Black holes, holography and strongly-coupled quark-gluon plasma

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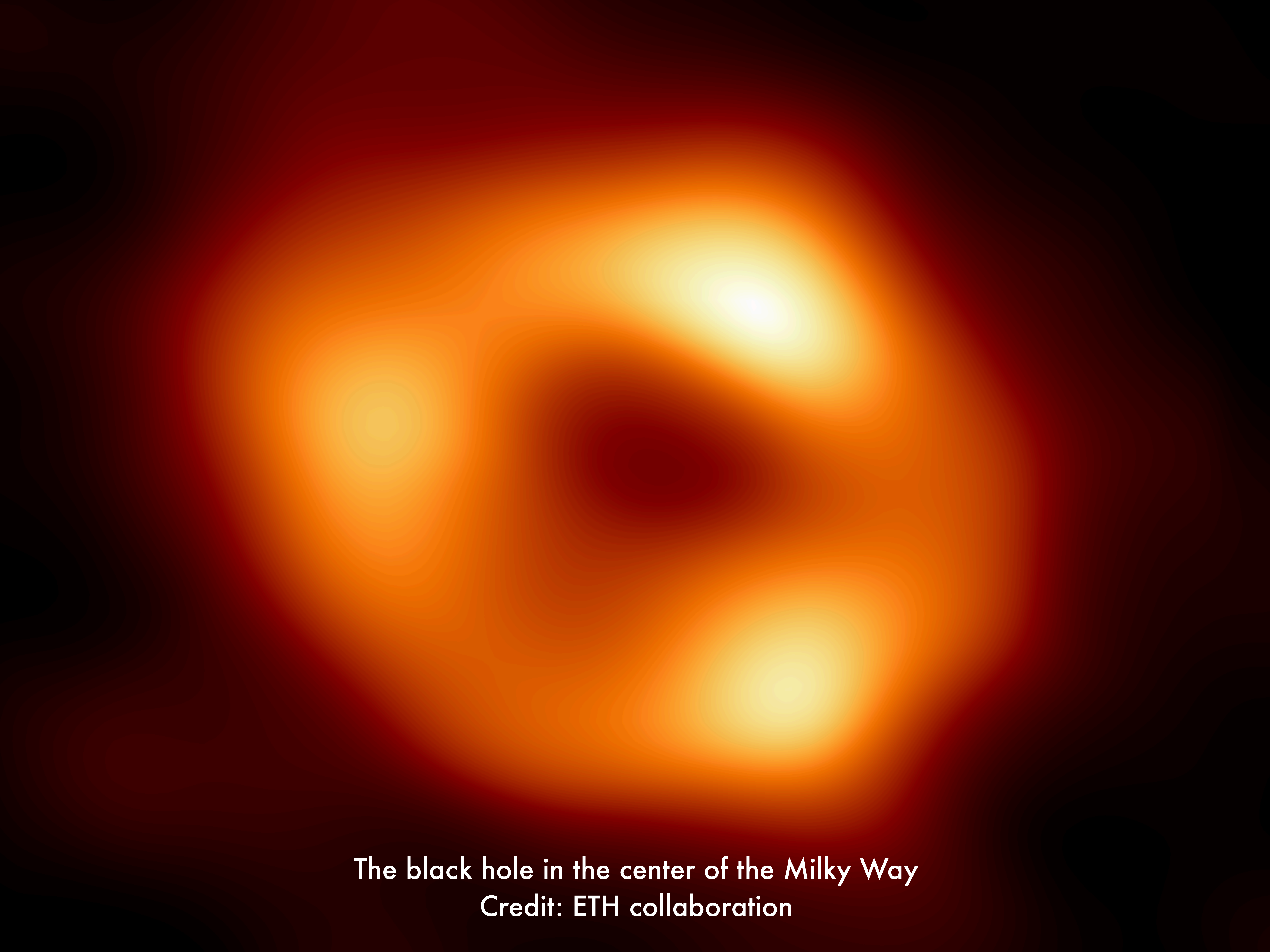
Outline

- Black holes
  - Solutions of Einstein theory of gravity
  - Surface gravity, temperature, entropy
  - Four laws of black hole mechanics
- Holographic duality
  - Holographic Principle
  - Holographic duality for strongly-coupled systems
- Quark-gluon plasma



The black hole in the center of the Milky Way
Credit: ETH collaboration

Black holes

- Einstein equations $R_{MN} - \frac{1}{2} g_{MN} R = \frac{8\pi G}{c^4} T_{MN}$

- Schwarzchild black hole (general non-rotating solution)

$$ds^2 = -\underbrace{\left(1 - \frac{r_h}{r}\right)}_{f(r)} c^2 dt^2 + \underbrace{\left(1 - \frac{r_h}{r}\right)^{-1}}_{f(r)^{-1}} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

