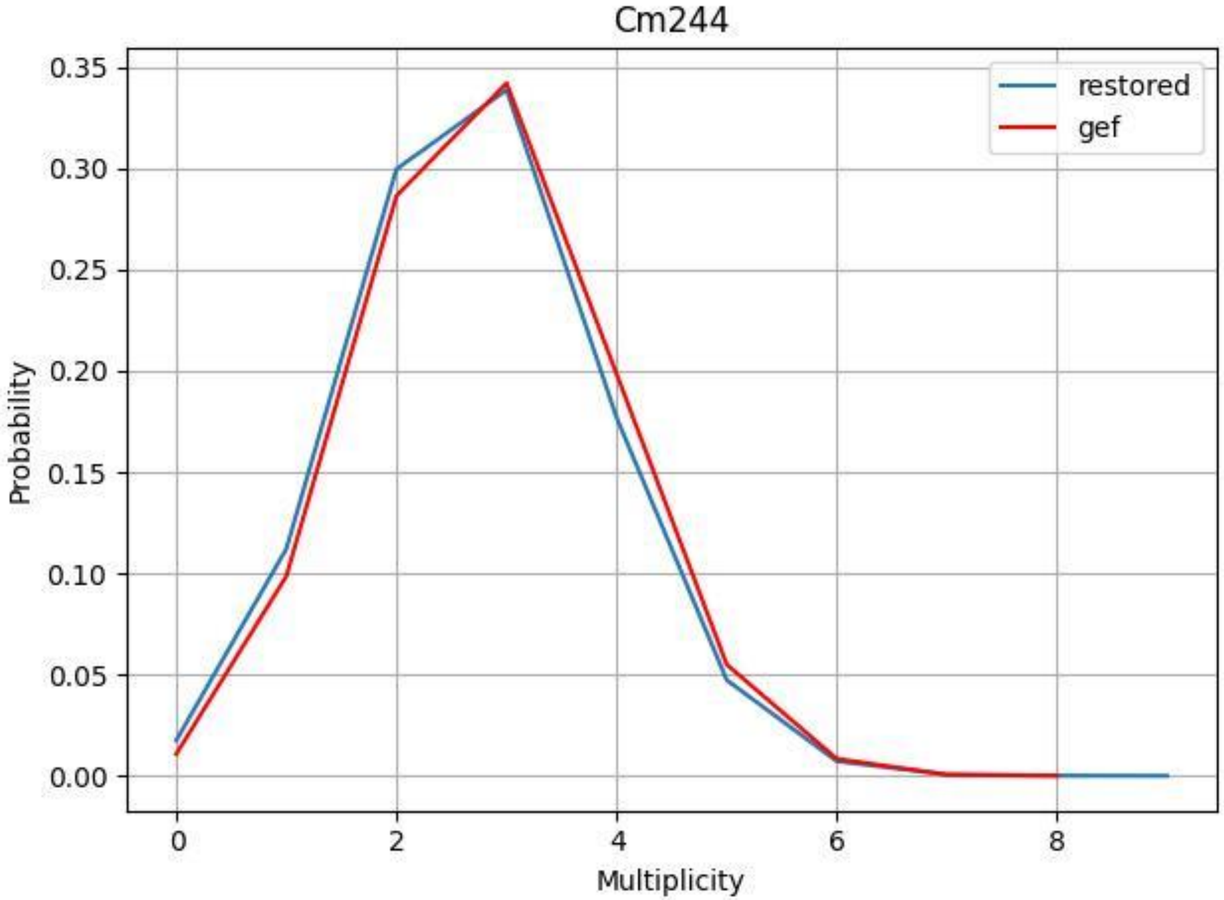
# Symmetric distributions

- Statistics: $> 10^6$ s.f. events
- A = 248
- Z = 96
- N = 152
- $\bar{\nu} = 3.133 \pm 0.009$
- $\sigma_\nu^2 = 1.34$



