

Rates of particle production in R^2 gravity

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Outline

- Cosmological evolution in R^2 -gravity
- Decay channels of the scalaron
 - Minimally coupled massless scalars mode
 - Conformally coupled massive scalars mode
 - Massive fermions mode
 - Gauge bosons mode
- SUSY dark matter
- Conclusions

General Relativity (GR):

$$S_{EH} = -\frac{M_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} R, \quad M_{Pl} = 1.22 \cdot 10^{19} \text{ GeV}$$

Beyond the frameworks of GR:

$$S_{tot} = -\frac{M_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{R^2}{6M_R^2} \right] + S_m$$

Magnitude of temperature fluctuations of CMB demands $M_R \approx 3 \cdot 10^{13} \text{ GeV}$.

Nonlinear R^2 -term:

- Leads to (quasi)exponential cosmological expansion (Starobinsky inflation).
- Creates considerable deviation from the Friedmann cosmology in the post-inflationary epoch. (EA, A.D. Dolgov, R. Singh, "Distortion of the standard cosmology in $R + R^2$ theory," JCAP **1807** (2018) no.07, 019)
- R becomes a dynamical variable \implies new gravitational scalar degree of freedom, **scalaron**, with the mass equal to M_R .

Cosmological evolution in R^2 -modified gravity: 4 distinct epochs

EA, A. Dolgov, R. Singh, *R^2 -Cosmology and New Windows for Superheavy Dark Matter*, Symmetry 13 (2021) 5, 877

- 1 **Inflation:** R slowly decreases from large value $R/M_R^2 \gtrsim 10^2$ down to zero
- 2 **Curvature oscillations:**

$$R(t) = 4M_R \frac{\cos(M_R t + \theta)}{t}$$

leading to efficient particle production through the scalaron decay and consequently to **the universe heating (scalaron dominated regime)**

- 3 Transition of the scalaron domination regime to the **dominance of the produced matter** of mostly relativistic particles
- 4 **Transition to the conventional cosmology** governed by the General Relativity.

We consider the epoch of the universe heating and calculate the rate of the production of different types of particles:

- massless minimally coupled scalars; conformally coupled massive scalars
- massive fermions
- massless gauge bosons

Modified Equations of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{3M_R^2} \left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}D^2 - D_\mu D_\nu \right) R = \frac{8\pi}{M_{Pl}^2} T_{\mu\nu}$$

- $D^2 = g^{\mu\nu} D_\mu D_\nu$, D_μ is the covariant derivative
- $T_{\mu\nu}$ is of the energy-momentum tensor of matter

Trace equation:

$$D^2 R + M_R^2 R = -\frac{8\pi M_R^2}{M_{Pl}^2} T^\mu_\mu$$

FLRW metric: $ds^2 = dt^2 - a^2(t) [dr^2 + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2]$

- $a(t)$ is the cosmological scale factor
- $H = \dot{a}/a$ is a Hubble parameter

For homogeneous field $R = R(t)$: $D^2 R = (\partial_t^2 + 3H\partial_t) R$

Energy density of the scalaron field

The effective action of the scalaron field leading to presented EoM:

$$A_R = \frac{M_{Pl}^2}{48\pi M_R^4} \int d^4x \sqrt{-g} \left[\frac{(DR)^2}{2} - \frac{M_R^2 R^2}{2} - \frac{8\pi M_R^2}{m_{Pl}^2} T^\mu{}_\mu R \right]$$

The canonically normalized scalar field:

$$\Phi = \frac{M_{Pl}}{\sqrt{48\pi} M_R^2} R$$

The energy density of the scalaron field:

$$\rho_R = \rho_\Phi = \frac{\dot{\Phi}^2 + M_R^2 \Phi^2}{2} = \frac{M_{Pl}^2 (\dot{R}^2 + M_R^2 R^2)}{96\pi M_R^4} = \frac{M_{Pl}^2}{6\pi t^2}$$

EA, A. D. Dolgov, and R. S. Singh, *Distortion of the standard cosmology in $R + R^2$ theory*, JCAP **07** (2018) 019 [arXiv:1803.01722 [gr-qc]].

