Moscow Institute of Physics and Technology
NRC "Kurchatov Institute" - ITEP

МФТИ

Kirill Gubarev

Generalizing 11D supergravity

supervisor: Edvard Musaev
in coop. with: Ilya Bakhmatov, Aybike Catal-Ozerb, Nihat Sadik Deger

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Motivation

- What are consistent backgrounds for the superstring?
  - 10D supergravity solutions (RNS string)
  - 10D generalized supergravity solutions (GS string)
- What are consistent backgrounds for the supermembrane?
  - 11D supergravity solutions (GS string)
  - some 11D generalized supergravity solutions? (GS string)

RNS string and Weyl invariance

RNS action

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}(G_{mn}\partial_J x^m\partial^J x^n + iB_{mn}\epsilon^{IJ}\partial_I x^m\partial_J x^n + \alpha'\Phi R^{(2)}), \tag{1}$$

to be Weyl invariant

$$< T^J{}_J >= -\frac{1}{2\alpha'}\beta_{mn}(G)\partial_J x^m\partial^J x^n - \frac{i}{2\alpha'}\beta_{mn}(B)\epsilon^{IJ}\partial_I x^m\partial_J x^n - \frac{1}{2}\beta(\Phi)R^{(2)} = 0, \tag{2}$$

one loop calculation reproduces NS-NS supergravity equations

$$\begin{aligned}
\beta^{1loop}(\Phi) = &\quad \alpha'(R - \frac{1}{12}H^2 + 4\nabla^m\nabla_m\Phi - 4(\nabla\Phi)^2) = 0\,,\\
\beta^{1loop}_{mn}(G) = &\quad \alpha'(R_{mn} - \frac{1}{4}H_{mkl}H_n{}^{kl} + 2\nabla_m\nabla_n\Phi) = 0\,,\\
\beta^{1loop}_{mn}(B) = &\quad \alpha'(\frac{1}{2}\nabla_k H^{kmn} - H^{kmn}\nabla_k\Phi) = 0\,,
\end{aligned} \tag{3}$$

where $H_{mnk} = 3\nabla_{[m}B_{nk]}$ and ∇_m covariant with respect to G_{mn}.

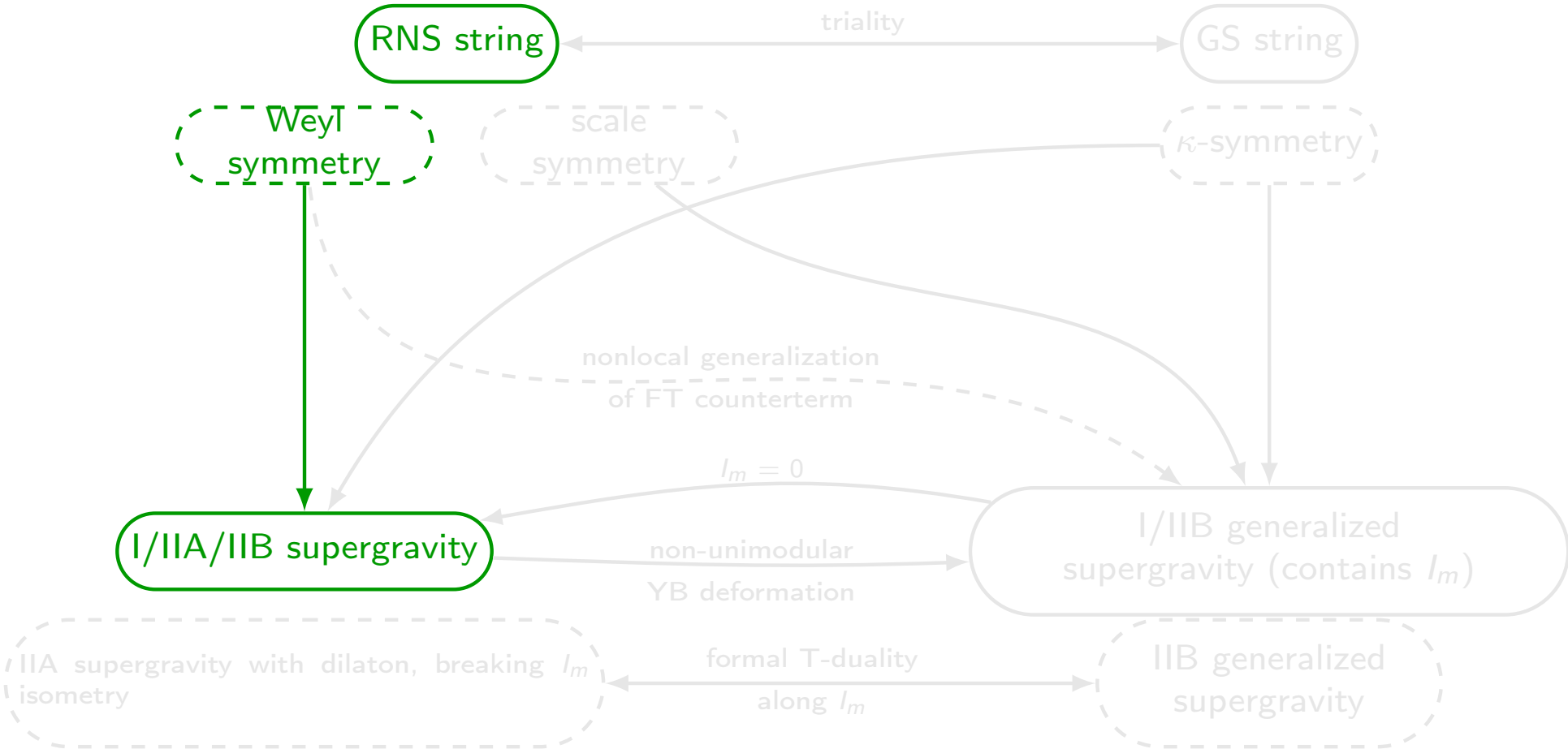


Рис.: Ordinary and generalized supergravities in 10D

GS string and κ-symmetry

GS action

$$S = \int d^2\sigma \sqrt{-G} - \int_\Sigma B\,, \qquad G = \det G_{IJ}\,, \tag{4}$$

$$G_{IJ} = E_I{}^a E_J{}^b \eta_{ab}\,, \qquad E_I{}^A = \partial_I z^M E_M{}^A(z)\,, \qquad z^M = (x^m, \theta^\mu)\,. \tag{5}$$

κ-symmetry transformation (IIB example)

$$\delta_\kappa z^M E_M{}^a = 0\,, \qquad \delta_\kappa z^M E_M{}^{\alpha i} = \frac{1}{2}(1+\Gamma)^{\alpha i}{}_{\beta j}\kappa^{\beta j}\,, \qquad \Gamma = \frac{1}{2\sqrt{-G}}\varepsilon^{IJ} E_I{}^a E_J{}^b \gamma_{ab}\sigma^3\,. \tag{6}$$

GS string and κ-symmetry

Constraints on the supertorsion $T^A = dE^A + E^B \wedge \Omega_B{}^A$ and $H = dB$

$$H_{\beta j \gamma k \alpha i}(1+\Gamma)^{\alpha i}{}_{\delta l} = 0\,, \tag{7}$$

$$E_l{}^a \left[\sqrt{-G}\, G^{IJ} T_{\alpha i \beta j a} - \varepsilon^{IJ} H_{a \alpha i \beta j}\right](1+\Gamma)^{\alpha i}{}_{\gamma k} = 0\,, \tag{8}$$

+ Bianchi identities are solved as [Tseytlin, Wulff (2016)]

$$R - \frac{1}{12} H^2 + 4\nabla^m X_m - 4 X_m X^m = 0\,, \tag{9}$$

$$R_{mn} - \frac{1}{4} H_{mkl} H_n{}^{kl} + \nabla_m X_n + \nabla_n X_m = 0\,, \tag{10}$$

$$\frac{1}{2}\nabla_k H^{kmn} - H^{kmn} X_k - \nabla^m X^n + \nabla^n X^m = 0\,, \tag{11}$$

where $X_m = I_m + \nabla_m \Phi - B_{mn} I^n$ и I_m is a Killing vector.