# Symmetric distributions

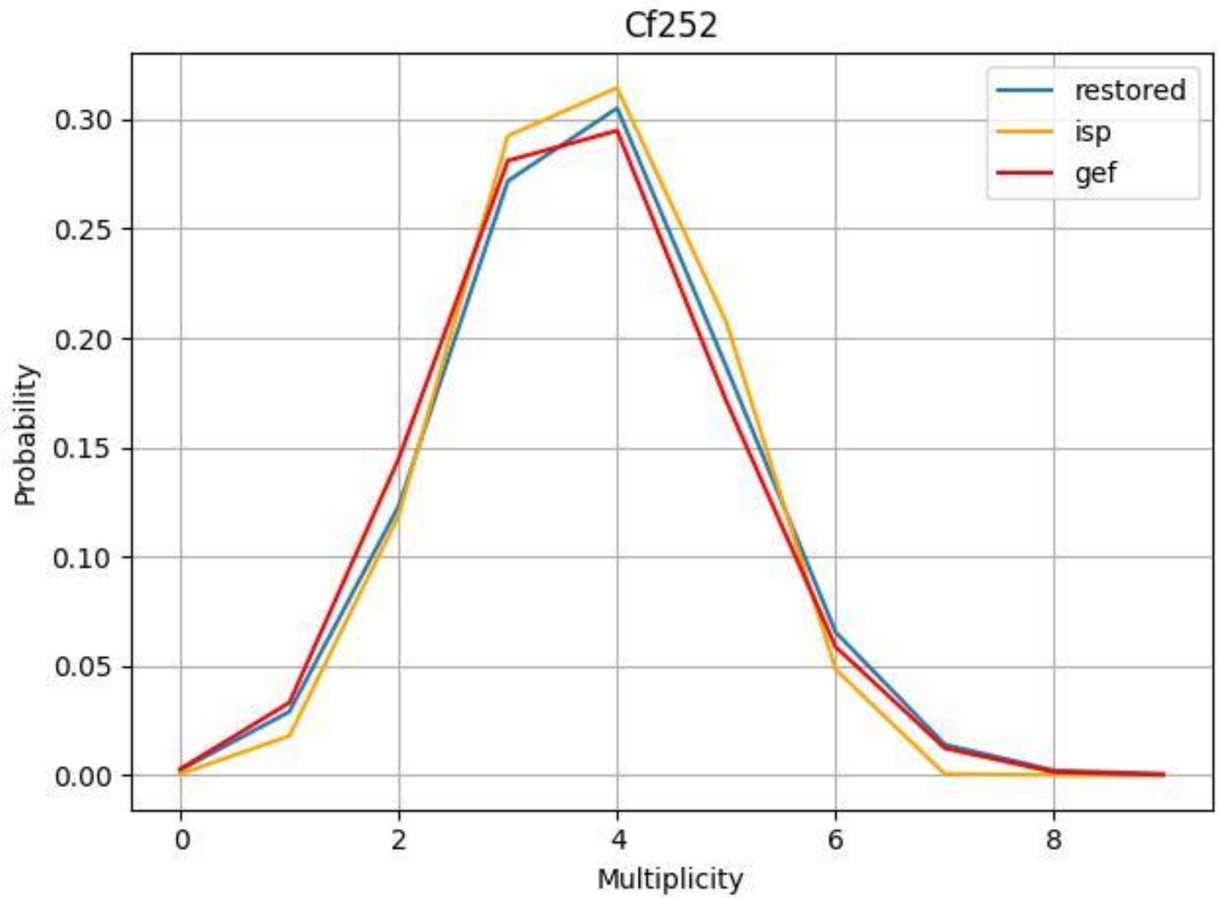
- A = 244
- Z = 96
- N = 148
- $\bar{\nu} = 2.72 \pm 0.06$ [5]
- $\sigma_\nu^2 = 1.27$




