Parton distribution functions in QED

Andrej Arbuzov

BLTP, JINR, Dubna

(with U. Voznaya)

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Outline

1. Intro
2. e^+e^- colliders
3. QED
4. Higher order logs
5. Outlook

Motivation

Motivation:

- Development of the physical program for future high-energy e^+e^- colliders
- Having high-precision theoretical description of basic e^+e^- processes is of crucial importance
- Two-loop calculations are in progress, but higher-order QED corrections are also important
- The formalism of QED parton distribution functions can give a fast estimate of the leading higher-order effects

Future e^+e^- collider projects

Linear Colliders

- ILC, CLIC
- ILC: technology is ready, not to be built in Japan (?)

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}$ cm^{-2}s^{-1}

Stat. uncertainty $\sim 10^{-3}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, TLEP
- CEPC
- $\mu^+\mu^-$ collider (μTRISTAN)

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}$ cm^{-2}s^{-1} (4 exp.)

Stat. uncertainty $< 10^{-3}$

Beam polarization: desirable

Super Charm-Tau Factory Projects

Budker Institute of Nuclear Physics + Sarov and/or China

Colliding electron-positron beams with c.m.s. energies from 2 to 7 GeV with unprecedented high luminosity $10^{35} cm^{-2} c^{-1}$

The electron beam will be longitudinally polarized

The main goal of experiments at the Super Charm-Tau factory is to study the processes charmed mesons and tau leptons, using a data set that is 2 orders of magnitude more than the one collected by BESIII

Estimated experimental precision

Now:

Quantity	Theory error	Exp. error
M_W [MeV]	4	15
$\sin^2\theta^l_{eff}[10^{-5}]$	4.5	16
Γ_Z [MeV]	0.5	2.3
$R_b[10^{-5}]$	15	66

Quantity	ILC	FCC-ee	CEPC	Projected theory error
M_W [MeV]	3–4	1	3	1
$\sin^2\theta^l_{eff}[10^{-5}]$	1	0.6	2.3	1.5
Γ_Z [MeV]	0.8	0.1	0.5	0.2
$R_b[10^{-5}]$	14	6	17	5–10

The estimated error for the theoretical predictions of these quantities is given, under the assumption that $O(\alpha\alpha_s^2)$, fermionic $O(\alpha^2\alpha_s)$, fermionic $O(\alpha^3)$, and leading four-loop corrections entering through the ρ-parameter will become available.

FCC-ee: The Tera-Z

Report on the 1st Mini workshop: Precision EW and QCD calculations for the FCC studies: methods and tools: A. Blondel et al., "Standard Model Theory for the FCC-ee: The Tera-Z," arXiv:1809.01830 [hep-ph].

Having high-precision luminosity measurements is crucial for extraction of electroweak quantities. The most sensitive are:
the cross section of $\sigma(e^+e^- \to hadrons)$
and the number of (light) neutrinos N_ν

In general, QED, EW, and QCD radiative corrections to cross-sections and angular distributions that are needed to get: couplings, masses, partial widths, asymmetries, etc.

Perturbative QED (I)

Fortunately, in our case the general perturbation theory can be applied:

$$\frac{\alpha}{2\pi} \approx 1.2 \cdot 10^{-3}, \quad \left(\frac{\alpha}{2\pi}\right)^2 \approx 1.4 \cdot 10^{-6}$$

Moreover, other effects: hadronic vacuum polarization, (electro)weak contributions, hadronic pair emission, etc. are small in Bhabha scattering and can be treated one-by-one separately

Nevertheless, there are some enhancement factors:

1) First of all, the large logarithm $L \equiv \ln \frac{\Lambda^2}{m_e^2}$ where $\Lambda^2 \sim Q^2$ is the momentum transferred squared, e.g., $L(\Lambda = 1\,\mathrm{GeV}) \approx 16$ and $L(\Lambda = M_Z) \approx 24$.