Massive Scalar Fields

Actions of the non-interacting, except for **coupling to gravity**, complex and real scalar fields with mass m :

$$S_c[\phi_c] = \int d^4x \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi_c^* \partial_\nu \phi_c - m^2 |\phi_c|^2 + \xi R |\phi_c|^2)$$

$$S_r[\phi_r] = \frac{1}{2} \int d^4x \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi_r \partial_\nu \phi_r - m^2 \phi_r^2 + \xi R \phi_r^2)$$

- If the constant $\xi = 0$, fields ϕ 's are called **minimally coupled to gravity**
- For $\xi = 1/6$ they are called **conformally coupled**, because the trace of the EM tensor of the fields ϕ 's becomes zero.

The equation of motion both for real and complex fields ϕ 's:

$$D^2 \phi + m^2 \phi - \xi R \phi = 0,$$

which in FLRW-metric transforms to

$$\ddot{\phi} - \frac{\Delta \phi}{a^2} + 3H\dot{\phi} + m^2 \phi - \xi R \phi = 0,$$

where Δ is the three-dimensional Laplace operator in flat 3D-space.

Energy Momentum tensor of the scalar field ϕ

The EM tensor is defined as the variation of the action over the metric tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}.$$

Correspondingly for the complex field:

$$T_{\mu\nu}^{(c)} = (\partial_\mu \phi_c^*)(\partial_\nu \phi_c) + (\partial_\nu \phi_c^*)(\partial_\mu \phi_c) - g_{\mu\nu} (g^{\alpha\beta} \partial_\alpha \phi_c^* \partial_\beta \phi_c - m^2 |\phi_c|^2) + \xi (2R_{\mu\nu} - g_{\mu\nu} R) |\phi_c|^2 - 2\xi (D_\mu D_\nu - g_{\mu\nu} D^2) |\phi_c|^2$$

The trace of this tensor:

$$T_{\mu}^{(c)\mu} = 2(6\xi - 1) \partial_\mu \phi_c^* \partial^\mu \phi_c + 2\xi(6\xi - 1) R |\phi_c|^2 + 4(1 - 3\xi) m^2 |\phi_c|^2$$

NB. For $\xi = 1/6$ and $m = 0$ the trace vanishes.

- For the real field ϕ_r the EM tensor has the twice smaller coefficients.

Fields ϕ 's enter the equation of motion for R via the traces of their EM tensors.

Decay into a pair of minimally coupled massless scalars

The action for the complex massless scalar field with minimal coupling to gravity:

$$S_c^{(00)}[\phi_c] = \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi_c^* \partial_\nu \phi_c$$

Corresponding equation of motion:

$$\ddot{\phi}_c + 3H\dot{\phi}_c - \frac{1}{a^2} \Delta \phi_c = 0$$

Conformally rescaled field and the conformal time:

$$\chi_c = a(t)\phi_c, \quad d\eta = dt/a(t)$$

The curvature scalar is expressed through the scale factor as

$$R = -6 \left(\dot{H} + 2H^2 \right) = -6a''/a^3$$

The equation of motion for the conformally rescaled field χ :

$$\chi_c'' - \Delta \chi_c + \frac{1}{6} a^2 R \chi_c = 0$$

Curvature evolution

Action in conformal time:

$$S_c^{(00)}[\chi_c] = \int d\eta d^3x \left(\chi_c'^* \chi_c' - \vec{\nabla} \chi_c^* \vec{\nabla} \chi_c - \frac{a^2 R}{6} |\chi_c|^2 \right)$$

Equation for the scalaron evolution:

$$\begin{aligned} R'' + 2 \frac{a'}{a} R' + a^2 M_R^2 R &= \\ &= \frac{16\pi}{a^2} \frac{M_R^2}{M_{Pl}^2} \left[\chi_c'^* \chi_c' - \vec{\nabla} \chi_c^* \vec{\nabla} \chi_c + \frac{a'^2}{a^2} |\chi_c|^2 - \frac{a'}{a} (\chi_c^* \chi_c' + \chi_c'^* \chi_c) \right] \end{aligned}$$

Our aim is to derive a closed equation for R taking the average value of the χ -dependent quantum operators in presence of classical curvature field $R(\eta)$.

Technique: A.D. Dolgov, S. Hansen, *Equation of motion of a classical scalar field with back reaction of produced particles*, Nucl. Phys. **B 548** (1999) 408 [hep-ph/9810428].

