False vacuum Skrymions revisited

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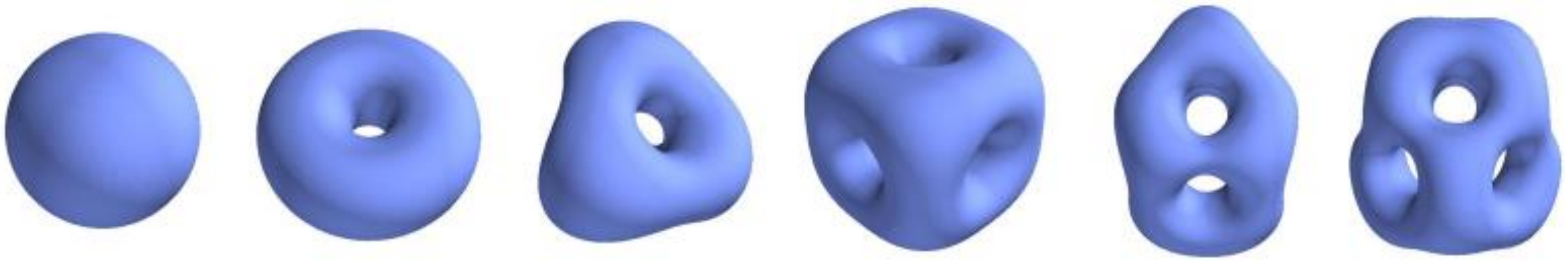
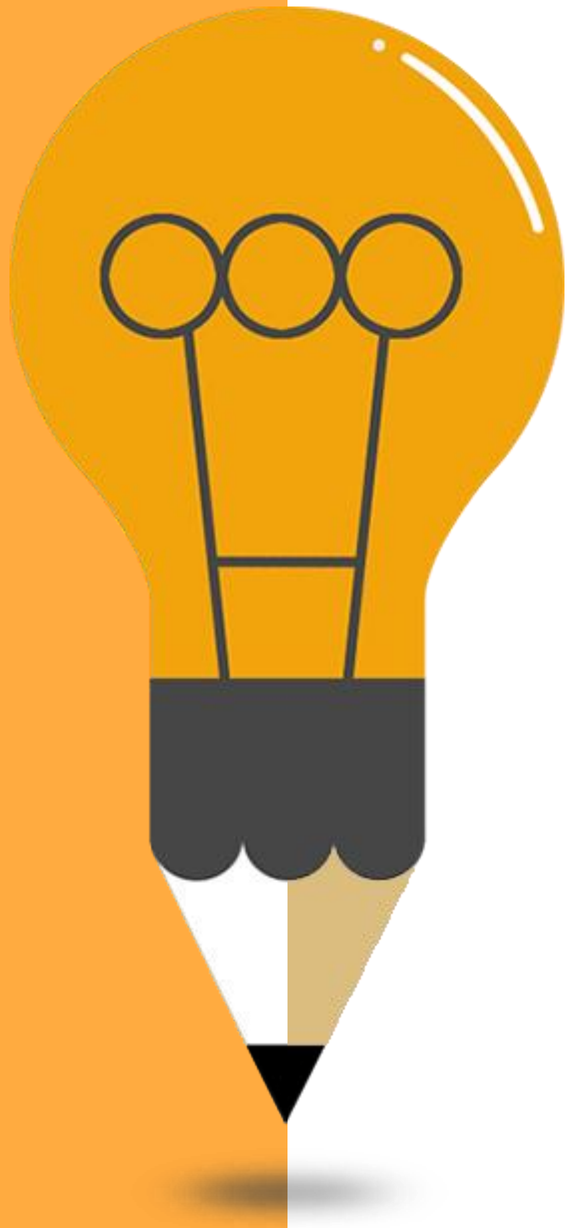


Supersymmetries and Quantum Symmetries – SQS'22
BLTP, JNIR

(8/9/2022)

Outline



1. The Skyrme model
2. A false vacuum potential
3. The vacuum Skyrmions with Q=1-6
4. The negative charge density structures of the Skyrmions and false vacuum Skyrmions
5. Conclusion

Standard Skyrme model

The Skyrme model is a nonlinear effective field theory at the low energy regime for the triplet of pions defined in the 3+1 Minkowski space, defined by the Lagrangian

$$L_{\text{Skyrme}} = \int d^3x \left[-\frac{1}{2} \operatorname{Tr}\left(R_\mu R^\mu\right) + \frac{1}{16} \operatorname{Tr}\left([R_\mu, R_\nu]\,[R^\mu, R^\nu]\right) \right]$$

where the energy and length units are $f_\pi/(4\,e)$ and $2/(e\,f_\pi)$, respectively.

Mauer Cartan form $R_\mu = \partial_\mu U U^\dagger \in su(2)$

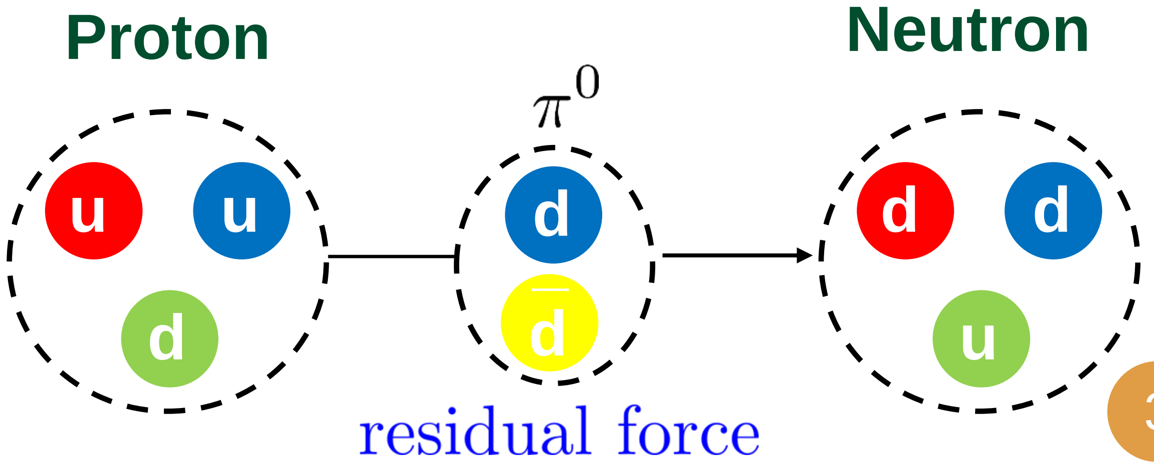
Skyrme field is an SU(2)-valued scalar field $U(\mathbf{x},\, t) = \phi_0 + i\,\vec{\phi} \cdot \vec{\tau} \in SU(2)$

- $\vec{\phi}$ → triplet of pion fields
- $\vec{\tau}$ → triplet of Pauli matrices
- ϕ_0 → field constraint $(\phi_0^2 + \vec{\phi} \cdot \vec{\phi} = 1)$