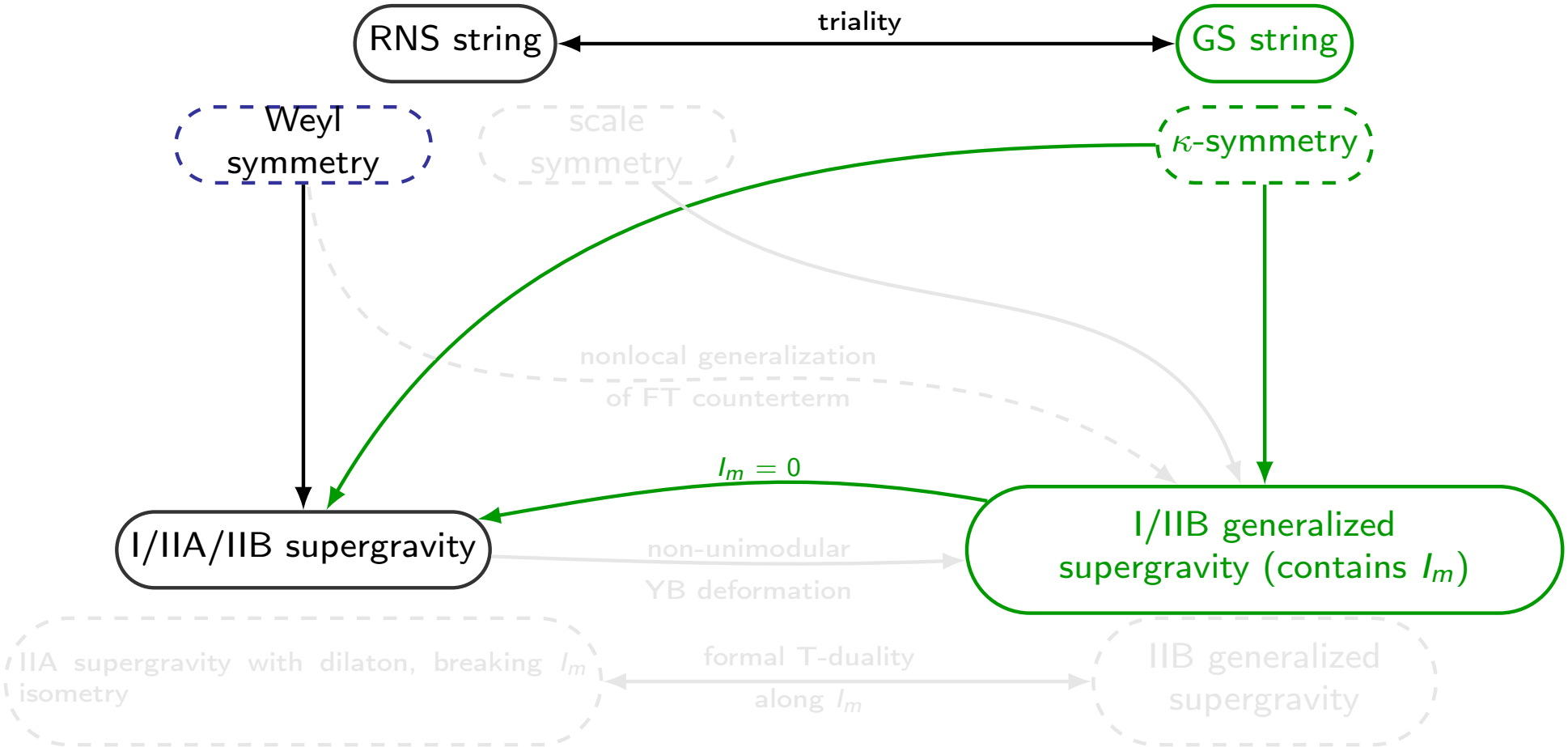


Рис.: Ordinary and generalized supergravities in 10D

RNS string and scale invariance

For the scale invariance of RNS string G_{mn} and B_{mn} must satisfy

[Arutynov, Frolov, Hoare, Roiban, Tseytlin (2015)]

$$\begin{aligned}\frac{1}{\alpha'}\beta^{1loop}_{mn}(G) = &\quad R_{mn} - \frac{1}{4}H_{mkl}H_n{}^{kl} = -\nabla_m X_n - \nabla_n X_m\,,\\ \frac{1}{\alpha'}\beta^{1loop}_{mn}(B) = &\quad \frac{1}{2}\nabla_k H^{kmn} = H^{kmn}X_k + \nabla^m Y^n - \nabla^n Y^m = 0\,,\end{aligned} \tag{12}$$

which is true for the G_{mn} and B_{mn} solving generalized supergravity equations ($Y_m = X_m$).

The stress energy tensor

$$< T^J{}_J >= 2\nabla^J(X_m\partial_J x^m + \epsilon_{IJ}Y_m\partial^I x^m) \tag{13}$$

that can be canceled only with non-local generalization of Fradkin-Tseytlin counterterm.

[Fernandez-Melgarejo, Sakamoto, Sakatani, Yoshida (2018)]

