# SANCphot - polarized photon-photon scattering 

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## Overview

- Motivation
- Photon-photon collisions in SANC
- Polarized gamma-gamma cross section
- Implementation in SANC


## Motivation

Growing interest to photon-photon collider physics in the scientific community:

- Higgs physics
- SM benchmarks
- New physics (SUSY)
- QCD tests (photon structure function, jet production)
- Flavor physics $\left(\eta_{c}\right)$

The gamma-gamma collisions may constitute significant background for $\mathrm{e}+\mathrm{e}-\mathrm{colliders}$ (equivalent photons, beamstrahlung)

Our study was initiated by our colleagues from Novosibirsk

## ILC physics

Table 3. Gold-plated processes at photon colliders.

| Reaction | Remarks |
| :--- | :--- |
| $\gamma \gamma \rightarrow h^{0} \rightarrow b \bar{b}$ | SM (or MSSM) Higgs, $M_{h^{0}}<160 \mathrm{GeV}$ |
| $\gamma \gamma \rightarrow h^{0} \rightarrow W W\left(W W^{*}\right)$ | SM Higgs, $140 \mathrm{GeV}<M_{h^{0}}<190 \mathrm{GeV}$ |
| $\gamma \gamma \rightarrow h^{0} \rightarrow Z Z\left(Z Z^{*}\right)$ | SM Higgs, $180 \mathrm{GeV}<M_{h^{0}}<350 \mathrm{GeV}$ |
| $\gamma \gamma \rightarrow H, A \rightarrow b \bar{b}$ | MSSM heavy Higgs, for intermediate tan $\beta$ |
| $\gamma \gamma \rightarrow \tilde{\tilde{f}}, \tilde{\chi}_{i}^{+} \tilde{\chi}_{i}^{-}, H^{+} H^{-}$ | large cross-sections, possible observations of FCNC |
| $\gamma \gamma \rightarrow S[\tilde{\tilde{t} t}]$ | $\tilde{\tilde{t}}$ stoponium |
| $\gamma e \rightarrow \tilde{e}^{-} \tilde{\chi}_{1}^{0}$ | $M_{\tilde{e}^{-}}<0.9 \times 2 E_{0}-M_{\tilde{\chi}_{1}^{0}}$ |
| $\gamma \gamma \rightarrow W^{+} W W^{-}$ | anomalous $W$ interactions, extra dimensions |
| $\gamma e^{-} \rightarrow W \nu_{e}$ | anomalous $W$ couplings |
| $\gamma \gamma \rightarrow W W W W^{-} W, W W Z Z$ | strong $W W$ scatt., quartic anomalous $W, Z$ couplings |
| $\gamma \gamma \rightarrow t \bar{t}$ | anomalous top quark interactions |
| $\gamma e^{-} \rightarrow \bar{t} b \nu_{e}$ | anomalous $W t b$ coupling |
| $\gamma \gamma \rightarrow$ hadrons | total $\gamma \gamma$ cross-section |
| $\gamma e^{-} \rightarrow e^{-} X$ and $\nu_{e} X$ | $\mathcal{N C}$ and $\mathcal{C C}$ structure functions (polarized and unpolarized) |
| $\gamma g \rightarrow q \bar{q}, c \bar{c}$ | gluon distribution in the photon |
| $\gamma \gamma \rightarrow J / \psi J / \psi$ | QCD Pomeron |

(taken from [Boos et.al. Nucl.Instrum.Meth.A472:100-120,2001]

## Photon-photon collisions in SANC

The unpolarized $\gamma \gamma \rightarrow \gamma \gamma, Z \gamma, Z Z$ SM processes through fermion and boson loops were calculated within the SANC framework by D. Bardin, L. Kalinovskaya and E. Uglov.

The computations take into account non-zero mass of loop particles and massive box diagrams.