Determination of the decay width of the scalaron

Closed integro-differential equation for R :

$$\begin{aligned}\ddot{R} + 3H\dot{R} + M_R^2 R &\simeq -\frac{1}{6\pi} \frac{M_R^2}{M_{Pl}^2} \frac{1}{a^4} \int_{\eta_0}^{\eta} d\eta_1 \frac{a^2(\eta_1) R''(\eta_1)}{\eta - \eta_1} \\ &\simeq -\frac{1}{6\pi} \frac{M_R^2}{M_{Pl}^2} \int_{t_0}^t dt_1 \frac{\ddot{R}(t_1)}{t - t_1}\end{aligned}$$

- This equation is naturally **non-local in time** since the effect of particle production depends upon all the history of the system evolution.
- Transforms into ordinary differential equation for harmonic oscillations of R .

EA, A.Dolgov, L. Reverberi, *Cosmological evolution in R^2 gravity*, JCAP **02** (2012) 049

Solution:

$$R = R_{amp} \cos(\omega t + \theta) \exp(-\Gamma t/2)$$

- R_{amp} is the slowly varying amplitude of R -oscillations, θ is a constant phase
- ω and Γ is to be determined from the equation.

Assuming that Γ is small and neglecting $3H\dot{R}$ -term, we obtain:

$$\begin{aligned} & [(-\omega^2 + M_R^2) \cos(\omega t + \theta) + \Gamma\omega \sin(\omega t + \theta)] e^{-\Gamma t/2} = \\ & \frac{1}{6\pi} \frac{\omega^2 M_R^2}{M_{Pl}^2} e^{-\Gamma t/2} \int_0^{t-t_0} \frac{d\tau}{\tau} [\cos(\omega t + \theta) \cos \omega\tau + \sin(\omega t + \theta) \sin \omega\tau] \end{aligned}$$

- **1st term:** logarithmically divergent \implies mass renormalization, included into M_R .
- **2nd term:** finite and can be analytically calculated at large ωt as:

$$\int_0^\infty \frac{d\tau}{\tau} \sin \omega\tau = \frac{\pi}{2}$$

Comparing the l.h.s. and r.h.s. we can conclude that $\omega = M_R$ and the width of the scalaron decay into a pair of "charged" massless minimally coupled scalars is:

$$\Gamma_c = \frac{M_R^3}{12M_{Pl}^2}$$

The width of the decay into a pair of neutral identical particles is twice smaller:

$$\Gamma_r = \frac{M_R^3}{24M_{Pl}^2}$$

Decay into conformally coupled massive scalars

Action of a real **massive** scalar field ϕ_r with **non-minimal coupling** to gravity:

$$S_r^{(m,\xi)}[\phi_r] = \frac{1}{2} \int d^4x \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi_r \partial_\nu \phi_r + \xi R \phi_r^2 - m^2 \phi_r^2)$$

leads to the equation of motion:

$$\ddot{\phi}_r + 3H\dot{\phi}_r - \frac{1}{a^2} \Delta \phi_r + (m^2 - \xi R) \phi_r = 0$$

Conformally rescaled field and the conformal time:

$$\chi = a(t)\phi, \quad d\eta = dt/a(t)$$

Correspondingly EoM transforms into:

$$\chi'' - \Delta \chi + \left(\frac{1}{6} - \xi \right) a^2 R \chi + m^2 a^2 \chi = 0$$

Particle production by external time-dependent field $V(t)$

- C. Bambi, A.D. Dolgov, *Introduction to Particle Cosmology*, Springer, 2015.

For the (inflaton) field with the harmonic dependence on time:

$$V(\eta) = V_0 \cos(\Omega_c \eta + \theta)$$

the field χ satisfies the equation:

$$\chi'' - \Delta\chi + V(\eta)\chi = 0.$$

- Since $dt = a d\eta$, and $\Omega dt = \Omega_c d\eta$, the conformal frequency is $\Omega_c = a\Omega = aM_R$.