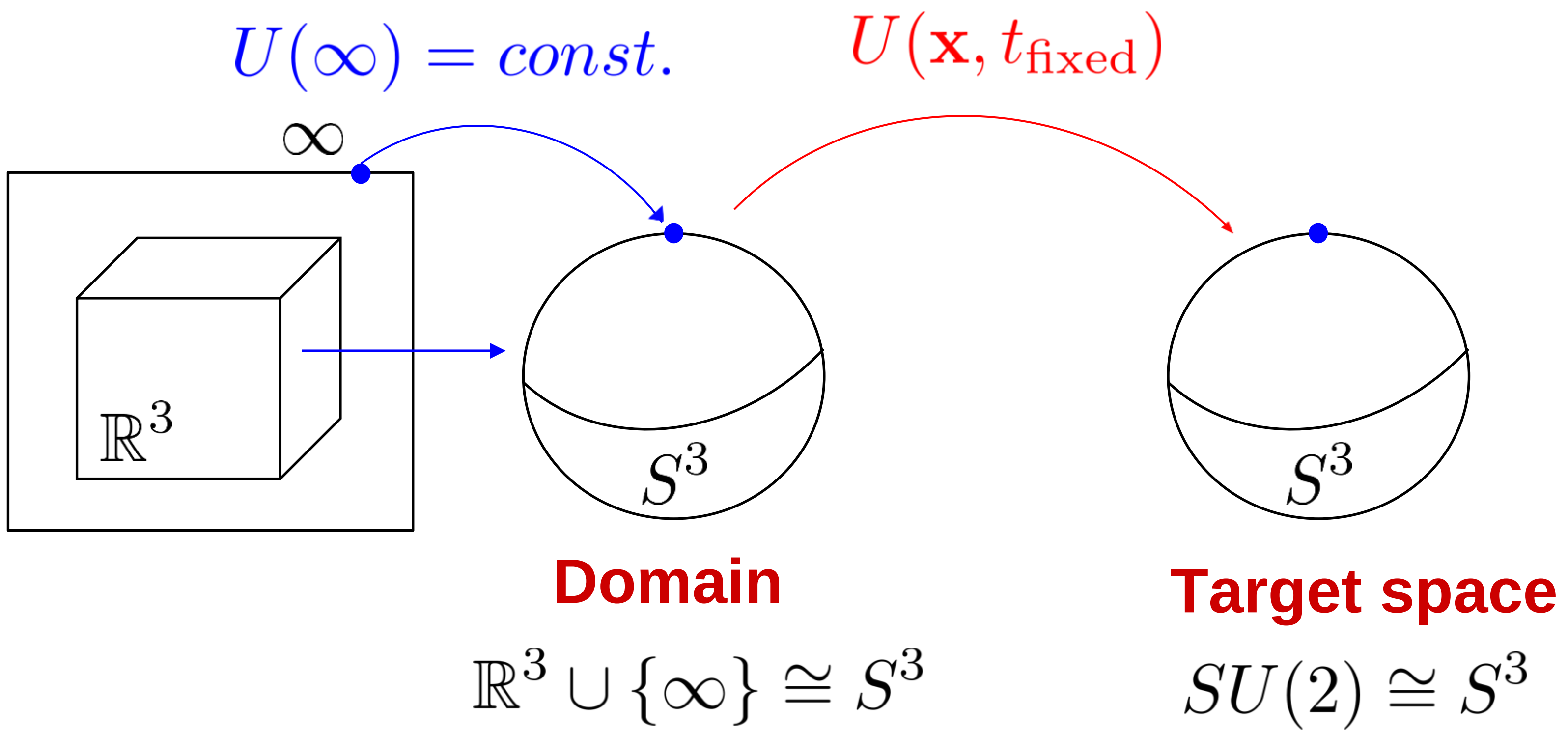
Internal symmetries

Global $SU(2)_L \times SU(2)_R$ chiral symmetry

$$U \to g_L\, U\, g_R, \quad \forall g_L,\, g_R \in SU(2)$$



Topological charge and pion mass potential



$$U(\mathbf{x}, t_{\text{fixo}}) : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \to SU(2) \cong S^3$$

Homotopy group $\pi_3\left(S^3\right) = \mathbb{Z}$

Topological (Skyrme) charge

(winding number of the U-fields)

$$Q = -\frac{1}{24\,\pi^2} \int d^3x\, \varepsilon_{ijk}\, \mathrm{Tr}\left(R_i\, R_j\, R_k\right)$$

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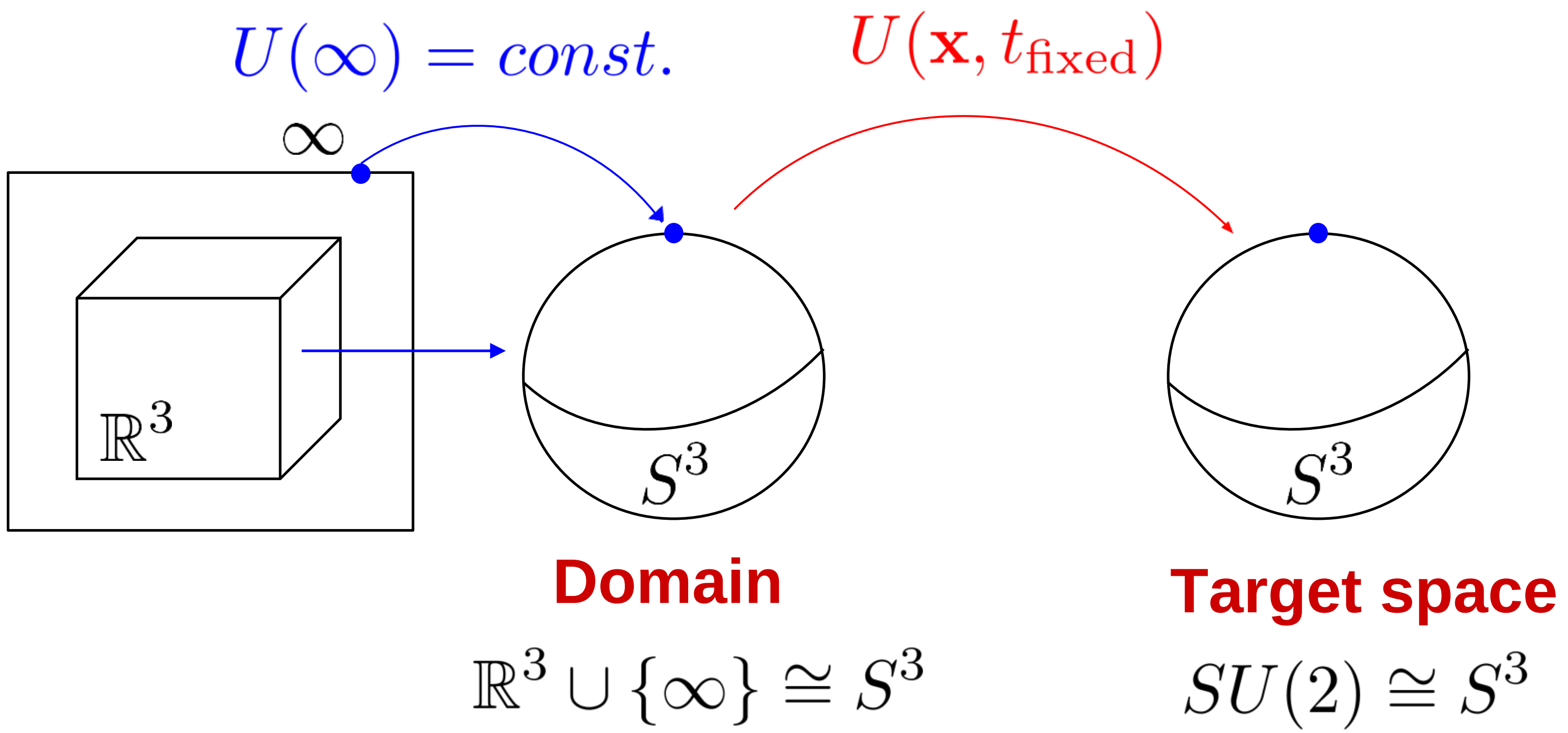

$$U(\mathbf{x}, t_{\text{fixo}}) : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \rightarrow SU(2) \cong S^3$$