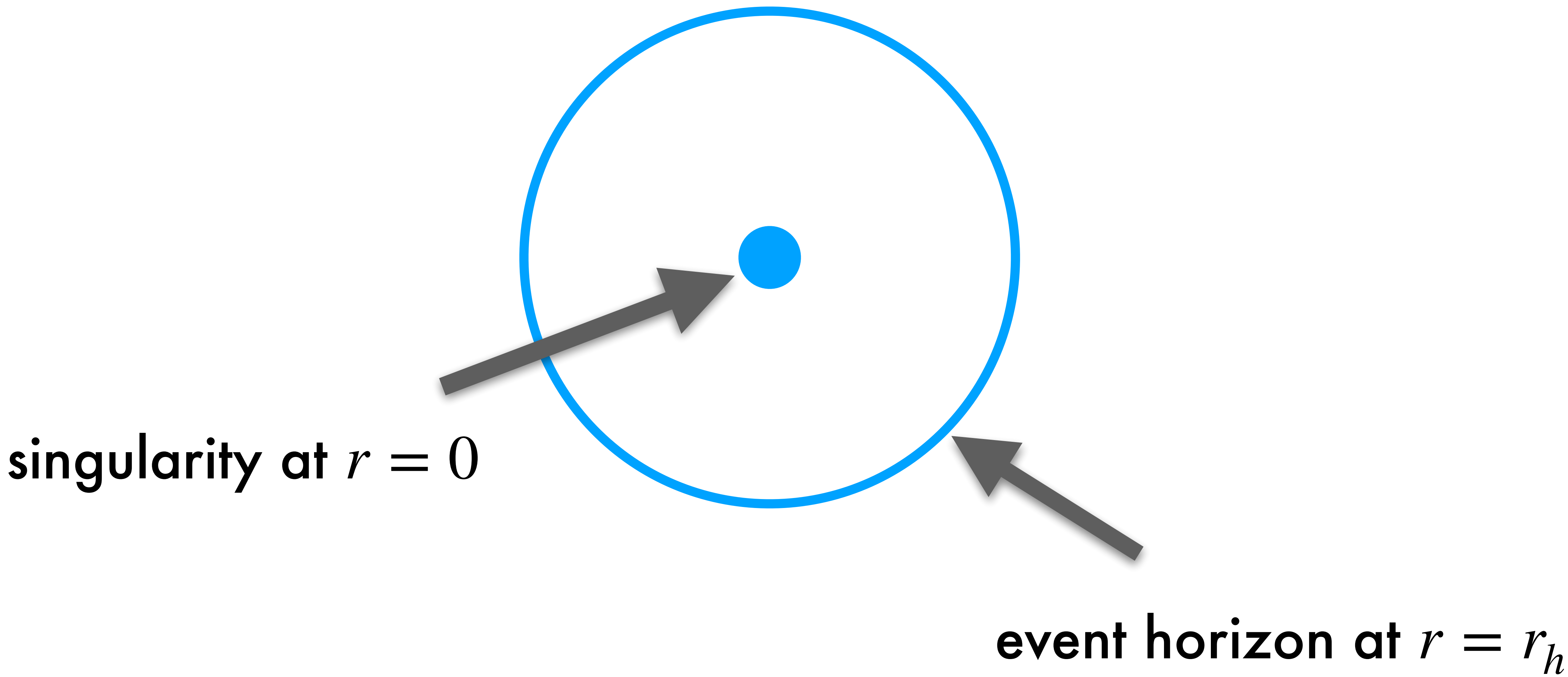
  - describes the metric of space-time OUTSIDE of a body of mass M
  - the Schwarzschild radius r_h defines the location of the horizon

$$r_h = \frac{2GM}{c^2}$$

Black holes

A black hole is a localized region of spacetime from which neither massive particles nor massless ones (photons) can escape.

Horizon is a boundary of no go.



Surface gravity, temperature

- Newton: the gravitational force (per unitmass) or the gravitational acceleration on the horizon is called the surface gravity.

$$a = \frac{GM}{r^2}$$

- Einstein: the surface gravity is the force (per unitmass) $a_\infty(r_h)$, which is necessary to hold the particle at the horizon by an asymptotic observer

$$\kappa = a_\infty(r_h) = \frac{f'(r)}{2}\Big|_{r_h}$$

- Hawking: the surface gravity defines the temperature of the black hole

$$T_H = \frac{\kappa}{2\pi}, \quad \left(T_H = \frac{\hbar}{8\pi k_B M}\right)$$

Entropy

- The area of black hole horizon

$$A = 4\pi r_h^2 = \frac{16\pi G^2 M^2}{c^4}$$

Classically nothing comes out from the black hole

- Bekenstein (1972): Black hole should have a well-defined entropy proportional to the horizon area $S \propto A$

- Hawking (1974):

$$S = \frac{A}{4G\hbar} k_B c^3 = \frac{1}{4}\frac{A}{l_P^2} k_B$$

The entropy of gravitational system is proportional to the AREA

The Four Laws of Black Hole Mechanics

	Thermodynamics	Black hole
Zeroth law	Temperature T is constant at equilibrium	Surface gravity is constant for a stationary solution
First law	$dE = TdS$	$dM = \frac{\kappa}{8\pi G} dA$
Second law	$dS \geq 0$	$dA \geq 0$
Third law	$S \to 0, \mathtt{as}\, T \to 0$	$S \to 0, \mathtt{as}\, T \to 0?$

Holographic principle

- The entropy of BH is proportional to the horizon area

$$S = \frac{A}{4G\hbar} k_B c^3$$

- The gravitational degrees of freedom in D dimensions are effectively described by a theory in (D-1) dimensions

- A principle can arise from some newly recognized pattern, an apparent law of physics that stands by itself, both uncontradicted and unexplained by existing theories.

