# Simulation of the dynamic aperture of the NICA booster synchrotron based on 

 magnetic measurement dataShandov M. M. ${ }^{\text {a }}$, Kostromin S. A. ${ }^{\text {a, }}$ b<br>a VBLHEP, Joint Institute for Nuclear Research, Dubna, Russia<br>a, b Saint-Petersburg State University, St. Petersburg, Russia

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## Topics

1 Introduction
2 Simulation

- Magnetic Measurements
- Tracking Parameters
- Simulation Modes

3 Results

- 2D Betatron Motion
- $3,2 \mathrm{MeV} / \mathrm{u}$
- $578 \mathrm{MeV} / \mathrm{u}$
- Experemental Results
- $\nu_{x / y}=f(x / y)$
- 4D Betatron Motion
- Working Point $\nu_{x} / \nu_{y}=4,925 / 4,667$

4 Conclusions
5 Spare Slides

- $\nu_{x / y}=f(x / y)$. CO Correction


## Introduction

## Introduction

NICA Booster is a new superconducting synchrotron with operating energies up to $578 \mathrm{MeV} / \mathrm{u}$. The first commissioning Run was successfully carried out in December 2020.


6 DFO-cells x 4 Superperiods


Woprking poing: $\nu_{x} / \nu_{y}=4,8 / 4,85$

## Introduction

| Parameter | Value |
| :---: | :---: |
| Ion ${ }_{31}^{197} \mathrm{Au}$ energy, $\mathrm{MeV} / u$ <br> Betatron tunes $\nu_{x} / \nu_{y}$ <br> Natural chromaticity $\xi_{x} / \xi_{y}$ <br> Ring acceptance, $\pi \cdot m m \cdot m r a d$ : <br> horizontal <br> vertical <br> Beam emittance, $\pi \cdot m m \cdot m r a d$ : <br> at the injection $\varepsilon_{x} / \varepsilon_{y}$ <br> at the end of acceleration $\varepsilon_{x} / \varepsilon_{y}$ <br> Number of ions in the bunch <br> Momentum spread $\Delta p / p$ : <br> at the injection <br> maximal <br> at the end of acceleration <br> Revolution time, $\mu s$ : <br> at the injection <br> at the end of acceleration | $\begin{array}{r} 3,2 \div 578 \\ 4,80 / 4,85 \\ -5,10 /-5,50 \\ 305 / 150 \\ 80 / 58 \\ \\ 15 \div 150 / 15 \\ 0,2 \div 3 / 0,2 \div 1,5 \\ 3 \cdot 10^{9} \\ \pm 10^{-3} \\ \pm 2,3 \cdot 10^{-3} \\ \pm 5 \cdot 10^{-4} \\ 8,51 \\ 0,89 \end{array}$ |

## Introduction

Dynamic aperture (or acceptance) (DA) - domain of the particles are located (coordinates) in 6D phase space, where their motion in electromagnetic field of the facility is stable under the influence perturbations. Here the perturbations are:

■ systematic and random errors of the lattice magnets and RF-stations;

- space charge force;
- impedance of the beam pipe;
$\square$ etc.
In this works, only transvers DA are considered (without the synchrotron motion).
Let us introduce the concept of the maximal initial amplitude of a particle oscillation:

$$
A_{x, y}(s) \Leftrightarrow\left(x, x^{\prime}, y, y^{\prime}\right): \forall N_{\text {turn }}, \exists A_{x, y}(s)>A_{x, y}^{\prime}(s), N_{\text {turn }} \in \mathbb{Z}^{+}
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$A_{x, y}^{\prime}(s)$ - the geometrical acceptance of the facility.

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The following tasks are necessary solved for checking this condition:

- find DA by numerical simulation;
- estimate the effect of the different perturbation on the DA value;
- compare this value with the acceptance of the facility ( 150 and $58 \pi \cdot \mathrm{~mm} \cdot \mathrm{mrad}$ for the horizontal and vertical plane, respectively).