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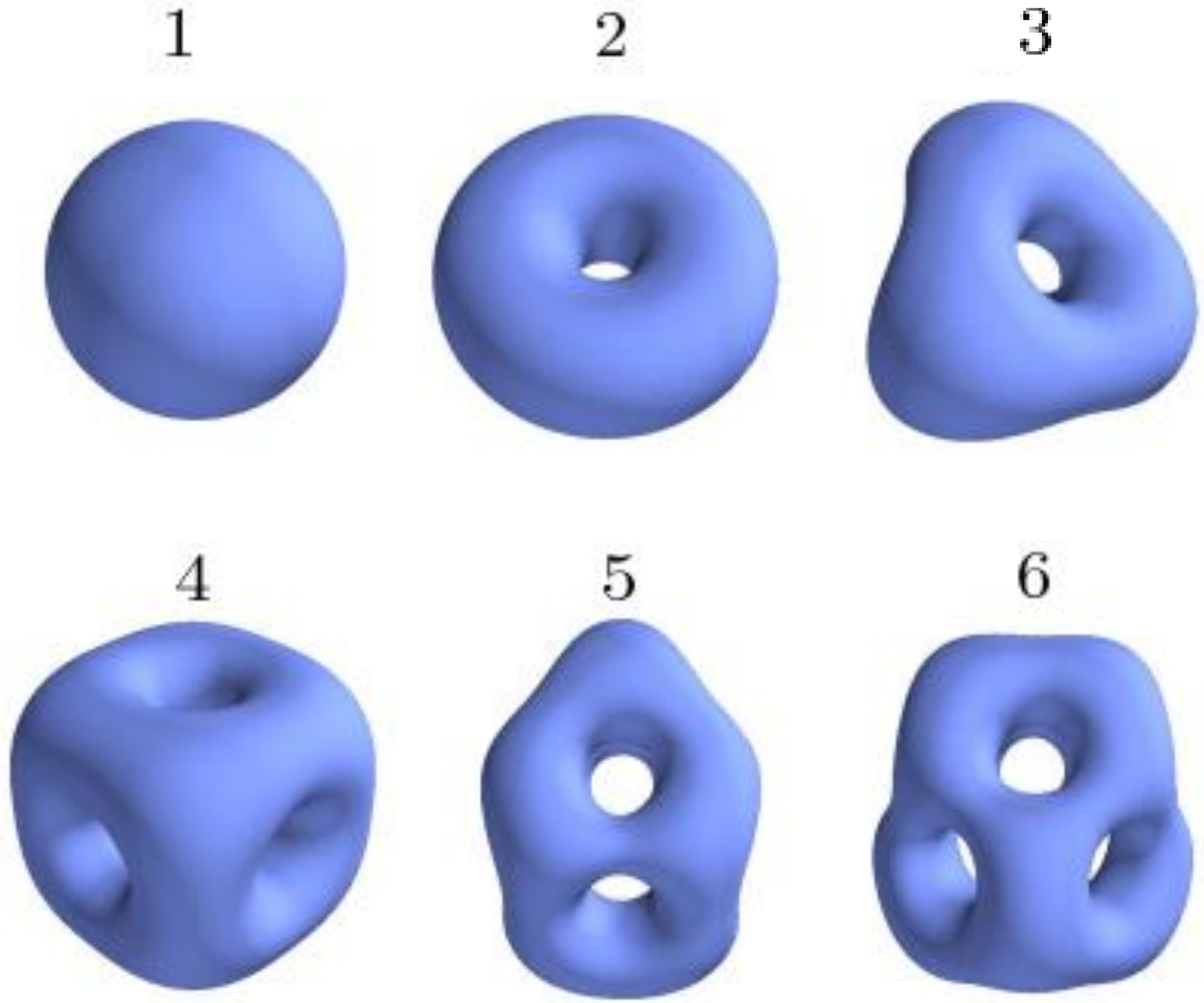
Usual pion mass potential $V(U) = m^2\ \mathrm{Tr}\ (\mathbb{1} - U) \geq 0 \Longrightarrow$ Vacuum conf. $U = \mathbb{1}$

The $SU(2)_L \times SU(2)_R$ symmetry breaks to its $SU(2)$ diagonal subgroup $U \rightarrow g\, U\, g^{-1}, \quad \forall g \in SU(2)$

Standard Skyrmions

By convenience, let us denote the static energy in topological charge units, i.e. $E \to E/(12\pi^2)$.

Isosurfaces of topological charge density



Q	Sym.	E/Q	E_{rational}/Q	E_{B}
1	$O(3)$	1.232	1.232	0
2	$D_{\infty h}$	1.1791	1.208	0.0531
3	T_d	1.1462	1.184	0.0860
4	O_h	1.1201	1.137	0.1121
5	O_{2d}	1.1172	1.147	0.1150
6	D_{4d}	1.1079	1.137	0.1243

$\sim 10\,\%\, E/Q$

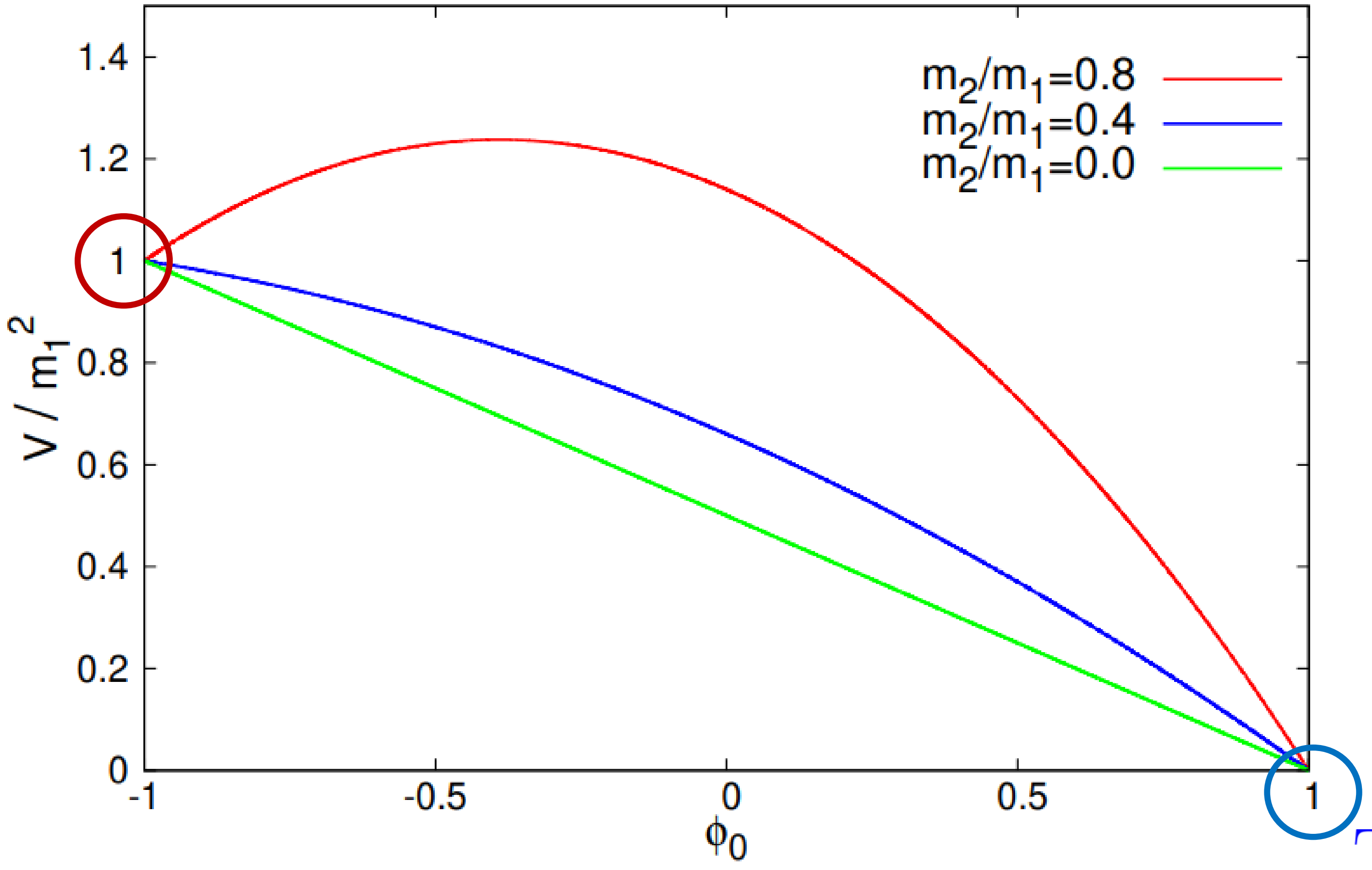
*Binding energy per topological charge unit: $E_{\mathrm{B}} \equiv E_{Q=1} - E/Q$