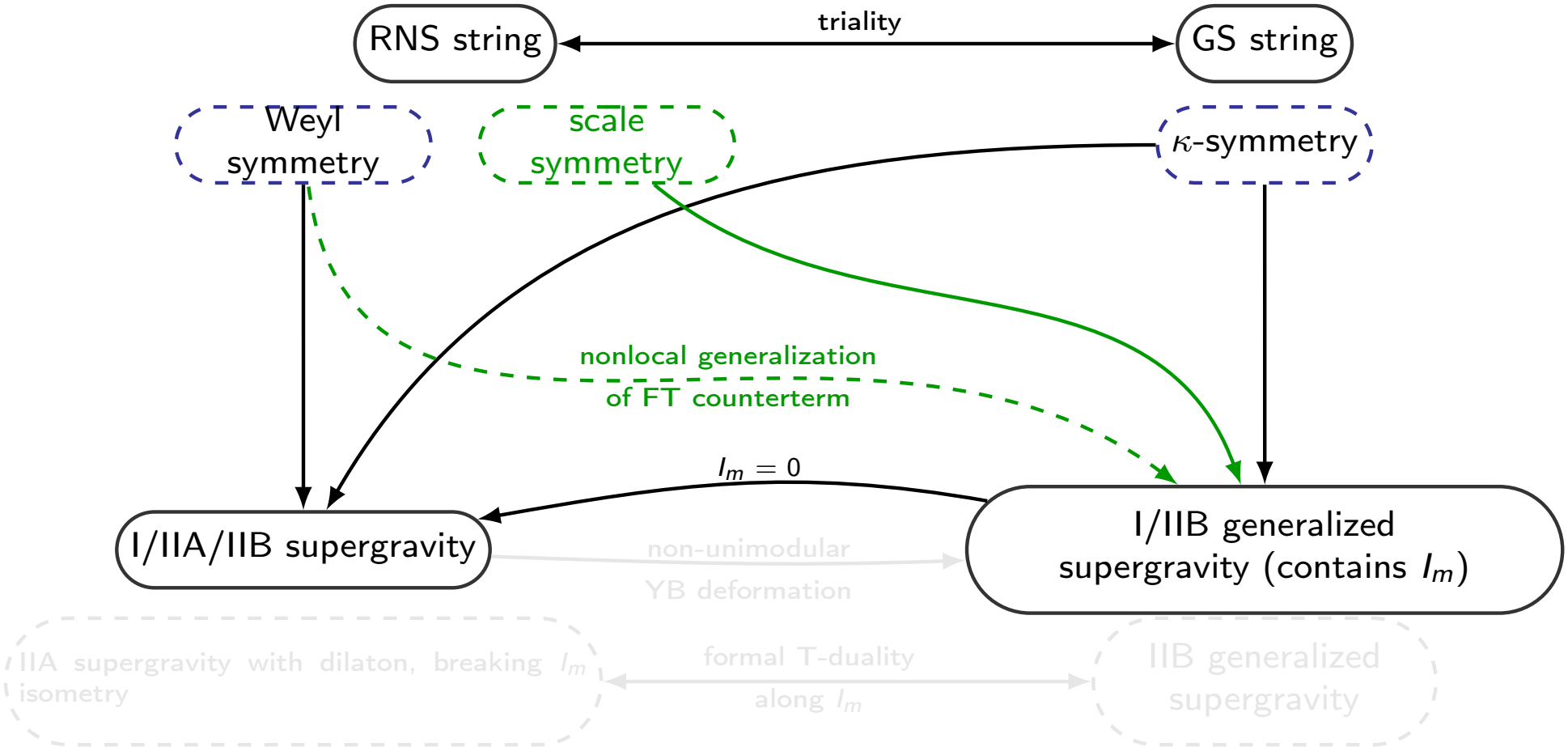


Рис.: Ordinary and generalized supergravities in 10D

η-deformation of $AdS_5 \times \mathbb{S}^5$ - the real way the generalized supergravity was discovered

- GS string on $AdS_5 \times \mathbb{S}^5$ is integrable [Bena, Polchinski, Roiban (2004)]
- GS string is integrable on Yang–Baxter (η-) deformed background $AdS_5 \times \mathbb{S}^5$ [Vicedo, Delduc, Magro (2013)]

$$S = -\frac{(1+\eta^2)^2}{2(1-\eta^2)} \int d\tau d\sigma \, P_-^{ab} \left[A_a \, . d \circ \frac{1}{1-\eta \, R_g \circ d} (A_b) \right] \tag{14}$$

η–deformed $AdS_5 \times \mathbb{S}^5$

- does not solve ordinary supergravity equations [Arutyunov, Borsato, Frolov (2015)]
- solves equations of generalized supergravity [Arutynov, Frolov, Hoare, Roiban, Tseytlin (2015)]
- formally T–dual to HT background (IIA solution) [Hoare, Tseytlin (2015)]

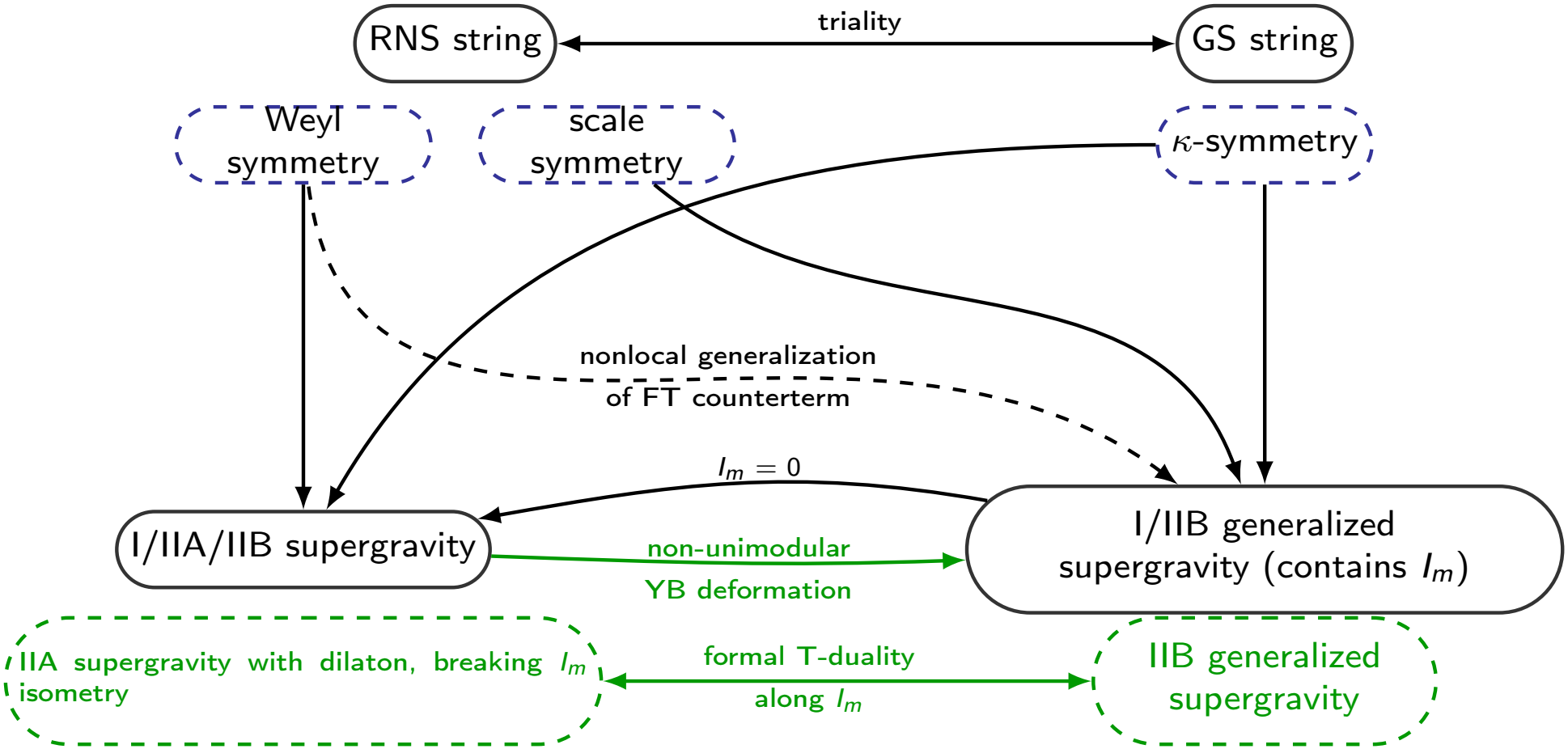


Рис.: Ordinary and generalized supergravities in 10D

Double field theory

DFT action

$$S_{DFT} = \int d^{20}\mathbb{X}\, e^{-2d}\, \mathbb{Q}, \tag{15}$$

where the DFT curvature

$$\mathbb{Q} = \mathcal{H}^{AB}\mathcal{F}_A\mathcal{F}_B + \mathcal{F}_{ABC}\mathcal{F}_{DEF}\left(\frac{1}{4}\mathcal{H}^{AD}\eta^{BE}\eta^{CF} - \frac{1}{12}\mathcal{H}^{AD}\mathcal{H}^{BE}\mathcal{H}^{CF}\right) - \mathcal{F}_A\mathcal{F}^A - \frac{1}{6}\mathcal{F}_{ABC}\mathcal{F}^{ABC}, \tag{16}$$

$$\mathcal{H}_{AB} = \begin{pmatrix} h_{ab} & 0 \\ 0 & h^{ab} \end{pmatrix}, \quad \eta_{AB} = \begin{pmatrix} 0 & \delta_a^b \\ \delta_b^a & 0 \end{pmatrix}, \tag{17}$$

and DFT fluxes

$$\mathcal{L}_{E_A} E_B{}^M = \mathcal{F}_{AB}{}^C E_C{}^M, \quad \mathcal{L}_{E_A} d = \frac{1}{2}\mathcal{F}_A, \tag{18}$$

$$\mathcal{F}_{ABC} = 3E_{N[C}\partial_A E^N{}_{B]}, \qquad \mathcal{F}_A = E_{NA}\partial^B E^N{}_B + 2\partial_A d. \tag{19}$$