5. A. S. Vorobyev, et al., AIP Conference Proceedings 769, 613 (2005)

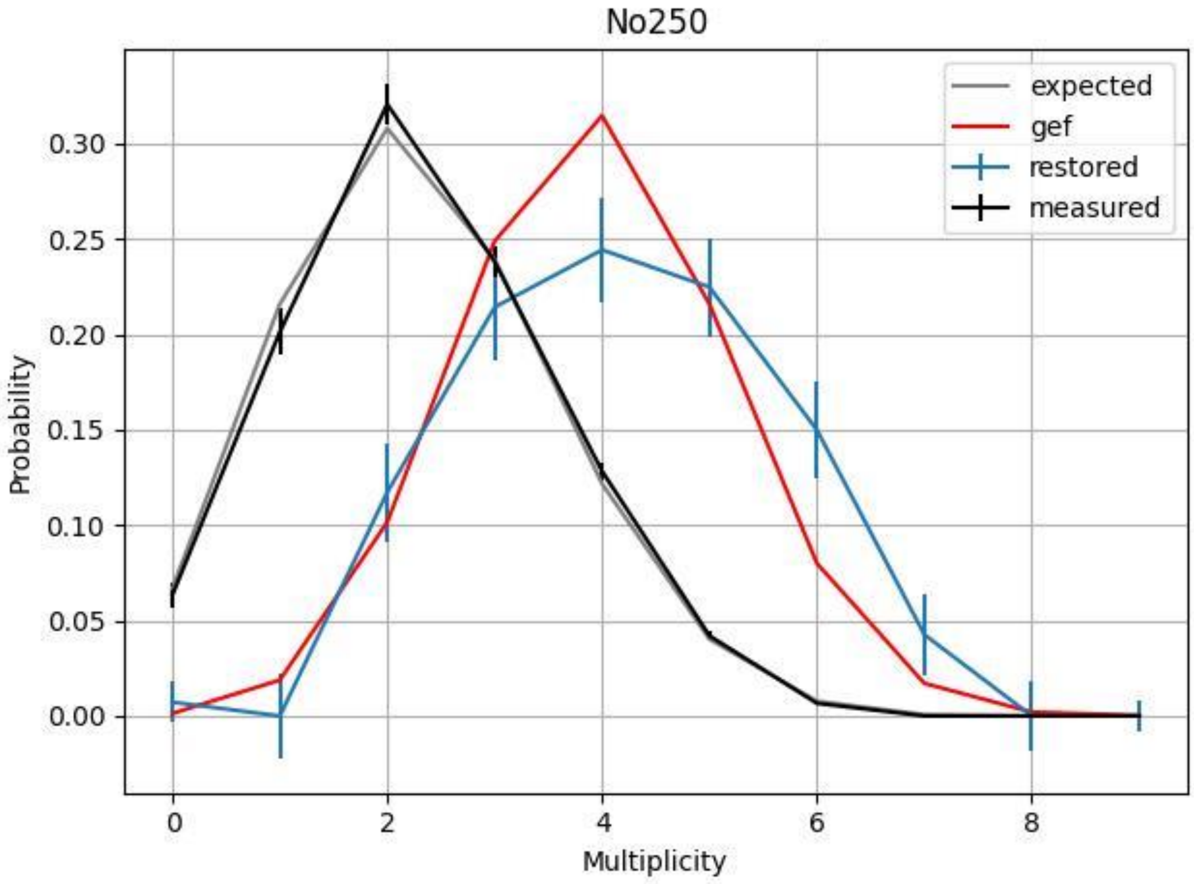
# Symmetric distributions

- A = 252
- Z = 98
- N = 154
- $\bar{\nu} = 3.76 \pm 0.03$ [5]
- $\sigma_\nu^2 = 1.62$



