Al- Farabi Kazakh National University,Almaty

The influence of low-lying resonances in the Coulomb breakup of ^{11}Be

Вклад низко-лежащих резонансных состояний в Кулоновский развал гало ядра ^{11}Be

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Overview:

- *The aim of investigation, the relevance of topic*
- *Halo nuclei*
- *Formulation of the problem*
- *Theoretical description: Stationary and Time-dependent SE*
- *Results: the contribution of resonance states into breakup cross section:*

  a) for intermediate beam energies (69 & 72 MeV/nucleon)

  b) for low beam energies (<30 MeV/nucleon)

- Contribution to breakup of nuclear interaction between projectile and target
- How good is linear-trajectory approach for projectile motion at low energies
- *Conclusion and further research*

- **The aim of work**

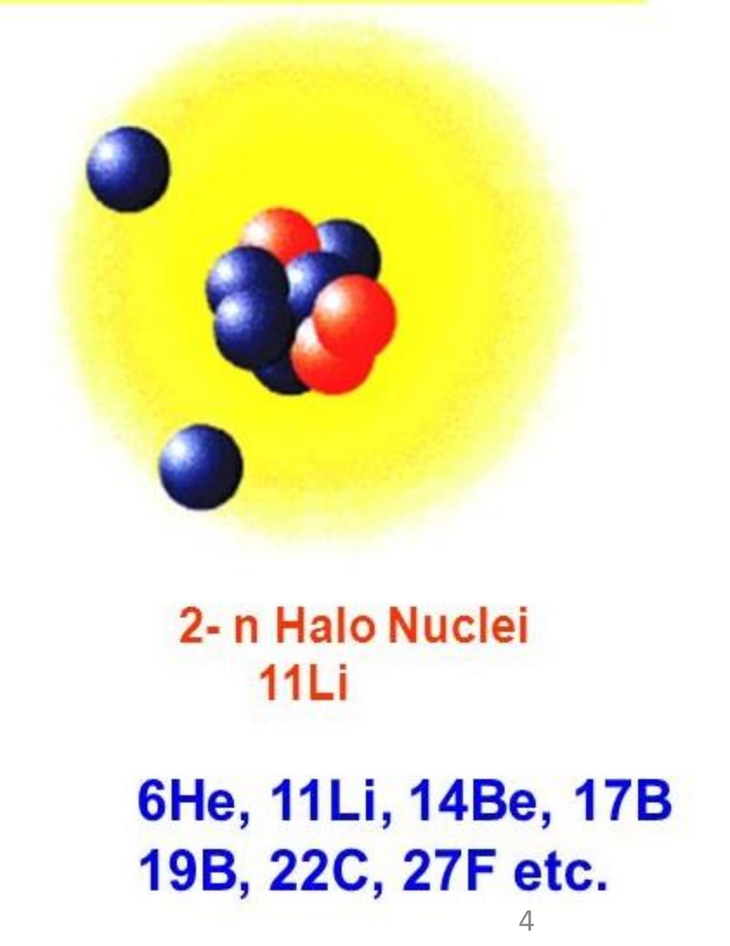
Investigation of low-lying resonances in **the Coulomb breakup of ^{11}Be** halo nuclei on heavy target (^{208}Pb) from intermediate (70 MeV/nucleon) to low energies (5 MeV/nucleon) within non-perturbative time-dependent approach.

- **Relevance of the research topic**

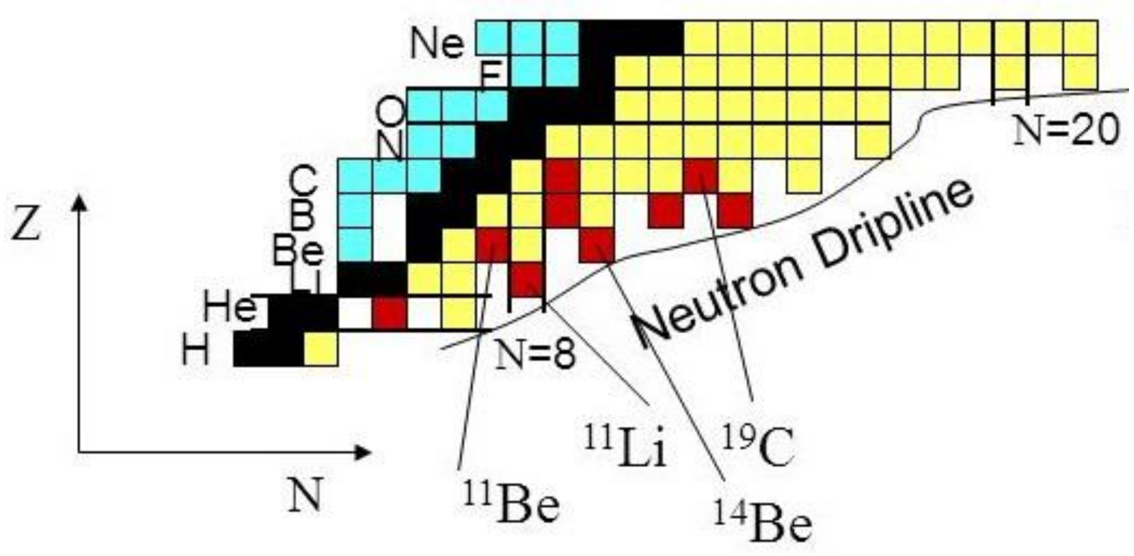
The **halo** is one of the most intensively studied objects in modern nuclear physics. **Coulomb breakup** is one of the main tools for studying the halo nuclei. The breakup could be considered as a transition of a neutron from halo nucleus to the continuum, due to varying Coulomb field between the nucleus and the target in collisions. **The breakup cross section** provides a useful information about the structure of the halo.

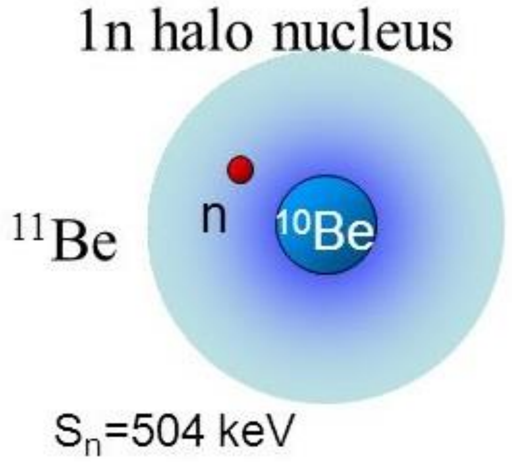
HALO

The neutron halo effect is caused by the presence of weakly bound states of neutrons located near the continuum. The small value of the binding energy of a neutron (or a group of neutrons) and the short-range nature of nuclear forces lead to the tunneling of neutrons into the outer peripheral region over large distances from the core of the nucleus. The mean radii of the orbits of certain nucleons of these nuclei may be larger than the range of nuclear interaction with other nucleons.



Among the halo nuclei, the **^{11}Be nucleus** is of particular importance, since the relative simplicity of its structure allows accurate theoretical studies. In fact, the bound states of the **^{11}Be nucleus** can be described quite well as a **^{10}Be core** and a weakly bound **neutron**.