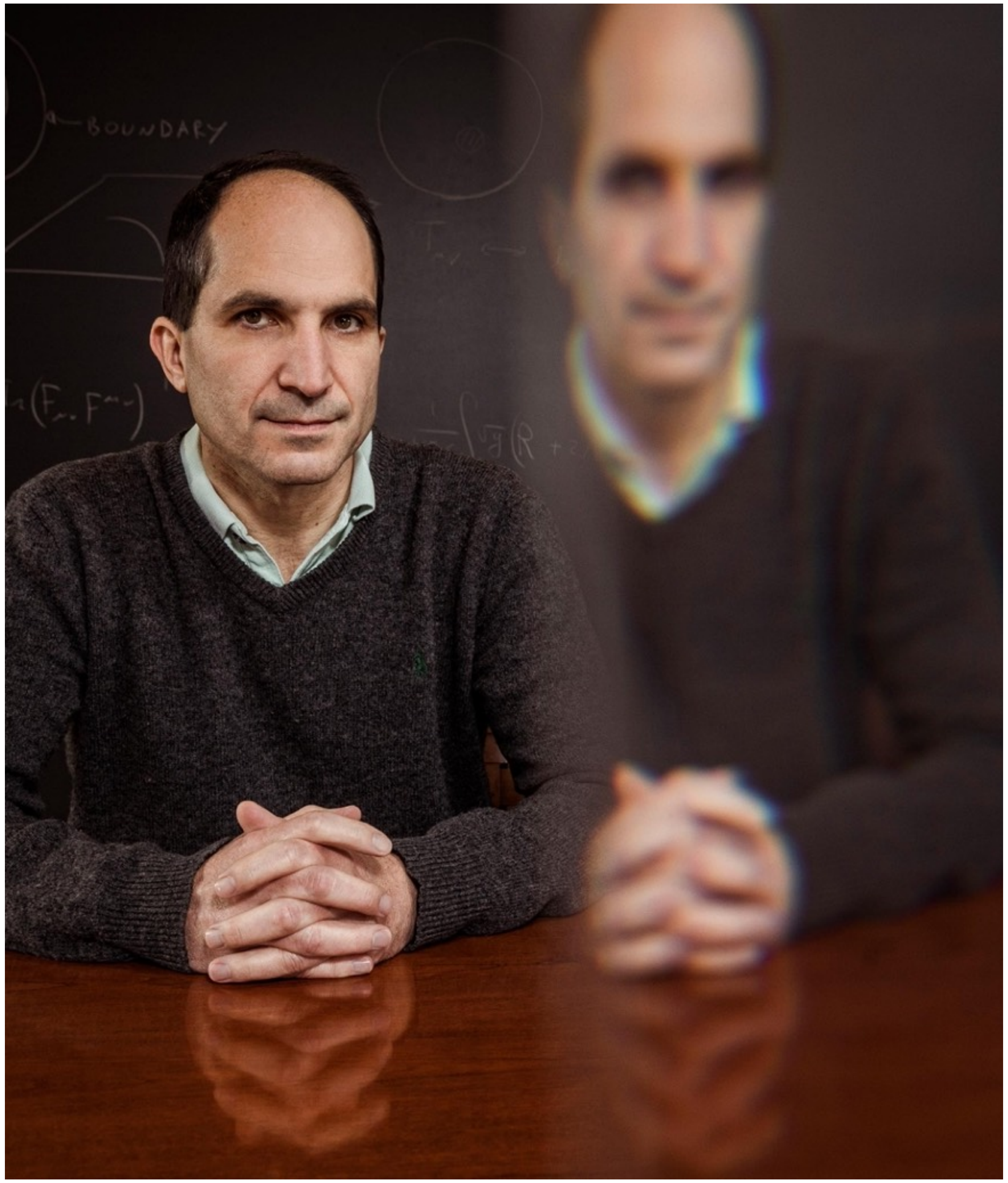
- 't Hooft (1993): The description of the volume of a space-time can be considered as a certain encoded region on the boundary of a lower dimension, i.e. such a light-like boundary as the gravitational horizon.