The number density of particles created per unit of conformal time is

$$n'_\chi = \frac{V_0^2}{32\pi}, \quad \text{where} \quad n_\chi \sim \chi \partial_\eta \chi \sim a^3 n_\phi$$

In physical time with $\chi = a\phi$:

$$\dot{n}_\phi = \frac{V_0^2}{32\pi a^4}, \quad \dot{\rho}_\phi = m\dot{n}_\phi, \quad \Gamma = \frac{\dot{\rho}_\phi}{\rho_\phi}$$

Decay into conformally coupled scalars: $\xi = 1/6$, $m \neq 0$, but $m \ll M_R$

The interaction leading to the particle production:

$$V = m^2 a^2(t)$$

Using the solution:

$$H = \frac{\dot{a}}{a} = \frac{2}{3t}(1 + \sin(M_R t + \theta))$$

we find

$$V = m^2 a^2(t) \approx m^2 t^{4/3} \exp\left(1 - \frac{4 \cos(M_R t + \theta)}{3tM_R}\right) \rightarrow \frac{4m^2 t^{1/3}}{3M_R} \cos(M_R t + \theta)$$

Decay width for conformal massive scalars:

$$\Gamma(\xi = 1/6, m \neq 0) = \frac{m^4}{6M_R M_{Pl}^2}$$

The energy release from ϕ decay into the primeval plasma:

$$\dot{\rho}_\phi = \Gamma(\xi = 1/6; m \neq 0) \rho_\phi = \frac{m^4}{36\pi t^2 M_R}$$

Decay into fermions

We consider fermion-antifermion pair production by oscillating scalar field

$$\Phi = \Phi_0 \cos M_R t = \Phi_0 \frac{e^{iM_R t} + e^{-iM_R t}}{2}, \quad \Phi = \frac{M_{Pl}}{\sqrt{48\pi} M_R^2} R$$

The density of fermions produced per unit conformal time:

$$n'_\psi = \frac{|g|^2 M_R^2 \Phi_0^2}{16\pi} = a^4 \dot{n}_\psi$$

Particle production is induced by the oscillation of the scale factor:

$$g\Phi_0 \rightarrow m_\psi a(t) \rightarrow \frac{2m_\psi a}{3tM_R}$$

The width of the scalaron decay into a pair of fermions with the mass m_ψ :

$$\Gamma_\psi = \frac{\dot{\rho}_\psi}{\rho_\Phi} = \frac{m_\psi^2 M_R}{36\pi t^2} \cdot \frac{6\pi t^2}{M_{Pl}^2} = \frac{m_\psi^2 M_R}{6M_{Pl}^2}$$

Decay into gauge bosons due to conformal anomaly

Under conformal transformation vector gauge bosons are not transformed:
 $A_\mu \rightarrow A_\mu \Rightarrow$ their EoM in conformal time is the same as in flat Minkowski metric.
 \Rightarrow Gauge bosons cannot be created by conformally flat gravitational field.

Conformal anomaly destroys this conclusion and allows for GB to be created.

A.D. Dolgov, *Conformal anomaly and the production of massless particles by a conformally flat metric*, Sov.Phys.JETP **54** (1981) 223 [Zh.Eksp.Teor.Fiz. **81**(1981)417].

Equation of motion of massless gauge field, A , with an account of the anomaly:

$$A'' - \Delta A + \alpha^2 \beta (\ln a)'' A' = 0$$

- α is the gauge coupling constant;
- For EM $U(1)$ -gauge group $\alpha = 1/137$ at low energies, for SUSY $\alpha \sim 10^{-2}$.

$$\beta = \frac{11}{3}N - \frac{2}{3}N_F,$$

N is the rank of the proper gauge group and N_F is the number of fermion families

The width of the decay into two gauge bosons

The number density of the produced gauge bosons per unit of physical time:

$$\dot{n}_g = \frac{\alpha^2 \kappa^2}{32\pi} \left(\frac{\ddot{a}}{a} \right)^2$$

Note that for quickly oscillating curvature R :

$$R = -6 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) \approx -6 \frac{\ddot{a}}{a}$$

In R^2 -cosmology we take the averaged value

$$\langle R^2(t) \rangle = (4M_R/t)^2 \langle \cos^2(M_R t) \rangle = 8M_R^2/t^2$$

In this case:

$$\dot{n}_g = \frac{\alpha^2 \kappa^2 M_R^2}{144\pi t^2}$$

The width of the scalaron decay into two gauge bosons:

$$\Gamma_g = \frac{\dot{\rho}_g}{\rho_\Phi} = \frac{\alpha^2 \kappa^2 M_R^3}{144\pi t^2} \cdot \frac{6\pi t^2}{M_{Pl}^2} = \frac{\alpha^2 \kappa^2 M_R^3}{24M_{Pl}^2}$$

SUSY Dark Matter

Dark matter:

- electrically neutral, since doesn't scatter light
- properties are practically unknown

Particles of many different types can be DM candidates.