There were a number of techniques developed for the calculation of Coulomb breakup for high energies $\gtrsim 70$ MeV/nucleon:

- **perturbative expansion**
  [S. Typel and G. Baur, Phys. Rev. C **50**, 2104 (1994)],
  [T. Kido, K. Yabana, and Y. Suzuki, Phys. Rev. C **50**, R1276 (1994)]
- **adiabatic approximation**
  [J.A. Tostevin, S. Rugmai, and R.C. Johnson, Phys. Rev. C **57**, 3225 (1998)]
- **coupled-channels with a discretized continuum (CDCC)**
  [M. Kamimura, M. Yahiro, Y. Iseri, H. Kameyama,et al., Prog. Theor. Phys. Suppl. **89**, 1 (1986)]
  [J. A. Tostevin, F. M. Nunes, and I. J. Thompson, Phys. Rev. C **63**, 024617 (2001)]
- **Coulomb wave Born approximation (CWBA)**
  [P. Banerjee,G.Baur, et.al., Phys. Rev. C **65**, 064602 (2002)]
- **dynamical eikonal approximation (DEA)**
  [D. Baye, P. Capel, and G. Goldstein, Phys. Rev. Lett. **95**, 082502 (2005)]

- **Non-perturbative: integration of 3D time-dependent Schrodinger equation (TDSE)**
  [V. S. Melezhik and D. Baye, Phys. Rev. C 59, 3232 (1999)]
  [V. S. Melezhik and D. Baye, Phys. Rev. C 64, 054612 (2001)]
  [P. Capel, D. Baye and V. S. Melezhik, Phys. Rev. C 68, 014612 (2003)]

Formulation and scheme of the solving problem

NON-STATIONARY PROBLEM

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

STATIONARY PROBLEM

$$H\psi = E\psi$$

Numerical resolution of 3D time-dependent SE (TDSE):

- *Solution of TDSE on 2D angular grid (discrete-variable representation (DVR) or Lagrange mesh) and 1D radial grid (quasiuniform finite-difference approximation).*
- *Splitting-up method for time evolution of the system*

Numerical methods of solving stationary SE:

- *Inverse iteration method in the subspace, sweep method, finite-difference method*

$$\begin{cases} \psi(0,t) = \psi(r_m,t) = 0, & r_m \to \infty \\ \psi(r,t_{in}) = \varphi_{1s}(r) \end{cases}$$

Solving the boundary value problem is the initial step of TDSE.

Stationary Schrodinger equation:

$$H\psi_{Nlm} = E_N\psi_{Nlm} \qquad (1)$$

with boundary conditions: $\begin{cases} \psi_{Nlm}(\boldsymbol{r}=\boldsymbol{0}) = \boldsymbol{const}; \\ \psi_{Nlm}(\boldsymbol{r}\to\infty)=\boldsymbol{0} \end{cases}$

The Hamiltonian of the interaction: $H_0(\boldsymbol{r}) = -\frac{\hbar^2}{2\mu}\Delta + V(\boldsymbol{r})$ **(2)**

$\mu = \frac{m_n \cdot m_c}{M}$ - reduced mass**;**

$$\psi_{Nlm}(r) = R_{Nl}(r)Y_{lm}(\theta,\varphi) \qquad (3)$$

the radial SE: $\left[-\frac{\hbar^2}{2\mu}\Delta + V(\boldsymbol{r}) + \frac{\hbar^2 l(l+1)}{2\mu\, r^2}\right]R_l(\boldsymbol{r}) = E\,R_l(\boldsymbol{r})$ **(4)**

Internal interaction: $V(\boldsymbol{r}) = V_l(\boldsymbol{r}) + V_l^s(\boldsymbol{r})\,\boldsymbol{l}\cdot\boldsymbol{s}$ **(5)**

Wood-Saxon potential: $V_l(\boldsymbol{r}) = -V_l\, f(\boldsymbol{r}, R_0, a)$

where $f(\boldsymbol{r}, R_0, a) = \left[1 + exp\left(\frac{\boldsymbol{r}-R_0}{a}\right)\right]^{-1}$ **(6)**

Spin-orbit interaction: $V_l^s(\boldsymbol{r}) = V_{ls}\frac{\boldsymbol{1}}{\boldsymbol{r}}\frac{\boldsymbol{d}}{\boldsymbol{dr}}\, f(\boldsymbol{r}, R_0, a)$ **(6’)**

*[P.Capel, D.Baye and V. S. Melezhik, Phys. Rev. C **68**, 014612 (2003)]*

Parameters of potential

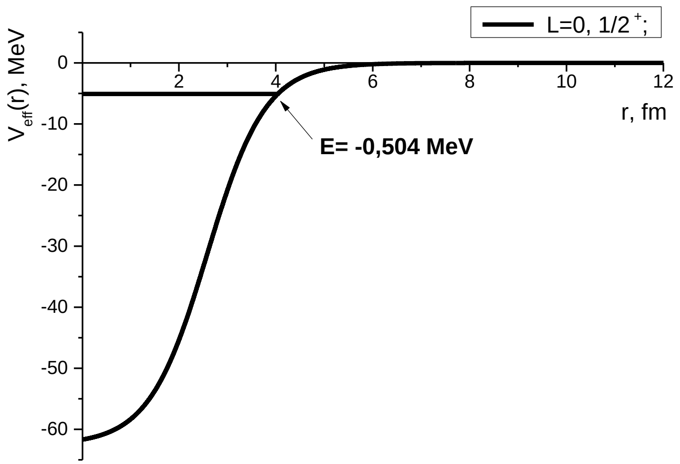
$V_{l\ even}$ (MeV)	$V_{l\ odd}$ (MeV)	V_{ls} (MeV fm^2)	a (fm)	R_0 (fm)	States	
62.52	39.74	21.0	0.6	2.585	$1/2^+$, $1/2^-$, $5/2^+$, $3/2^+$	[1]
-	6.8*	21.0	0.35*	2.5*	$3/2^-$	It was found in the present investigation

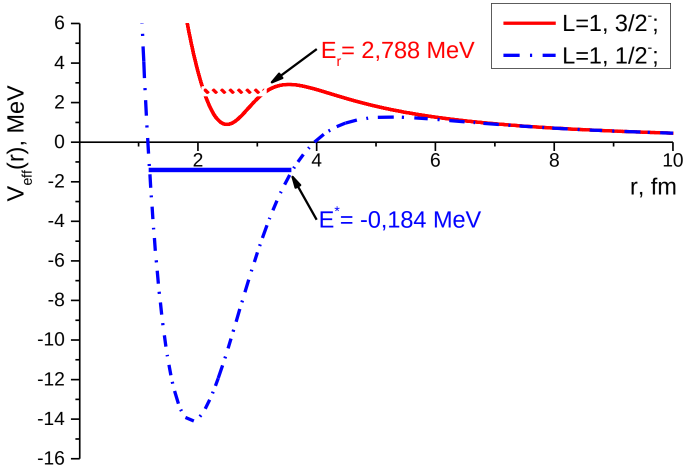
[1] P. Capel, G. Goldstein, and D. Baye, Phys. Rev. C 70, 064605 (2004).

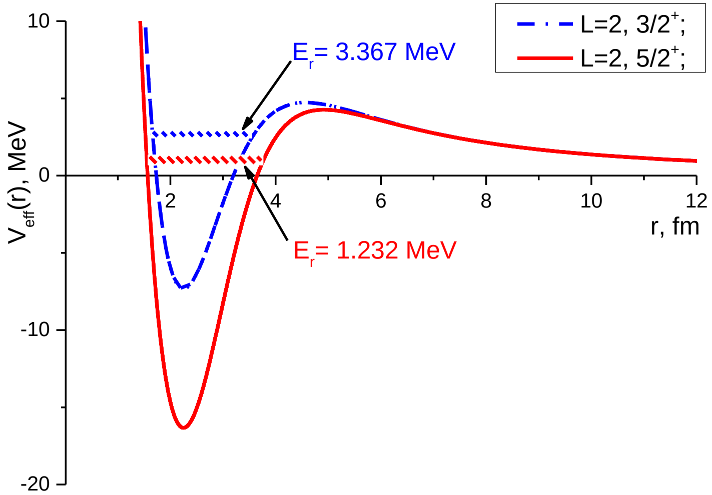
$$V_{eff}(\mathbf{r}) = V_l(\mathbf{r}) + V_l^S(\mathbf{r})\mathbf{l}\cdot\mathbf{s} + \frac{\hbar^2 l(l+1)}{2\mu r^2},$$

Woods-Saxon form $V_l(\mathbf{r}) = -V_l\, f(\mathbf{r}, R_0, a);\ f(\mathbf{r}, R_0, a) = [1 + \exp(\frac{\mathbf{r}-R_0}{a})]^{-1}$

Spin-orbit interaction $V_l^S(\mathbf{r}) = V_{ls}\frac{\mathbf{1}}{\mathbf{r}}\frac{\mathbf{d}}{\mathbf{dr}} f(\mathbf{r}, R_0, a)$