The results are presented as the covariant and helicity amplitudes for these processes with some particular cases of D0 and C0 Passarino-Veltman functions. Reference: Phys.Atom.Nucl.73:1878-1888,2010

However the form of helicity amplitudes allows to implement the same calculations taking into account the photon polarizations

## Polarized gamma-gamma cross section

The differential photon-photon cross section taking into account polarizations can be taken from [Gounaris et.al. Eur.Phys.J.C10:499-513,1999]:

$$
\begin{align*}
\frac{d \sigma}{d \tau d \cos \vartheta^{*}}= & \frac{d \bar{L}_{\gamma \gamma}}{d \tau}\left\{\frac{d \bar{\sigma}_{0}}{d \cos \vartheta^{*}}+\left\langle\xi_{2} \xi_{2}^{\prime}\right\rangle \frac{d \bar{\sigma}_{22}}{d \cos \vartheta^{*}}+\left\langle\xi_{3}\right\rangle \cos 2 \phi \frac{d \bar{\sigma}_{3}}{d \cos \vartheta^{*}}+\left\langle\xi_{3}^{\prime}\right\rangle \cos 2 \phi^{\prime} \frac{d \bar{\sigma}_{3}^{\prime}}{d \cos \vartheta^{*}}\right. \\
& +\left\langle\xi_{3} \xi_{3}^{\prime}\right\rangle\left[\frac{d \bar{\sigma}_{33}}{d \cos \vartheta^{*}} \cos 2\left(\phi+\phi^{\prime}\right)+\frac{d \bar{\sigma}_{33}^{\prime}}{d \cos \vartheta^{*}} \cos 2\left(\phi-\phi^{\prime}\right)\right] \\
& \left.+\left\langle\xi_{2} \xi_{3}^{\prime}\right\rangle \sin 2 \phi^{\prime} \frac{d \bar{\sigma}_{23}}{d \cos \vartheta^{*}}-\left\langle\xi_{3} \xi_{2}^{\prime}\right\rangle \sin 2 \phi \frac{d \bar{\sigma}_{23}^{\prime}}{d \cos \vartheta^{*}}\right\}, \tag{10}
\end{align*}
$$

## Polarized gamma-gamma cross section

$$
\begin{array}{rlr}
\frac{d \bar{\sigma}_{0}}{d \cos \vartheta^{*}} & = & N \sum_{\lambda_{3} \lambda_{4}}\left[\left|\mathcal{H}_{++\lambda_{3} \lambda_{4}}\right|^{2}+\left|\mathcal{H}_{+-\lambda_{3} \lambda_{4}}\right|^{2}\right] \\
\frac{d \bar{\sigma}_{22}}{d \cos \vartheta^{*}} & = & N \sum_{\lambda_{3} \lambda_{4}}\left[\left|\mathcal{H}_{++\lambda_{3} \lambda_{4}}\right|^{2}-\left|\mathcal{H}_{+-\lambda_{3} \lambda_{4}}\right|^{2}\right] \\
\frac{d \bar{\sigma}_{3}}{d \cos \vartheta^{*}} & = & -2 N \sum_{\lambda_{3} \lambda_{4}} \operatorname{Re}\left[\mathcal{H}_{++\lambda_{3} \lambda_{4}} \mathcal{H}_{-+\lambda_{3} \lambda_{4}}^{*}\right] \\
\frac{d \bar{\sigma}_{3}^{\prime}}{d \cos \vartheta^{*}}= & -2 N \sum_{\lambda_{3} \lambda_{4}} \operatorname{Re}\left[\mathcal{H}_{++\lambda_{3} \lambda_{4}} \mathcal{H}_{--\lambda_{3} \lambda_{4}}^{*}\right] \\
\frac{d \bar{\sigma}_{33}}{d \cos \vartheta^{*}}= & N \sum_{\lambda_{3} \lambda_{4}} \operatorname{Re}\left[\mathcal{H}_{++\lambda_{3} \lambda_{4}} \mathcal{H}_{--\lambda_{3} \lambda_{4}}^{*}\right] \\
\frac{d \bar{\sigma}_{33}^{\prime}}{d \cos \vartheta^{*}}= & N \sum_{\lambda_{3} \lambda_{4}} \operatorname{Re}\left[\mathcal{H}_{++\lambda_{3} \lambda_{4}} \mathcal{H}_{--\lambda_{3} \lambda_{4}}^{*}\right] \\
\frac{d \bar{\sigma}_{23}}{d \cos \vartheta^{*}}= & 2 N \sum_{\lambda_{3} \lambda_{4}} \operatorname{Im}\left[\mathcal{H}_{++\lambda_{3} \lambda_{4}} \mathcal{H}_{+-\lambda_{3} \lambda_{4}}^{*}\right], \text { where } N=\frac{\beta_{z}}{64 \pi \hat{s}}
\end{array}
$$

