Cubic Interacting Vertices for unconstrained Integer Higher Spin Fields

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based on I.L. Buchbinder, A.R, General Cubic Interacting Vertex for Massless Integer Higher Spin Fields, Physics Letters B 820 (2021) 136470, [arXiv:2105.12030],
A.R. Towards the structure of a cubic interaction vertex for massless integer higher spin fields [arXiv:2205.00488],
C. Burdik, A.A.R, BRST-BV Quantum Actions for Constrained Totally-Symmetric Integer HS Fields, Nuclear Physics B 965 (2021) 115357, [arXiv:2010.15741]

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Abstract

We consider the massless theory of higher spin fields within the framework of the BRST approach and construct a (off-shell) general covariant cubic interaction vertex corresponding to the irreducible fields of higher helicities s_1, s_2, s_3 on the d-dimensional flat spacetime. As compared to previous works on covariant cubic vertices, which did not take into account sequentially traceless constraints, we use the complete BRST operator, which includes trace constraints necessary to describe irreps with a certain integer helicity. As a result, we generalize the covariant cubic vertex found in [arXiv:1205.3131 [hep-th]] and calculate new contributions to the vertex that contain additional terms with fewer space-time derivatives of fields, as well as terms without derivatives.. BRST approach is extended to construct BRST-BV quantum action for interacting theory.

Contents

- Interaction vertices in the gauge theories
- Lagrangians for HS fields from SFT
- BRST-BFV approach for Lagrangians for HS fields
- BRST approach with complete BRST operator to Lagrangians for free massless HS fields on $R^{1,d-1}$;
- Including interaction through systems of equations for cubic vertices;
- General solution for the cubic vertices for unconstrained of helicities s_1, s_2, s_3 HS fields
  1. BRST-closed linear on oscillators operators $\mathcal{L}^{(i)}_{k_i}$;
  2. BRST-closed cubic on oscillators operators $\mathcal{Z}$;
  3. BRST-closed trace operators $U^{(s_i)}_{j_i}$;
- General (covariant) and partial solutions for the cubic vertices;
- Correspondence with Metsaev's results on cubic vertices ;
- Conclusion & Outlook

Known results on cubic vertices

- metric formalism F. Berends, J. Van Reisen, NPB164 (1980), Berends, G. Burgers, H Van Dam, Nucl. Phys. B271 (1986); A. K. H. Bengtsson, I. Bengtsson, L. Brink, NPB (1983), E.S. Fradkin, M.A. Vasiliev, NPB 291 (1987), R. Manvelyan, K. Mkrtchyan, W. Ruhl, PLB 696 (2011), [arXiv:1009.1054 [hep-th]], E. Joung, M. Taronna, NPB 861 (2012) 145, arXiv:1110.5918[hep-th], I. Buchbinder, V. Krykhtin, M. Tsulaia, D. Weissman, Cubic Vertices for $\mathcal{N}=1$, NPB 967 (2021), arXiv:2103.08231;
- (half)integer spin for $ISO(1,d-1)$ in the light-cone R.R. Metsaev, NPB 759 (2006) hep-th/0512342, NPB 859 (2012) [arXiv:0712.3526[hep-th]]; 4d [arXiv:2206.13268[hep-th]] ;
- within constrained (with algebraic constraints, cov.) BRST approach for integer spins -R.R. Metsaev, PL B 720 (2013) arXiv:1205.3131 [hep-th];
- first, in BRST approach for reducible reprs $ISO(1,d-1)$, $SO(2,d-1)$ I.L Buchbinder, A. Fotopoulos, A. Petkou, M. Tsulaia, PRD 74 (2006) 105018, [arXiv:hep-th/0609082];
- in frame-like approach M. Vasiliev, Cubic Vertices for Symmetric higher spin Gauge Fields in (A)dS$_d$, NPB 862 (2012) 341 , arXiv:1108.5921[hep-th] arXiv:2208.02004, M. Khabarov, Yu. Zinoviev. JHEP 02 (2021);
- *Cov. cubic vertex in BRST approach for unconstrained HS fields (irreps) not found*

Gauge theory of 1 -stage reducibility

$S_0[A]$ – `classical action of fields` $A^i, i=1,...,n=(n_+,n_-),\ \varepsilon(A^i)=\varepsilon_i,$

$$\mathrm{rank}\left\|\overrightarrow{\partial}_j\overleftarrow{\partial_i}S_0\right\|\,|_{\partial_i S_0=0}=\boxed{n-m=N},\quad \partial_j\equiv\frac{\partial}{\partial A^j}:$$

$$\delta_0 S_0=0,\ \delta_0 A^i=R_0{}^i_{\alpha_0}\xi^{\alpha_0},\ \alpha_0=1,...,m_0=(m_{0+},m_{0-}),\Longrightarrow$$

$$\overleftarrow{\partial}_i S_0 R_0{}^i_{\alpha_0}=0\ \ \mathrm{rank}\left\|R_0{}^i_{\alpha_0}\right\|\,|_{\partial_i S_0=0}=m<m_0,\Rightarrow\delta_0^{(0)}\xi^{\alpha_0}=Z_0{}^{\alpha_0}_{\alpha_1}\xi^{\alpha_1}:$$

`so that` $R_0{}^i_{\alpha_0}Z_0{}^{\alpha_0}_{\alpha_1}|_{\partial_i S_0=0}=0$ и , $\alpha_1=1,...,m_1=(m_{1+},m_{1-})=(m_0-m)$

$$\varepsilon(\xi^{\alpha_s})=\varepsilon_{\alpha_s},\quad s=0,1$$

Deformation of k copies of LF for fields $A^{i(p)}$, $p=1,..,k$ with quadratic $\sum_p S_0^{(p)}[A^{(p)}]$ of free fields $A^{i(p)}$ with rank condition $\boxed{N=k(n-m)}$

$$S_{int}=\sum_{p=1}^k S_0^{(p)}[A^{(p)}]+g^1 S_1+g^2 S_2+\ldots g^r S_r,\ \boxed{deg_A S_r=r+2},$$

$$\delta_{[l]}A^{i(p)}=\delta_0 A^{i(p)}+g\delta_1 A^{i(p)}+\ldots+g^l\delta_1 A^{i(p)}=R_{[l]}{}^{i(p)}_{\alpha_0(t)}\xi^{\alpha_0(t)},\ \boxed{deg_A R_l{}^{i(p)}_{\alpha_0(t)}=l}$$

$$\delta_{[l]}^{(0)}\xi^{\alpha_0(p)}=\left\{\delta_0^{(0)}+g\delta_1^{(0)}+\ldots+g^l\delta_l^{(0)}\right\}\xi^{\alpha_0(p)}=Z_{[l]}{}^{\alpha_0(p)}_{\alpha_1(t)}\xi^{\alpha_1(t)},\ \boxed{deg_A Z_l{}^{\alpha_0(p)}_{\alpha_1(t)}=l},$$

$$R_0{}^{i(p)}_{\alpha_0(t)}\equiv R_0{}^i_{\alpha_0}\delta_t^p,\quad Z_0{}^{\alpha_0(p)}_{\alpha_1(t)}\equiv Z_0{}^{\alpha_0}_{\alpha_1}\delta_t^p.$$