The MAD-X software using the PTC (Polymorphic tracking code) symplectic tracking algorithm was used for the DA simulation.

## Simulation

## Magnetic Measurements

| Parameter | Dipole magnet |  | Quadrupole magnet |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Mean | SD | Mean | SD |
| At the injection energy ( $3,2 \mathrm{MeV} / \mathrm{u}$ ) |  |  |  |  |
| $\alpha$ (tilt), mrad | -1,3 | 1,7 | - | 0,1 |
| $d X=d Y, m m$ | - | - | - | 0,1 |
| $B_{0} L\left(B_{1} L\right), T \cdot m m$ | - | 0,19 | - | 0,06 |
| $b_{2} \cdot 10^{-4}$ | -0,8 | 0,6 | -2,1 | 4,6 |
| $a_{2} \cdot 10^{-4}$ | - | - | 1,3 | 4,3 |
| $b_{3} \cdot 10^{-4}$ | - | - | 7,6 | 3,1 |
| $b_{5} \cdot 10^{-4}$ | - | - | 4,9 | 0,6 |
| At the extraction energy ( $578 \mathrm{MeV} / \mathrm{u}$ ) |  |  |  |  |
| $\alpha$ (tilt), mrad | -1,2 | 1,6 | - | 0,1 |
| $d X=d Y, m m$ | - | - | - | 0,1 |
| $B_{0} L\left(B_{1} L\right), T \cdot m m$ | - | 2,19 | - | 0,7 |
| $b_{2} \cdot 10^{-4}$ | 8,5 | 0,7 | -2,0 | 4,7 |
| $a_{2} \cdot 10^{-4}$ | - | - | 1,3 | 4,0 |
| $b_{3} \cdot 10^{-4}$ | - | - | 7,1 | 3,1 |
| $b_{5} \cdot 10^{-4}$ | - | - | 4,2 | 0,8 |

$$
\begin{gathered}
b(a)_{n}=\frac{1}{n!} \frac{r_{\text {ref }}^{n}}{B_{\text {ref }}} \frac{\partial^{n} B_{y(x)}}{\partial x^{n}} \\
b(a)_{n}^{i n t}=\frac{\int_{-\infty}^{\infty} b(a)_{n}(s)}{L_{e f f}}
\end{gathered}
$$

$n$ is the harmonics number (starting from $n=0$ ); $B_{\text {ref }}$ is the main component of the field at $r_{r e f}=30 \mathrm{~mm}$; $L_{e f f}$ is the effective length of the magnet.

The dependence of DA on the number of turns can be estimated as:

$$
D A\left(N_{t u r n}\right)=A+\frac{B}{\log _{10}\left(N_{t u r n}\right)} ; A, B=\text { const. }
$$

- number of turns for tracking is chosen, on the one hand, providing an acceptable deviation of the DA value from the asymptotic value ( $A$ ), and, on the other hand, as small as possible to reduce the simulation time;
$\square$ to calculate the tune values (see bellow), the nuber of turns has to be proportional to $2^{N}, N \in \mathbb{Z}^{+}$;
- To find the parameters $A$ and $B$ and hence, the asymptotic value of the DA, a long-term simulations are performed which is a separete study.


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Calculation for 2D and 4D betatron motion are presented:
■ the particle tracking $2^{10}$ turns to build-up the phase-plane plot ( $x, p_{x}$ ) and ( $y, p_{y}$ ) and correspondence the tune vs. initial amplitude $\nu_{x / y}=f(x / y)$ (2D DA);

- the particle tracking 5000 turns to calculate 4D DA $\left(A_{x, y}\right)$.


## Simulation Modes



Simulation modes:

- without errors: No Errors;
- misalignment errors (rotation around the longitudinal axis and displacement of the magnetic axis of the element relative to geometric axis): Allign;
- misalignment errors (see above) and deviation of the integral value of the main magnetic field (dipole or quadrupole) component of the lattice elements: Allign+Int;
- only mean values of all error types are included: Total;
- the mean values and SD of all error types are included: Total+Rnd (2000 sampling).