## Polarized gamma beams at e+e- collider


(picture taken from slides of V.Serbo "Basics of photon collider" )

## Density matrix

After a Compton scattering of $e^{ \pm}$off a laser photon, the electron (positron) beam looses most of its energy and a beam of "backscattered photons" is produced, moving essentially along the direction of the original $e^{ \pm}$momentum and characterized, in its helicity basis, by the density matrix:

$$
\rho^{B N}=\frac{1}{2}\left(\begin{array}{cc}
1+\xi_{2}(x) & -\xi_{3}(x) e^{-2 i \phi} \\
-\xi_{3}(x) e^{+2 i \phi} & 1-\xi_{2}(x)
\end{array}\right)
$$

where $x=\omega / E$ is the back-scattered photon energy fraction

## Stocks parameters

$$
\begin{aligned}
\xi_{2}(x) & =\frac{P_{e} f_{2}(x)+P_{\gamma} f_{3}(x)}{C(x)} \\
\xi_{3}(x) & =\frac{2 r^{2}(x) P_{t}}{\mathcal{C}(x)}
\end{aligned}
$$

with $C(x)=f_{0}(x)+P_{e} P_{\gamma} f_{1}(x)$, and

$$
\begin{aligned}
f_{0}(x) & =\frac{1}{1-x}+1-x-4 r(1-r), \\
f_{1}(x) & =\frac{x}{1-x}(1-2 r)(2-x), \\
f_{2}(x) & =x_{0} r\left[1+(1-x)(1-2 r)^{2}\right] \\
f_{3}(x) & =(1-2 r)\left(\frac{1}{1-x}+1-x\right), \\
r(x) & =\frac{x}{x_{0}(1-x)},
\end{aligned}
$$

## $\gamma \gamma$ scattering in SANC

We reuse the MCSANC code to create a stand-alone SANCphot package to implement the $\gamma \gamma \rightarrow \gamma \gamma, Z \gamma$, and $Z Z$ processes relying on the Fortran modules created within the SANC framework. arXiv:2201.04350 [hep-ph]

The Monte-Carlo calculation requires:

- Initial electron beam energy, laser energy
- Laser and electron polarizations, with laser polarization could be both longitudinal and transverse
- Final Z particle polarization

And the integration variables are:

- $\cos \theta^{*}-\operatorname{cosine}$ of the final particle azimuthal angle
- $\tau=\frac{s_{\gamma \gamma}}{s_{e e}}$
- $x=\omega / E$


## SANCphot numerical results

The cross sections were calculated in $\alpha(0)$ scheme with the following cuts: $20 \mathrm{GeV}<M_{i n v}<1 \mathrm{TeV} ; \quad\left|\cos \theta^{*}\right|>\cos \left(30^{\circ}\right)$ The initial laser energy was set to 1.26120984 Ev , giving optimal $x_{0} \equiv 4 E \omega_{0} / m_{e}^{2}=4.83$