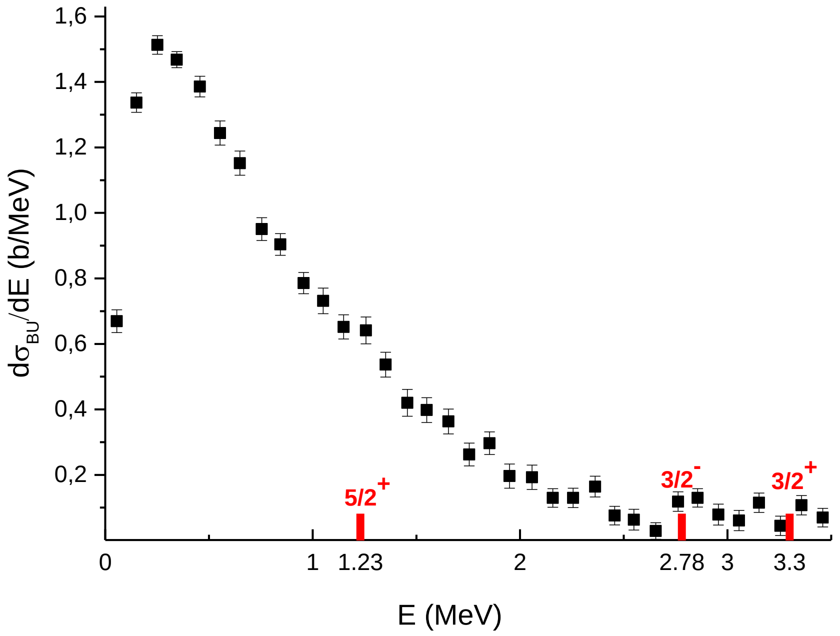




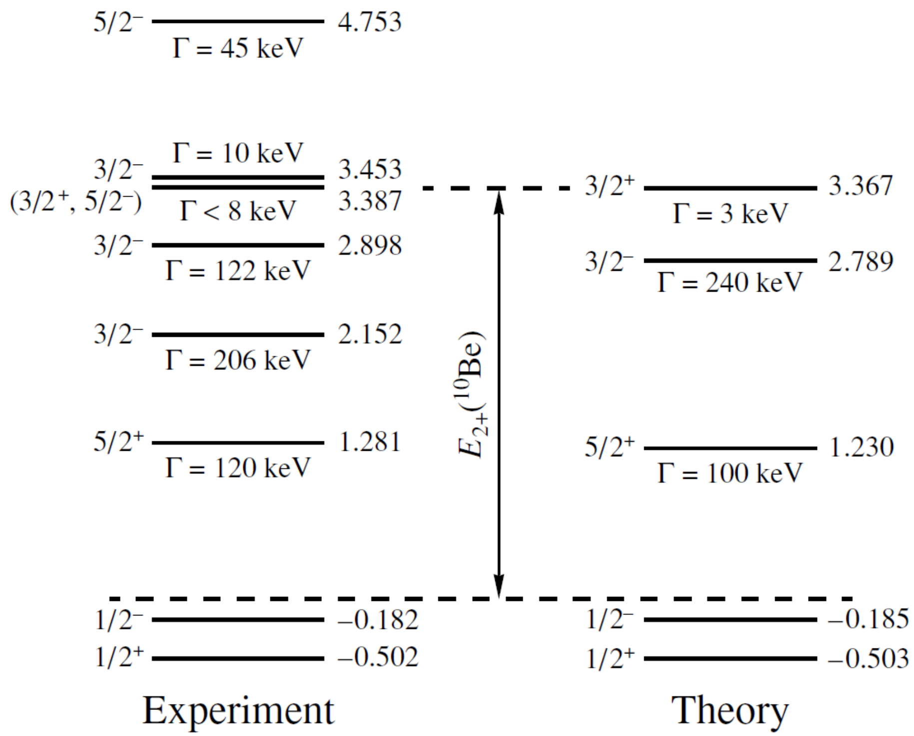


Influence of resonance states to the breakup cross section of ^{11}Be

	5/2+		3/2-		3/2+	
	E, MeV	Γ, keV	E, MeV	Γ, keV	E, MeV	Γ, keV
Theory [1]	1.230	100	2.789	240	3.367	3
Exp. [2]	1.281	120	2.898	122	2.387	< 8



Experimental data of breakup cross sections of ^{11}Be+^{208}Pb → ^{10}Be+n+^{208}Pb at 69 MeV/nucleon
[*N. Fukuda, et. al., Phys. Rev. C 70, 054606 (2004)*]



[1] S.N. Ershov, J.S. Vaagen and M.V. Zhukov, Phys. of Atomic Nucl. 77(8), 989 (2014).
[2] National Nuclear Data Center, https://www.nndc.bnl.gov/

Time dependent Schrodinger equation

In order to describe the breakup reaction **^{11}Be+^{208}Pb →^{10}Be+n+^{208}Pb** we write TDSE:

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = H(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) = [H_0(\boldsymbol{r}) + V_C(\boldsymbol{r},t)]\Psi(\boldsymbol{r},t) \qquad (7)$$

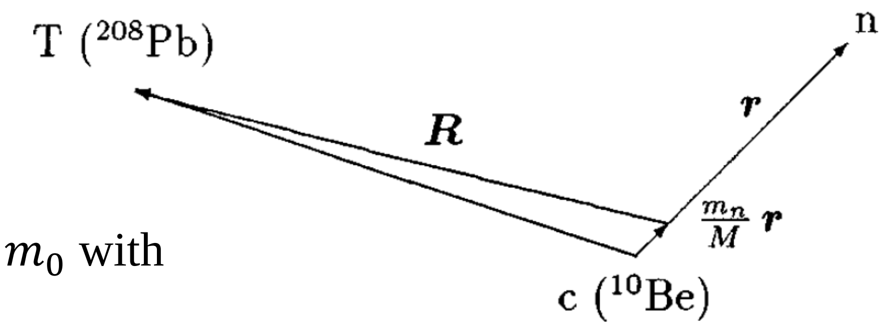
Internal Hamiltonian: $H_0(\boldsymbol{r}) = -\frac{\hbar^2}{2\mu}\Delta_r + V(\boldsymbol{r})$ (8)

TD Coulomb potential: $V_C(\boldsymbol{r},t) = \frac{Z_C Z_T e^2}{\left|\frac{m_n r}{M}+\boldsymbol{R}(t)\right|} - \frac{Z_C Z_T e^2}{\boldsymbol{R}(t)}$ (9)

here Z_C and $\mathrm{Z_T}$ - charge numbers of the **core** and **target**

$$\boldsymbol{R}(t) = \boldsymbol{b} + \boldsymbol{v_0}t$$

***R(t)* –** relative coordinate between the target and projectile



As an initial condition at $t = -\infty$, the system is in its ground state $l_0 j_0 m_0$ with energy E_0<0,

$$\psi^{m_0}(\boldsymbol{r},-\infty) = \varphi_{l_0 j_0 m_0}(E_0,\boldsymbol{r})$$

*[V. S. Melezhik and D. Baye, Phys. Rev. C **59**, 3232 (1999)]*

Angular-subspace discretization

For solving 3D TDSE we seek a solution $\psi(\boldsymbol{r},t)$ in spherical coordinates (r,Ω)≡ (r,θ,φ) as an expansion

$$\Psi(\boldsymbol{r},t) = \frac{1}{r}\sum_{s}\sum_{\nu j}^{N}\varphi_{\nu}(\Omega)(\varphi^{-1})_{\nu j}\psi_{j}^{s}(r,t) \qquad (10)$$

$\varphi_{\nu}(\Omega)$-is a 2D basis, they are related with spherical harmonics at lower l values

$$\varphi_{\nu}(\Omega) = \sum_{\nu'=\{l\prime,m\prime\}} C_{lm}^{l'm'}P_{l'}^{m'}(\theta)e^{im'\phi} \qquad (11)$$

$C_{lm}^{l'm'} = \delta_{ll'}\delta_{mm'}$; $\Omega_{j} = \left(\theta_{j_\theta},\varphi_{j_\varphi}\right)$ is equal to the basis functions in (10).