False vacuum potential

$$V(U) = \frac{1}{4}\Big[\underbrace{m_1^2\ \mathrm{Tr}\ (\mathbb{1} - U)}_{\textbf{usual mass term}} + \underbrace{m_2^2\ \mathrm{Tr}\ (\mathbb{1} - U^2)}_{\textbf{quadratic in } U}\Big] = \frac{m_1^2}{2}\left[(1-\phi_0)\ +\ 2\left(\frac{m_2}{m_1}\right)^2 (1-\phi_0^2)\right] \geq 0$$

$$U = \phi_0 + i\,\vec{\phi}\cdot\vec{\tau}, \quad \phi_0^2 + \vec{\phi}\cdot\vec{\phi} = 1$$

Local minimum at $\phi_0 = -1$ $(U = -\mathbb{1})$ iff $m_1^2 < 4\,m_2^2$. Global maximum at $\phi_0 = 1$ $(U = \mathbb{1})$.



True vacuum

False vacuum potential

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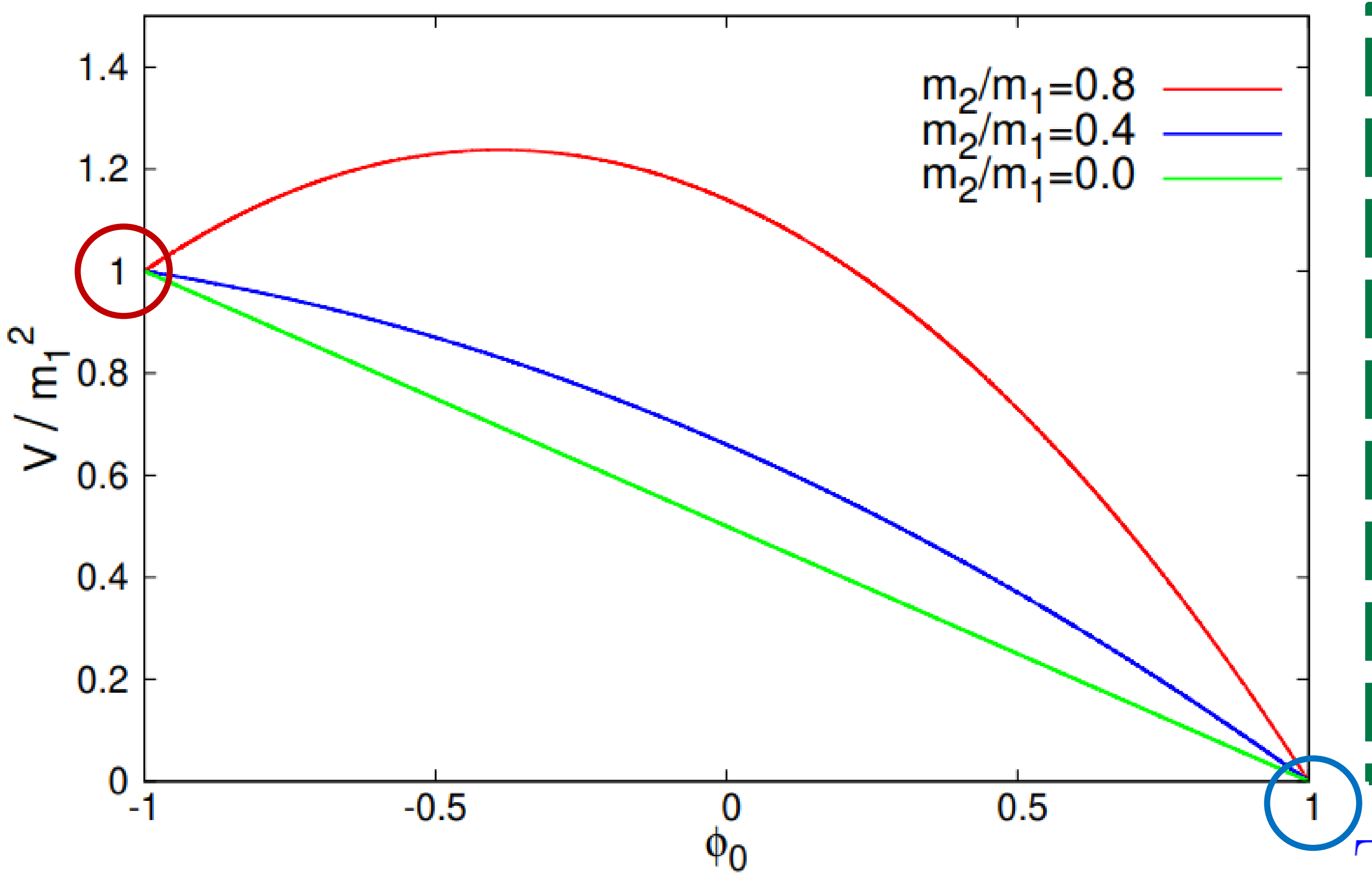

True vacuum

Fate of the false monopoles: Induced vacuum decay, Phys. Rev. D 82, 025022 (2010).

Field conf. obtained inside the rational map ansatz

Q = 1 : Solutions of the E.L. were obtained inside the rational map ansatz.

Q ≥ 2 : Approx. of the false vacuum Skyrmions were constructed by minimizing the Hamiltonian inside the rational map ansatz.