Noether's identities as system in powers of g from $\delta_\Sigma S_{int} = 0$: $\boxed{\delta_\Sigma \equiv \sum_{l=0}^{\infty} \delta_l}$

$$
\begin{aligned}
g^1 &: \quad \delta_0 S_1 + \delta_1 \overline{S}_0 = 0, \\
g^2 &: \quad \delta_0 S_2 + \delta_1 S_1 + \delta_2 \overline{S}_0 = 0, \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
g^l &: \quad \delta_0 S_l + \sum_{p=1}^{l-1} \delta_p S_{l-p} + \delta_l \overline{S}_0 = 0, \quad \overline{S}_0 \equiv \sum_{p=1}^{k} S_0^{(p)}
\end{aligned}
\tag{1}
$$

for the gauge transforms of 0 level from $\delta_\Sigma^{(0)} \delta_\Sigma A^{i(p)}|_{\partial S_{int}=0} = 0$:

$$
\begin{aligned}
g^1 &: \quad \left(\delta_1^{(0)} \delta_0 A^{i(p)} + \delta_0^{(0)} \delta_1 A^{i(p)}\right)|_{\partial S_{[1]}=0} = 0, \\
g^2 &: \quad \left(\delta_2^{(0)} \delta_0 + \delta_1^{(0)} \delta_1 + \delta_0^{(0)} \delta_2\right) A^{i(p)}|_{\partial S_{[2]}=0} = 0, \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
g^l &: \quad \left(\delta_l^{(0)} \delta_0 + \sum_{p=1}^{l} \delta_{1-p}^{(0)} \delta_p\right) A^{i(p)}|_{\partial S_{[l]}=0} = 0
\end{aligned}
\tag{2}
$$

for the cubic vertex for GTh of 1 reducibility level

$$S_{int} = \sum_{p=1}^{3} S_0^{(p)}[A^{(p)}] + g^1 S_1, \ \boxed{deg_A S_1 = 3},$$

$$\delta_{[1]} A^{i(p)} = \delta_0 A^{i(p)} + g\delta_1 A^{i(p)} = R_{[1]}{}^{i(p)}_{\alpha_0(t)} \xi^{\alpha_0(t)}, \quad (3)$$

$$\delta_{[1]}^{(0)} \xi^{\alpha_0(p)} = \left\{\delta_0^{(0)} + g\delta_1^{(0)}\right\}\xi^{\alpha_0(p)} = Z_{[1]}{}^{\alpha_0(p)}_{\alpha_1(t)} \xi^{\alpha_1(t)}. \quad (4)$$

The equations (1)–(2) pass to

$$g^1: \qquad \delta_0 S_1 + \delta_1 \sum_{p=1}^{3} S_0^{(p)} = 0, \quad (5)$$

$$g^1: \qquad \left(\delta_1^{(0)}\delta_0 A^{i(p)} + \delta_0^{(0)}\delta_1 A^{i(p)}\right)|_{\partial S_{[1]}=0} = 0, \ p = 1,2,3. \quad (6)$$

In 4d for spins $(0, s_1), (0, s_2), (0, s_3)$... **in light-cone formalism**, *first classification* :
A. Bengtsson, I. Bengtsson, L. Brink, NPB (1983), A. Bengtsson, I. Bengtsson, N. Linden (1987)
$\sqrt{2}x^{\pm} = (x^0 \pm x^3)$, $\sqrt{2}x = (x^1 + ix^2)$ cubic (self)interaction for $s_1 = s_2 = s_3 = s$

for $s=2k,\ k\in\mathbb{N}_0$

$$S_{[}1]=\int d^4x\Big\{\frac{1}{2}\bar{\varphi}\Box\varphi+g\Big[\bar{\varphi}\sum_{n=0}^{s}(-1)^n\binom{s}{n}(\partial^+)^s\Big(\frac{\bar{\partial}^{s-n}}{\partial^{+s-n}}\varphi\frac{\bar{\partial}^n}{\partial^{+n}}\varphi\Big)+c.c.\Big]\Big\},$$

for $s=2k+1,\ k\in\mathbb{N}_0$ appears structure constants f^{abc} which close non-linear realized Lorentz algebra on φ^a

$$S_{[}1]=\int d^4x\Big\{\frac{1}{2}\bar{\varphi}^a\Box\varphi^a+gf^{abc}\Big[\bar{\varphi}^a\sum_{n=0}^{s}(-1)^n\binom{s}{n}(\partial^+)^s\Big(\frac{\bar{\partial}^{s-n}}{\partial^{+s-n}}\varphi^b\frac{\bar{\partial}^n}{\partial^{+n}}\varphi^c\Big)+c.c.\Big]\Big\}$$

$[g]=[l]^{s-1}$ dim. coupling constant. Lorentz algebra is realized in light cone non-linearly:

$$\delta_{j^+}\varphi=\delta^0_{j^+}\varphi-igx^+\delta_H\varphi+O(g^2)$$
$$\delta_{j^-}\varphi=\delta^0_{j^-}\varphi+ig\big(x\delta_H\varphi+\delta_s\varphi\big)+O(g^2)$$
$$\delta_{\bar{j}^-}\varphi=\delta^0_{\bar{j}^-}\varphi+ig\big(\bar{x}\delta_H\varphi+\delta_{\bar{s}}\varphi\big)+O(g^2)$$

How to construct **covariant cubic vertex** for irreducible HS fields of spins $(0,s_1),(0,s_2),(0,s_3)...$ $[(m_1,s_1),(m_2,s_2),(m_3,s_3)...]$ on $\mathbb{R}^{1,d-1}$ within Lagrangian formulation for totally symmetric tensor fields without holonomic constraints?

A construction it is suggested within BRST approach with complete BRST operator (following from SFT).