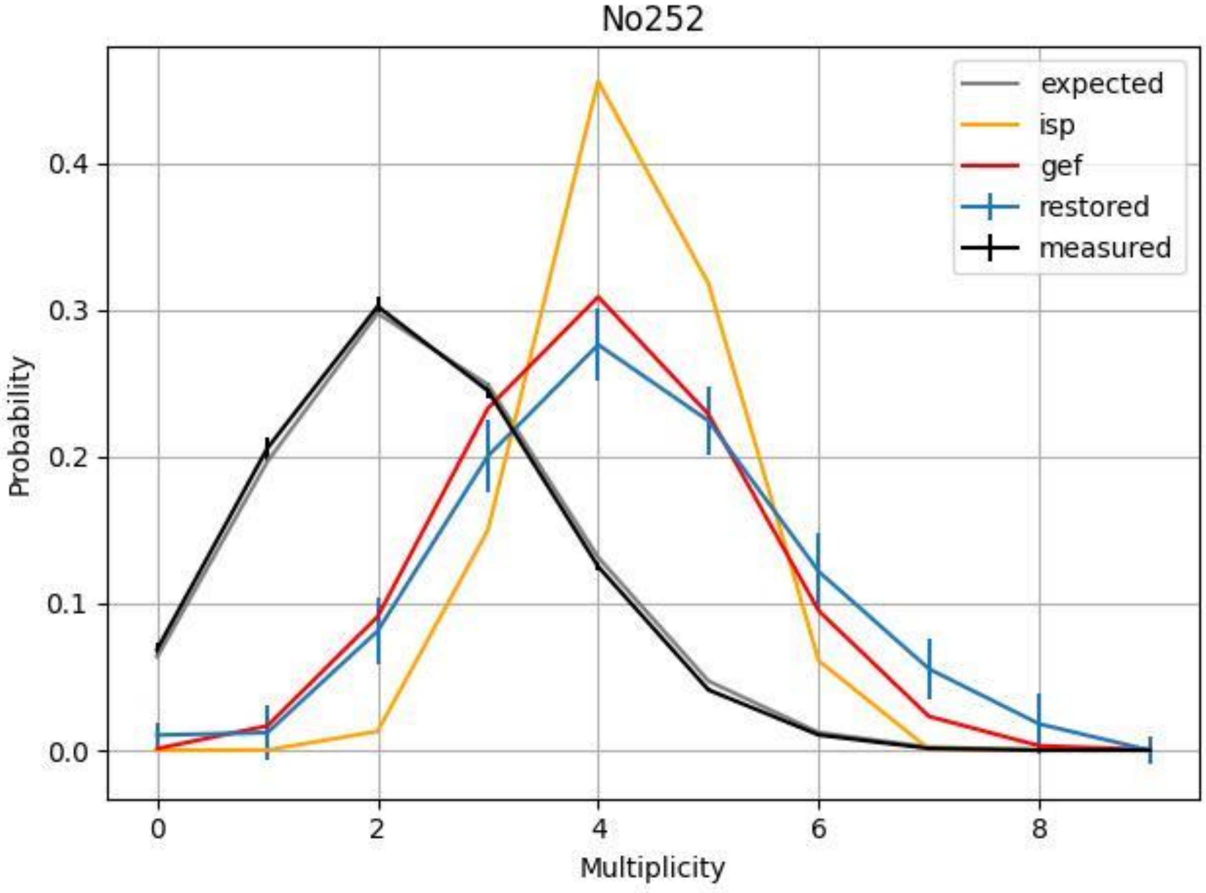
# Symmetric distributions

- 1357 s.f. events
- A = 250
- Z = 102
- N = 148
- $\bar{\nu} = 4.17 \pm 0.16$
- $\sigma_\nu^2 = 2$