2) The energy region at the Z boson peak ($s \sim M_Z^2$) requires a special treatment since factor M_Z/Γ_Z appears in the annihilation channel

Perturbative QED (II)

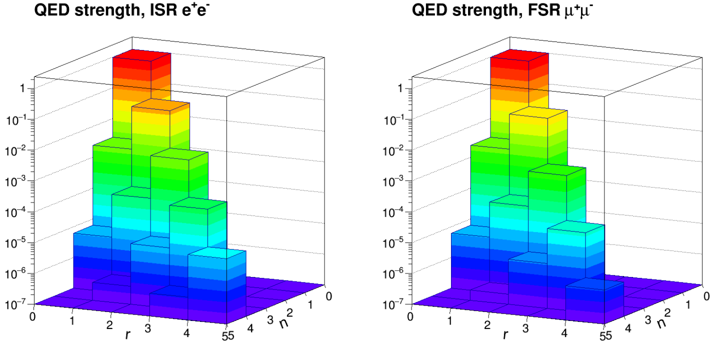


Fig.: The parameter γ_{nr} characterizing the size of the QED corrections,

$$\gamma_{nr} = \left(\frac{\alpha}{\pi}\right)^n \left(2\ln\frac{M_Z^2}{m_f^2}\right)^r, \qquad 1 \le r \le n$$

Figure from [S.Jadach and M.Skrzypek, arXiv:1903:09895]

Perturbative QED (III)

Methods of resummation of QED corrections

- Resummation of vacuum polarization corrections (geometric series)
- Yennie–Frautschi–Suura (YFS) soft photon exponentiation and its extensions, see, e.g., PHOTOS
- Resummation of leading logarithms via QED structure functions or QED PDFs (E.Kuraev and V.Fadin 1985; A. De Rujula, R. Petronzio, A. Savoy-Navarro 1979)

N.B. Resummation of real photon radiation is good for inclusive observables...

Example: Bhabha scattering in two loops

The complete $\mathcal{O}\left(\alpha^2 L\right)$ analytic result was first received in A.A., V. Fadin, E. Kuraev, L. Lipatov, N. Merenkov, L. Trentadue [Nucl.Phys.B '1997]

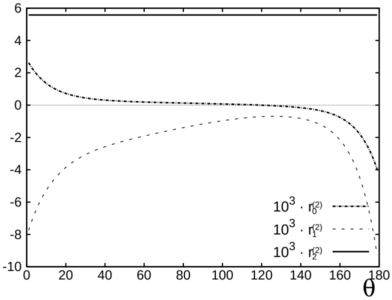
Two-loop virtual pure QED RC were computed by A. Penin [PRL'2005, NPB'2006] and [T. Becher, K. Melnikov, JHEP'2007]

Emission of one or two real photons was also added, see e.g. C. Carloni Calame, H. Czyz, J. Gluza, M. Gunia, G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek
NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories, JHEP 1107 (2011) 126

A. Penin and G. Ryan, Two-loop electroweak corrections to high energy large-angle Bhabha scattering, JHEP'2011

Size of second order RC

Let's look at soft + virtual $\mathcal{O}\left(\alpha^2\right)$ RC [A. Penin, PRL'2005, NPB'2006]: $r_k^{(2)} = \left(\frac{\alpha}{2\pi}\right)^2 C_k \ln^k \frac{M^2}{m_e^2}, \qquad k = 0, 1, 2$



Soft and virtual second order photonic relative radiative corrections in permil versus the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $M = \sqrt{s}$

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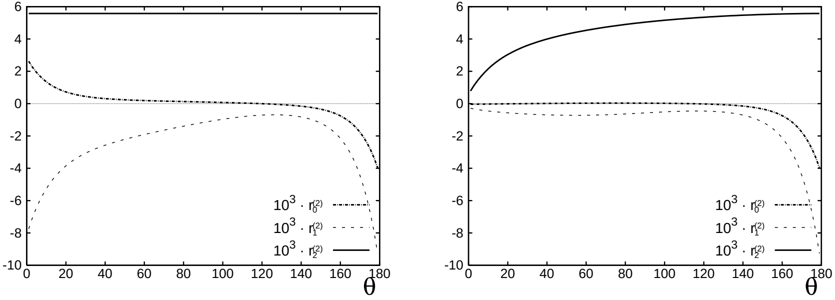


Soft and virtual second order photonic relative radiative corrections in permil versus the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $M = \sqrt{s}$ on the left side and $M = \sqrt{-t}$ on the right side.