E. Wigner, M. Fierz, W. Pauli; V. Ginzburg; E.Fradkin; L.Singh, C.Hagen, C. Fronsdall, J.Fang, M. Vasiliev, R. Metsaev, Labastida.Yu. Zinoviev, J.Buchbinder, A.Pashnev, V.Kryhktin HS fields $(0, \mathsf{s})$ as $ISO(1, d-1)$ irreps $\mathsf{s} = (s_1, s_2, ..., s_k)$, $k \leq [\frac{d-2}{2}]$

$$\Phi_{\mu(s_1),\nu(s_2),...,\rho(s_k)} \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|c|}\hline \mu_1 & \mu_2 & \cdot\quad\cdot\quad\cdot & \cdot & \cdot & \cdot & \cdot & \mu_{s_1} \\ \hline \nu_1 & \nu_2 & \cdot\quad\cdot\quad\cdot & \cdot & \cdot & \nu_{s_2} \\ \cline{1-6} \cdots & \cdots & \cdots\quad\cdot\quad\cdot & \cdot & \cdots \\ \cline{1-5} \end{array} = Y(s_1, ..., s_k) \ .$$

$$\left(\partial^2, \partial^{\mu^i_{k_i}}\right)\Phi_{\mu^1(s_1),\mu^2(s_2),...,\mu^k(s_k)} = (0,0)\,, \quad \eta^{\mu^i_{l_i}\mu^j_{l_j}}\Phi_{\mu^1(s_1),\mu^2(s_2),...,\mu^k(s_k)} = 0 \qquad (7)$$

$$\Phi_{\mu^1(s_1),...,\{\mu^i(s_i)\underbrace{,\,...,\,\mu^j_1...}\mu^j_{l_j}\}...\mu^j_{s_j},...,\mu^k(s_k)} = 0, \quad i \leq j, \ 1 \leq l_j \leq n_j,$$

основной

Including of $\Phi_{\mu^1(s_1),...,\mu^k(s_k)}$ in string vector $([a^\mu_k, a^{+\nu}_l] = -\delta_{kl}\eta^{\mu\nu})$

$$|\Phi\rangle = \sum_{s_1=0}^{\infty}\sum_{s_2=0}^{s_1}\cdots\sum_{s_k=0}^{s_{k-1}} \frac{\imath^{\sum_i s_i}}{\prod_i s_i!}\Phi_{\mu^1(s_1),...,\mu^k(s_k)}\prod_{i=1}^{k}\prod_{l_i=1}^{s_i} a_i^{+\mu^i_{l_i}}|0\rangle,$$

$$\Big(l_0|\Phi\rangle, l_i|\Phi\rangle, l_{ij}|\Phi\rangle, t_{rs}|\Phi\rangle\Big) = 0, \quad \Big(l_i, l_{ij}, t_{rs}\Big) = \Big(-i\partial_\mu a^\mu_i, \tfrac{1}{2}a^\mu_i a_{j\mu}\,, a^+_{\mu r}a^\mu_s\Big),$$

Lagrangians for HS fields from SFT

A connection of massless HS fields with SFT : (E. Witten (1986); C. Thorn(1989)) $\Rightarrow$ follows from tensionless limit of BRST operator $\mathcal{Q}$ for $(\alpha' \to \infty)$: (A. Bengtsson (1983) G.Bonelli (2003), A. Sagnotti, M. Tsulaia, NPB (2004)).

$$\boxed{\Longrightarrow \mathcal{Q} \stackrel{\alpha'\to\infty}{\longrightarrow} \{\infty\} \texttt{ set of HS fields in srting spectrum}}$$

In the tensionless limit for open bosonic string BRST $\mathcal{Q}$ for Virasoro algebra and string GI string EoM turn in nilpotent $\forall d$ BRST Q_c & EoM for HS fs for reducible $ISO(1,d-1)$ reps: (in representation on $\mathcal{H}$: $(a^\mu_k, \mathcal{P}_0, \mathcal{P}_k, \eta_k)|0\rangle = 0$)

$$\boxed{\lim_{\alpha'\to\infty} \mathcal{Q} \ = Q_c = \eta_0 l_0 + \sum_{k>0}^{\infty}\Big[\eta^+_k l_k + \eta_k l^+_k + \imath \eta^+_k \eta_k \mathcal{P}_0\Big] = \eta_0 l_0 - \imath \mathcal{P}_0 M + \Delta Q_c,}$$

$$(l_k,\, l_k^+) = -i\partial_\mu\big(a^\mu_k,\, a^{+\mu}_k\big);\ [l_0, l_k^{(+)}] = 0,\ [l_k, l_k^+] = l_0 :\ [a^\mu_k, a^{+\nu}_l] = -\delta_{kl}\eta^{\mu\nu}$$

$$\boxed{(d=26:)\ \mathcal{Q}|\chi\rangle = 0,\ \delta|\chi\rangle = \mathcal{Q}|\Lambda_0\rangle,\ \delta|\Lambda_0\rangle = \mathcal{Q}|\Lambda_1\rangle, \ldots; (\varepsilon, gh)|\Lambda_k\rangle = (k+1, -k-1)}$$

$$\boxed{(\forall d:)\ Q_c|\chi\rangle = 0,\ \delta|\chi\rangle = Q_c|\Lambda_0\rangle,\ \delta|\Lambda_0\rangle = Q_c|\Lambda_1\rangle, \ldots}$$

decomposing Q_c and string fields $|\chi\rangle$, $|\Lambda_0\rangle$, ... in η_0

$$|\chi\rangle = |\varphi_1\rangle + \eta_0|\varphi_2\rangle, \quad |\Lambda_k\rangle \ = |\Lambda^0_k\rangle + \eta_0|\Lambda^1_k\rangle, \ldots \tag{8}$$

EoM in terms of η_0-independent equations and gauge symmetries looks

$$\boxed{\begin{array}{c} l_0|\varphi_1\rangle - \Delta Q_c|\varphi_2\rangle = 0, \quad \Delta Q_c|\varphi_1\rangle - M|\varphi_2\rangle = 0, \\ \delta|\varphi_1\rangle \ = \Delta Q_c|\Lambda^0_0\rangle - M|\Lambda^1_0\rangle, \quad \delta|\varphi_2\rangle \ = l_0|\Lambda^0_0\rangle - \Delta Q_c|\Lambda^1_0\rangle \end{array}} \tag{9}$$

(9) - Lagrangian, determines free gauge theory of $(k-1)$-reducibility stage & follows from

$$\boxed{S[\Phi, ..] = \int d\eta_0 \langle\chi|Q_c|\chi\rangle, \ \delta|\chi\rangle = Q_c|\Lambda_0\rangle, \ldots \delta|\Lambda_{k-1}\rangle = Q_c|\Lambda_k\rangle, \mathrm{gh}(|\chi\rangle, |\Lambda_i\rangle) = (0, -i-1)} \tag{10}$$

For TS $\Phi_{\mu(s)}$ due to gh-homogeneity

$$\begin{aligned} |\varphi_1\rangle_s &= \Phi_{\mu(s)} a^{\mu_1+} ... a^{\mu_s+}|0\rangle + \eta_1^+ \mathcal{P}_1^+ D_{\mu(s-2)} a^{\mu_1+} ... a^{\mu_{s-2}+}|0\rangle, \\ |\varphi_2\rangle_s &= \mathcal{P}_1^+ C_{\mu(s-1)} a^{\mu_1+} ... a^{\mu_{s-1}+}|0\rangle, \quad |\Lambda_0\rangle_s \ = \mathcal{P}_1^+ \Lambda_{\mu(s-1)} a^{\mu_1+} ... a^{\mu_{s-1}+}|0\rangle. \end{aligned}$$