Effective potential

Static energy density $\mathcal{E} = -\frac{1}{2}\,\mathrm{Tr}\,(R_i\,R_i) - \frac{1}{16}\,\mathrm{Tr}\,([R_i,\,R_j]\,[R_i,\,R_j]) + V\,(U)$

To have $\mathcal{E}(\infty) = 0$ with $U(\infty) = -\mathbb{1}$: $V \to V - m_1^2 \xrightarrow[\mathbf{x}\to\infty]{} 0$

$$V_{\text{eff}} \equiv \begin{cases} V, & U(\infty) = +\mathbb{1}\ \text{(Skyrmions)} \\ V - m_1^2, & U(\infty) = -\mathbb{1}\ \text{(False vacuum Skyrmions, for } \mathrm{m}_1^2 < 4\,\mathrm{m}_2^2) \end{cases}$$

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Euler-Lagrange eq $0 = \partial_\mu \left(R^\mu + \frac{1}{4}\,[R_\nu,\,[R^\nu,\,R^\mu]] \right) + \frac{1}{8}\,m_1^2\,(U - U^\dagger) + \frac{m_2^2}{4}\,(U^2 - U^{\dagger 2})$

Field excitations arround the vacuum and false vacuum:

$U \sim (-1)^l\ \mathbb{1} + i\,\vec{v}\cdot\vec{\tau} + \mathcal{O}\left(v_a^2\right)$

Excitations of the pion fields

$$l \equiv \begin{cases} 0, & U(\infty) = +\mathbb{1} \\ 1, & U(\infty) = -\mathbb{1}\ (m_1^2 < 4\,m_2^2) \end{cases}$$

Effective potential

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$\partial_\mu \partial^\mu \vec{v} + m_{\text{eff}}^2\,(l)\;\vec{v} = 0\,,\qquad m_{\text{eff}}\,(l) \equiv \sqrt{m_2^2 + (-1)^l\,\frac{m_1^2}{4}} \;>\; 0 \text{ iff } \begin{cases} l = 0 \text{ and } m_1^2 \neq 0 \neq m_2^2 \\ l = 1 \text{ and } m_1^2 < 4\,m_2^2 \end{cases}$

If $m_1^2 > 4\,m_2^2$, then m_{eff} is a pure imaginary number.

$U = -\mathbb{1}$ is local minima of V

3D simulations

Static sector: $E_{\text{static}} = -L_{\text{static}}$ (metric $\eta = \text{diag}\,(1, -1, -1, -1)$)

Minimize E_{static} using 3D simulated annealing (SA)

SA 1 — **Colletive** rotation in the target space of points belonging to an random box in physical space — efficiency_1

SA 2 — **Singular** rotation in the target space of random points (not neighbors) of the physical space — efficiency_2

loop

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Derrick-type virial identities $E_4 = E_2 + 3\,E_0$

$$E_0 = \frac{1}{12\,\pi^2}\int d^3x\; V_{\text{eff}}\,, \quad E_2 = -\frac{1}{24\,\pi^2}\int d^3x\;\text{Tr}\,(R_i\,R_i)\,, \quad E_4 = -\frac{1}{192\,\pi^2}\int d^3x\;\text{Tr}\,([R_i,\,R_j]\,[R_i,\,R_j])$$

Topological charge density

$$\mathcal{Q} = -\frac{1}{24\,\pi^2}\,\varepsilon_{ijk}\,\text{Tr}\,(R_i\,R_j\,R_k)$$

RMS radius

$$\sqrt{\langle r^2\rangle} \equiv \sqrt{\frac{1}{Q}\int d^3x\,r^2\,\mathcal{Q}}$$

Derrick's factor

$$\mathcal{D} = \frac{E_4 - E_2 - 3E_0}{E_4 + E_2 + E_0}$$

False vacuum Skyrmions $(m_1 = 0.5)$

m_2, m_{eff}, E/Q increases and $\sqrt{\langle r^2 \rangle}$ decreases

E/Q decreases
$\sqrt{\langle r^2 \rangle}$ increases

$m_2 = 0.5$

Q	E/Q	$\mathcal{D}$	$\sqrt{\langle r^2 \rangle}$
1*	1.2716	−0.0016	0.9655
1	1.2761	0.0213	0.9510
2	1.2146	0.0155	1.3204
3	1.1761	0.0186	1.5510
4	1.1464	0.0219	1.7355
5	1.1426	0.0282	1.9343
6	1.1317	0.0322	2.0830

$m_{\text{eff}} = 0.433$

$m_2 = 2.5$

Q	E/Q	$\mathcal{D}$	$\sqrt{\langle r^2 \rangle}$
1*	1.6384	0.0001	0.6288
1	1.6381	−0.0025	0.6332
2	1.5591	−0.0025	0.8764
3	1.5075	−0.0020	1.0259
4	1.4650	−0.0022	1.1507
5	1.4588	−0.0021	1.2870
6	1.4425	−0.0021	1.3899

$m_{\text{eff}} = 2.487$

$m_2 = 10$

Q	E/Q	$\mathcal{D}$	$\sqrt{\langle r^2 \rangle}$
1*	2.6055	0.0000	0.3609
1	2.6047	−0.0022	0.3634
2	2.4920	−0.0020	0.5023
3	2.4233	−0.0017	0.5821
4	2.3601	−0.0018	0.6489
5	2.3528	−0.0015	0.7236
6	2.3293	−0.0015	0.7786

$m_{\text{eff}} = 9.997$

$Q = 1^*$-Skyrmions were obtained through 1D minimization inside the rational map ansatz.

False vacuum Skyrmions $(m_1 = 0.5)$

m_2, m_{eff}, E/Q increases and $\sqrt{\langle r^2 \rangle}$ decreases

E/Q decreases, $\sqrt{\langle r^2 \rangle}$ increases

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Type	m_1	m_2	m_{eff}	$\mathcal{Q}_-$ (10^{-3})	$\mathcal{Q}_+$
Standard $Q = 3$ Skyrmion	0.0	0.0	0.0000	−3.50	0.245
Massive $Q = 3$ Skyrmion	0.5	0.0	0.2500	−3.65	0.255
False vac. $Q = 3$ Skyrmion	0.5	0.5	0.4330	−3.56	0.262
False vac. $Q = 3$ Skyrmion	0.5	2.5	2.4875	−6.99	0.790
False vac. $Q = 3$ Skyrmion	0.5	10.0	9.9969	−15.22	4.036