Leading and next-to-leading logs in QED

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc. for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

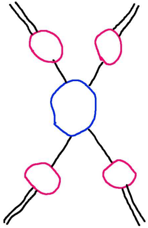
with $n = 3$ are required for future e^+e^- colliders

In the collinear approximation we can get them within the NLO QED structure function formalism

- F.A.Berends, W.L. van Neerven, G.J.Burgers, NPB'1988
- A.A., K.Melnikov, PRD'2002; A.A. JHEP'2003

QED NLO master formula

The NLO Bhabha cross section reads



$$
\begin{aligned}
d\sigma \;=\; & \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\mathrm{str}}_{ae}(z_1)\mathcal{D}^{\mathrm{str}}_{b\bar{e}}(z_2) \\
\times \; & \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right] \\
\times \; & \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\mathrm{frg}}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\mathrm{frg}}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right) \\
+ \; & \mathcal{O}\left(\alpha^n L^{n-2}, \frac{m_e^2}{s}\right)
\end{aligned}
$$

$\alpha^2 L^1$ terms are completely reproduced [A.A., E.Scherbakova, JETP Lett. 2006; PLB 2008]

High-order ISR in e+e- annihilation

$$
\begin{aligned}
\frac{d\sigma_{e^+e^-}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \Bigg[& \mathcal{D}_{e^+e^+}\left(N,\frac{\mu^2}{m_e^2}\right)\tilde{\sigma}_{e^+e^-}\left(N,\frac{s'}{\mu^2}\right)\mathcal{D}_{e^-e^-}\left(N,\frac{\mu^2}{m_e^2}\right) \\
& +\mathcal{D}_{\gamma e^+}\left(N,\frac{\mu^2}{m_e^2}\right)\tilde{\sigma}_{e^-\gamma}\left(N,\frac{s'}{\mu^2}\right)\mathcal{D}_{e^-e^-}\left(N,\frac{\mu^2}{m_e^2}\right) \\
& +\mathcal{D}_{e^+e^+}\left(N,\frac{\mu^2}{m_e^2}\right)\tilde{\sigma}_{e^+\gamma}\left(N,\frac{s'}{\mu^2}\right)\mathcal{D}_{\gamma e^-}\left(N,\frac{\mu^2}{m_e^2}\right) \\
& +\mathcal{D}_{\gamma e^+}\left(N,\frac{\mu^2}{m_e^2}\right)\tilde{\sigma}_{\gamma\gamma}\left(N,\frac{s'}{\mu^2}\right)\mathcal{D}_{\gamma e^-}\left(N,\frac{\mu^2}{m_e^2}\right)\Bigg]
\end{aligned}
$$

J. Ablinger, J. Blümlein, A. De Freitas and K. Schönwald,
"Subleading Logarithmic QED Initial State Corrections to $e^+e^- \to \gamma^*/Z^{0^*}$ to $O(\alpha^6 L^5)$," NPB 955 (2020) 115045

Contributions from $\mathcal{D}_{e^-e^+}$ and $\mathcal{D}_{e^+e^-}$ are missed. They are relevant starting from $\mathcal{O}(\alpha^4 L^4)$.