Consequences:

1) From (9) *triplet formulation* in terms $\Phi_{\mu(s)}, C_{\mu(s-1)}, D_{\mu(s-2)}$ $(m=0)$ reducible $ISO(1,d-1)$ representations (Francia, Sagnotti 2003) with HS fs $(s, s-2, ..., 1/0)$ in oscillator form,

$$\boxed{\begin{array}{c} l_0|\Phi\rangle_s - l_1^+|C\rangle_{s-1} = 0, \quad l_1|\Phi\rangle_s - l_1^+|D\rangle_{s-2} = |C\rangle_{s-1}, \\ l_0|D\rangle_{s-2} - l_1|C\rangle_{s-1} = 0 \\ \delta\Big(|\Phi\rangle_s, |C\rangle_{s-1}, |D\rangle_{s-2}\Big) \ = \Big(l_1^+, l_0, l_1\Big)|\Lambda\rangle_{s-1} \end{array}} \tag{11}$$

sign "s" in $|\Phi\rangle_s$, $|\chi\rangle_s$: $\sigma_c|\chi\rangle_s = (s + \frac{d-2}{2})|\chi\rangle_s$ *for the spin operator*

$$\boxed{\sigma_c = \left(g_0 + \eta_1^+\mathcal{P}_1 - \eta_1\mathcal{P}_1^+\right)}, \ g_0 = -\frac{1}{2}\{a_\mu^+ . a^\mu\}$$

Lagrangian property (11) follows from the action (given on Fock space $\mathcal{H}$):

$$\boxed{S[\Phi, C, D] \ = \int d\eta_{0\,s}\langle\chi|Q_c|\chi\rangle_s, \delta|\chi\rangle_s = Q_c|\Lambda_0\rangle_s, \quad \mathrm{gh}(|\chi\rangle, |\Lambda_0\rangle) = (0, -1)} \tag{12}$$

2) LF for HS field from **irreducible representation** $(m=0,s)$ includes trace condition $(l_{11}|\Phi\rangle_s=0)$ in the form of BRST-extended constraint $\mathcal{L}_{11}$ ($[Q_c,\mathcal{L}_{11}]=0$, $[Q_c,\sigma_c]=0$) imposed on $|\chi\rangle, |\Lambda_0\rangle$

$$\boxed{\begin{array}{c} \mathcal{S}_{0|s}[\Phi,C,D] \ = \int d\eta_{0\,s}\langle\chi|Q_c|\chi\rangle_s, \quad \delta|\chi\rangle_s = Q_c|\Lambda_0\rangle_s, \\ \mathcal{L}_{11}\Big(|\chi\rangle,|\Lambda_0\rangle\Big) = \Big(l_{11}+\eta_1\mathrm{P}_1\Big)\Big(|\chi\rangle,|\Lambda_0\rangle\Big) = (0,0), \ \text{с} \ \ l_{11}=1/2a^{\mu}a_{\mu} \end{array}}. \qquad (13)$$

(13) $\Longrightarrow$ ***Fronsdal formualtion*** with $\Lambda_{(\mu)_{s-1}}$ $(\Lambda^{\mu}{}_{\mu\ldots}=0)$ & $\Phi_{(\mu)_s}$ $(\Phi^{\mu\nu}{}_{\mu\nu\ldots}=0)$.
(13) - *constrained BRST Lagrangian formulation* with external holonomic constraint (Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev,Tipunin, 2008).

$$S_{[1]}[\chi^{(1)}_c,\chi^{(2)},\chi^{(3)}] \ = \ \sum_{i=1}^{3}\mathcal{S}_{0|s_i} + g\int\prod_{e=1}^{3}d\eta^{(e)}_0\Big({}_{s_e}\langle\chi^{(e)}\big|V^{M(3)}\rangle_{(s)_3}+h.c.\Big),$$

$$\delta_{[1]}\big|\chi^{(i)}_c\rangle_{s_i} = Q^{(i)}_c\big|\Lambda^{(i)}_c\rangle_{s_i} - g\int\prod_{e=1}^{2}d\eta^{(i+e)}_0\Big({}_{s_{i+1}}\langle\Lambda^{(i+1)}_c\big|{}_{s_{i+2}}\langle\chi^{(i+2)}_c\big|$$

$$+(i+1\leftrightarrow i+2)\Big)\big|V^{M(3)}\rangle_{(s)_3}, \quad \boxed{\Big(Q^{tot},\ \mathcal{L}^{(i)}_{11}\Big)\big|V^{M(3)}\rangle_{(s)_3}=0.}$$

Inclusion into the system a $(l_{11}|\Phi\rangle{=}0)$ equally with the rest differential constraints, in order to the all irrep conditions extracting the particle $(m=0,s)$;

$$\partial^2\Phi_{\mu\nu\ldots}=0, \quad \partial^{\mu}\Phi_{\mu\nu\ldots}=0, \quad \eta^{\mu\nu}\Phi_{\mu\nu\ldots}=0 \qquad (14)$$

should follow from LF with complete BRST Q without any external constraints,

BRST-BFV approach for Lagrangians for HS fields

A. Pashnev, M. Tsulaia, MPLA (1998);
I. Buchbinder, A. R., NPB 2012, [arXiv:1110.5044[hep-th]]

$$\mathcal{S}_{0|s}[\Phi, \phi_1, ...] = \mathcal{S}_{0|s}[|\chi\rangle_s] = \int d\eta_{0\,s} \langle\chi|KQ|\chi\rangle_s, \quad (15)$$

$$\delta|\chi\rangle_s = Q|\Lambda\rangle_s, \;\; \delta|\Lambda\rangle_s = Q|\Lambda^1\rangle_s, \quad \delta|\Lambda^1\rangle_s = 0. \quad (16)$$

An equivalence of the Lagrangian formulations with incomplete & complete BRST operators for any irrep with discrete spin on $\mathbb{R}^{1,d-1}$ is established in A. R, JHEP (2018) arXiv:1803.04678[hep-th], but for interacting theory of the same HS field this question has not yet been solved .

Basic result for the cubic vertex for TS HS fields on $\mathbb{R}^{1,d-1}$ was obtained for 5d, 6d in light-cone by R. Metsaev (hep-th/0512342, arxiv:0712.3526), for covariant form in constrained BRST approach in -R. Metsaev, PLB 2013.

Remarks: Almost all LFs for HS fields on $R^{1,d-1}$, AdS(d) in terms frame-like formalism are LFs with algebraic constraints (E.Skvortsov, M.Vasiliev, Yu.Zinoviev, M.Grigoriev, D.Ponomarev).