Holography

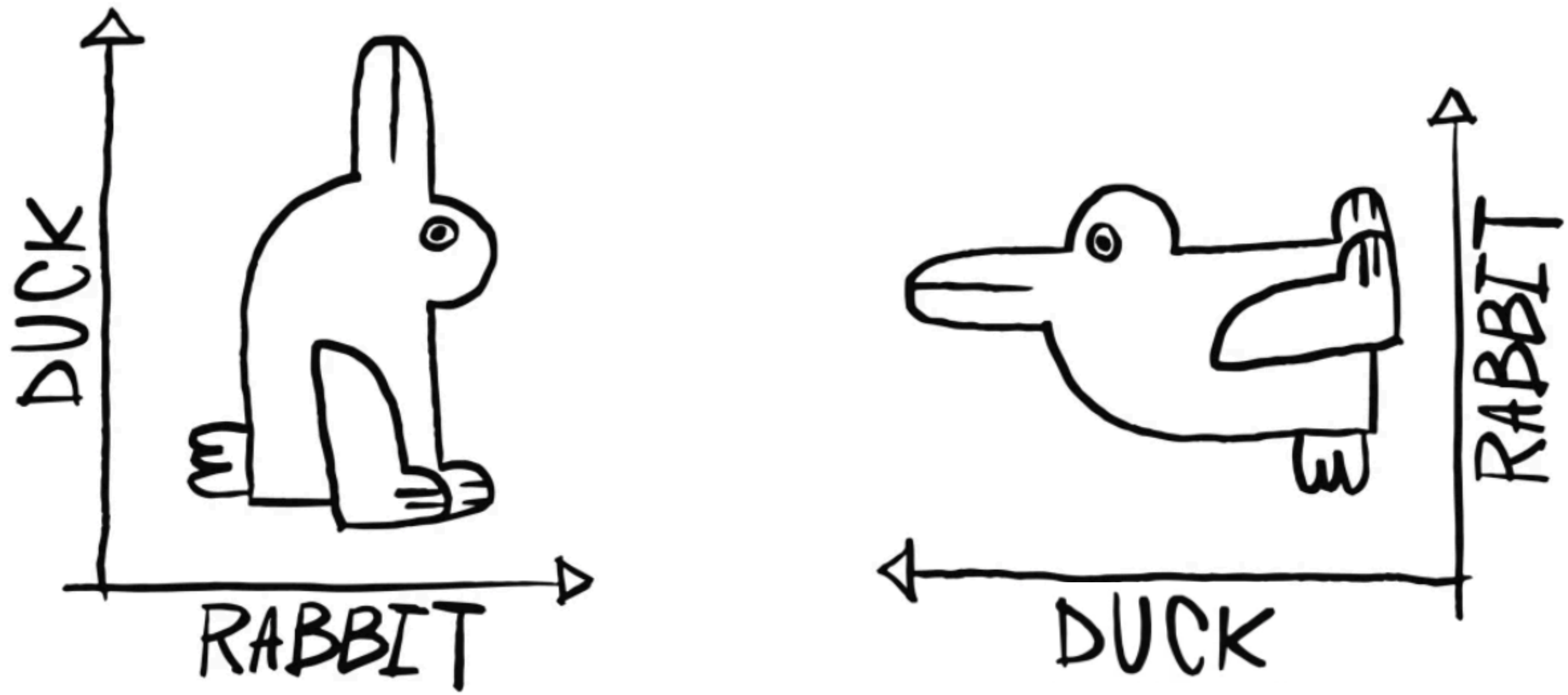
A two-dimensional image which stores information about all three dimensions of the object it represents. The two images here are photographs of a single hologram taken from different angles.

Holographic duality

- Maldacena (1998): a strongly coupled quantum field theory in a D-dimensional spacetime are dynamically equivalent to a D+1-dimensional classical gravity in a special space-time



Duality



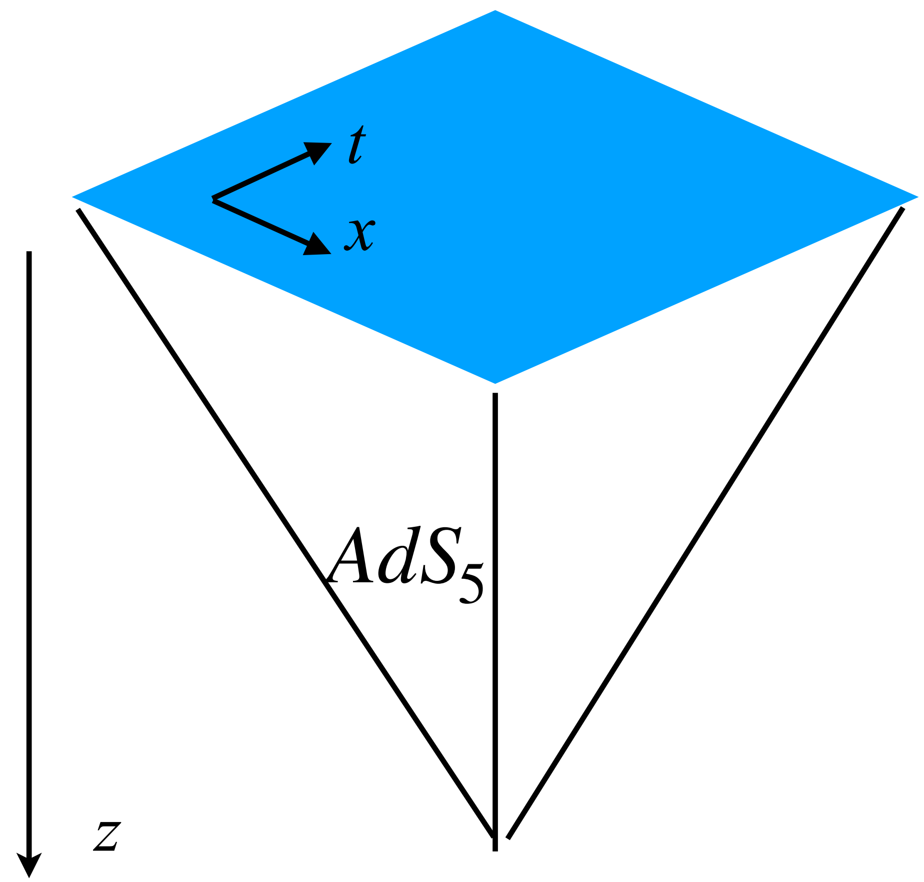
by Ludwig Wittgenstein,"seeing that" vs "seeing as"

Holographic duality

- Special spacetime means anti-De Sitter space-time in (D+1)-dimensions

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- Certain quantum field theory means Conformal field theory
- Gravity in AdS_5 ⟷ CFT in 4D Minkowski spacetime
- Field theory "lives" on the boundary of the gravity background