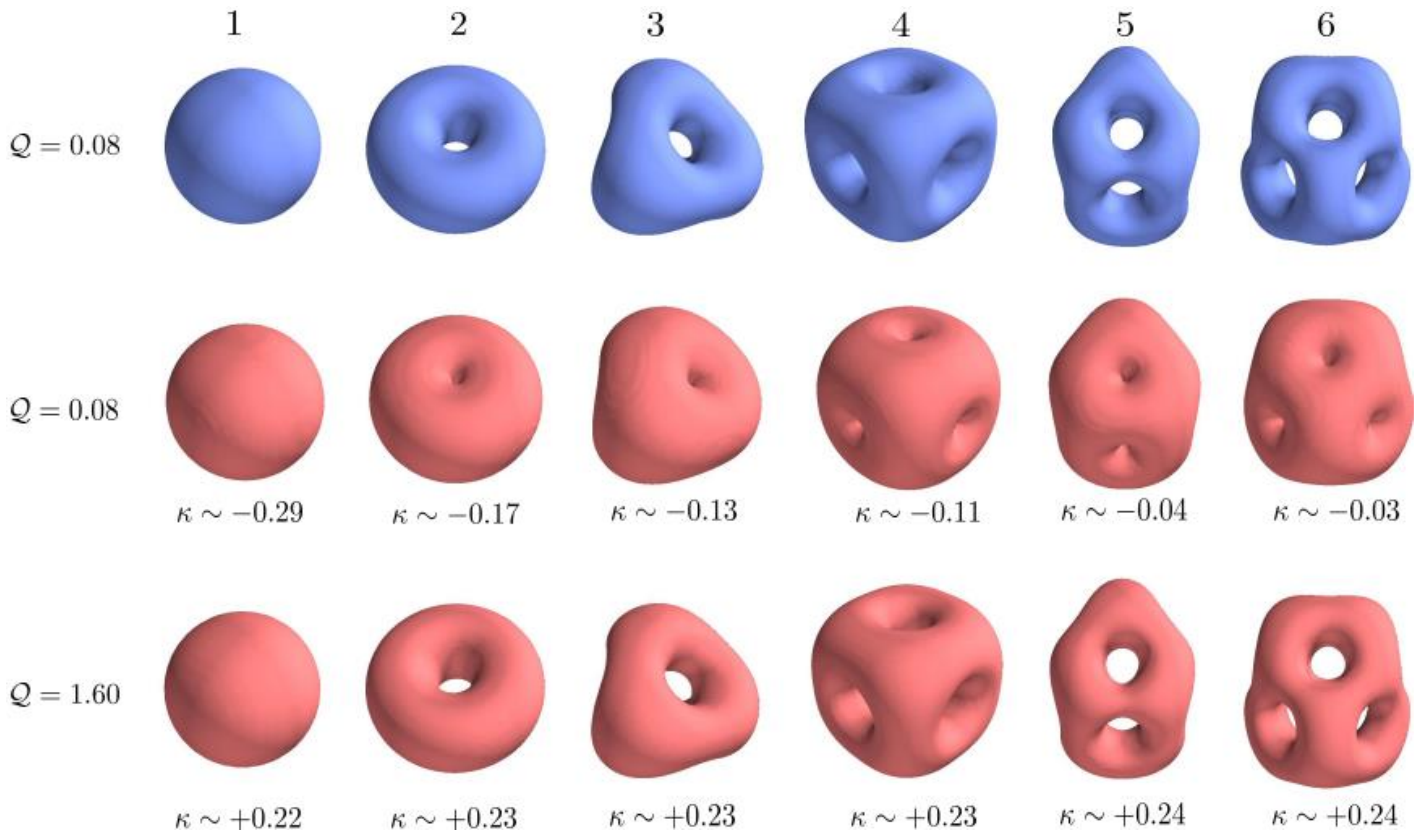
max **min**

$\mathcal{Q}_+ = \max_{\mathbb{R}^3}(\mathcal{Q})$, $\mathcal{Q}_- = \min_{\mathbb{R}^3}(\mathcal{Q})$

For both cases: m_{eff} increases

$\mathcal{Q}_+$ and $|\mathcal{Q}_-|$ increases

Skyrmions and false vacuum Skyrmions



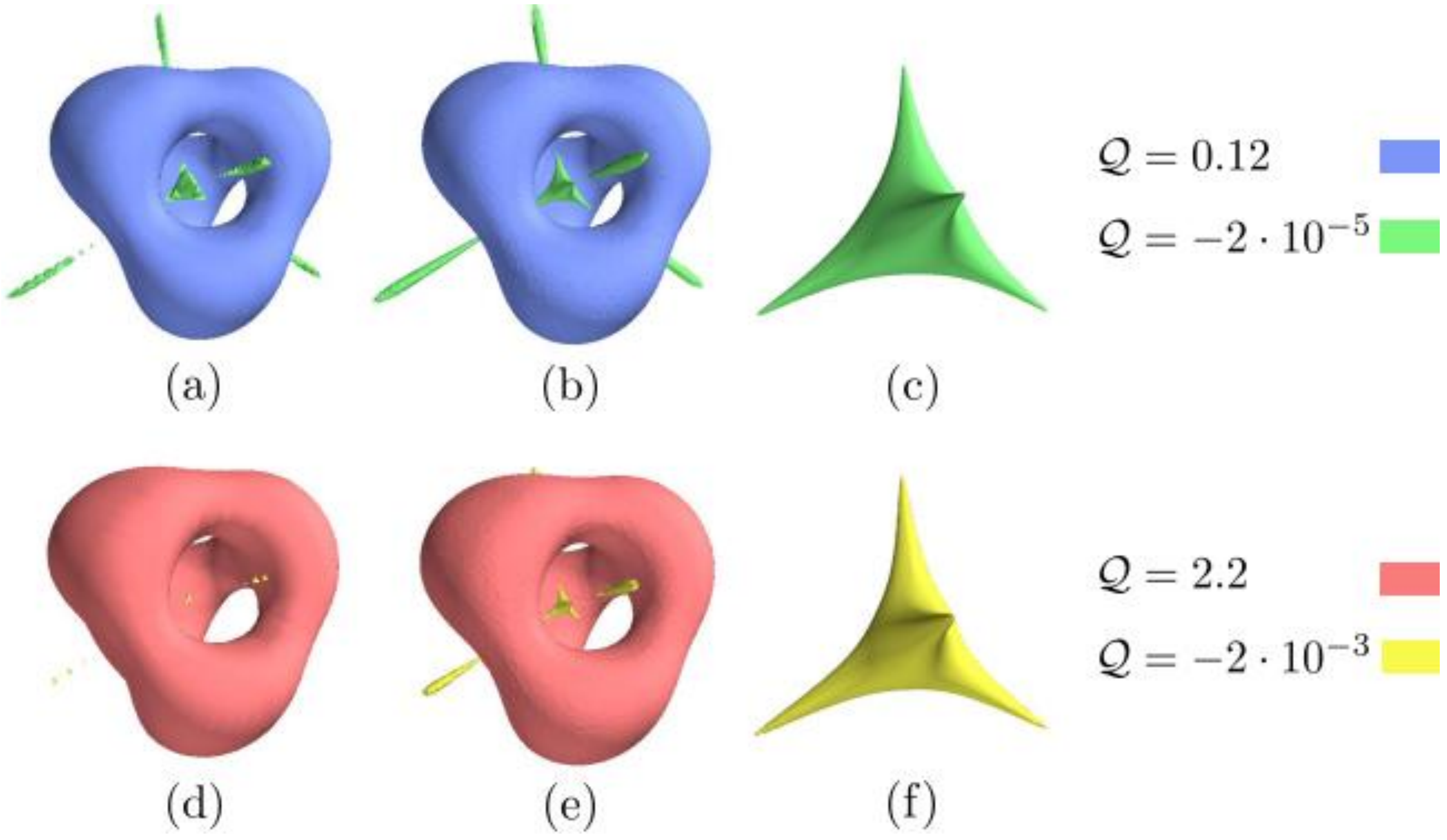
Skyrmions
$(m_1 = 0.5,\ m_2 = 0)$

False vacuum Skyrmions
$(m_1 = 0.5,\ m_2 = 10)$

*κ is a zoom factor between the Skyrmions (in blue) and the false vacuum Skyrmions (in red).
For $\kappa > 0$ ($\kappa < 0$) we are more close (distant) of the center of the soliton.