Electroweak parameters were set to:

$$
\begin{array}{ll}
\alpha=1 / 137.035990996, & G_{F}=1.13024 \times 10^{-5} \mathrm{GeV}^{-2}, \\
M_{W}=80.45149 \mathrm{GeV}, & M_{Z}=91.18670 \mathrm{GeV}, \\
M_{H}=125 \mathrm{GeV}, & \\
m_{e}=0.51099907 \mathrm{MeV}, & m_{\mu}=0.105658389 \mathrm{GeV}, \\
m_{\tau}=1.77705 \mathrm{GeV}, & \\
m_{u}=0.062 \mathrm{GeV}, & m_{d}=0.083 \mathrm{GeV}, \\
m_{c}=1.5 \mathrm{GeV}, & m_{s}=0.215 \mathrm{GeV}, \\
m_{t}=173.8 \mathrm{GeV}, & m_{b}=4.7 \mathrm{GeV}
\end{array}
$$

## Polarization setups

The numeric tests were conducted using the following three polarization combinations (same as in [Gounaris et.al., Eur.Phys.J.C9:673-686,1999]):

- set 1: $P_{e}=P_{e}^{\prime}=0.8, P_{\gamma}=P_{\gamma}^{\prime}=-1, P_{t}=P_{t}^{\prime}=0$
- set 2 : $P_{e}=P_{e}^{\prime}=0, P_{\gamma}=P_{\gamma}^{\prime}=0, P_{t}=P_{t}^{\prime}=1, \phi=\pi / 2$
- set 3: $P_{e}=0.8, P_{e}^{\prime}=0, P_{\gamma}=-1, P_{\gamma}^{\prime}=0, P_{t}=0, P_{t}^{\prime}=1, \phi=\pi / 2$


## Integrated cross sections 1

| $\sqrt{s_{e e}}$ | 250 GeV | 500 GeV | 1 TeV | 2 TeV |
| ---: | ---: | ---: | ---: | ---: |
| set 1 | $22.527(2)$ | $9.4431(9)$ | $6.9401(7)$ | $4.3934(4)$ |
| set 2 | $22.939(2)$ | $8.0162(8)$ | $6.8517(7)$ | $4.6385(5)$ |
| set 3 | $20.980(2)$ | $7.9847(8)$ | $6.7887(7)$ | $4.4438(4)$ |

Table: Integrated cross sections $\sigma(\gamma \gamma)$ [fb] for the process $\gamma \gamma \rightarrow \gamma \gamma$ in different polarization setups.

| $\sqrt{s_{e e}}$ | 250 GeV | 500 GeV | 1 TeV | 2 TeV |
| :--- | ---: | ---: | ---: | ---: |
| set 1 | $0.8335(8)$ | $15.725(2)$ | $33.500(3)$ | $24.475(2)$ |
| set 2 | $0.5125(5)$ | $19.375(2)$ | $38.278(4)$ | $28.887(3)$ |
| set 3 | $0.5299(5)$ | $17.015(2)$ | $34.528(3)$ | $26.571(3)$ |

Table: Integrated cross sections $\sigma(\gamma \gamma)$ [fb] for the process $\gamma \gamma \rightarrow \gamma Z$ in different polarization setups.

## Integrated cross sections 2

| $\sqrt{s_{e e}}$ | 500 GeV | 1 TeV | 2 TeV |
| ---: | ---: | ---: | ---: |
| set 1 | $32.339(3)$ | $59.671(6)$ | $45.898(5)$ |
| set 2 | $24.431(2)$ | $59.476(6)$ | $48.761(5)$ |
| set 3 | $26.995(3)$ | $58.864(6)$ | $46.416(5)$ |

Table: Integrated cross sections $\sigma(\gamma \gamma)$ [fb] for the process $\gamma \gamma \rightarrow Z Z$ in different polarization setups.

## Differential distributions $M_{i n v}$

$M_{\gamma \gamma}$ distributions for $\gamma \gamma$ final state for $250,500,1000,2000 \mathrm{GeV}$





The raise of cross section at high $M_{\gamma \gamma}$ is due to behaviour of the Stocks parameters convolution.