The sum over ν is equivalent to the double sum

$$\sum_{\nu=1}^{N} = \sum_{l=0}^{\sqrt{N}-1}\ \sum_{m=-l}^{l} \qquad (12)$$

- θ_{j_θ} –is chosen from the zeros of $P_{\sqrt{N}}(\cos\theta_{j_\theta})$, for φ_{j_φ}=$\pi(2j_\varphi - 1)/\sqrt{N}$ $(where\ \ N = N_\theta \times N_\varphi)$
- $(\varphi^{-1})_{\nu j}$-are the elements of the matrix NxN inverse to the matrix with the elements $\varphi_{j\nu}$=$\varphi_{\nu}\left(\Omega_{j}\right)$.
- $\varphi_{\nu}\left(\Omega_{j}\right)$ functions are orthogonal at the Gauss approximation:

$$\int \varphi_{\nu}^{*}(\Omega)\varphi_{\nu'}(\Omega)d\Omega = \sum_{j}\lambda_{j}\,\varphi_{\nu j}^{*}\varphi_{\nu' j} = \delta_{\nu\nu'} \qquad (13)$$

- λ_{j} =$2\pi/\sqrt{N}$ are the products of the standard Gauss−Legendre weights.

[1] [V. S. Melezhik and D. Baye, Phys. Rev. C 59, 3232 (1999)]
[2] Numerical recipes in Fortran 77 : the art of scientific computing/ William H. Press I.,[et al.]. – 2nd ed.1992

The components $\psi_j^s(r,t)$ correspond to $\psi\left(r,\Omega_j,t\right)|s>$ where $|s>=|\pm\frac{1}{2}>$ is a spin state and $\psi\left(r,\Omega_j,t\right)$ is a complex function defined on the angular grid Ω_j. Let us introduce the 2N-component vector $\boldsymbol{\Psi}(r,t)=\{\lambda^{\frac{1}{2}}{}_j\psi_j^s(r,t)\}$.

With respect to expansion $\psi(\boldsymbol{r},t)=\frac{1}{r}\sum_{\nu j}^{N}\varphi_\nu(\Omega)(\varphi^{-1})_{\nu j}\,\psi_j^s(r,t)$ the problem is reduced to a system of SE:

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{\Psi}(r,t)=\left[\widehat{H}_0(r)+\hat{h}(r,t)\right]\boldsymbol{\Psi}(r,t) \qquad (14)$$

$\widehat{H}_0(r)$ and $\hat{h}(r,t)$ are 2Nx2N matrix operators on the grid.

The elements of $\widehat{H}_0(r)$ are defined as:

$$H_{0kj}^{ss'}(r)=\left\{-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2}\delta_{kj}+(\lambda_k\lambda_j)^{-\frac{1}{2}}\sum_{\nu=\{l,m\}}^{N}(\varphi^{-1})_{k\nu}\right.$$
$$\left.\times\left[V_l^s(r)+V_l(r)+\frac{\hbar^2 l(l+1)}{2\mu r^2}\right](\varphi^{-1})_{\nu j}\right\}\delta_{ss'} \qquad (15)$$

TD Coulomb operator is diagonal:

$$h_{kj}^{ss'}(r,t)=\left[V_C(r,\Omega_k,t)\delta_{ss'}\right]\delta_{kj} \qquad (16)$$

and does not require multipole expansion as in some approaches.

*[1] V. S. Melezhik and D. Baye, Phys. Rev. C **59**, 3232 (1999).*
*[2] P.Capel, D.Baye and V. S. Melezhik, Phys. Rev. C **68**, 014612 (2003).*

The breakup component is obtained by eliminating the bound states from the calculated wave packet

$$|\Psi_{\mathrm{bu}}(\boldsymbol{r},t)\rangle = \left(1 - \sum_{\vartheta\in\mathrm{bound}} |\phi_\vartheta(\boldsymbol{r})\rangle\langle\phi_\vartheta(\boldsymbol{r})|\right)|\Psi(\boldsymbol{r},t)\rangle, \qquad (17)$$

where the sum runs over two bound states of ^{11}Be obtained from $H_0\phi_{ljm}(E,\mathbf{r}) = E\phi_{ljm}(E,\mathbf{r})$.
The total breakup cross section is calculated as a function of the energy E of the relative motion between the emitted neutron and the core nucleus by the formula

$$\frac{d\sigma_{bu}}{dE}(E) = \frac{4\mu k}{\hbar^2}\int_{b_{min}}^{b_{max}} \sum_{j=l+s}\sum_{lm} \left|\int j_l(kr)Y_{lm}(\hat{r})\Psi_{\mathrm{bu}}(\mathbf{r},T_{\mathrm{out}})d\boldsymbol{r}\right|^2 bdb \qquad (18)$$

- The integral is calculated numerically over the whole interval from b_{min}=12 fm to b_{max}=400 fm.

The choice of edges of integration b_{min} and b_{max} must be carefully tested.

- $j_l(kr)$- spherical Bessel functions, k-is the wave number, $k = \frac{\sqrt{2\mu E}}{\hbar}$;

*[V. S. Melezhik and D. Baye, Phys. Rev. C **59**, 3232 (1999).]*

$$\frac{d\sigma_{bu}}{dE}(E) = \frac{4\mu k}{\hbar^2}\int_{b_{min}}^{b_{max}} \sum_{j=l+s}\sum_{lm} \left| \int j_l(kr) Y_{lm}(\hat{r}) \Psi_{\rm bu}(\mathbf{r}, T_{\rm out}) d\mathbf{r} \right|^2 bdb \qquad (18)$$

- Time evolution starts at initial time $T_{\rm in}$ and stops at final time $T_{\rm out}$ by iteration over N_T time steps Δt as explained in [1]. The initial (final) time $T_{\rm in}$ ($T_{\rm out}$) has to be sufficiently big $|T_{in}|, T_{out} \to +\infty$ so as to allow the time-dependent potential $V_C(\boldsymbol{r}, t)$ to be negligible at the beginning (end) of the evolution process: **$T_{\rm in}$=-20$\hbar$/MeV and $T_{\rm out}$=20$\hbar$/MeV. The time step Δ*t* is fixed equal to 0.01 $\hbar$/MeV.**

- For discretizing with respect to the radial variable *r*, a sixth-order (seven point) finite-difference approximation on a quasiuniform grid has been used on the interval $r \in [0, r_m]$ with **$\boldsymbol{r_m}$=1200 fm**. The grid has been realized by the mapping $r \to x$ of the initial interval onto $x \in [0,1]$ by the formula $\boldsymbol{r = r_m(e^{\alpha x} - 1)/(e^{\alpha} - 1)}$ **, α=8** [2].

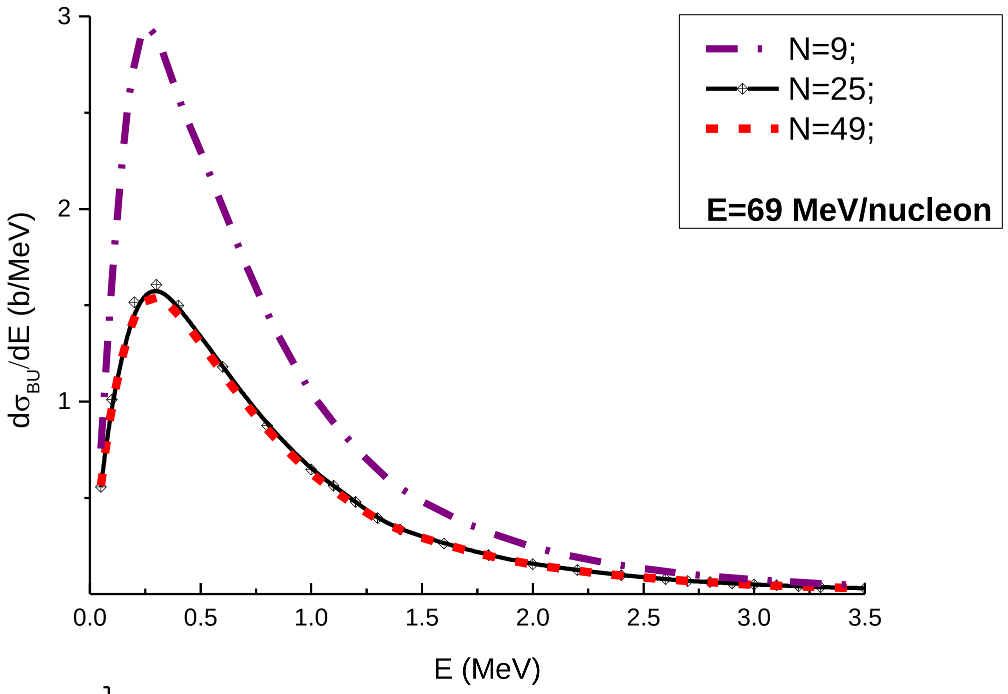
- The lower bound $b_{\rm min}$ is a cutoff related to the range of nuclear effects. The upper bound is in practice replaced by some value $b_{\rm max}$ whose choice must be carefully tested. In our calculations the edges of integration are chosen as **$b_{\rm min}$=12 fm, $b_{\rm max}$=400 fm** numerically, which give convergent result for this integral with accuracy about few percent.