Q=3

Obtain the Skyrmion → select a lattice region → 3D interpolation → SA



Skyrmions
$(m_1 = 0.5,\, m_2 = 0)$

False vacuum Skyrmions
$(m_1 = 0.5,\, m_2 = 10)$

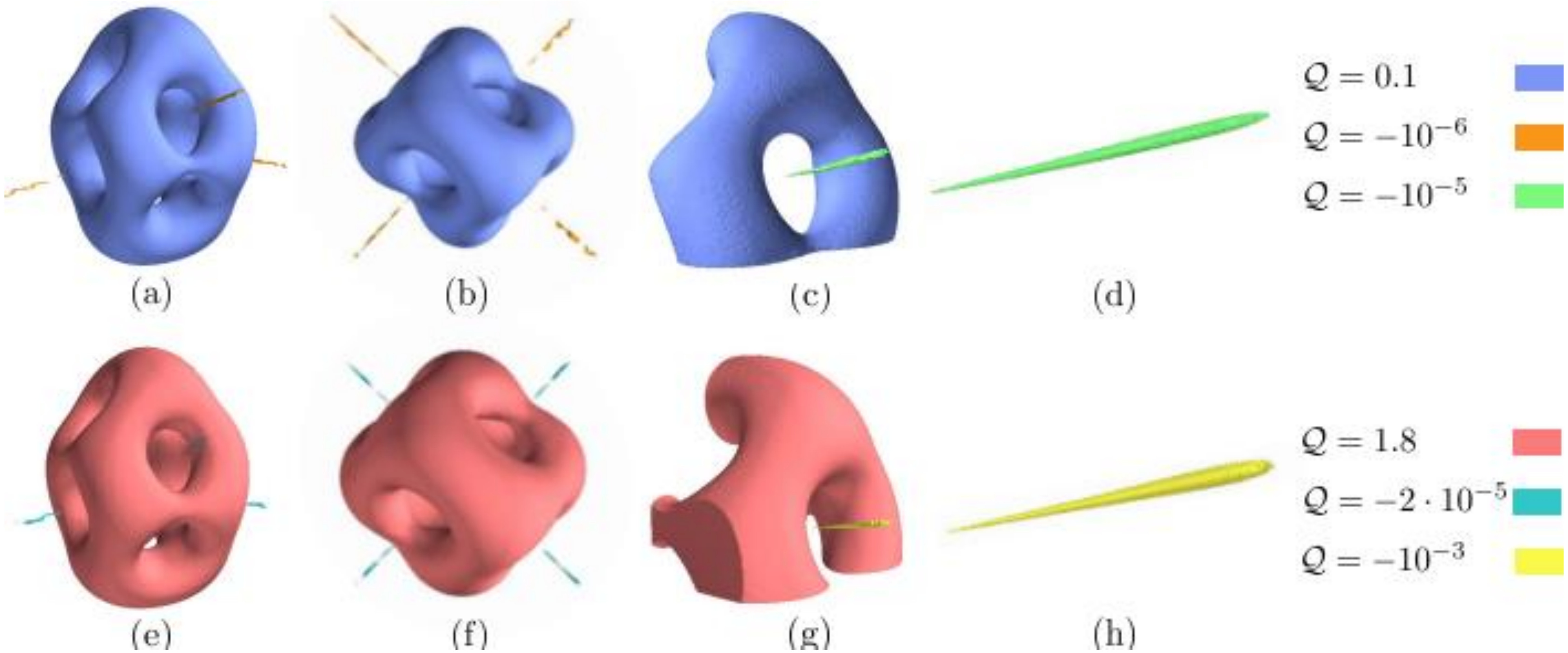
Spacing Δx ⟶ $\Delta x/4$ ⟶ $\Delta x/16$

4 tubes

For the $Q = 3$-**Skyrmions** such structure were previously observed in: J. Phys. A 46, 265401 (2013)