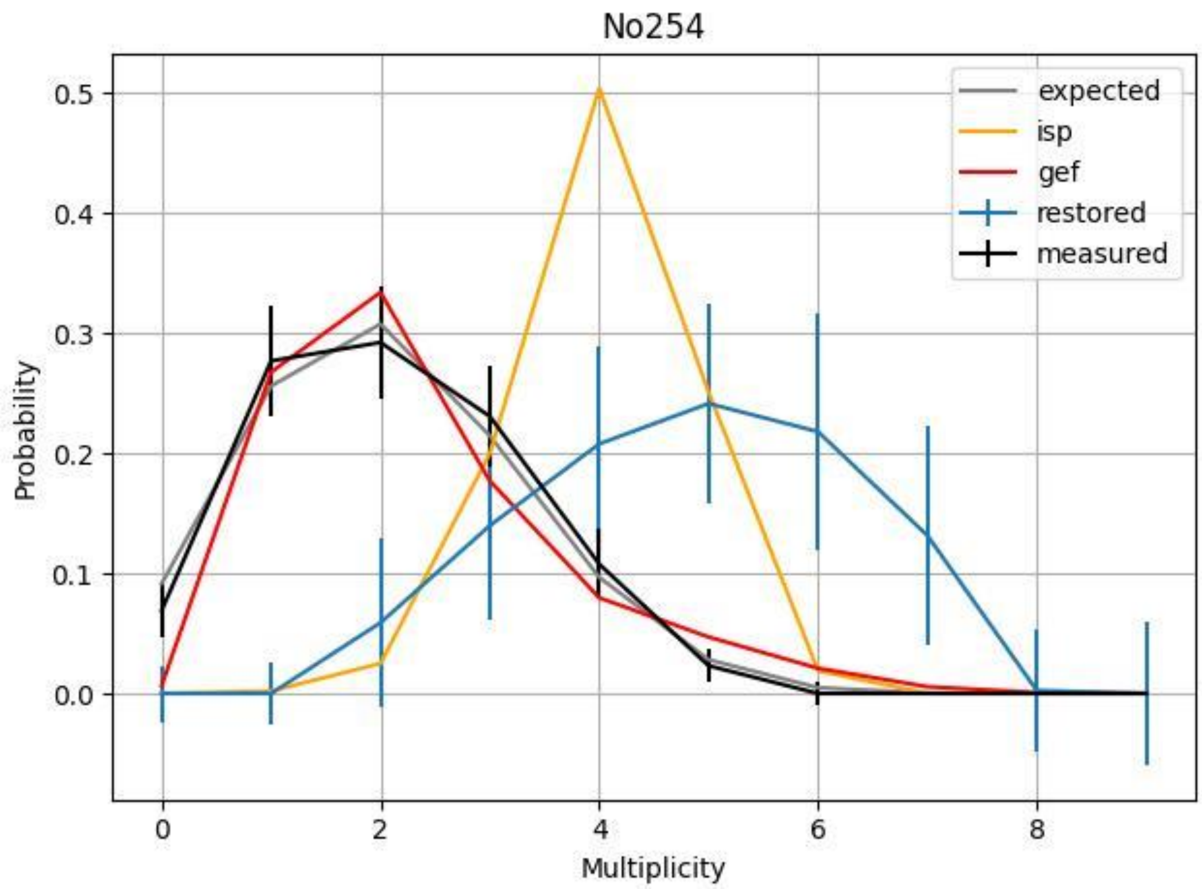
# Symmetric distributions

- 3260 s.f. events
- A = 252
- Z = 102
- N = 150
- $\bar{\nu} = 4.25 \pm 0.09$
- $\sigma_\nu^2$ =2.1