LFs without constraints for (half-integer HS fields on $R^{1.d-1}$ Campoleoni A, Francia D, Mourad J, Sagnotti A, 2009, 2010 NPB

BRST-BFV approach for Lagrangians for HS fields

In BRST-BFV approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, I. Buchbinder, V. Krykhtin, A.R.) It is developed an algorithm instead of direct problem of generalized canonical quantization for dynam. system subject to constraints inverse problem of constructing GI LF for HS fields with (m, s)

$$\boxed{\begin{array}{c}\texttt{irreps conditions}\\ \text{ISO(1,d-1), SO(2,d-1)}\end{array}} \xrightarrow{\texttt{SFT}} \boxed{\begin{array}{c}\texttt{(super)algebra}\{o_I(x)\} : \mathcal{H}\\ [o_I, o_J\} = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0)\end{array}}$$

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$$\xrightarrow[\texttt{A.Pashnev, J.Buchbinder, A.R.}]{\texttt{conversion}} \boxed{\begin{array}{c}O_I = o_I + o'_I : \mathcal{H} \bigotimes \mathcal{H}'\\ [O_I, O_J\} = F^K_{IJ}(o', O)O_K\end{array}}$$

$$\xrightarrow[\texttt{M.Henneaux}]{\texttt{BFV}} \boxed{\begin{array}{c}\texttt{BRST operator for } \{O_I\}{:}Q'(x)\\ Q' = C^I O_I + \frac{1}{2}C^I C^J F^K_{IJ}\mathcal{P}_K + \text{more}\end{array}}$$

BRST-BFV approach for Lagrangians for HS fields

In BRST-BFV approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, I. Buchbinder, V. Krykhtin, A.R.) It is developed an algorithm instead of direct problem of generalized canonical quantization for dynam. system subject to constraints inverse problem of constructing GI LF for HS fields with (m, s)

$$\boxed{\begin{array}{c}\texttt{irreps conditions}\\ \text{ISO(1,d-1), SO(2,d-1)}\end{array}} \xrightarrow{\texttt{SFT}} \boxed{\begin{array}{c}\texttt{(super)algebra}\{o_I(x)\}:\mathcal{H}\\ [o_I, o_J\} = f_{IJ}^K(o)o_k + \Delta_{ab}(g_0)\end{array}}$$

$$\xrightarrow[\texttt{A.Pashnev, J.Buchbinder, A.R.}]{\texttt{conversion}} \boxed{\begin{array}{c}O_I = o_I + o'_I : \mathcal{H} \bigotimes \mathcal{H}'\\ [O_I, O_J\} = F_{IJ}^K(o', O)O_K\end{array}}$$

$$\xrightarrow[\texttt{M.Henneaux}]{\texttt{BFV}} \boxed{\begin{array}{c}\texttt{BRST operator for } \{O_I\}{:}Q'(x)\\ Q' = C^I O_I + \frac{1}{2}C^I C^J F_{IJ}^K \mathcal{P}_K + \text{more}\end{array}}$$

$$\xrightarrow{\text{ЛФ}} \boxed{\begin{array}{c} Q' = Q + (g_0 + h + \text{more})C_g + \ldots : Q^2|_{\sigma|\chi\rangle = 0} = 0\\ \texttt{mass-shell}: Q|\chi\rangle = 0, \text{gh}(|\chi\rangle) = 0 \Rightarrow \texttt{action}: S = \int d\eta_0 \langle\chi|KQ|\chi\rangle\\ \texttt{spin}\ \ (g_0 + \text{more})_{|\chi\rangle, |\Lambda\rangle, \ldots} = -h(|\chi\rangle, |\Lambda\rangle, \ldots)\\ \texttt{gauge symmetries:}\ \ \delta|\chi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \ldots\end{array}}$$

Q - BRST operator for 1-st class constraints $\{O_\alpha\} \subset \{O_I\}$ without invertible g_0. on 2-3 stages gauge and auxiliary fields introduced automaticallywhen getting LF for the basic field

For (constrained) BRST approach there are not 2-nd class constraints in the isometry algebra,no conversion, $K = \hat{1}$, spectrum of component fields less but there are difficulties with quantization!
Usually one says on **BRST-BV (Batalin-Vilkovissky) approach** when $gh(\Phi_{(\mu)_s....}, \Phi_{(\mu)_s....}) = 0$ for the component fields in $|\Phi\rangle, |\Lambda\rangle, ..$ is weakened (as compared to BRST-BFV approach):

$$gh(C_{\mu(s-1),....}, ...) > 0 - \texttt{ghost fields}\ gh(\Phi^*_{\mu(s)....}, C^*_{\mu(s-1)....}, ...) < 0\ \texttt{antifields}$$
$$\texttt{jointly in}\ |\Phi_{\rm gen}\rangle \Rightarrow S_{BV} = \int d\eta_0 \langle\Phi_{\rm gen}|KQ|\Phi_{\rm gen}\rangle = S(\Phi) + more$$

minimal BV action with (free) class. action$S(\Phi)$ & gauge algebra in "more".
Up to now BRST-BV action in constrained form was known for integer spin Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev,Tipunin, 2008, 2011; R.Metsaev 2012-2017
for HS fields with half-integer spin it was solved in
A. R, , JHEP 1809 (2018), arXiv:1803.04678[hep-th] and Phys.Part.Nucl. 49 (2018) 952, arXiv:1803.05173 [hep-th]

BRST approach with complete BRST operator to Lagrangians for free massless HS fields on $R^{1,d-1}$

$$\big(\partial^\nu\partial_\nu,\, \partial^{\mu_1},\, \eta^{\mu_1\mu_2}\big)\phi_{\mu(s)} = (0,0,0) \quad \Longleftrightarrow \tag{17}$$
$$\big(l_0,\, l_1,\, l_{11}, g_0 - d/2\big)|\phi\rangle = (0,0,0,s)|\phi\rangle.$$

basic vector$|\phi\rangle$ & $l_0,\, l_1,\, l_{11}, g_0$ are determined:

$$|\phi\rangle = \sum_{s\geq 0} \frac{\imath^s}{s!}\phi^{\mu(s)}\prod_{i=1}^{s} a^+_{\mu_i}|0\rangle, \tag{18}$$

$$\big(l_0,\, l_1,\, l_{11}, g_0\big) = \big(\partial^\nu\partial_\nu,\, -\imath a^\nu\partial_\nu,\, \frac{1}{2}a^\mu a_\mu, -\frac{1}{2}\{a^+_\mu,\, a^\mu\}\big).$$

In BRST approach a dynamic of free field of helicity s is determined in the extended configuration spave with GI action by $\phi_{\mu(s)}$ and auxiliary fields $\phi_{1\mu(s-1)}, \dots$. All of them are included in $|\chi\rangle_s$ described

$$\mathcal{S}_{0|s}[\phi, \phi_1, ...] = \mathcal{S}_{0|s}[|\chi\rangle_s] = \int d\eta_0{}_s\langle\chi|KQ|\chi\rangle_s, \tag{19}$$