*Spacing $\Delta x_{(a)} = 0.08$ and $\Delta x_{(e)} = 0.04$

Q=5



Skyrmions

$(m_1 = 0.5,\ m_2 = 0)$

False vacuum Skyrmions

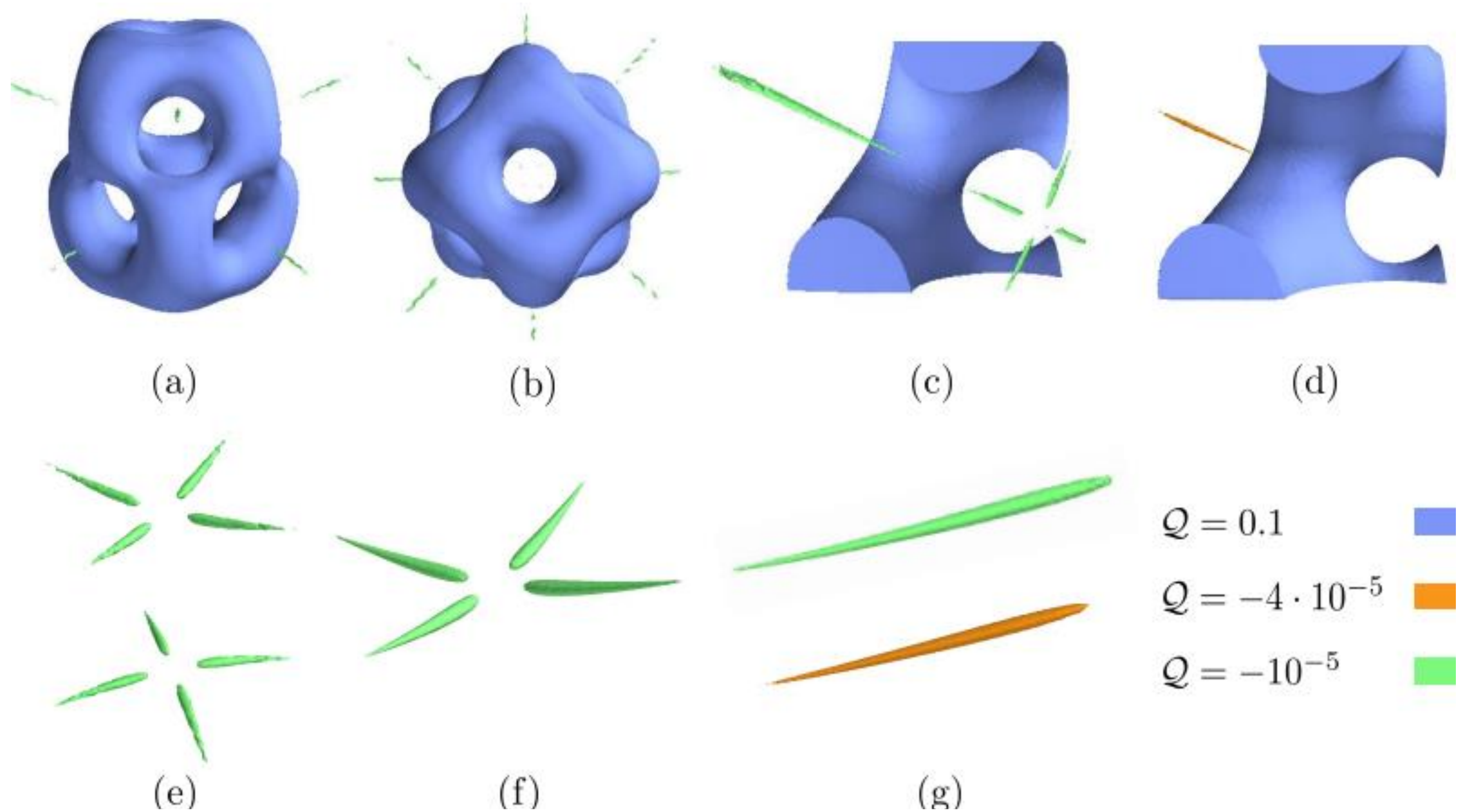
$(m_1 = 0.5,\ m_2 = 10)$

$\Delta x \longrightarrow \Delta x/4 \longrightarrow \Delta x/16$

4 tubes

*Spacing $\Delta x_{(a)} = 0.08$ and $\Delta x_{(e)} = 0.04$

Q=6-Skyrmion



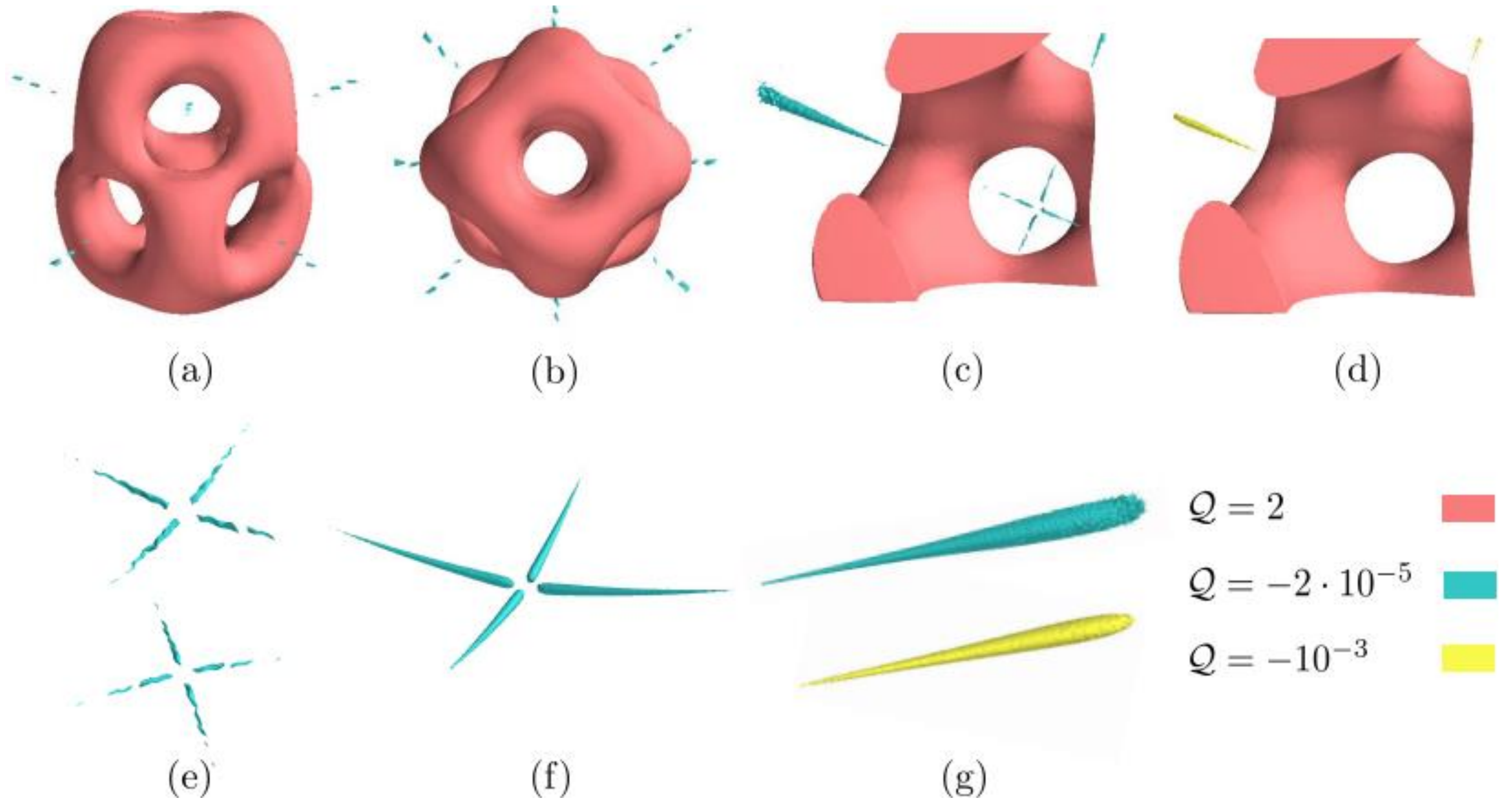
Skyrmions

$(m_1 = 0.5,\, m_2 = 0)$

8 tubes

*Spacing $\Delta x_{(a),(b)} = 0.08$, $\Delta x_{(c),(d),(e)} = 0.02$ and $\Delta x_{(f),(g)} = 0.005$

False vacuum Q=6-Skyrmion

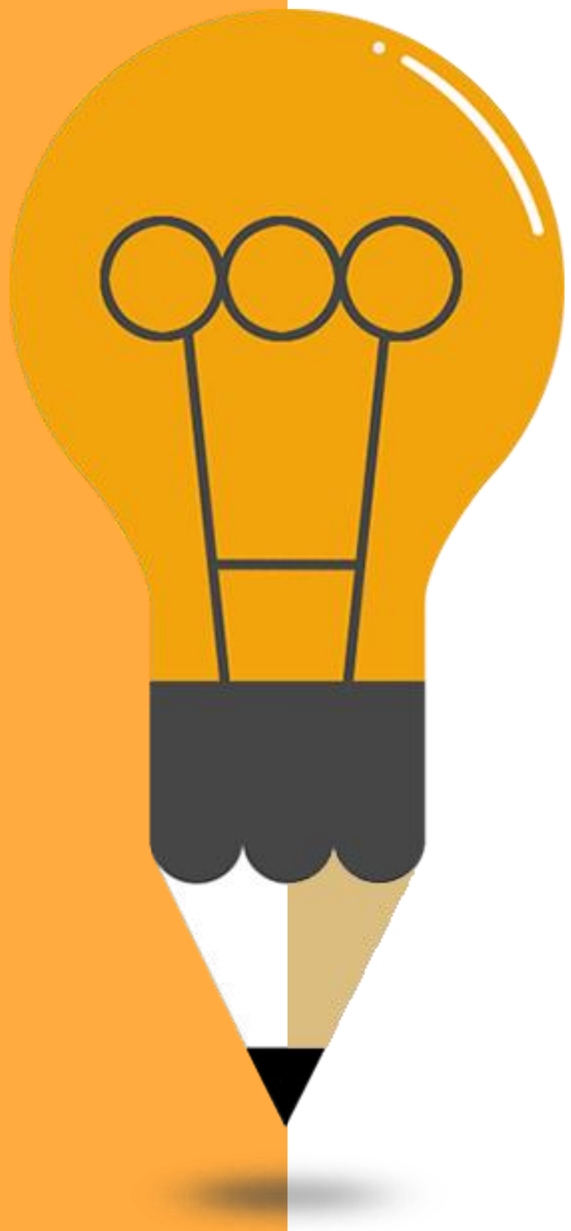


False vacuum Skyrmions
$(m_1 = 0.5,\ m_2 = 10)$

8 tubes

*Spacing $\Delta x_{(a),(b)} = 0.04$, $\Delta x_{(c),(d),(e)} = 0.01$, $\Delta x_{(f)} = 0.0025$ and $\Delta x_{(g)} = 0.005$

Conclusion



1 We have obtained false vacuum Skyrmions with Q=1-6 using 3D simulated annealing method.

2 As the effective mass remains positive, the shapes of the soliton solutions for both the Skyrmions and false vacuum Skyrmions are qualitative similar.

3 We explored numerically very small regions of negative topological density which appear for the solutions with degrees Q = 3, 5, 6. For Q=1, 2, 4 such structures were not found.

4 **Interesting ways to proceed**

- False vacuum Skyrmions of higher degrees.
- Introduction of the sixtic term in the space-time derivatives.
- Study false vacuum Skyrmions stability.
- The U(1) gauged Skyrme model + false vacuum potential.

Thank you