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- only mean values of all error types are included: Total;
- the mean values and SD of all error types are included: Total+Rnd (2000 sampling).
- The harmonics of the magnetic field were determined as absolute thin multipole elements at each of the magnet edges. The harmonics value for each of these elements was defined as half of the integral value.
- The initial tracking point in the center of the straight section of the first superperiod (the injection point into the Booster) were chosen.
- The initial state vector in phase-space was defined as: $\mathbf{s}_{0}=\left(x_{0}, p_{x 0}, y_{0}, p_{y 0}\right)^{T}$


## Results

## 2D Betatron Motion

The Poincaré and Hénon mapping (sections) is a common technique for analyzing complex dynamical systems.
$■$ The transformation of coordinates in the phase-space $\mathbf{x}_{0}=\left(x_{0}, p_{x 0}\right) \mapsto \mathbf{x}_{1}=\left(x_{1}, p_{x 1}\right)\left(\mathbb{R}^{2} \mapsto \mathbb{R}^{2}\right)$ is described using the Poincaré map $(\mathcal{M}):\left\{\mathcal{M}: \mathbf{x}_{n+1}=\mathcal{M}^{(1)}\left(\mathbf{x}_{n}\right), \forall n \in \mathbb{Z}^{+}\right\}$.
■ Introduce the concept of a one-turn map to describe the transformation of the phase-space coordinates when passing through all elements of the accelerator: $\mathbf{M}=\mathcal{M}^{n} \circ \mathcal{M}^{n-1} \circ \ldots \circ \mathcal{M}^{2} \circ \mathcal{M}^{1}$.
■ The Poincaré section is the set of all of the one-turn maps: $\left\{\mathbf{M}_{1}, \mathbf{M}_{2}, \ldots, \mathbf{M}_{n}\right\}$. For a periodical lattice structure: $\mathbf{x}\left(s_{0}\right)=\mathbf{x}\left(s_{L}\right)=\mathbf{x}\left(s_{0}+L\right) \Rightarrow$ for many turns passing through the elements: $\mathbf{x}\left(s_{L}\right)=\mathbf{M}^{N}\left(\mathbf{x}\left(s_{0}\right)\right)$.

The Hénon map (section) and, also, action-angle variables $(J, \psi)$ could be obtained by transformation to the Normal (Jordan) form of the Floquet solution (Floquet transform):

$$
\left(\hat{x}, \hat{p}_{x}\right)=\mathcal{A}^{-1} \circ\left(x, p_{x}\right), \mathcal{A}=\left(\begin{array}{cc}
\sqrt{\beta} & 0 \\
-\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}}
\end{array}\right),
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\end{array}\right)
$$

- the radius of the phase-space trajectory: $R=\sqrt{2 J}$;
- one-turn mapping is corresponded to a rotation in the phase-space by an angle $\psi=2 \pi \nu, \nu$ is the corresponding betatron tune;
- the axis of rotation is an elliptic stable point and the closed phase trajectories are the one-dimensional Kolmogorov-Arnold-Moser (KAM) tori.


## 2D Betatron Motion

1 Particles tracking with with the corresponding errors of the lattice elements. The initial amplitude of the particles varied $[1 ; 46] \mathrm{mm}$ with a step 5 mm .
2 Transformation from the Poincaré to section the Hénon section and the action-angle variables.
3 the DA value is defined as the area inside the phase trajectory:

$$
A_{\psi}=\int_{0}^{2 \pi} \int_{0}^{R(\psi)} d R \cdot d \psi=\frac{1}{2 \pi} \int_{0}^{2 \pi} R(\psi)^{2} \cdot d \psi \rightarrow \lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^{N}\left[R_{n}(\psi)\right]^{2} \equiv\langle R(\psi)\rangle(\pi \cdot \mathrm{m} \cdot \mathrm{rad})
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$$