[1] P. Capel, D. Baye, and V. S. Melezhik, Phys. Rev. C **68**, 014612 (2003).
[2] V. S. Melezhik, Phys. Lett. A **230**, 203 (1997).

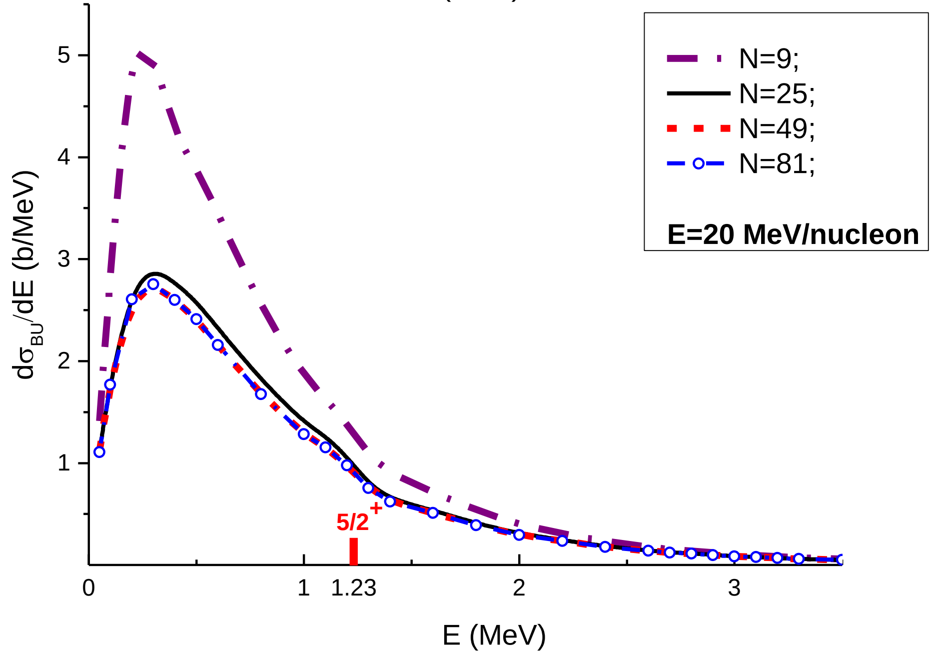
Convergence of the method for the breakup cross section $d\sigma(E, b_{max})/dE$ in (b/MeV) as a function of the upper bound of impact parameter b_{max} (fm) and relative energy E (MeV)

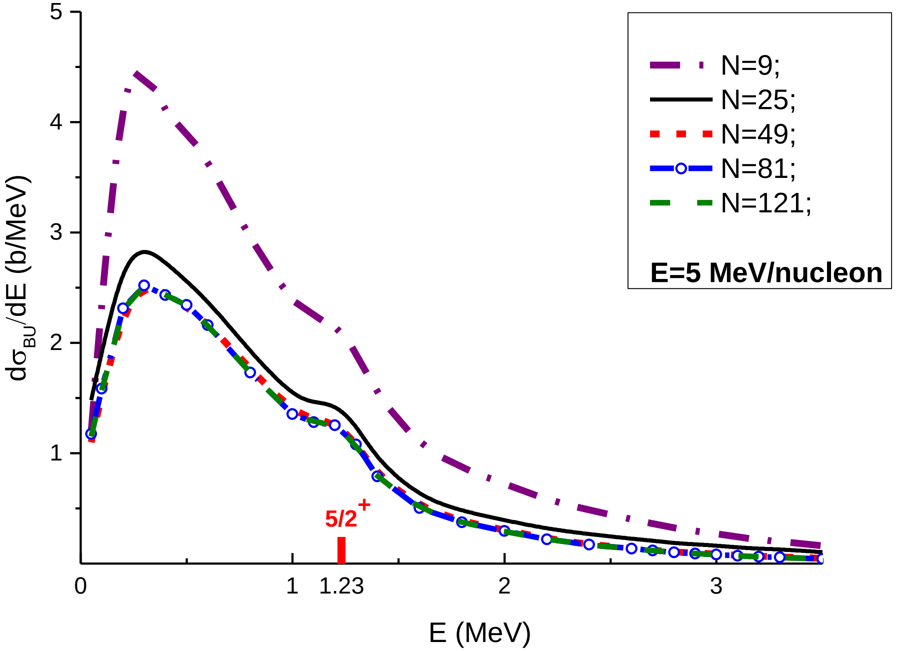
The total breakup cross section calculated for energy of 72 MeV/nucleon taking into account three resonance states . As it can be seen, the increasing of the value b_{max} gives a good convergence of cross section. The calculation is performed with N=25 ($N_\theta = 5, N_\varphi = 5$).

b_{max}	E=0.1	E=0.4	E=0.8	E=1.2	E=1.6	E=2.0	E=2.7	E=2.8	E=3.0	E=3.3
12	0.021	0.052	0.046	0.032	0.019	0.013	0.007	0.006	0.005	0.004
20	0.183	0.397	0.309	0.200	0.122	0.080	0.040	0.037	0.030	0.023
50	0.561	1.024	0.689	0.407	0.234	0.143	0.066	0.060	0.048	0.035
100	0.816	1.335	0.819	0.456	0.254	0.151	0.067	0.061	0.049	0.036
200	0.950	1.436	0.841	0.461	0.255	0.151	0.067	0.061	0.049	0.036
300	0.972	1.443	0.841	0.461	0.255	0.151	0.067	0.061	0.049	0.036
400	0.976	1.444	0.841	0.461	0.255	0.151	0.067	0.061	0.049	0.036



The convergence of the calculated breakup cross section $\mathrm{d}\sigma(\mathrm{E}, b_{max})/\mathrm{dE}$ over the number N of angular grid points with including three resonance states at 69, 20 and 5 MeV/nucleon.



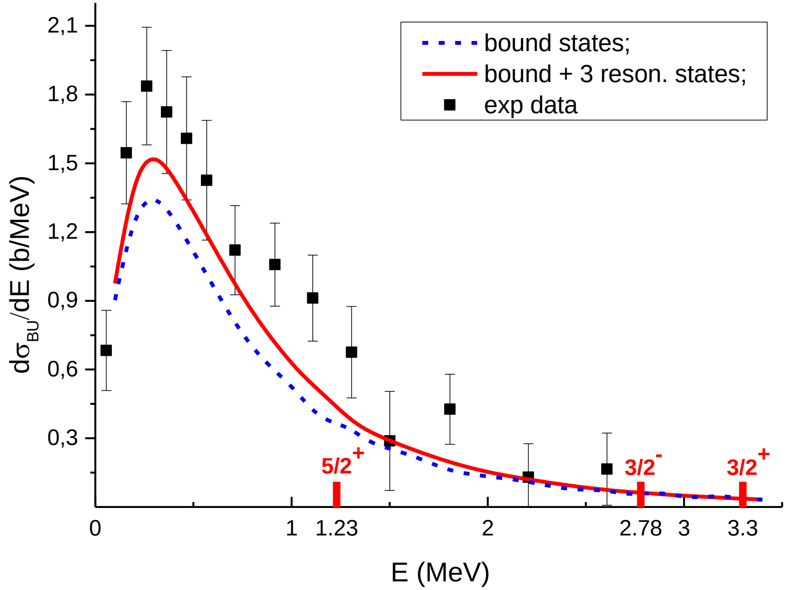


Influence of resonance states to the breakup cross section of ^{11}Be at 69 and 72 MeV/nucleon.