$\mathcal{S}_{0|s}[|\chi\rangle_s]$ invariant w,r.t. reducible gauge transforms

$$\delta|\chi\rangle_s = Q|\Lambda\rangle_s,\ \delta|\Lambda\rangle_s = Q|\Lambda^1\rangle_s,\ \ \delta|\Lambda^1\rangle_s = 0. \tag{20}$$

with $|\Lambda\rangle_s$, $|\Lambda^1\rangle_s$ gauge parameter vectors of 0- & 1-levels in Abelian gauge transforms

BRST approach with complete BRST operator to Lagrangians for free massless HS fields on $R^{1,d-1}$

Q - BRST operator constructed w.r.t. HS symmetry algebra $\{l_0,\, l_1,\, l_1^+,\, l_{11},\, l_{11}^+ = \frac{1}{2}a^{+\nu}a^+_\nu\}$ ith Grassmann-odd ghost operators $\eta_0,\, \eta_1^+,\, \eta_1,\, \eta_{11}^+,\, \eta_{11},\, \mathcal{P}_0,\, \mathcal{P}_1,\, \mathcal{P}_1^+,\, \mathcal{P}_{11},\, \mathcal{P}_{11}^+,$

$$\begin{aligned} Q &= \eta_0 l_0 + \eta_1^+ l_1 + l_1^+\eta_1 + \eta_{11}^+\widehat{L}_{11} + \widehat{L}_{11}^+\eta_{11} + \imath\eta_1^+\eta_1\mathcal{P}_0 \\ &= \eta_0 l_0 + \Delta Q + \imath\eta_1^+\eta_1\mathcal{P}_0, \end{aligned} \tag{21}$$

где

$$\begin{aligned} \big(\widehat{L}_{11},\, \widehat{L}_{11}^+\big) &= \big(L_{11} + \eta_1\mathcal{P}_1,\, L_{11}^+ + \mathcal{P}_1^+\eta_1^+\big). \\ L_{11} = l_{11} + (b^+b+h)b,&\quad L_{11}^+ = l_{11}^+ + b^+ \end{aligned} \tag{22}$$

и $(\epsilon, gh)Q = (1,1)$. Algebra l_0 ,$l_1,\, l_1^+, L_{11}, L_{11}^+, G_0$ is determined

$$\begin{aligned} &[l_0, l_1^{(+)}] = 0,\ [l_1, l_1^+] = l_0 \qquad [L_{11}, L_{11}^+] = G_0,\ [G_0, L_{11}^+] = 2L_{11}^+, \\ &[l_1, L_{11}^+] = -l_1^+, \quad [l_1, G_0] = l_1 \end{aligned}$$

with extended

$$G_0 = g_0 + 2b^+b + h, \qquad b,\, b^+\ [b,\, b^+] = 1$$

- auxiliary conversion oscillators generating $\mathcal{H}'$.

conversion parameter when constructing Verma module for $so(1,2)$
$h = h(s) = -s - \frac{d-6}{2}$.

$$\{\eta_0, \mathcal{P}_0\} = \imath, \quad \{\eta_1, \mathcal{P}_1^+\} = \{\eta_1^+, \mathcal{P}_1\} = \{\eta_{11}, \mathcal{P}_{11}^+\} = \{\eta_{11}^+, \mathcal{P}_{11}\} = 1.$$

theory is characterized by spin operator σ:

$$\boxed{\sigma = G_0 + \eta_1^+\mathcal{P}_1 - \eta_1\mathcal{P}_1^+ + 2(\eta_{11}^+\mathcal{P}_{11} - \eta_{11}\mathcal{P}_{11}^+).} \tag{23}$$

σ selects vectors with definite spin s

$$\sigma(|\chi\rangle_s,\ |\Lambda\rangle_s,\ |\Lambda^1\rangle_s) = (0, 0, 0), \tag{24}$$

with periodic ε and decreasing gh $(0,0)$, $(1,-1)$, $(0,-2)$ respectively.
All operators act in total Hilbert space $\mathcal{H}_{tot} = \mathcal{H} \otimes \mathcal{H}_{gh} \otimes \mathcal{H}'$ c

$$\langle\chi|\psi\rangle = \int d^dx \langle 0|\chi^*(a, b; \eta_1, \mathcal{P}_1, \eta_{11}, \mathcal{P}_{11})\psi(a^+, b^+; \eta_1^+, \mathcal{P}_1^+, \eta_{11}^+, \mathcal{P}_{11}^+)|0\rangle.$$

Operators Q, σ supercommute and Hermitian (see, I.L. Buchbinder, A. Pashnev, M. Tsulaia, PLB (2001), I. Buchbinder, A. R., NPB (2012) arXiv:1110.5044)

$$Q^2 = \eta_{11}^+\eta_{11}\sigma, \qquad [Q,\ \sigma\} = 0; \tag{25}$$

$$(Q^+, \sigma^+)K = K(Q, \sigma), \qquad K = \sum_{n=0}^{\infty} \frac{1}{n!}(b^+)^n|0\rangle\langle 0|b^n \prod_{i=0}^{n-1}(i + h(s)),$$

$Q^2|_{\sigma\widetilde{\mathcal{H}}_{tot}=0} = 0$ is nilpotent on subspace with zero. values for σ (24).
Field $|\chi\rangle_s$, $|\Lambda\rangle_s$ $|\Lambda^1\rangle_s$ (as the result of spin condition) are defined in the form

$$|\chi\rangle_s = |\Phi\rangle_s + \eta_1^+\Big(\mathcal{P}_1^+|\phi_2\rangle_{s-2} + \mathcal{P}_{11}^+|\phi_{21}\rangle_{s-3} + \eta_{11}^+\mathcal{P}_1^+\mathcal{P}_{11}^+|\phi_{22}\rangle_{s-6}\Big) \qquad (26)$$
$$+ \eta_{11}^+\Big(\mathcal{P}_1^+|\phi_{31}\rangle_{s-3} + \mathcal{P}_{11}^+|\phi_{32}\rangle_{s-4}\Big) + \eta_0\Big(\mathcal{P}_1^+|\phi_1\rangle_{s-1} + \mathcal{P}_{11}^+|\phi_{11}\rangle_{s-2}$$
$$+ \mathcal{P}_1^+\mathcal{P}_{11}^+\Big[\eta_1^+|\phi_{12}\rangle_{s-4} + \eta_{11}^+|\phi_{13}\rangle_{s-5}\Big]\Big),$$
$$|\Lambda\rangle_s = \mathcal{P}_1^+|\xi\rangle_{s-1} + \mathcal{P}_{11}^+|\xi_1\rangle_{s-2} + \mathcal{P}_1^+\mathcal{P}_{11}^+\Big(\eta_1^+|\xi_{11}\rangle_{s-4} \qquad (27)$$
$$+ \eta_{11}^+|\xi_{12}\rangle_{s-5}\Big) + \eta_0\mathcal{P}_1^+\mathcal{P}_{11}^+|\xi_{01}\rangle_{s-3},$$
$$|\Lambda^1\rangle_s = \mathcal{P}_1^+\mathcal{P}_{11}^+|\xi^1\rangle_{s-3}. \qquad (28)$$