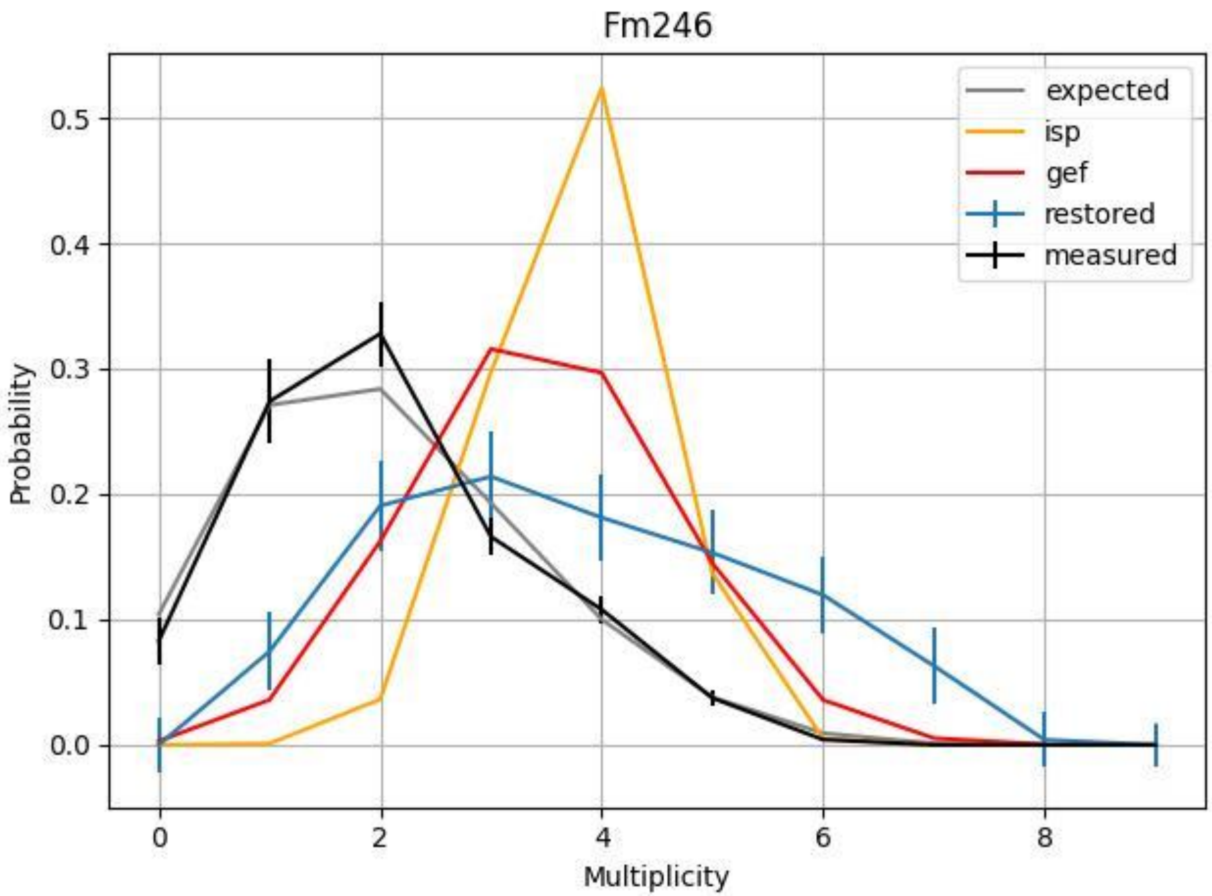
# Symmetric distributions

- 174 s.f. events
- A = 254
- Z = 102
- N = 152
- $\bar{\nu} = 4.88 \pm 0.53$
- $\sigma_\nu^2 = 2$