Low energy minimal SUSY model:

- Predicts the existence of stable LSPs with mass $M_{LSP} \sim 100\text{--}1000$ GeV
- No manifestation at LHC \implies restricted parameter space open for SUSY

The LSP's energy density

$$\rho_{LSP} \sim \rho_{DM}^{(obs)} (M_{LSP}/1 \text{ TeV})^2, \quad \rho_{DM}^{(obs)} \approx 1 \text{ keV}/\text{cm}^3$$

- For $M_{LSP} \sim 1$ TeV, ρ_{LSP} is of the order of the observed DM energy density
- For larger masses LSPs would overclose the universe.

LSPs are practically excluded as DM particles in the conventional cosmology.

In $(R + R^2)$ -gravity the energy density of LSPs may be much lower \implies it reopens for them the chance to be the dark matter, if $M_{LSP} \geq 1000\text{TeV}$.

Evolution of X -particles in thermal plasma

Freezing of massive species $X \implies$ Zeldovich Eq., 1965 (Lee-Weinberg, 1977):

$$\dot{n}_X + 3Hn_X = -\langle\sigma_{ann}v\rangle (n_X^2 - n_{eq}^2), \quad n_{eq} = g_s \left(\frac{M_X T}{2\pi}\right)^{3/2} e^{-M_X/T}$$

- $\langle\sigma_{ann}v\rangle$ is the thermally averaged annihilation cross-section of X -particles
- n_{eq} is their equilibrium number density, g_s is the number of spin states.

For annihilation of the non-relativistic particles:

$$\langle\sigma_{ann}v\rangle = \sigma_{ann}v = \frac{\pi\alpha^2\beta_{ann}}{2M_X^2} \quad (\text{S-wave}),$$

$$\langle\sigma_{ann}v\rangle = \frac{3\pi\alpha^2\beta_{ann}}{2M_X^2} \frac{T}{M_X} \quad (\text{P-wave, Majorana fermions})$$

- M_X is a mass of X -particle, α is a coupling constant, in SUSY theories $\alpha \sim 0.01$
- β_{ann} is a numerical parameter \sim the number of annihilation channels, $\beta \sim 10$.

We assume that direct X -particle production by $R(t)$ is suppressed in comparison with inverse annihilation of light particles into $X\bar{X}$ -pair.

Scaloron decay into a pair of fermions

- EA, A. D. Dolgov, R. S. Singh, "Dark matter in $R + R^2$ cosmology," JCAP **04** (2019) 014, arXiv:1811.05399 [astro-ph.CO]

The decay width and the energy density:

$$\Gamma_f = \frac{M_R m_f^2}{6M_{Pl}^2}, \quad \rho_f = \frac{M_R m_f^2}{120\pi t}$$

The largest contribution into the cosmological energy density at scalaron dominated regime is presented by the decay into the heaviest fermion species.

We assume:

- The mass of the LSP is considerably smaller than the masses of the other decay products, $m_X < m_f$, at least as $m_X \lesssim 0.1m_f$.
- The direct production of X -particles by $R(t)$ can be neglected.

In such a case LSPs are dominantly produced by the secondary reactions in plasma, which was created by the scalaron production of heavier particles.

Kinetic equation for freezing of fermionic species

$$\frac{df}{dx} = -\frac{\alpha^2 \beta_{ann}}{2\pi^3 g_*} \frac{n_{in} M_R m_f^2}{m_X^6} \frac{f^2 - f_{eq}^2}{x^5}, \quad n_X = n_{in} \left(\frac{a_{in}}{a}\right)^3 f, \quad x = \frac{m_X}{T}$$

$n_{in} = 0.09 g_s m_X^3$ is the initial number density of X -particles at $T \sim m_X$.

$$\rho_X = m_X n_\gamma \left(\frac{n_X}{n_{rel}}\right)_{now} = 7 \cdot 10^{-9} \frac{m_f^3}{m_X M_R} \text{ cm}^{-3}$$

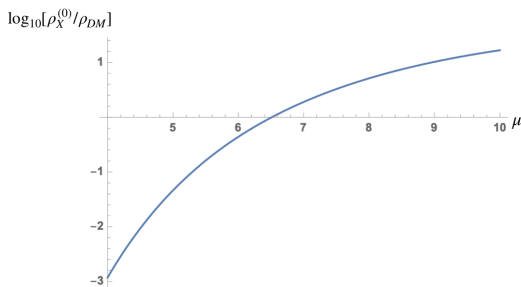
- $\alpha = 0.01$, $\beta_{ann} = 10$, $g_* = 100$, $n_\gamma \approx 412/\text{cm}^3$, $n_{rel} \approx \rho_{rel}/3T$
- If we take $m_f = 10^5$ GeV and $m_X = 10^4$ GeV, then $\rho_X \ll \rho_{DM}$.