QED NLO evolution equations

$$\mathcal{D}_{ba}(x,\mu_f,\mu_0) = \sum_{c=e,\gamma,\bar{e}} \int\limits_{\mu_0^2}^{\mu_f^2} \frac{dt\alpha(t)}{2\pi} \int\limits_x^1 \frac{dy}{y} P_{bc}(y) \mathcal{D}_{ca}\left(\frac{x}{y},\mu_f,\mu_0\right)$$

where $\mathcal{D}_{ba}^{\text{ini}}$ is the initial approximation in iterations for $\mathcal{D}_{ba}$

$$\mathcal{D}_{ee}^{\text{ini}}(x,\mu_0,m_e) = \delta(1-x) + \frac{\bar{\alpha}(\mu_0)}{2\pi} d_{ee}^{(1)}(x,\mu_0,m_e) + \mathrm{O}(\alpha^2)$$

$$\mathcal{D}_{\gamma e}^{\text{ini}}(x,\mu_0,m_e) = \frac{\bar{\alpha}(\mu_0)}{2\pi} P_{\gamma e}^{(0)}(x) + \mathrm{O}(\alpha^2)$$

$$d_{ee}^{(1)}(x,\mu_0,m_e) = \left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_0^2}{m_e^2} - 2\ln(1-x) - 1\right)\right]_+$$

QED splitting functions

The perturbative splitting functions are

$$P_{ba}(z, \bar{\alpha}(\mu_f)) = \frac{\bar{\alpha}(\mu_f)}{2\pi} P_{ba}^{(0)}(z) + \left(\frac{\bar{\alpha}(\mu_f)}{2\pi}\right)^2 P_{ba}^{(1)}(z) + \mathcal{O}(\alpha^3)$$

They come from direct loop calculations, see, e.g., review “Partons in QCD” by G. Altarelli. For instance, $P_{ba}^{(1)}(z)$ comes from 2-loop calculations.

The splitting functions can be obtained by reduction of the ones known in QCD to the abelian case of QED.

NLO matching

The expansion of the master formula for ISR gives

$$d\sigma = d\sigma^{(0)}_{ab\to cd} + \frac{\alpha}{2\pi}\left\{2LP^0 \otimes d\sigma^{(0)} + 2d^{(1)}_{ee} \otimes d\sigma^{(0)} + d\bar{\sigma}^{(1)}_{ab\to cd}\right\} + \mathcal{O}\left(\alpha^2\right)$$

We know the massive and massless ($m_e = 0$ with $\overline{\text{MS}}$ subtraction) results in $\mathcal{O}(\alpha)$. That gives $d^{(1)}_{ee}(x)$.

A scheme dependence appears here.

Note so-called massification procedure.

Running coupling constant

Running of $\alpha_{\rm QED}$ is known, e.g., P.Baikov, K.Chetyrkin et al., NPB 867 (2013) 182

$$\alpha(\mu_f^2) = \frac{\alpha(\mu_0^2)}{1+\Pi(\mu_f^2)}, \qquad \alpha \equiv \alpha(\mu_0^2 = m_e^2) \approx \frac{1}{137.036}$$

$$\Pi(\mu_f^2) = \frac{\alpha}{\pi}\left(\frac{5}{9} - \frac{1}{3}L\right) + \left(\frac{\alpha}{\pi}\right)^2\left(\frac{55}{48} - \zeta(3) - \frac{1}{4}L\right) + \left(\frac{\alpha}{\pi}\right)^3\left(-\frac{1}{24}L^2 + \ldots\right) + \mathcal{O}(\alpha^4)$$

The same $\overline{\rm MS}$ scheme is used here, $L \equiv \ln(\mu_f^2/\mu_0^2)$.

Note that only electron loops are taken into account. Other contributions can be added.