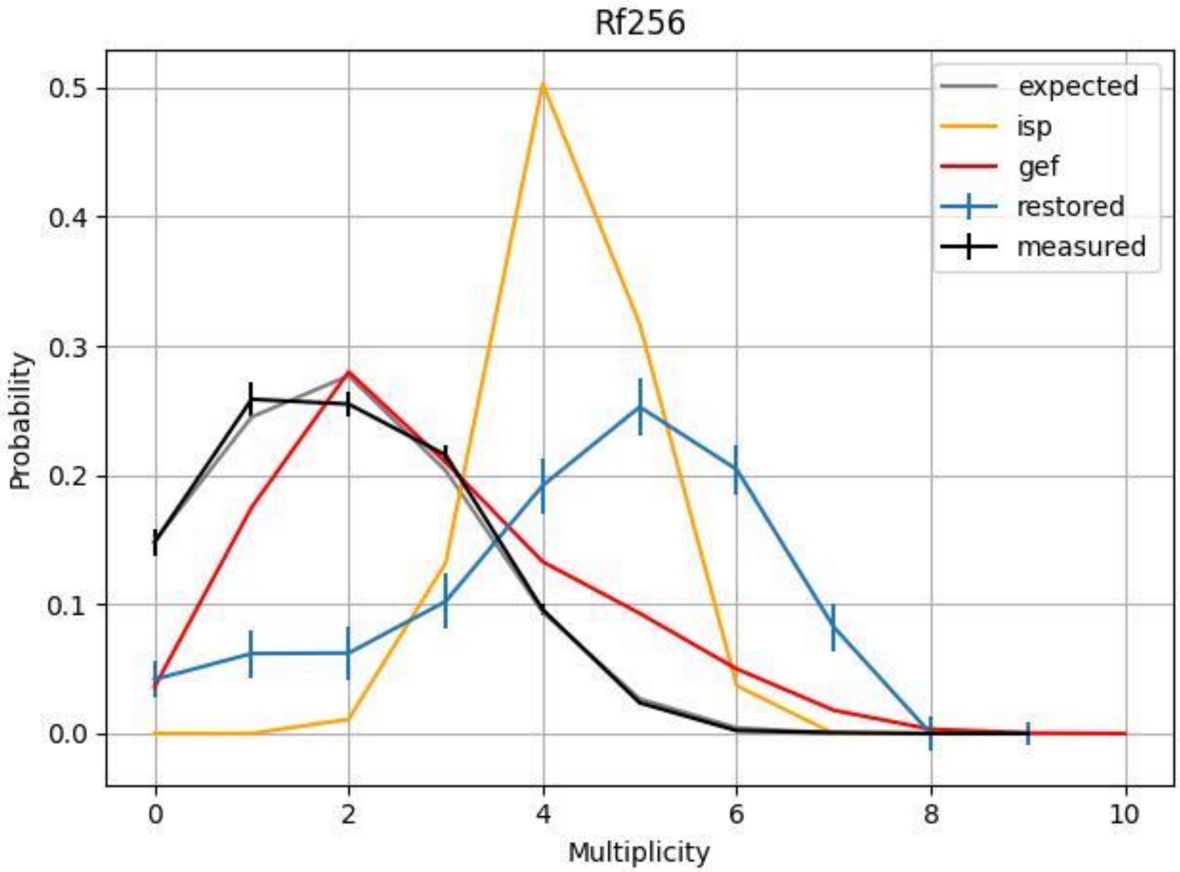
# Asymmetric distributions

- 235 s.f. events
- A = 246
- Z = 100
- N = 146
- $\bar{\nu} = 3.8 \pm 0.3$
- $\sigma_\nu^2 = 2.8$