For the consistency $\forall$ DFT fields f, g must satisfy section constraints

$$\eta^{MN}\partial_M f \partial_N g = 0, \quad \eta^{MN}\partial_M\partial_N f = 0. \tag{20}$$

10D supergravity from DFT

Supergravity parametrization

$$E_M{}^A = \begin{pmatrix} e^a_m & 0 \\ -e^k_a B_{km} & e^m_a \end{pmatrix}, \qquad d = \phi - \frac{1}{4} log\ G, \tag{21}$$

$$\partial_M = (\partial_m, \partial^m \equiv 0), \quad \partial_A = E^M_A \partial_M. \tag{22}$$

The DFT EOMs $\Leftrightarrow$ 10D supergravity EOMs

$$\delta d: \qquad R - \frac{1}{12} H^2 + 4\nabla^m \nabla_m \phi - 4(\nabla\phi)^2 = 0, \tag{23}$$

$$\delta e^m{}_a: \qquad R_{mn} - \frac{1}{4} H_{mkl} H_n{}^{kl} + \nabla_m \nabla_n \phi + \nabla_n \nabla_m \phi = 0, \tag{24}$$

$$\delta b_{mn}: \qquad \frac{1}{2} \nabla_k H^{kmn} - H^{kmn} \nabla_k \phi = 0. \tag{25}$$

Yang–Baxter deformation

Yang–Baxter deformation — local O(d,d) rotation

$$E'_M{}^A = O[\beta]_M{}^N E_N{}^B, \qquad O_M{}^N = \begin{pmatrix} \delta^n_m & -\beta^{nm} \\ 0 & \delta^m_n \end{pmatrix} = exp(\beta^{mn} T_{mn}), \tag{26}$$

with bi-Killing ansatz

$$\beta^{mn} = r^{ab} k^m_a k^n_b. \tag{27}$$

Fluxes deform as

$$\delta\mathcal{F}_{ABC} \propto CYBE, \quad \delta\mathcal{F}_a = 2I^m b_{mn} e^n_a, \quad \delta\mathcal{F}^a = 2I^a, \tag{28}$$

where

$$I^m = \nabla_k \beta^{km} = \frac{1}{2} r^{ab} f_{ab}{}^c k_c{}^m, \quad f_{de}{}^{[a} r^{b|d|} r^{c]e} = 0 \,(\text{CYBE}). \tag{29}$$

DFT Bianchi identities and constraints on I_m

DFT Bianchi identities

$$
\begin{aligned}
0 &= \partial_{[A}\mathcal{F}_{BCD]} - \frac{3}{4}\mathcal{F}_{[AB}{}^{E}\mathcal{F}_{CD]E}, \\
0 &= 2\partial_{[A}\mathcal{F}_{B]} + \partial^{C}\mathcal{F}_{CAB} - \mathcal{F}^{C}\mathcal{F}_{CAB}, \\
0 &= \partial^{A}\mathcal{F}_{A} - \frac{1}{2}\mathcal{F}^{A}\mathcal{F}_{A} + \frac{1}{12}\mathcal{F}^{ABC}\mathcal{F}_{ABC}.
\end{aligned}
\tag{30}
$$

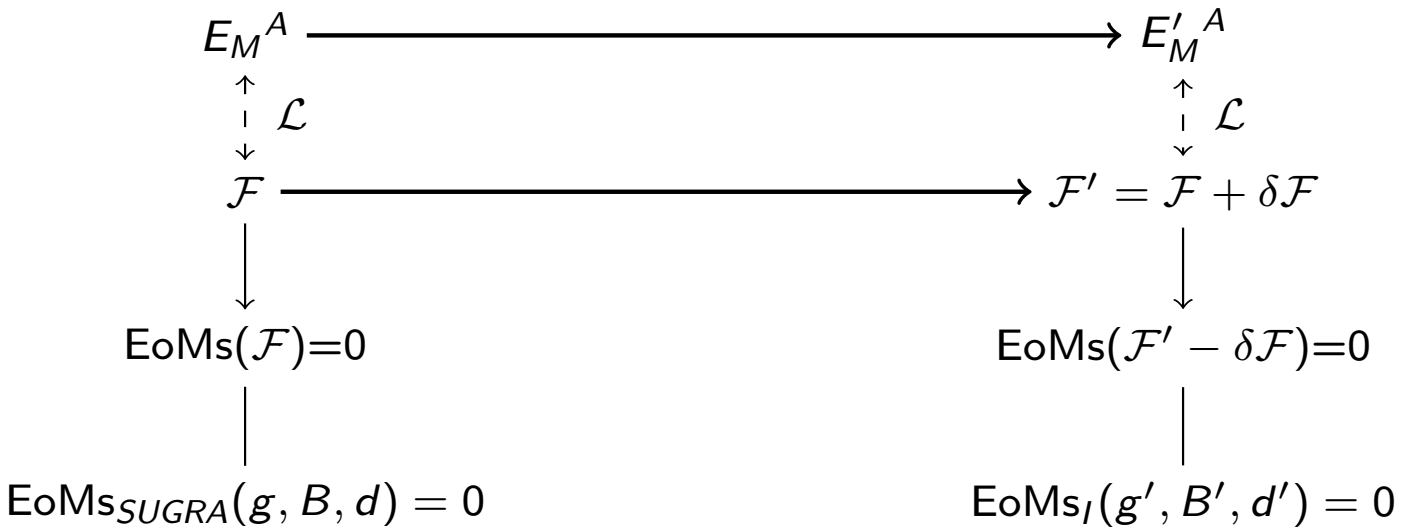
After the deformation

$$
\begin{aligned}
\mathcal{F}'_{ABC} &= \mathcal{F}_{ABC}, \\
\mathcal{F}'_{A} &= \mathcal{F}_{A} + X_{A},
\end{aligned}
\tag{31}
$$

for the BIs to be satisfied X_M must obey

$$
\mathcal{L}_X E'_A{}^M = 0, \quad \mathcal{L}_X d' = 0, \quad X_M X^M = 0. \tag{32}
$$

10D generalized supergravity from flux shift in DFT



10D generalized supergravity EOMs

$$\delta d: \qquad R - \frac{1}{12} H^2 + 4\nabla^m X_m - 4 X_m X^m = 0\,, \tag{33}$$

$$\delta e^m{}_a: \qquad R_{mn} e^{na} - \frac{1}{4} H_{nkm} H^{nk}{}_l e^{la} + \nabla_m X_n e^{na} + \nabla_n X_m e^{na} = 0, \tag{34}$$

$$\delta b_{mn}: \qquad \frac{1}{2}\nabla_k H^{kmn} - H^{kmn} X_k - \nabla^m X^n + \nabla^n X^m = 0\,, \tag{35}$$

where $X_m = I_m + \nabla_m \Phi - B_{mn} I^n$ и I_m is a Killing vector.