ρ_X becomes comparable with the energy density of the cosmological DM, $\rho_{DM} \approx 1 \text{ keV}/\text{cm}^3$, if $m_X \sim 10^6$ GeV, $m_f \sim 10^7$ GeV:

$$\rho_X = 0.23 \left(\frac{m_f}{10^7 \text{ GeV}}\right)^3 \left(\frac{10^6 \text{ GeV}}{m_X}\right) \frac{\text{keV}}{\text{cm}^3}$$

Scalaron decay into gauge bosons due to conformal anomaly

- X, \bar{X} are Majorana fermions \implies direct production by scalaron is forbidden.
- $X\bar{X}$ -pairs are produced through the inverse annihilation of relativistic particles in thermal plasma, created by the scalaron decay into gauge bosons



Log of the ratio of the energy density of X -particles to the observed energy density of DM as a function of $\mu = M_R/M_X$ calculated through [the Zeldovich equation](#).

X -particles may be viable candidates for the carriers of the cosmological dark matter, if their mass $M_X \approx 5 \cdot 10^{12}$ GeV.

Possible observations

According to our results, the mass of DM particles, with the interaction strength typical for supersymmetric ones, can be in the range from 10^6 to 10^{13} GeV.

Possibilities to make X-particles visible:

- 1 Annihilation effects in clusters of dark matter in galaxies and galactic halos, in which, according to
 - V. S. Berezinsky, V. I. Dokuchaev and Y. N. Eroshenko, *Small-scale clumps of dark matter*, *Phys. Usp.* **57** (2014) 1 [arXiv:1405.2204]the density of DM is many times higher than DM cosmological density.
- 2 The decay of superheavy DM particles, which could have a lifetime long enough to manifest themselves as stable DM, but at the same time lead to the possibly observable contribution to the UHECR spectrum.
- 3 Furthermore, instability of superheavy DM particles can arise due to Zeldovich mechanism through virtual black holes formation.

The existence of stable particles with interaction strength typical for SUSY and heavier than several TeV is in tension with conventional Friedmann cosmology.

R^2 -gravity opens a way to save life of such X -particles, because in this theory the density of heavy relics with respect to the plasma entropy could be noticeably diluted by radiation from the scalaron decay.

The range of allowed masses of X -particles to form cosmological DM depends upon the dominant decay mode of scalaron.

Dominant decay channel of the scalaron	Allowed M_X to form DM
Minimally coupled scalars mode: $\Gamma_s = \frac{M_R^3}{24M_{Pl}^2}$	$M_X \gtrsim M_R \approx 3 \cdot 10^{13} \text{ GeV}$
Massive fermions mode: $\Gamma_f = \frac{m_f^2 M_R}{6M_{Pl}^2}$	$M_X \sim 10^6 \text{ GeV}$
Gauge bosons mode: $\Gamma_g = \frac{\alpha^2 \kappa^2 M_R^3}{24M_{Pl}^2}$	$M_X \sim 5 \cdot 10^{12} \text{ GeV}$

The END

Thank You for Your Attention

Some comments

Two possible channels to produce massive stable X -particles:

- Directly through the scalaron decay into a pair $X\bar{X}$,
- By inverse annihilation of relativistic particles in thermal plasma.

Direct production of $X\bar{X}$ -pair by scalaron gives

$$\rho_X^{(0)} \approx \rho_{DM} \approx 1 \text{keV}/\text{cm}^3, \text{ if } M_X \approx 10^7 \text{ GeV}$$

"Catch-22":

- For such small mass thermal production results in too large ρ_X .
- For larger masses $\rho_X^{(0)}$ would be unacceptably larger than ρ_{DM} .

A possible way out:

- Since oscillating curvature scalar creates particles only in symmetric state, the direct production of X -particles is forbidden, if they are Majorana fermions, which must be in antisymmetric state.