Fundamental forces and couplings constants

- Electromagnetic interaction (QED) $\alpha_e = \frac{e^2}{\hbar} = 0.0073$ for wide range of energies
- Weak interaction (EW theory) $\alpha_W = 0.03 - 0.04$
- Gravitation $\alpha_G(G_N) = 0.5 - 10^{-38}$
- Strong interaction (QCD)
  - $\alpha_s(0.1fm) \approx 0.31, \quad 1fm = 10^{-15}m$
  - $\alpha_s(0.001fm) \approx 0.105$
  - $\alpha_s(1fm) \approx 1$, non-perturbative regime

Perturbative and non-perturbative regimes

- The partition function $Z[\alpha] = \int [d\Phi] e^{-S[\Phi,\alpha]}$
- Toy model $Z[\alpha] = \int_0^1 dx \frac{1}{\sqrt{1-\alpha^2 x^2}}$
- Prediction for small α: $Z[\alpha] \sim 1, \quad \alpha \ll 1$
- Exact: $\alpha x = \sin y$, $Z[\alpha] = \frac{1}{\alpha}\int_0^{\arcsin\alpha} \frac{\cos y}{\cos y} dy = \frac{\arcsin\alpha}{\alpha}$

Quasi-conformal behaviour of QCD, T>300 MeV

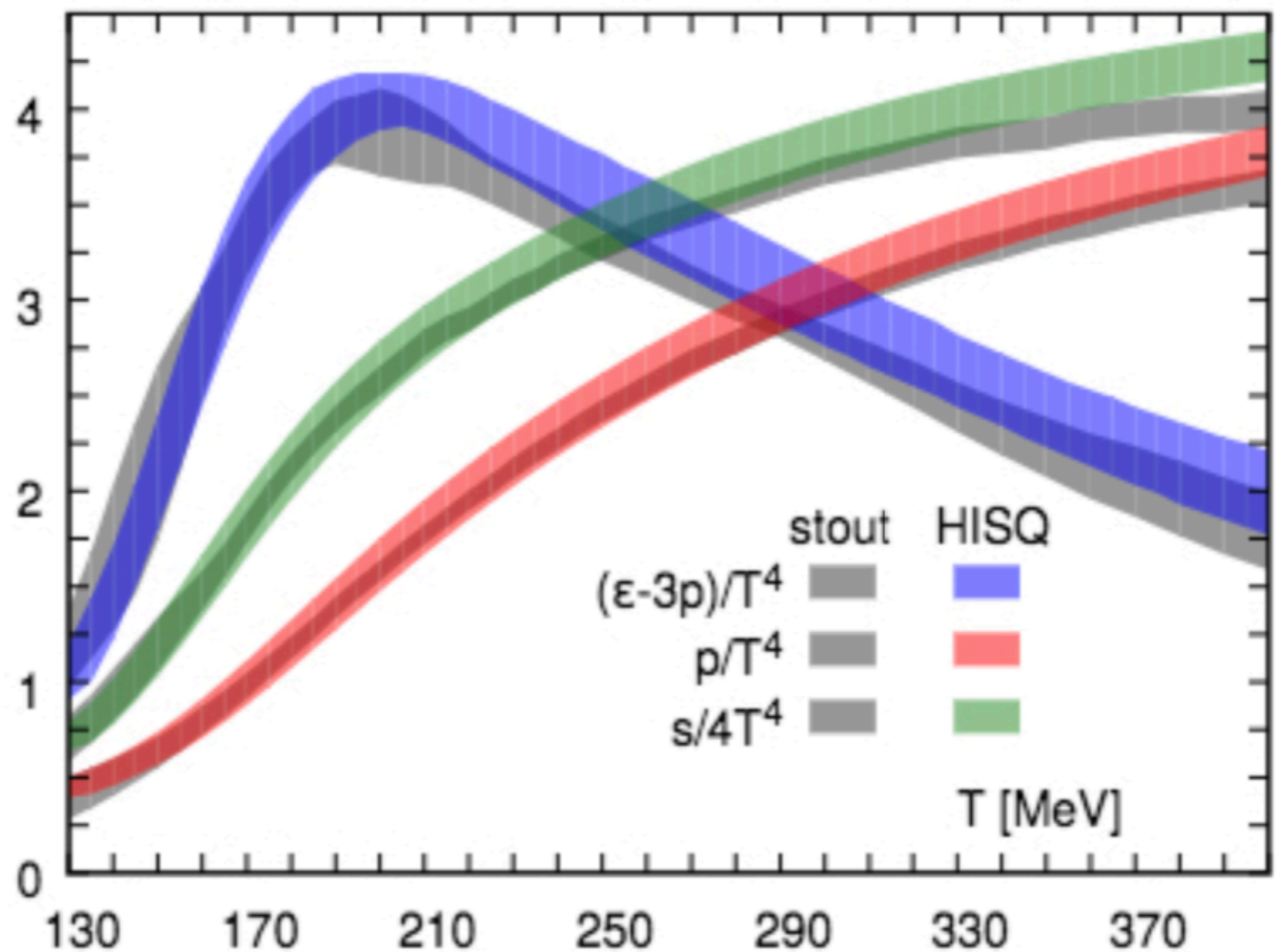
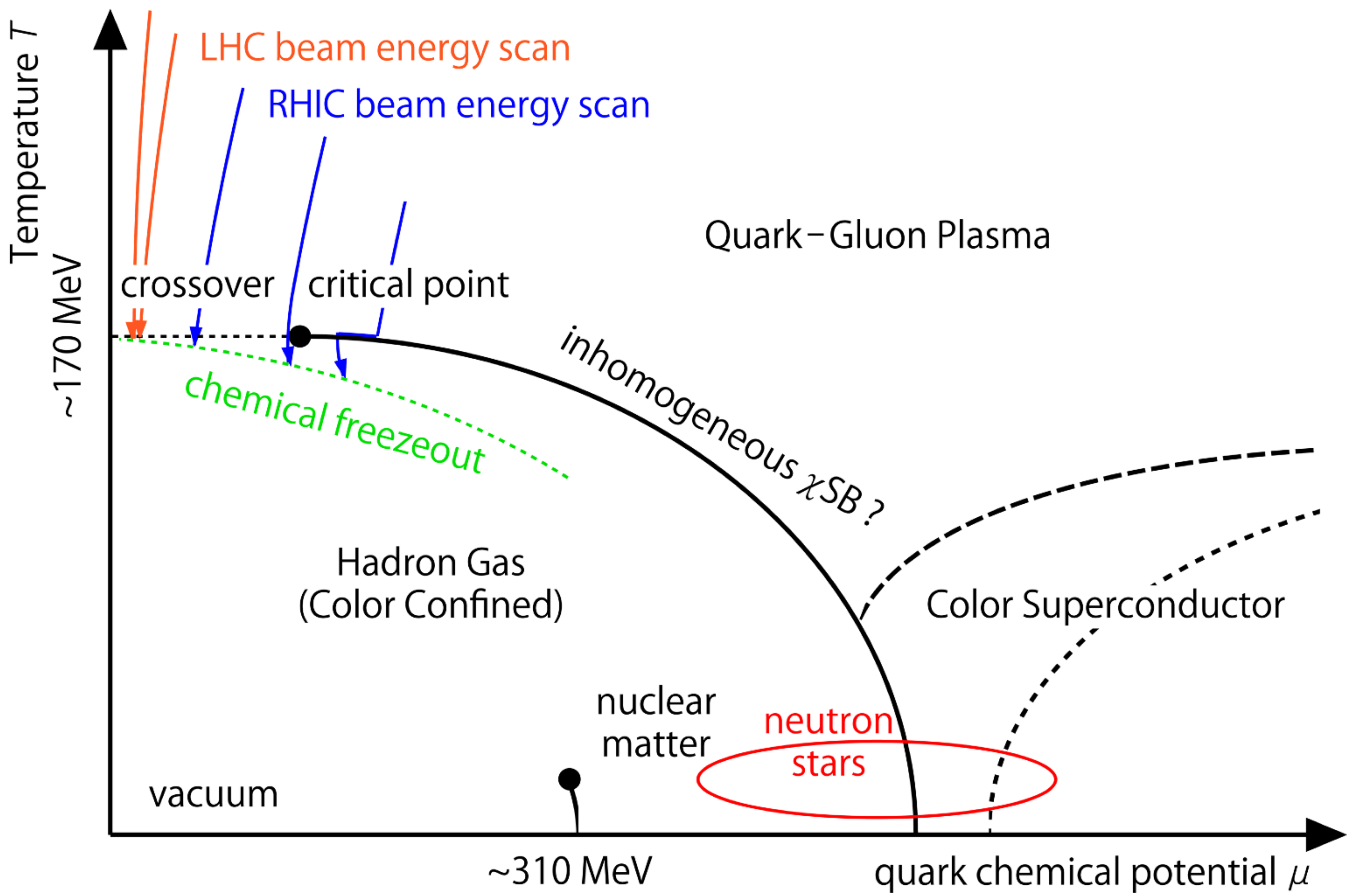