Generalization to 11D case

In string theory, the appearance of generalized supergravity can be associated with a violation

Weyl invariance supergravity $\Longrightarrow$ scale invariance generalized supergravity

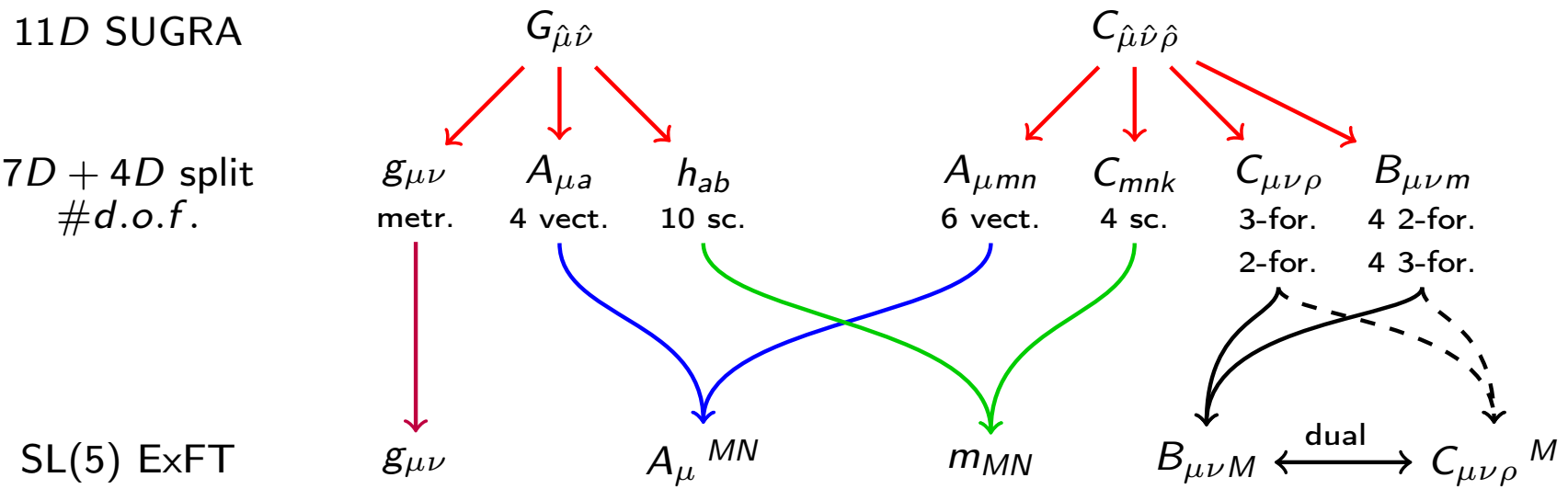
In M-theory, there is no Weyl symmetry for membranes. This was one of the arguments why 11D generalized supergravity should not exist. However, when the supergravity is considered in a split form, there is a violation

GL(11) invariance supergravity $\Longrightarrow$ GL(d)$\times$GL(11-d) invariance generalized supergravity

that allows to build generalization.

Exceptional field theory

Exceptional field theory (ExFT) - this is a generalization of 11-dimensional supergravity in split form $d + (11 - d)$, explicitly covariant with respect to the U-duality group.



$$m_{MN} = \mathcal{E}_M{}^A \mathcal{E}_N{}^B m_{AB} \tag{36}$$

Parametrization that corresponds to supergavity

$$m_{MN} = \mathcal{E}_M{}^A \mathcal{E}_N{}^B m_{AB}, \quad \mathcal{E}_M{}^A = e^{\frac{1}{10}} \begin{bmatrix} e^{-\frac{1}{2}} e_m{}^a & e^{\frac{1}{2}} V^a \\ 0 & e^{\frac{1}{2}} \end{bmatrix}, \quad e = \det(e^a_m), \quad V^m = \frac{1}{3!} \varepsilon^{mnkl} C_{nkl}, \tag{37}$$

section condition

$$\epsilon^{MNKLP} \epsilon_{MQRST} \partial_{NK} \bullet \otimes \partial_{LP} \bullet = 0, \quad \partial_{5m} = \partial_m, \quad \partial_{mn} = 0 \quad (SUGRA), \tag{38}$$

Ansatz for fields

We consider theory on the background $M_{11} = M_7 \times M_4$

$$\begin{aligned} g_{\mu\nu} &= e^{-2\phi(x^m)} e^{\frac{2}{5}} \bar{g}_{\mu\nu}(y^\mu), \quad & m_{MN} &= e^{-\phi} e^{\frac{1}{5}} M_{MN}, \\ A_\mu{}^{MN} &= 0, & B_{\mu\nu M} &= 0. \end{aligned} \tag{39}$$

Lagrangian

$$\begin{aligned} m\,\mathcal{L}' = {} & Y_{AB} Y_{CD} m^{AC} m^{BD} - \frac{1}{2} Y_{AB} Y_{CD} m^{AB} m^{CD} + 32 Z^{ABC} Z^{DEF} m_{AD} m_{BE} m_{CF} \\ & + 32 Z^{ABC} Z^{DEF} m_{AC} m_{BD} m_{EF} - \frac{7}{3} \theta_{AB} \theta_{CD} m^{AC} m^{BD} + \bar{e} \mathcal{R}[\bar{g}_{(7)}], \end{aligned} \tag{40}$$

Fluxes

$$\mathcal{L}_{E_{AB}} E^M{}_C = \mathcal{F}_{ABC}{}^D E^M{}_D, \tag{41}$$

$$\mathcal{F}_{ABC}{}^D = \frac{3}{2} E_N{}^D \partial_{[AB} E^N{}_{C]} - E^M{}_C \partial_{MN} E^N{}_{[B} \delta^D{}_{A]} - \frac{1}{2} E^M{}_{[B|} \partial_{MN} E^N{}_{|A]} \delta^D{}_C, \tag{42}$$

$$\mathcal{F}_{ABC}{}^D = \frac{3}{2} Z_{ABC}{}^D - \frac{1}{2} \theta_{[AB} \delta_{C]}{}^D + \delta_{[A}{}^D Y_{B]C}, \tag{43}$$

Generalized Yang-Baxter deformation

$$E'_M{}^A = O[\Omega]_M{}^N E_N{}^B, \qquad O[\Omega] = \begin{bmatrix} \delta_m{}^n & 0 \\ \frac{1}{3!}\epsilon_{mpqr}\Omega^{pqr} & 1 \end{bmatrix}. \tag{44}$$

For the deformation parameters, we use the poly-Killing ansatz

$$\Omega^{mnk} = \frac{1}{6}\rho^{\alpha\beta\gamma} k_\alpha{}^m k_\beta{}^n k_\gamma{}^k, \tag{45}$$

satisfying the condition (a generalization of the Yang-Baxter equation)

$$6\rho^{[i2|i7j1}\rho^{|i3i4|j2} f_{j1j2}{}^{|i5]} + \rho^{j1j2[i2}\rho^{i3i4i5]} f_{j1j2}{}^{i7} = 0. \tag{46}$$

Under such deformations, the fluxes transform as

$$\delta\mathcal{F}_{ABC}{}^D = \frac{1}{4} E^m{}_C E^n{}_A E^k{}_B E_l{}^E J^{lp}\epsilon_{kmnp}, \quad J^{mn} = k_{i_1}{}^m k_{i_4}{}^n \rho^{i_1 i_2 i_3} f_{i_2 i_3}{}^{i_4}, \tag{47}$$

that under reduction gives

$$J^{mn} = \rho^{i_1 i_2 i_3} f_{i_2 i_3}{}^{i_4} k_{i_1}{}^m k_{i_4}{}^n \underset{i=(*,\alpha),\ m=(*,\bar{m})}{\longrightarrow} I^{\bar{m}} \equiv J^{*\bar{m}} = \rho^{*\alpha\beta} f_{\alpha\beta}{}^\gamma k_\gamma{}^{\bar{m}}\ (gCYBE). \tag{48}$$