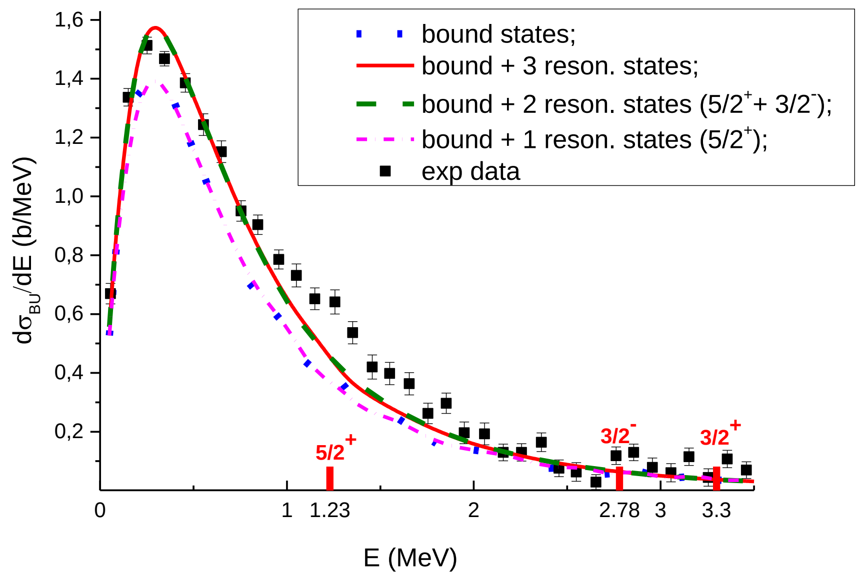
Energy, MeV	$d\sigma(E, b_{max})/dE$, b/MeV						
	72 MeV/nucleon		69 MeV/nucleon				
	bound states	b.s.+ 3 res	bound states (b.s.)	b.s.+5/2$^+$	b.s. + 5/2$^+$+3/2$^+$	b.s. + 5/2$^+$+3/2$^-$	b.s. + 3 res
E=0.1	0.903	0.976	0.936	0.936	1.007	1.011	1.011
E=0.3	1.369	1.549	1.420	1.421	1.595	1.606	1.606
E=0.8	0.676	0.841	0.704	0.708	0.865	0.875	0.875
E=1.0	0.534	0.623	0.555	0.562	0.639	0.647	0.648
E=1.2	**0.368**	**0.461**	**0.383**	**0.378**	**0.473**	**0.479**	**0.479**
E=2.0	0.133	0.151	0.138	0.137	0.155	0.156	0.156
E=2.7	**0.054**	**0.067**	**0.056**	**0.056**	**0.07**	**0.069**	**0.069**
E=2.8	0.060	0.061	0.061	0.062	0.064	0.063	0.063
E=3.0	0.042	0.049	0.043	0.044	0.051	0.050	0.050
E=3.3	**0.035**	**0.036**	**0.036**	**0.036**	**0.037**	**0.037**	**0.037**
E=3.4	0.032	0.033	0.032	0.033	0.035	0.034	0.034

Theoretical calculations of breakup cross sections with taking into account only bound states, with including one 5/2$^+$, two 3/2$^-$ and 5/2$^+$ and three resonances (5/2$^+$, 3/2$^-$, 3/2$^+$) in comparison with experimental data at 72 MeV/nucleon [1] and 69 MeV/nucleon [2] .

The calculations are performed for angular grid N=25 ($N_\theta = 5, N_\varphi = 5$).

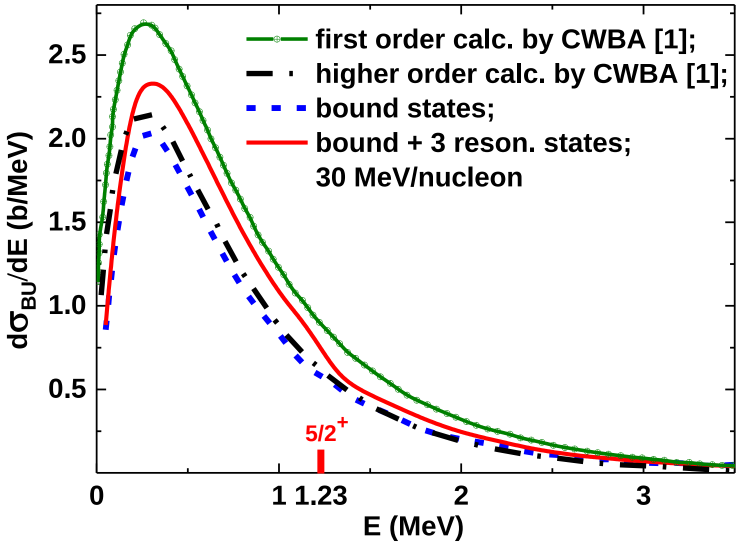


[1] T. Nakamura, et.al., Phys. Lett. B 331, 296 (1994).

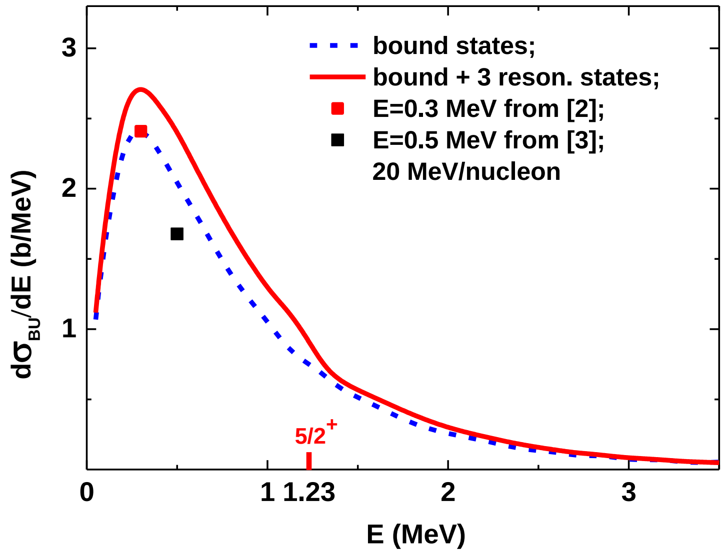


[2] N. Fukuda, et. al., Phys. Rev. C 70, 054606 (2004).

Breakup cross section of ^{11}Be at low beam energies



Comparing of our results of differential breakup cross section with the calculations of **[1]**, where authors investigated the breakup cross section of ^{11}Be by **Coulomb wave Born approximation (CWBA)** (resonance states were not including). Our calculations for the bound state coincide with that of the finite range **CWBA** of **[1]** at the beam energy of 30 MeV/nucleon.



Calculations of breakup cross section taking into account only bound states and with adding three resonances at a beam energy of 20 MeV/nucleon. Also, the points of total differential cross section at E=0.3 MeV from **[2]** and E=0.5 MeV from the calculation of **[3]** is presented, which was calculated within **dynamic eikonal approximation** (without including resonances).