Figure: from Bazavov et al, PRD 90 (2014) 094503

QCD phase diagram



QCD phase diagram: questions

- Confinement-deconfinement phase transitions at non-zero temperature and non-zero baryonic density
- Phenomena at high and at low temperatures
- Quark-qluon plasma produced in Heavy-Ion Collisions: thermalization and evolution
- Hadron spectrum

Conventional picture of QGP dynamics

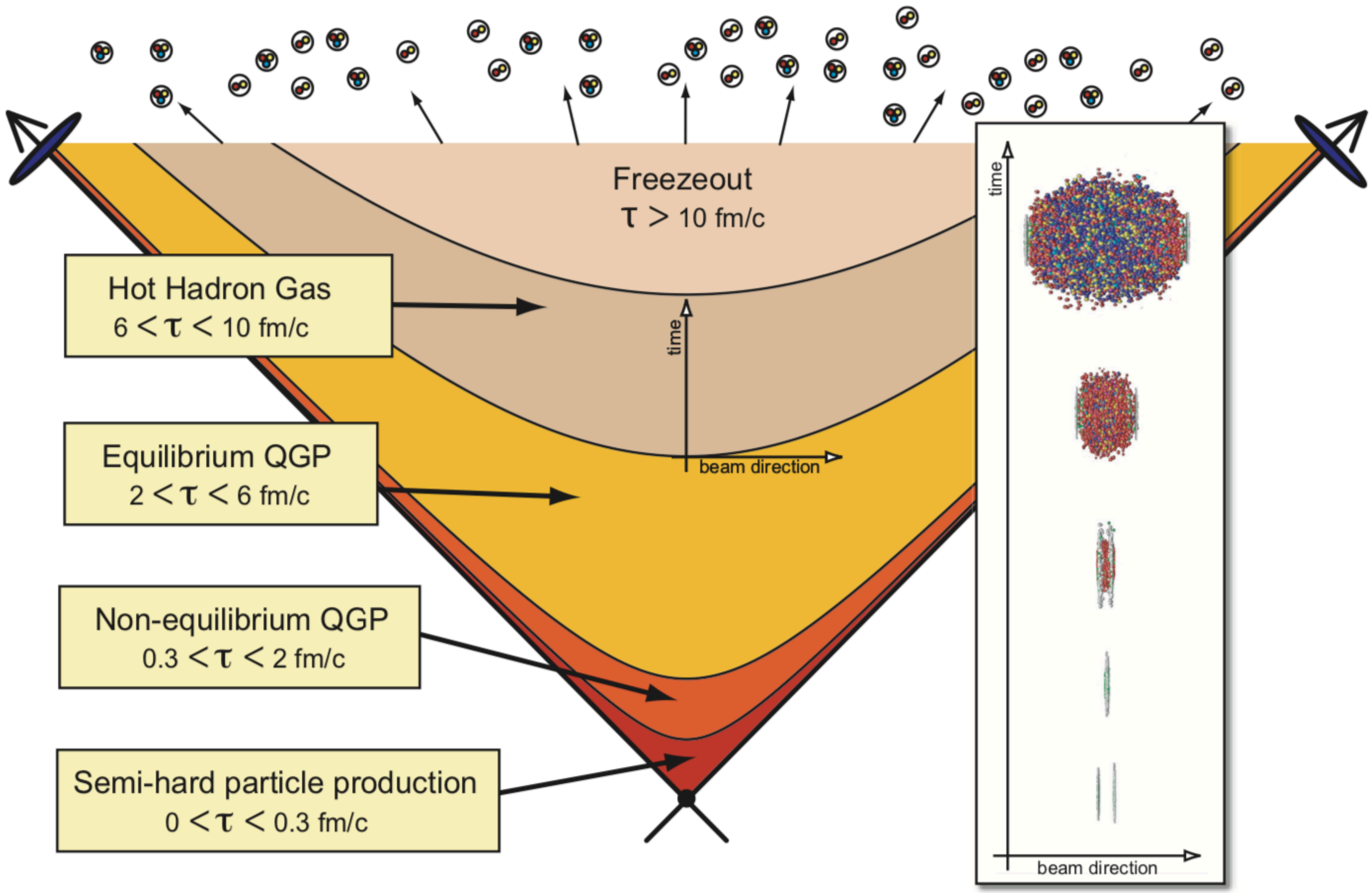


Figure: from Strickland 1410.5786

Holographic picture of QGP

- 4d QGP ⟷ 5d black hole in AdS spacetime
- The scenario of HIC ⟷ a gravity shock wave collision in which trapped surface is formed. (Yaffe, Shuryak, Arefeva)
- After the collision the shocks slowly decay, leaving the plasma described by hydrodynamics Kovtun,Policastro,Son, Starinets'02-05

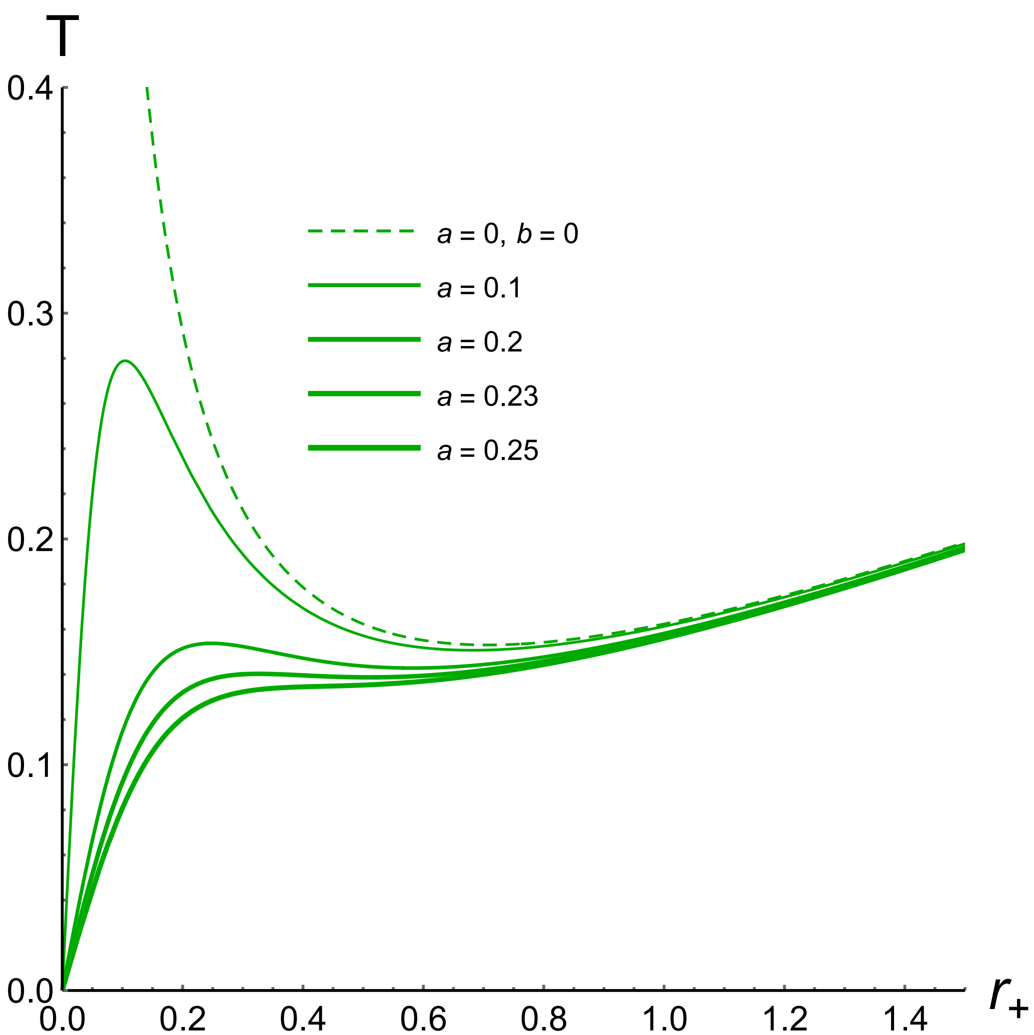
$$\frac{\eta}{s} = \frac{1}{4\pi}$$ (confirmed at RHIC, 2008)