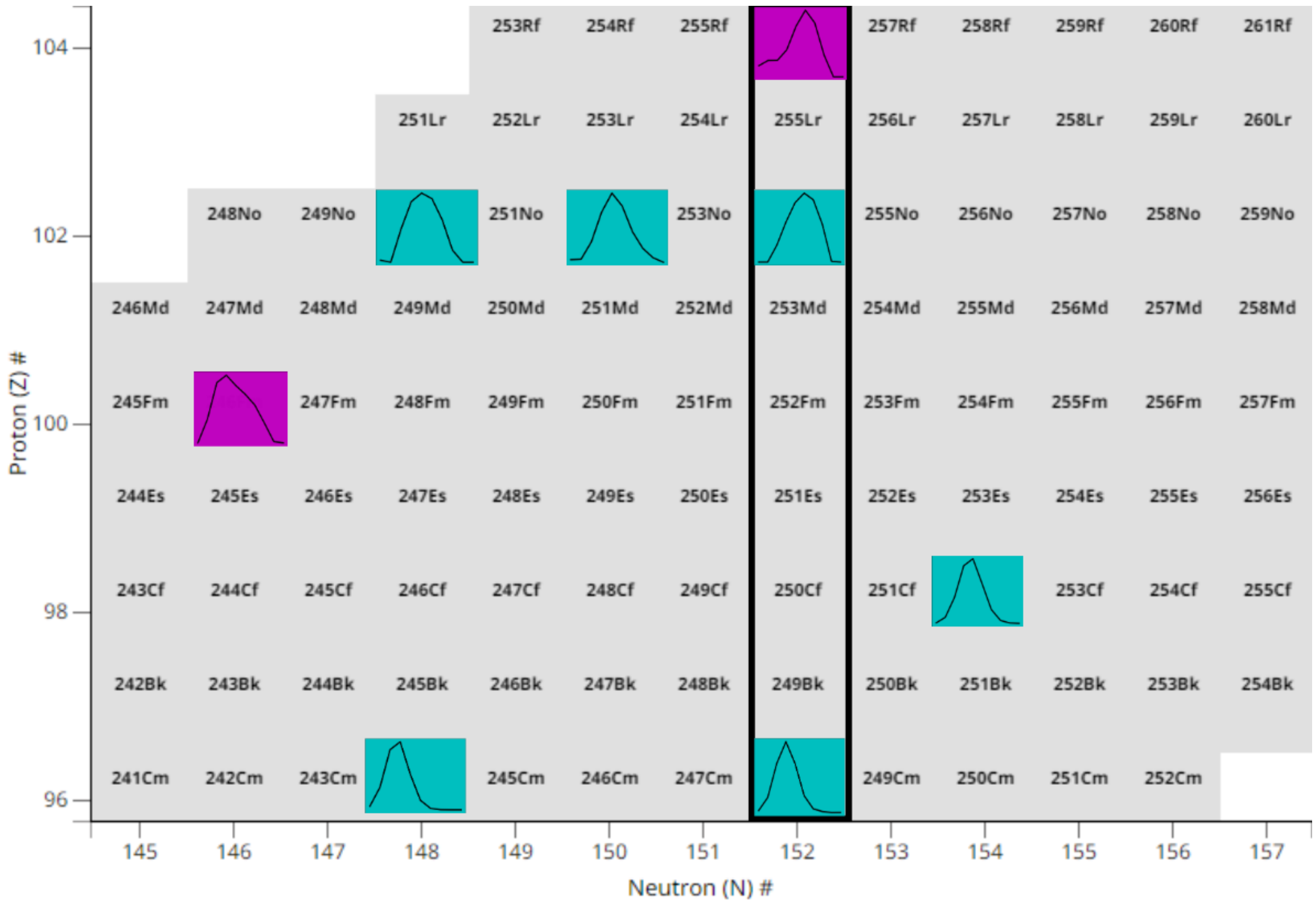


# Asymmetric distributions

- 1345 s.f. events
- A = 256
- Z = 104
- N = 152
- $\bar{\nu} = 4.3 \pm 0.2$
- $\sigma_\nu^2 = 3.2$



# Summary and outlook



# Summary and outlook

- Using restoring technique it possible to analyze not just moments of distribution (mean and variance) but shapes too.
- Emission probability distributions of spontaneous fission prompt neutrons for 8 isotopes was observed.
- 6 of them were classified as “symmetric” and 2 as an “asymmetric”
- “Asymmetric” distributions could point at exotic spontaneous decay modes
- We are looking for theoretical support to plan further experiments and clear interpretation of already carried one.
- We are ready to offer experimental support for theoretical models development

# Thank you for your attention!