## Differential distributions $M_{i n v}$

$M$ distributions for $\gamma Z$ final state


The distribution is constrained by kinematic thresholds defined by limit on energy transfer from electron to laser photon

## Differential distributions: $\cos \theta$

$\cos \theta$ distributions for $\gamma \gamma \rightarrow \gamma Z$


Asymmetric distributions are observed for set 2 and 3 due to $\pi / 2$ rotation of the laser polarization

## Validation

The $\sigma_{0,3,22,33, \text { etc. }}$ components were thoroughly cross checked against $Z Z$ code [Diakonidis et.al. Eur.Phys.J.C50:47-52,2007] $\left(\sqrt{s_{\gamma \gamma}}=500 \mathrm{GeV}\right)$ :







## Contribution to the integrated cross section, $\gamma \gamma \rightarrow Z Z$.



Contribution to the integrated $\sigma_{0}$ cross section. Process $\gamma \gamma \rightarrow Z Z$. Black line is SANC result, green line is result from paper [G.J. Gounaris, J. Layssac, P.I. Porfyriadis and F.M. Renard, Eur.Phys.J. C13 (2000) 79-97]

## Contribution to the integrated cross section, $\gamma \gamma \rightarrow Z Z$.



Contribution to the integrated $\sigma_{22}$ cross section. Process $\gamma \gamma \rightarrow Z Z$. Black line is SANC result, green line is result from paper [G.J. Gounaris, J. Layssac, P.I. Porfyriadis and F.M. Renard, Eur.Phys.J. C13 (2000) 79-97]

## Contribution to the integrated cross section, $\gamma \gamma \rightarrow Z Z$




Contribution to the integrated $\sigma_{33}^{\prime}$ and $\sigma_{3}$ cross section. Black line is SANC result, green line is result from paper [G.J. Gounaris, J. Layssac, P.I. Porfyriadis and F.M. Renard, Eur.Phys.J. C13 (2000) 79-97]

## Summary

- The calculations for polarized photon-photon collisions were implemented in the SANCphot code.
- The paper is about to be submitted, preprint can be seen at arXiv:2201.04350 [hep-ph]
- Different combinations of polarizations are available (transverse, longitudinal)
- The integrator has user-friendly interface and can be used for photon-photon collider simulations
- Currently the tool calculates cross sections for photons produced from (polarized) e-beam. However there is a demand for event generator with explicit photon spectrum.


## Thank you

## Covariant Amplitude

The Covariant Amplitude expression In terms of Lorentz structures is:

$$
\begin{gathered}
\mathcal{A}_{\gamma \gamma \rightarrow \gamma \gamma(\gamma z, z z)}= \\
4 e^{4} Q_{f}^{4} \sum_{i=1}\left[\mathcal{F}_{i}^{(f e r)}(s, t, u)+\mathcal{F}_{i}^{(b o s)}(s, t, u)\right] T_{i}^{\alpha \beta \mu \nu}
\end{gathered}
$$

where $e$ - is the electron charge, $Q_{f}$ - is the charge of loop fermion in unitss of electron charge, $T_{i}^{\alpha \beta \mu \nu}$ are tensors, $\mathcal{F}_{i}$ - are Form Factorss, i.e scalar coefficients in front of tensor structures of the covariant amplitude (depended on invariants $s, t, u$ and also on fermion mass and Passarino Velman functions).

## Differential Cross Section

The differential cross section of the $\gamma \gamma \rightarrow \gamma \gamma(\gamma Z, Z Z)$ processes has the following form:

$$
d \sigma_{\gamma \gamma \rightarrow \gamma \gamma(\gamma Z, z Z)}=\frac{e^{8}}{8 \pi \omega^{2}} \sum_{\text {spins }}\left|\mathcal{H}_{\text {spins }}\right|^{2} d \cos \theta
$$

where $\omega$ is the photon frequency, $\theta$ is the scattering angle in the CMS, and helicity amplitudes are expressed in terms of FFs. All dependence on Mandelstam invariants and loop fermion masses, also on Passarino Veltman functions, are includes in these FFs.