Iterative solution

The NLO “electron in electron” distribution function reads

$$
\begin{aligned}
\mathcal{D}_{ee}(x,\mu_f,m_e) &= \delta(1-x) + \frac{\alpha}{2\pi} L P^{(0)}_{ee}(x) + \frac{\alpha}{2\pi} d^{(1)}_{ee}(x,\mu_0,m_e) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left(\frac{1}{2} P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \frac{1}{2} P^{(0)}_{ee}(x) + \frac{1}{2} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma e}(x) \right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L \left(P^{(0)}_{e\gamma} \otimes d^{(1)}_{\gamma e}(x,\mu_0,m_e) + P^{(0)}_{ee} \otimes d^{(1)}_{ee}(x,\mu_0,m_e) - \frac{10}{9} P^{(0)}_{ee}(x) + P^{(1)}_{ee}(x) \right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^3 \left(\frac{1}{6} P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \frac{1}{6} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma\gamma} \otimes P^{(0)}_{\gamma e}(x) + \ldots \right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^2 \left(P^{(0)}_{ee} \otimes P^{(1)}_{ee}(x) + P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes d^{(1)}_{ee}(x,\mu_0,m_e) + \frac{1}{3} P^{(1)}_{ee}(x) - \frac{10}{9} P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \ldots \right) \\
&+ \mathcal{O}(\alpha^2 L^0, \alpha^3 L^1)
\end{aligned}
$$

The large logarithm $L \equiv \ln \frac{\mu_f^2}{\mu_0^2}$ with factorization scale $\mu_f^2 \sim s$ or $\sim -t$; and renormalization scale $\mu_0 = m_e$.

Required convolution integrals are listed in [A.A. hep-ph/0304063]

Электрон также неисчерпаем, как атом (В.И. Ленин ’1908)

Example of convolution (I)

$$P_{ee}^{(0)}(x) = \left[\frac{1+x^2}{1-x}\right]_+$$

$$P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \delta(1-x)\left(\frac{3}{8} - 3\zeta(2) + 6\zeta(3)\right)$$
$$+\frac{1+x^2}{1-x}\left(-2\ln x\ln(1-x) + \ln^2 x + 2\mathrm{Li}_2(1-x)\right)$$
$$-\frac{1}{2}(1+x)\ln^2 x + 2\ln x + 3 - 2x$$

Convolution

$$f \otimes g(x) = \int_0^1 dz \int_0^1 dz' \delta(x - zz') f(z) g(z')$$

Plus prescription

$$\int_{x_{\min}}^1 dx\,[f(x)]_+ \,g(x) = \int_0^1 dx\,f(x)\left[g(x)\Theta(x - x_{\min}) - g(1)\right]$$

Example of convolution (II)

$$P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \Bigg[\frac{1+x^2}{1-x}\Bigg(-4\mathrm{S}_{12}(1-x) + 4\mathrm{Li}_2(1-x)(\ln(1-x) - \ln(x))$$

$$-4\ln(x)\ln^2(1-x) + 4\ln^2(x)\ln(1-x) - \frac{2}{3}\ln^3(x) + 3\mathrm{Li}_2(1-x)$$

$$-3\ln(x)\ln(1-x) + \frac{3}{2}\ln^2(x) + 4\zeta(2)\ln(x) + 6\zeta(3) - 3\zeta(2) + \frac{3}{8}\Bigg)$$

$$+4(1+x)\mathrm{S}_{12}(1-x) + \frac{1+x}{2}\ln^3(x) - 2(1+x)\ln^2(x)\ln(1-x)$$

$$+(6x-2)\mathrm{Li}_2(1-x) + (6-2x)\ln(x)\ln(1-x) + \left(\frac{11}{4}x - \frac{9}{4}\right)\ln^2(x)$$

$$+(6-4x)\ln(1-x) + (5x-3)\ln(x) + 2x - \frac{1}{2}\Bigg]_+$$

This is a part of universal collinear radiation factors

Outlook

- Having high theoretical precision for the normalization processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, and $e^+e^- \to 2\gamma$ is crucial for future e^+e^- colliders, especially for the Tera-Z mode
- There are several two-loop QED results, but leading higher order corrections are also numerically important
- New Monte Carlo codes are required
- But semi-analytic codes are relevant for cross-checks and benchmarks
- $O(\alpha^3 L^2)$ collinear radiator factors are derived
- Comparisons with recent results of Blümlein et al., show some deviations. It will be investigated.
- Our results will be implemented into ZFITTER code for annihilation processes (ISR)
- We also will compute $O(\alpha^3 L^2)$ corrections to μe scattering for MUonE experiment.

Thank You for attention!