The 2D DA $\left(A_{\psi}\right)$ calculation results $(\pi \cdot \mathrm{mm} \cdot \mathrm{mrad})$

| Simulation type | ( $\hat{x}, \hat{p}_{x}$ ) |  | ( $\hat{y}, \hat{p}_{y}$ ) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Sextupole correctors off | Sextupole correctors on | Sextupole correctors off | Sextupole correctors on |
| At the injection energy |  |  |  |  |
| No Errors | $400(\infty)$ | $400(\infty)$ | $400(\infty)$ | $400(\infty)$ |
| Total | 310 | 140 | 340 | 320 |
| Total+Rnd | 310 | 160 | 330 | 260 |
| At the extraction energy |  |  |  |  |
| No Errors | 400 ( $\infty$ ) | 400 ( $\infty$ ) | $400(\infty)$ | 400 ( $\infty$ |
| Total | 190 | 220 | 320 | 250 |
| Total+Rnd | 180 | 180 | 310 | 250 |

## 3,2 MeV/u







## $578 \mathrm{MeV} / u$



## Experemental Results

$$
\nu_{x / y}=f(x / y)
$$



## 4D Betatron Motion

1 The initial position of the particle was set on the polar grid: $x_{0}=r_{i} \cos \theta_{j}, y_{0}=r_{i} \sin \theta_{j}, 0 \geq \theta_{j} \geq \pi$.
$\boxed{2}$ The corresponding beam emittance values: $\varepsilon_{x}=\frac{x_{0}^{2}}{\beta_{x}}, \varepsilon_{y}=\frac{y_{0}^{2}}{\beta_{y}}$.
3 An ensemble of particles with the maximum initial amplitude was selected, when the motion was stable in each azimuth: $r(\theta) \approx r\left[\theta_{j}\right] \longrightarrow \max _{i}\left\{r_{i}\left(\theta_{j}, N\right)\right\}$.
4 the DA is calculated as

$$
A_{r, \theta}=\frac{1}{\pi} \int_{0}^{\pi} r(\theta) d \theta \rightarrow \lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^{N}\left[r_{n}(\theta)\right]^{2} \equiv\langle r(\theta)\rangle(m) .
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The 4D DA ( $A_{r, \theta}$ ) calculation results ( mm )

| Simulation type | At the injection energy <br> Sextupole <br> correctors off |  | Sextupole <br> correctors on | At the extraction energy <br> Sextupole <br> correctors off |
| :--- | :---: | :---: | :---: | :---: |
| No Errors | $79,0(\infty)$ | 63,3 | $79,0(\infty)$ | Sextupole <br> correctors on |
| Total | 37,4 | 34,7 | 38,3 | 33,3 |
| Total+Rnd | 37,2 | 35,0 | 38,6 | 36,3 |

## 4D Betatron Motion



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The influence of the errors of the NICA Booster lattice elements on the transverse DA was carried out at the injection and extraction energies. The DA was estimated in 2D and 4D phase space.
$\checkmark$ The Booster lattice model was developed in the MAD-X software. The model allows to take into account the distribution of element parameters based on the results of magnetic measurements and influence of the sextupole correctors.

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$\checkmark$ The working point $\nu_{x} / \nu_{y}=4,925 / 4,667$ is located in the vicinity of the 3rd order resonance $3 \nu_{y}=14$ was considered to test the created Booster lattice model and methodology. The sensitivity of the model to the working point position in the vicinity of vertical and horizontal resonances of different orders and the consistency with the motion of particles in the phase space were obtained in simulation results.

## Conclusions

The objects of further research are:

- taking into account the arrangement of the Booster lattice elements;
- simulation at the energy of electron cooling;
- the 6D DA calculation (includes synchrotron motion);
- long-term simulations (to find the asymptotic value of the DA) at three energies (injection, electron cooling, extraction)
- calculation and simulation of the sequential Booster tuning (closed orbit, coupling of the horizontal and vertical motions, natural chromaticity and DA value corrections) and verification the result and the developed technique with the experimental data. In progress

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