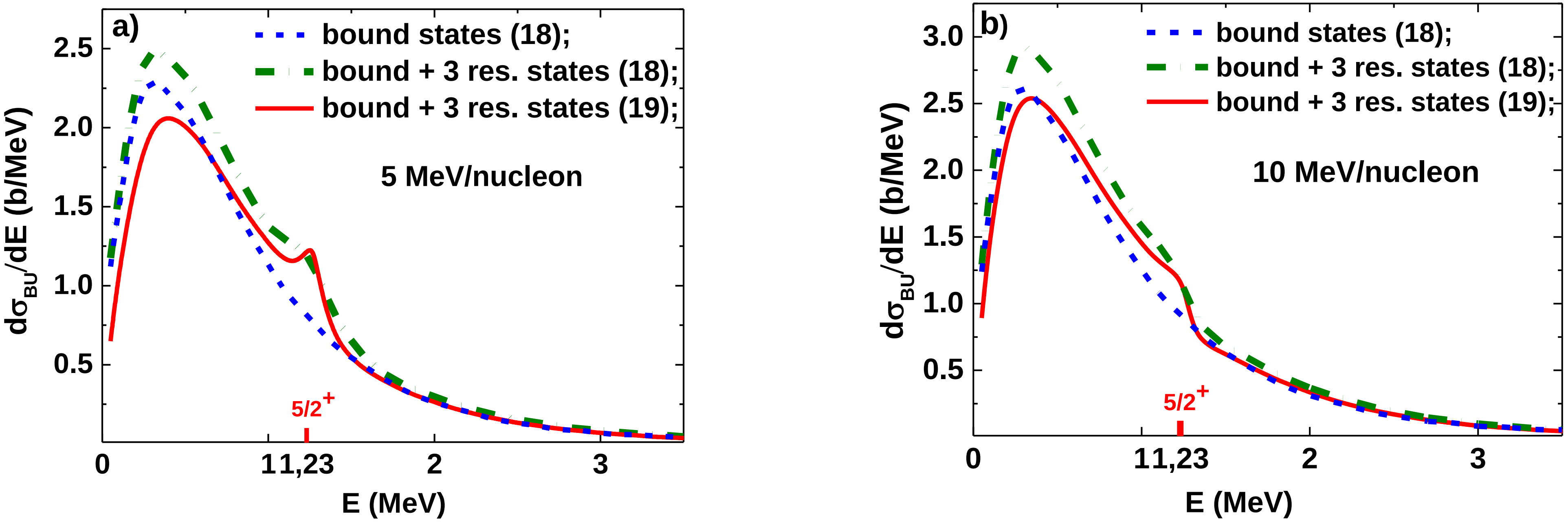
[1] P. Banerjee, G.Baur, et.al. Phys. Rev. C 65, 064602 (2002).
[2] G. Goldstein, D. Baye, and P. Capel, Phys. Rev. C 73, 024602 (2006).
[3] D. Baye, P. Capel, and G. Goldstein, Phys. Rev. Lett. 95, 082502 (2005).

Alternative formula for the breakup cross section including neutron interaction with the core in the final state of the breakup process :

$$\frac{d\sigma_{bu}}{dE}(E) = \frac{4\mu k}{\hbar^2}\int_{b_{min}}^{b_{max}} \sum_{j=l+s}\sum_{lm} \left| \int \varphi_{ljm}(k,r) Y_{lm}(\hat{r}) \Psi(\mathbf{r}, T_{\mathrm{out}}) d\mathbf{r} \right|^2 bdb \qquad (19)$$

- Here $\varphi_{ljm}(k,r)$ is the radial part of the eigenfunction of the Hamiltonian $H_0(r)$ $(H_0\phi_{ljm}(E,\mathbf{r}) = E\phi_{ljm}(E,\mathbf{r}))$ in the continuum spectrum $(E = \frac{k^2\hbar^2}{2\mu} > 0)$, normalized to $j_l(kr)$ as $kr \rightarrow \infty$ if $V(r)=0$.
- To find the states of the continuous spectrum of problem , we used the method of reducing the scattering problem to a boundary value problem, described in the work *[V. S. Melezhik and Chi-Yu Hu, PRL 90, 083202 (2003)]*.
- Summation over (l,m) in (19) includes all 16 partial waves up to $l_{max} = 3$ inclusive, as in (18). Since the wave functions $\varphi_{ljm}(k,r)$ of the continuum spectrum of the Hamiltonian are orthogonal to the states of the discrete spectrum of the same Hamiltonian, the elimination (17) of the bound states from the neutron wave packet after collision with the target is not required here.

Breakup cross section of ^{11}Be at low beam energies



The contribution to the breakup cross section of the resonant states and the neutron interaction with the core in the continuum at beam energies of 5 and 10 MeV/nucleon, a and b sides, respectively. The calculations are performed at N = 81.

$$\frac{d\sigma_{bu}}{dE}(E) = \frac{4\mu k}{\hbar^2}\int_{b_{min}}^{b_{max}} \sum_{j=l+s}\sum_{lm} \left| \int j_l(kr) Y_{lm}(\hat{r}) \Psi_{\rm bu}(\mathbf{r}, T_{\rm out}) d\boldsymbol{r} \right|^2 bdb \qquad (18)$$

$$\frac{d\sigma_{bu}}{dE}(E) = \frac{4\mu k}{\hbar^2}\int_{b_{min}}^{b_{max}} \sum_{j=l+s}\sum_{lm} \left| \int \varphi_{ljm}(k,r) Y_{lm}(\hat{r}) \Psi(\mathbf{r}, T_{\rm out}) d\boldsymbol{r} \right|^2 bdb \qquad (19)$$

Contribution to breakup of nuclear interaction between projectile and target

Following the approach of optical potential for the nuclear part $\Delta V_N(\boldsymbol{r}) = V_{CT}(r_{CT}) + V_{nT}(r_{nT})$ between the target and projectile nuclei interaction:

$$V(\boldsymbol{r},t) = V_C(\boldsymbol{r},t) + \Delta V_N(\boldsymbol{r}) \qquad (20)$$

Here r_{cT} *and* r_{nT} are the core-target $\boldsymbol{r}_{cT}(t) = \boldsymbol{R}(t) + m_n\boldsymbol{r}/M$ and neutron-target $\boldsymbol{r}_{nT} = \boldsymbol{R}(t) - m_C\boldsymbol{r}/M$ и relative variables and optical potentials V_{cT} и V_{nT} have the form:

$$V_{xT}(r_{xT}) = -V_x f(r_{xT}, R_R, a_R) - i\, W_x f(r_{xT}, R_I, a_I) \qquad (21)$$

with Woods–Saxon form factors $f(r_{xT}, R_R, a_R) = 1/(1+\exp(r_{xT} - R)/a)$, where x stands for either core or neutron. We use here the parameters of the optical potentials (21) from the paper [*Capel P, Baye D and Melezhik V (2003) PRC 68 014612*], which are given in Table:

c or n	$V_x(MeV)$	$W_x(MeV)$	$R_R(fm)$	$R_I(fm)$	$a_R(fm)$	$a_I(fm)$
^{10}Be	70.0	58.9	7.43	7.19	1.04	1.00
n	28.18	14.28	6.93	7.47	0.75	0.58

How good is linear-trajectory approach for projectile motion at low energies

In the **hybrid quantum-quasiclassical approach** simultaneously with the time-dependent Schrödinger equation (7) for the halo-nucleon wave function Ψ(r, t) we integrate the set of Hamilton equations:

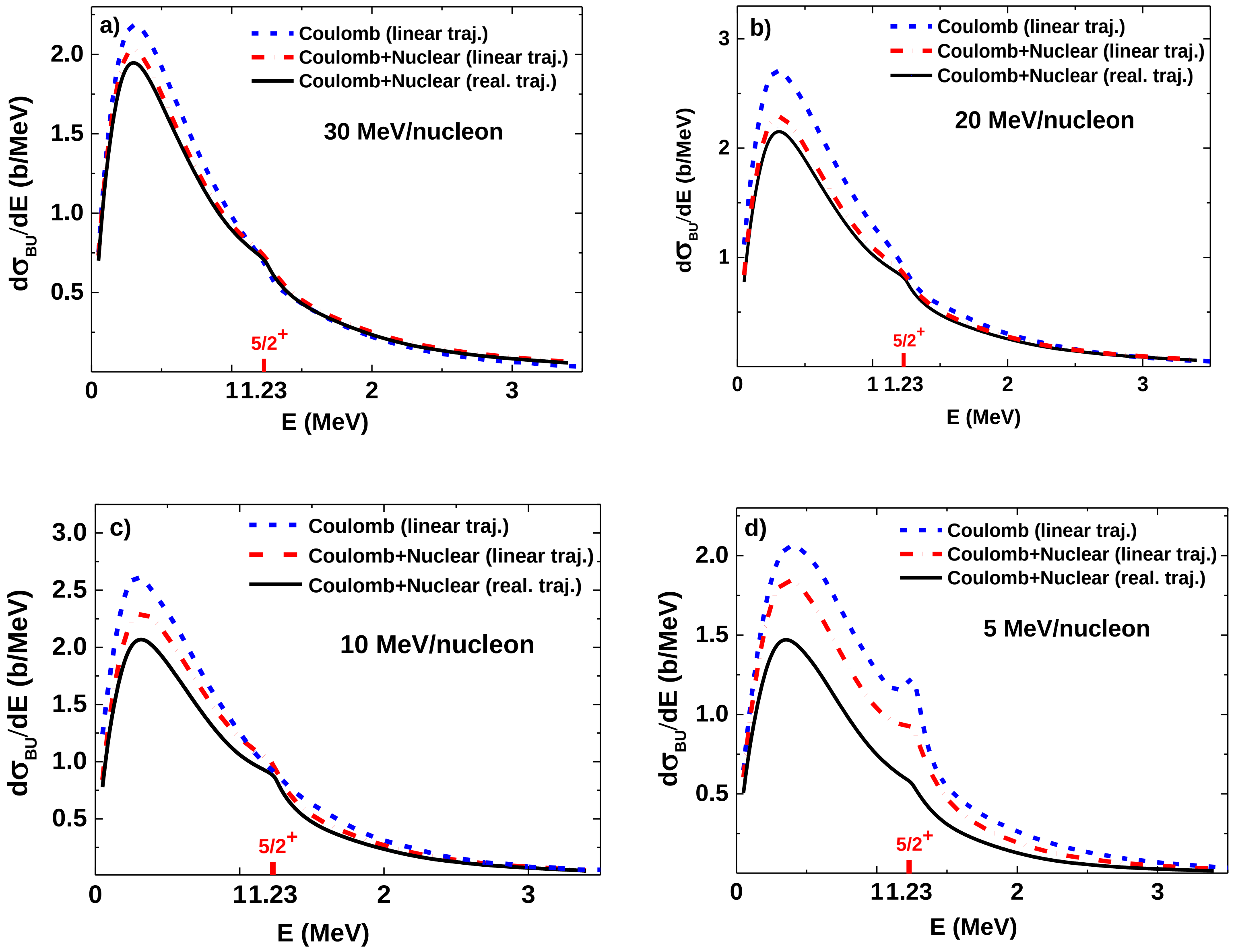
$$\frac{d}{dt}\boldsymbol{P} = -\frac{\partial}{\partial \boldsymbol{R}} H_{BP}\left(\boldsymbol{P},\boldsymbol{R},t\right),\ \frac{d}{dt}\boldsymbol{R} = -\frac{\partial}{\partial \boldsymbol{P}} H_{BP}\left(\boldsymbol{P},\boldsymbol{R},t\right) \tag{22}$$

describing relative projectile-target dynamics. Here, the classical Hamiltonian $H_{BP}(\mathbf{P},\mathbf{R},t)$ is given by

$$H_{BP}\left(\boldsymbol{P},\boldsymbol{R},t\right) = \frac{\boldsymbol{P}^2}{2M} + \left\langle \Psi(\boldsymbol{r},t) \middle| \frac{Z_C Z_T e^2}{\left|\frac{m_n r}{M} + \boldsymbol{R}(t)\right|} + \Delta V_N(\boldsymbol{r},t \Big) \middle| \Psi(\boldsymbol{r},t) \right\rangle \tag{23}$$

where the last term $\langle\Psi(\mathbf{r},t)|\ldots|\Psi(\mathbf{r},t)\rangle$ represents the quantum-mechanical average of the projectile-target interaction over the halo-nucleon density instantaneous distribution $|\Psi(\mathbf{r},t)|^2$ during the collision.

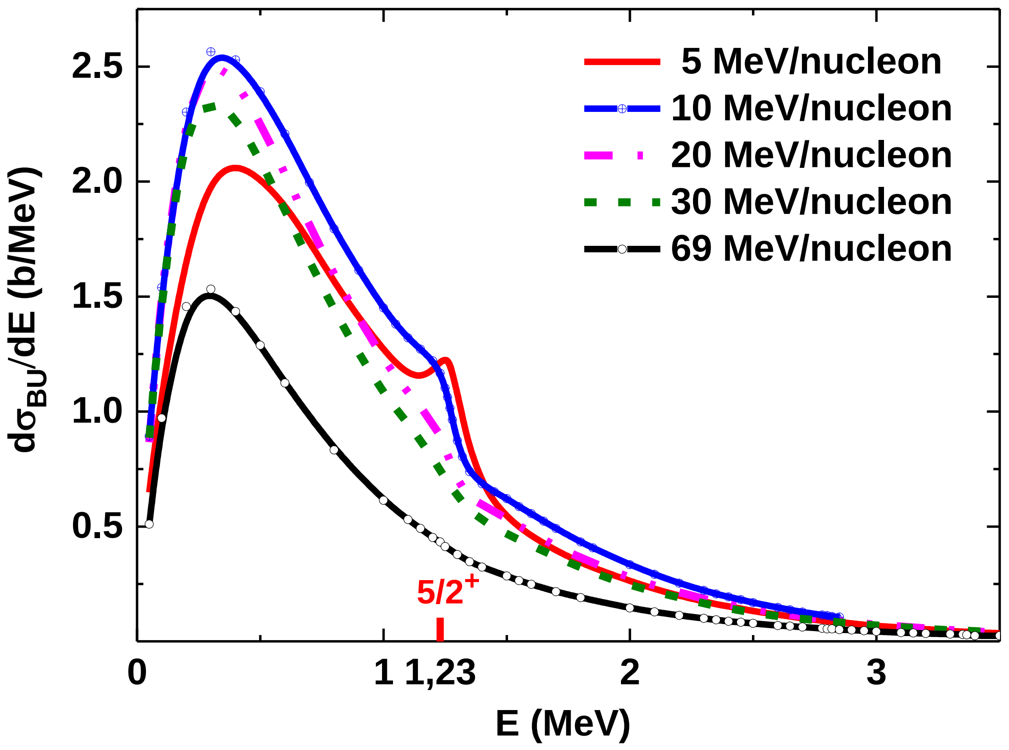
The inclusion of the strong coupling between the projectile and the target in the computational scheme insures that the effect of deformation and “vibration” of the projectile trajectory, as well as the transfer of energy from the target to the projectile and vice versa, are taken into account at the moment of collision.



Breakup cross sections calculated with semiclassical approach using
linear trajectories of the projectile and with quantum-quasiclassical approach
for beam energies 30 (side a), 20 (side b), 10 (side c) and 5 MeV/nucleon (side d).

Summary

The relative energy spectra of the fragments (neutron and core) were calculated for the Coulomb breakup of ^{11}Be on a ^{208}Pb target at the range of beam energies 5-70 MeV/nucleon. We presented new calculations taking into account an influence of resonance states ($5/2^+$, $3/2^-$ and $3/2^+$) to the breakup cross section of ^{11}Be nucleus.



In the numerical calculations performed for the incident beam energies at 5–30 MeV/nucleon region, the contribution of the $5/2^+$ resonance state of ^{11}Be to the breakup cross sections is clearly visible, while at energies of 69 and 72 MeV/nucleon, resonant states $3/2^-$ and $3/2^+$ make the largest contribution to breakup cross sections of ^{11}Be.

Valiolda D., Janseitov D., Melezhik V.S., *Investigation of low-lying resonances in breakup of halo nuclei within the time-dependent approach*
***European Physical Journal A*, 2022, 58(2), 34**

Conclusion and further research

- ✓ A quantitative model has been developed to describe the Coulomb breakup of one-neutron halo nuclei in a wide range of collision energies, including both excited and low-lying resonant states of ^{11}Be within the non-perturbative semiclassical and quantum quasiclassical time-dependent approaches. The obtained results are in good agreement with existing experimental data at 72 and 69 MeV/nucleon.

- ✓ The convergence of the computational scheme is demonstrated in all considered range of the energy including the low-lying resonances in different partial and spin states of ^{11}Be. The numerical technique allows an accurate and straightforward modelling of the nuclear interaction between the nucleon and the ^{10}Be - core in a wide range of beam energies (5 - 70 MeV/nucleon).

- ✓ The developed computational scheme opens new possibilities in investigation of Coulomb, as well as nuclear, breakup of other halo nuclei on heavy, as well as, light targets. This theoretical model can potentially be useful for interpretation and planning of low-energy experiments in studying the halo structure of the nuclei.

Thank you for your attention!