with $|\phi_{...}\rangle_{...} \equiv |\phi(a^+,b^+)_{...}\rangle_{...}$: . $|\Phi\rangle_s|_{(b^+=0)} = |\phi\rangle_s$

the most complete classification for covariant cubic vertices for $m = 0, m \neq 0$, (s_1, s_2, s_2) R.R. Metsaev, Phys. Lett. B 720 (2013) arXiv:1205.3131 [hep-th] for the fields (with $Y(s_1, ..., s_k)$) was initially given in light-cone coordinates in $5d, 6d$ R.R. Metsaev, NPB 759 (2006), hep-th/0512342 whereas for $4d$ by A. K. H. Bengtsson, I. Bengtsson & N. Linden,(1987) in LC with realization in BRST approach with algebraic constraints -R.R. Metsaev, PLB 720 (2013) arXiv:1205.3131

$$\boxed{\begin{aligned} \mathcal{S}_0[\phi, C, D] \ &= \int d\eta_{0\,s} \langle\chi_c|Q_c|\chi_c\rangle_s, \quad \delta|\chi_c\rangle_s = Q_c|\Lambda_{0c}\rangle_s, \\ \mathcal{L}_{11}\Big(|\chi_c\rangle, |\Lambda_{0c}\rangle\Big) &= \Big(l_{11} + \eta_1 \mathrm{P}_1\Big)\Big(|\chi_c\rangle, |\Lambda_{0c}\rangle\Big) = (0, 0), \end{aligned}}$$

$$(m_i = 0, s_1, s_2, s_3): \ |\mathbf{V}^{\mathbf{M(3)}}\rangle_{\mathbf{(s)_3}} \ = \ \sum\nolimits_{\mathbf{k}} \mathbf{t_k} \mathbf{Z}^{\mathbf{1/2\{s-k\}}} \prod\nolimits_{\mathbf{i=1}}^{\mathbf{3}} \big(\mathbf{L}^{(\mathbf{i})}\big)^{\mathbf{s_i - 1/2(s-k)}} |\mathbf{0}\rangle, \tag{29}$$

$$s \ = \ \sum\nolimits_i s_i, \ |0\rangle \equiv |0_1\rangle|0_2\rangle|0_3\rangle. \tag{30}$$

and enumerated by natural k subject to

$$s - 2s_{\min} \leq k \leq s, \quad k = s - 2p, \ p \in \mathbb{N}_0.$$

with Q_c-BRST-closed differential forms ($[i + 3 \simeq i]$)

$$L^{(i)} \ = \ (p_\mu^{(i+1)} - p_\mu^{(i+2)})a^{(i)\mu+} - \imath\big(\mathcal{P}_0^{(i+1)} - \mathcal{P}_0^{(i+2)}\big)\eta_1^{(i)+}, \tag{31}$$

$$Z \ = \ L_{11}^{(12)+}L^{(3)} + L_{11}^{(23)+}L^{(1)} + L_{11}^{(31)+}L^{(2)}. \ (\text{см. } (38)) \tag{32}$$

but $L_{11}^i|V^{M(3)}\rangle_{(s)_3} \neq 0$ with destroying irreducibility for the interacting fields for deformed gauge transforms: $L_{11}^i(|\chi_c^{(j)}\rangle_{s_j} + \delta|\chi_c^{(j)}\rangle_{s_j}) \neq 0$.

Cubic vertex for HS fields (s_1, s_2, s_3) within BRST approach includes 3 copies of vectors $|\chi^{(i)}\rangle_{s_i}$, $|\Lambda^{(i)}\rangle_{s_i}$, $|\Lambda^{(i)1}\rangle_{s_i}$ with $|0\rangle^i$ and oscillators $a^{(i)\mu+}$..., $i = 1, 2, 3$. Deformed action and gauge transformations

$$S_{[1]|(s)_3}[\chi^{(1)}, \chi^{(2)}, \chi^{(3)}] \; = \; \sum_{i=1}^{3} \mathcal{S}_{0|s_i} + g \int \prod_{e=1}^{3} d\eta_0^{(e)} \Big({}_{s_e}\langle \chi^{(e)} K^{(e)} \big| V^{(3)} \rangle_{(s)_3} + h.c. \Big),$$

$$\delta_{[1]} \big| \chi^{(i)} \rangle_{s_i} = Q^{(i)} \big| \Lambda^{(i)} \rangle_{s_i} - g \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{s_{i+1}}\langle \Lambda^{(i+1)} K^{(i+1)} \big| {}_{s_{i+2}}\langle \chi^{(i+2)} K^{(i+1)} \big| + (i+1 \leftrightarrow i+2) \Big) \big| \widetilde{V}^{(3)} \rangle_{(s)_3},$$

$$\delta_{[1]} \big| \Lambda^{(i)} \rangle_{s_i} = Q^{(i)} \big| \Lambda^{(i)1} \rangle_{s_i} - g \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{s_{i+1}}\langle \Lambda^{(i+1)1} K^{(i+1)} \big| {}_{s_{i+2}}\langle \chi^{(i+2)} K^{(i+1)} \big| + (i+1 \leftrightarrow i+2) \Big) \big| \widehat{V}^{(3)} \rangle_{(s)_3}$$

with unknown $\big| V^{(3)} \rangle_{(s)_3}$, $\big| \widetilde{V}^{(3)} \rangle_{(s)_3}$, $\big| \widehat{V}^{(3)} \rangle_{(s)_3}$.