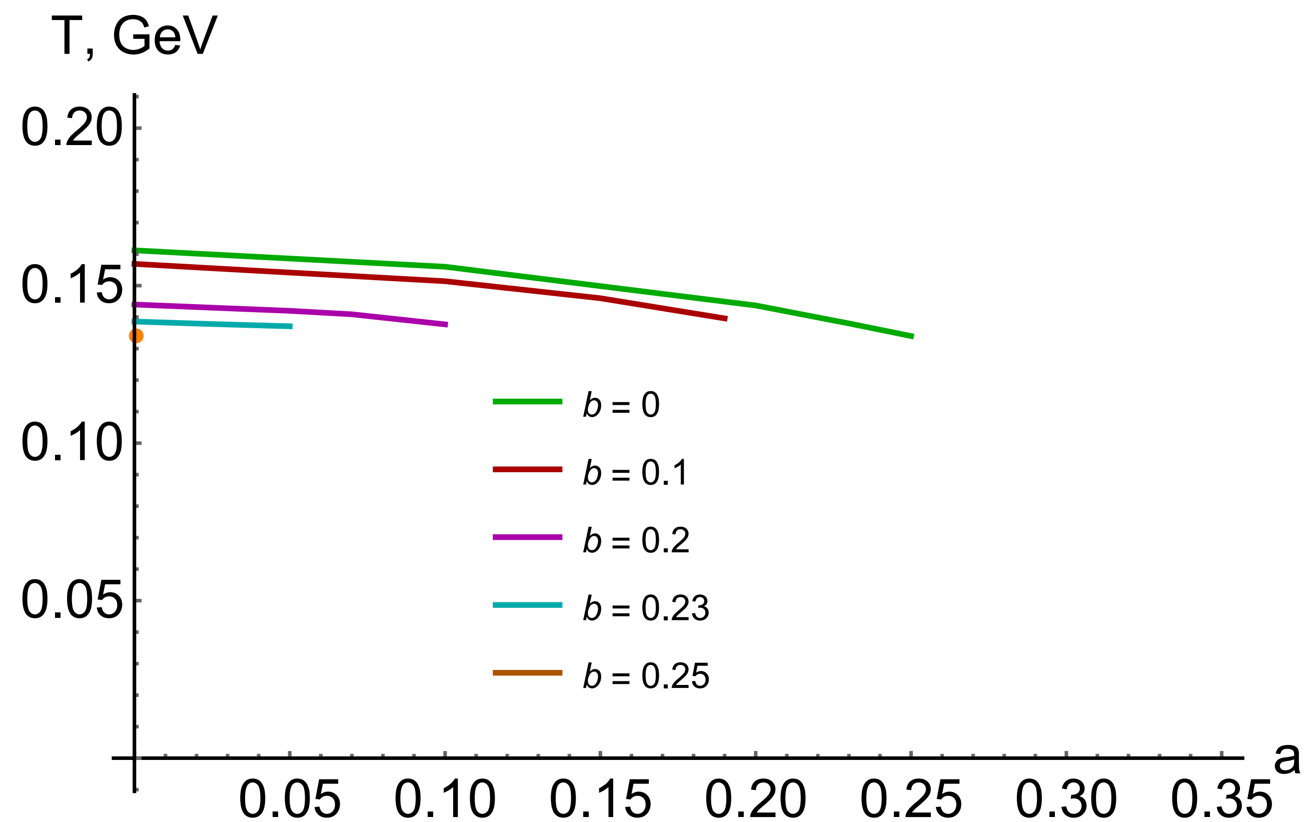
- Thermalization stage – infalling shell (non-stationary solutions)

Holographic observables

- Temperature: $T_H \Leftrightarrow T_{QGP}$
- Temperature of confinement/deconfinement $T_c \Leftrightarrow T_{HP}$
- Free energy: $F_H \Leftrightarrow F_{QGP}$
- 4d Multiplicity in HIC $\Leftrightarrow$ BH Entropy
- Thermalization time of QGP $\Leftrightarrow$ BH formation time

Temperature of phase transition in rotating QGP






So long
and thanks for all the fish!