Constraints from SL(5) ExFT BI

SL(5) ExFT Bianchi identities

$$\begin{aligned}&\frac{1}{2}\partial_{AB}\mathcal{F}_{DFC}{}^{E}+\frac{1}{2}\partial_{BC}\mathcal{F}_{DFA}{}^{E}-\frac{1}{2}\delta^{E}_{A}\partial_{CG}\mathcal{F}_{DFB}{}^{G}-\frac{1}{4}\delta^{E}_{C}\partial_{BG}\mathcal{F}_{DFA}{}^{G}+\frac{1}{4}\delta^{E}_{C}\partial_{AG}\mathcal{F}_{DFB}{}^{G}+\frac{1}{2}\delta^{E}_{B}\partial_{CG}\mathcal{F}_{DFA}{}^{G}\\&-\frac{1}{2}\partial_{AC}\mathcal{F}_{DFB}{}^{E}-\mathcal{F}_{BGC}{}^{E}\mathcal{F}_{DFA}{}^{G}+\mathcal{F}_{AGC}{}^{E}\mathcal{F}_{DFB}{}^{G}+\mathcal{F}_{ABG}{}^{E}\mathcal{F}_{DFC}{}^{G}-\mathcal{F}_{ABC}{}^{G}\mathcal{F}_{DFG}{}^{E}-\frac{1}{2}\partial_{DF}\mathcal{F}_{ABC}{}^{E}=0\end{aligned}\tag{49}$$

also satisfied after the deformation

$$\mathcal{F}'_{ABC}{}^{D}=\mathcal{F}_{ABC}{}^{D}+\delta\mathcal{F}_{ABC}{}^{D},\tag{50}$$

if

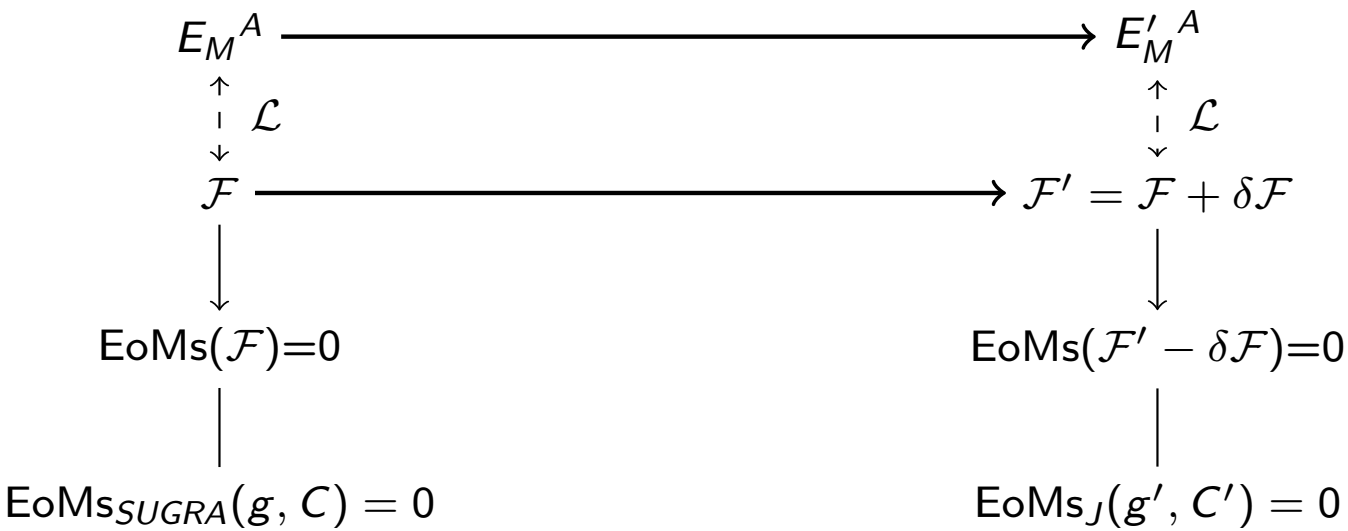
$$\begin{aligned}L_{e_a}J^{kl}+J^{nl}\partial_n\phi\, e_a{}^{k}=0,\qquad & J^{mn}\partial_n\phi=0,\\ \nabla_m\left(e^{-\phi}I^{mn}\right)=0,\qquad & J^{m[n}J^{kl]}=0,\end{aligned}\tag{51}$$

$$\begin{aligned}\nabla_{[m}Z_{n]}-\frac{1}{3}J^{kl}F_{mnkl}=0,\\ \nabla_k\left(e^{-\phi}J^{k[l}V^{p]}\right)=0,\\ \nabla_k(J^{(pl)}V^{k})-\nabla_k(V^{(p}J^{l)k})=0.\end{aligned}\tag{52}$$

where

$$Z_m=\partial_m\phi-\frac{2}{3}\varepsilon_{mnkl}I^{nk}V^{l}.\tag{53}$$

Construction of generalized supergravity equations



flux deformation

$$\delta\mathcal{F}_{mnk}{}^{l} = \frac{1}{4}\epsilon_{mnkp}J^{lp}. \tag{54}$$

Generalization of 11D supergravity

Generalized equations

$$
\begin{aligned}
0 &= \mathcal{R}_{mn}[h_{(4)}] - 7\,\tilde{\nabla}_{(m}Z_{n)} - \frac{1}{3}h_{mn}(\nabla V) + 8(1+V^2)\Big(S_{mn}J^{k}{}_{k} - 2J^{k}{}_{(m}J_{n)k}\Big) \\
&\quad + 4V_mV_n\Big(J^{kl}J_{kl} - 2J^{kl}J_{lk}\Big) + 4V_kV_l\Big(4J_{(m}{}^{k}J_{n)}{}^{l} - J^{k}{}_{(m}J^{l}{}_{n)} - 2S^{kl}S_{mn}\Big) \\
&\quad + 8V_kV_{(m}\Big(2J^{l}{}_{n)}J^{k}{}_{l} - 2S_{n)}{}^{k}J^{l}{}_{l} + J^{kl}J_{n)l}\Big), \\
0 &= \frac{1}{7}e^{2\phi}\,\mathcal{R}[\bar{g}_{(7)}] + \frac{1}{6}\,(\nabla V)^2 + \tilde{\nabla}^{m}Z_m - 6Z_mZ^m - 2J^{mn}J_{mn} + \frac{4}{3}J_{mn}J^{nm}, \\
0 &= \tilde{\nabla}^{m}F_{mnkl} - 6Z^{m}F_{mnkl} + 6\Big(2J^{pm}C_{m[nk}J_{l]p} - J^{pm}J_{p[n}C_{kl]m}\Big), \\
0 &= \mathcal{R}_{\mu\nu}[\bar{g}_{(7)}] - \frac{1}{7}\bar{g}_{\mu\nu}\mathcal{R}[\bar{g}_{(7)}],
\end{aligned}
\tag{55}
$$

where $S^{mn} = J^{(mn)}$, $F_{mnkl} = 4\partial_{[m}C_{nkl]}$ and

$$
\tilde{\nabla}_m = \nabla_m - \partial_m\phi\,. \tag{56}
$$

Example of solution - deformed $AdS_4 \times S^7$

AdS_4 Killing vectors

$$\begin{aligned} P_a &= \partial_a, & K_a &= x^2 \partial_a + 2x_a D, \\ D &= -x^m \partial_m, & M_{ab} &= x_a \partial_b - x_b \partial_a, \end{aligned} \tag{57}$$

где $a, b = 0, 1, 2$ и $m, n = 0, 1, 2, z$, $x^2 = \eta_{mn} x^m x^n$, $x_a = \eta_{ab} x^b$.

deformation along $D \wedge M \wedge M$:

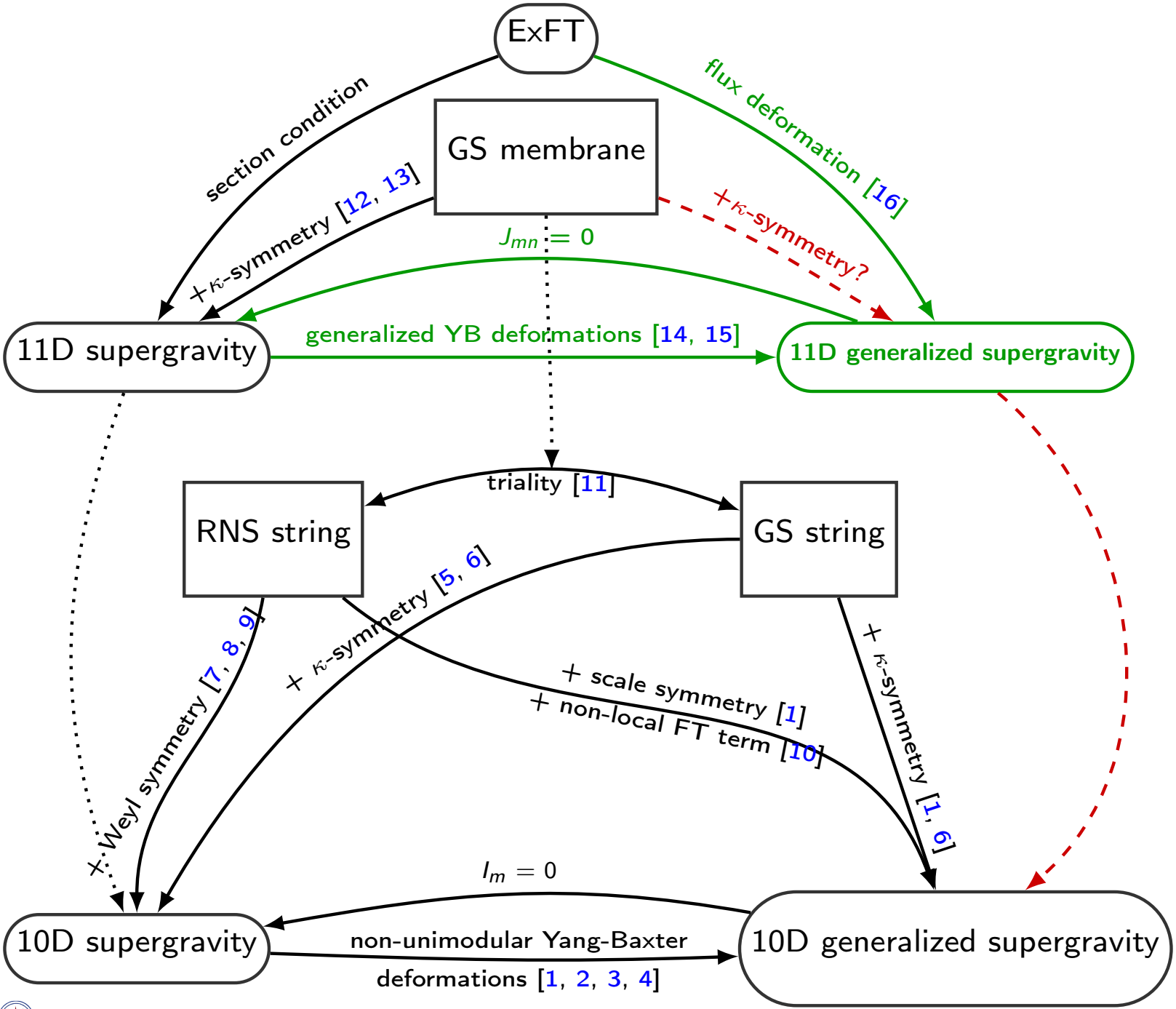
$$\Omega = \frac{4}{R^3} \rho_a \epsilon^{abc} D \wedge M_{bd} \wedge M_c{}^d$$

gives deformed background

$$\begin{aligned} ds^2 &= \frac{R^2}{4z^2} K^{\frac{2}{3}} \left\{ dx_a dx^a + \frac{1}{z^2} \rho_a x^a x^b dx_b dz + \left(1 - \frac{x_a x^a \rho_b x^b}{z^3}\right) dz^2 \right\} + R^2 K^{-\frac{1}{3}} d\Omega^2_{(7)}, \\ F &= -\frac{3}{8} \frac{R^3}{z^4} K^2 \left(1 + \frac{1}{12} \frac{x_a x^a \rho_b \rho_c x^b x^c}{z^4}\right) dx^0 \wedge dx^1 \wedge dx^2 \wedge dz. \end{aligned} \tag{58}$$

solving generalized equations if

$$GYBE = \rho^a \rho_a = 0. \tag{59}$$


ExFT
section condition
flux deformation [16]
GS membrane
+ κ-symmetry [12, 13]
+ κ-symmetry?
$J_{mn} = 0$
11D supergravity
generalized YB deformations [14, 15]
11D generalized supergravity
triality [11]
RNS string
GS string
+ Weyl symmetry [7, 8, 9]
+ κ-symmetry [5, 6]
+ scale symmetry [1]
+ non-local FT term [10]
+ κ-symmetry [1, 6]
$I_m = 0$
10D supergravity
non-unimodular Yang-Baxter
deformations [1, 2, 3, 4]
10D generalized supergravity

Thank you for your attention!

Рис.: "Deformations open the way to the world of new knowledge"

Equations of motion

From C-frame lagrangian

$$\bar{e}^{-1} h^{-\frac{1}{2}} \mathcal{L} = e^{-5\phi} \mathcal{R}[\bar{g}_{(7)}] + e^{-7\phi} \left(\mathcal{R}[h_{(4)}] + 42 h^{mn} \partial_m \phi \partial_n \phi + \frac{1}{2} \nabla_m V^m \nabla_n V^n \right), \tag{60}$$

we obtain the equations of motion for dynamic fields ϕ, h_{mn} и V_m

$$\begin{aligned}
\delta\phi: \quad & \frac{5}{7} e^{2\phi} \mathcal{R}[\bar{g}_{(7)}] + \mathcal{R}[h_{(4)}] + 12 \nabla_m \nabla_n \phi \; h^{mn} - 42 \nabla_m \phi \nabla_n \phi \, h^{mn} + \frac{1}{2} (\nabla V)^2 = 0, \\
\delta V^m: \quad & \partial_m (\nabla V) - 7 (\nabla V) \partial_m \phi = 0, \\
\delta h^{mn}: \quad & \mathcal{R}_{mn}[h_{(4)}] - 7 \partial_m \phi \, \partial_n \phi + 7 \nabla_m \nabla_n \phi \\
& + h_{mn} \left(-\frac{1}{2} e^{2\phi} \mathcal{R}[\bar{g}_{(7)}] - \frac{1}{2} \mathcal{R}[h_{(4)}] + 28 \, \partial_k \phi \, \partial_l \phi \, h^{kl} - 7 \nabla_k \nabla_l \phi \, h^{kl} + \frac{1}{4} (\nabla V)^2 \right) = 0,
\end{aligned} \tag{61}$$

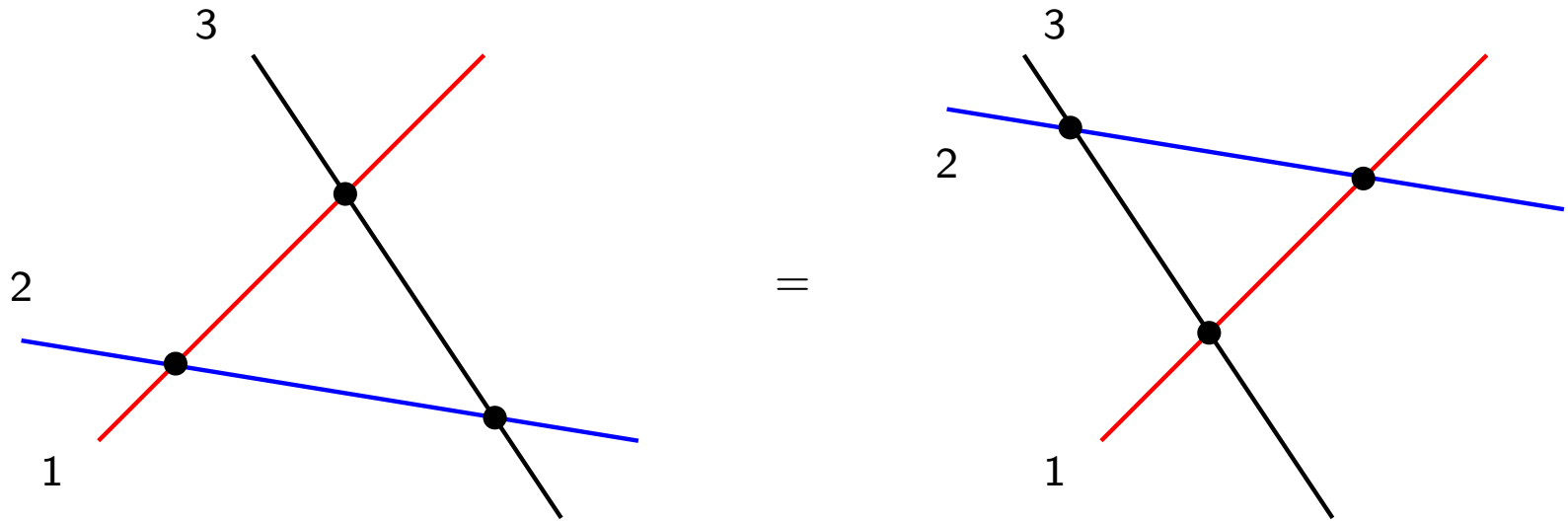
Generalized Lie derivatives

DFT

$$\begin{aligned}\mathcal{L}_\Lambda V^M &= \Lambda^N \partial_N V^M - V^N \partial_N \Lambda^M + \eta^{MN}\eta_{KL}\partial_N \Lambda^K V^L, \\ \mathcal{L}_\Lambda d &= \Lambda^M \partial_M d - \frac{1}{2}\partial_M \Lambda^M,\end{aligned} \tag{62}$$

ExFT

$$\mathcal{L}_\Lambda V^M = \frac{1}{2}\Lambda^{KL}\partial_{KL} V^M - V^L \partial_{LK}\Lambda^{MK} + \frac{1}{4} V^M \partial_{KL}\Lambda^{KL} + \lambda_V \partial_{KL}\Lambda^{KL} V^M, \tag{63}$$



QYBE

$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u-v). \tag{64}$$

In classical limit

$$R_{ij} = \mathbb{1} + \epsilon\, r_{ij} + \mathcal{O}(\epsilon^2). \tag{65}$$

CYBE

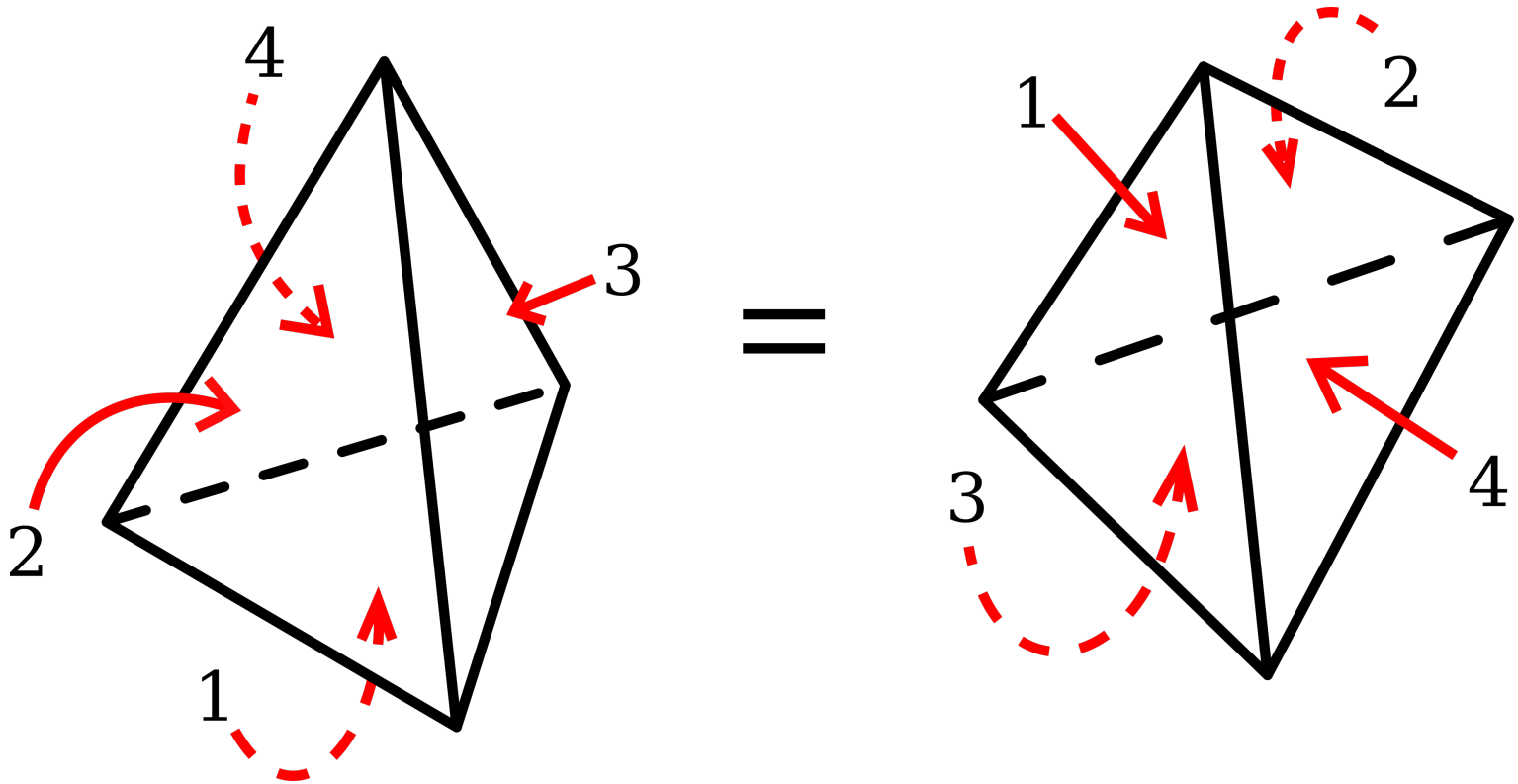
$$[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0. \tag{66}$$

using

$$[r_{12}, r_{13}] = r^{\alpha_1\beta_1} r^{\alpha_1\beta_1} \cdot [e_{\alpha_1}, e_{\alpha_2}] \otimes e_{\beta_1} \otimes e_{\beta_2}, \tag{67}$$

we obtain

$$r^{\alpha_1[\beta_1|} r^{\alpha_2|\beta_2} f_{\alpha_1\alpha_2}{}^{\gamma]} = 0. \tag{68}$$



Tetrahedron equation

$$R_{123}R_{124}R_{134}R_{234} = R_{234}R_{134}R_{124}R_{123}. \tag{69}$$

In classical limit

$$R_{ijk} = \mathbb{1} + \epsilon\, r_{ijk} + \mathcal{O}(\epsilon^2). \tag{70}$$

classical tetrahedron equation

$$\epsilon^2 \cdot | \qquad [r_{123}, r_{124}] + [r_{123}, r_{134}] + [r_{123}, r_{234}] + [r_{124}, r_{134}] + [r_{124}, r_{234}] + [r_{134}, r_{234}] = 0. \tag{71}$$

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