x-locality $\big|V^{(3)}\rangle$, $\big|\widetilde{V}^{(3)}\rangle$, $\big|\widehat{V}^{(3)}\rangle$ and momenta conversation law:

$$\big|V^{(3)}\rangle_{(s)_3} = \prod_{i=2}^{3}\delta^{(d)}\big(x_1 - x_i\big)V^{(3)}\prod_{j=1}^{3}\eta_0^{(j)}|0\rangle, \quad |0\rangle \equiv \otimes_{e=1}^{3}|0\rangle^e$$

$$p_\mu^{(1)} + p_\mu^{(2)} + p_\mu^{(3)} = 0. \quad (s)_3 \equiv (s_1, s_2, s_3) \quad [i+3 \simeq i]$$

invariance $S_{[1]}$ w.r.t $\delta_{[1]}\big|\chi^{(i)}\rangle_{s_i}$, $i = 1,2,3$, $\rightarrow$ system of equations:

$$g^0: \qquad Q^{(i)}Q^{(i)}|\Lambda^{(i)}\rangle_{s_i} = \eta_{11}^{(i)+}\eta_{11}^{(i)}\sigma^{(i)}|\Lambda^{(i)}\rangle_{s_i} \equiv 0, \quad i = 1,2,3,$$

$$g^1: \qquad \int\prod_{e=1}^{3} d\eta_0^{(e)}{}_{s_j}\langle\Lambda^{(j)}K^{(j)}\big|_{s_{j+1}}\langle\chi^{(j+1)}K^{(j+1)}\big|_{s_{j+2}}\langle\chi^{(j+2)}K^{(j+2)}\big|\mathcal{Q}(V^3,\widetilde{V}^3) = 0,$$

$$\mathcal{Q}(V^3,\widetilde{V}^3) = \sum_{k=1} Q^{(k)}\big|\widetilde{V}^{(3)}\rangle_{(s)_3} + Q^{(j)}\Big(\big|V^{(3)}\rangle_{(s)_3} - \big|\widetilde{V}^{(3)}\rangle_{(s)_3}\Big), \; j = 1,2,3.$$

preservation the form of gauge transformations $\big|\chi^{(i)}\rangle_{s_i}$ w.r.t $\delta_{[1]}\big|\Lambda^{(i)}\rangle_{s_i}$ c $\big|\Lambda^{(i)1}\rangle_{s_i} \rightarrow$

$$g^0: Q^{(i)}Q^{(i)}|\Lambda^{(i)1}\rangle_{s_i} = \eta_{11}^{(i)+}\eta_{11}^{(i)}\sigma^{(i)}|\Lambda^{(i)1}\rangle_{s_i} \equiv 0, \quad i = 1,2,3,$$

$$g^1: \int\prod_{e=1}^{2} d\eta_0^{(e)}{}_{s_{j+1}}\langle\Lambda^{(j+1)1}K^{(j+1)}\big|_{s_{j+2}}\langle\chi^{(j+2)}K^{(j+2)}\big|\Big(\mathcal{Q}(\widetilde{V}^3,\widehat{V}^3) - Q^{(j+2)}|\widehat{V}^{(3)}\rangle\Big) = 0,$$

c $$\mathcal{Q}(\widetilde{V}^3,\widehat{V}^3) = \sum_{k=1} Q^{(k)}\big|\widehat{V}^{(3)}\rangle_{(s)_3} + Q^{(j)}\Big(\big|\widetilde{V}^{(3)}\rangle_{(s)_3} - \big|\widehat{V}^{(3)}\rangle_{(s)_3}\Big), \; j = 1,2,3.$$

should fulfill spin condition as consequence of spin equations on each vectors (24) $|\chi^{(i)}\rangle_{s_i}$, $|\Lambda^{(i)}\rangle_{s_i}$, $|\Lambda^{(i)1}\rangle_{s_i}$:

$$\sigma^{(i)}\Big(\big|V^{(3)}\rangle_{(s)_3},\, \big|\widetilde{V}^{(3)}\rangle_{(s)_3},\, \big|\widehat{V}^{(3)}\rangle_{(s)_3}\Big) = 0,\ i = 1,2,3 \tag{33}$$

deformed gauge transformations should form closed algebra

$$[\delta^{\Lambda_1}_{[1]}, \delta^{\Lambda_2}_{[1]}]|\chi^{(i)}\rangle \ = \ -g\delta^{\Lambda_3}_{[1]}|\chi^{(i)}\rangle \ , \tag{34}$$

where Grassmann-odd $\Lambda_3 = \Lambda_3(\Lambda_1, \Lambda_2)$ explicitly calculated as

$$\big|\Lambda_3\rangle \sim \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big\{ \Big(\langle \Lambda_2^{(i+1)} K \big| \langle \Lambda_1^{(i+2)} \big| K + \big(i+1 \leftrightarrow i+2\big) \Big) - \big(\Lambda_1 \leftrightarrow \Lambda_2\big) \Big\} \big|\widetilde{V}^{(3)}\rangle.$$

Introduce $Q^{tot} = \sum_{k=1} Q^{(k)}$.

General solution for the cubic vertices for unconstrained of helicities s_1, s_2, s_3 HS fields

For simplicity we suppose $\big|\widetilde{V}^{(3)}\rangle_{(s)_3} = \big|V^{(3)}\rangle_{(s)_3} = \big|\widehat{V}^{(3)}\rangle_{(s)_3}$. then

$$\left\{ \begin{array}{l} {}_{s_2}\langle\Lambda^{(1)}K^{(1)}\big|_{s_2}\langle\chi^{(2)}K^{(2)}\big|_{s_3}\langle\chi^{(3)}K^{(3)}\big|\mathcal{Q}(V^3,V^3) = 0 \\ {}_{s_{j+1}}\langle\Lambda^{(j+1)1}K^{(j+1)}\big|_{s_{j+2}}\langle\chi^{(j+2)}K^{(j+1)}\big|\mathcal{Q}(V^3,V^3) = 0 \end{array}\right. \Longrightarrow \boxed{Q^{tot}\big|V^{(3)}\rangle_{(s)_3} = 0.}$$

we seek (Q^{tot}-BRST- closed) solution in the form of products of specific homogeneous in oscillators operators

, (1), Q^{tot}-BRST- closed forms $\mathcal{L}^{(i)}_{k_i}$, $i = 1,2,3,\ k_i = 1,...,s_i$ & $\mathcal{Z}$ constructed from $L^{(i)}$, Z R.R. Metsaev, (2013) arXiv:1205.3131 [hep-th], where $deg_{(a^+,\eta^+)}L^{(i)} = 1$, $deg_{(a^+,\eta^+)}Z = 3$,

$$\mathcal{L}^{(i)}_{k_i} = (L^{(i)})^{k_i-2}\Big((L^{(i)})^2 - \frac{\imath k_i!}{2(k_i-2)!}\eta^{(i)+}_{11}\left[2\mathcal{P}^{(i+1)}_0 + 2\mathcal{P}^{(i+2)}_0 - \mathcal{P}^{(i)}_0\right]\Big), \quad (35)$$

$$L^{(i)} = (p^{(i+1)}_\mu - p^{(i+2)}_\mu)a^{(i)\mu+} - \imath\big(\mathcal{P}^{(i+1)}_0 - \mathcal{P}^{(i+2)}_0\big)\eta^{(i)+}_1, \quad p^{(i)}_\mu = -i\partial^{(i)}_\mu \quad (36)$$

$$Z = L^{(12)+}_{11}L^{(3)} + L^{(23)+}_{11}L^{(1)} + L^{(31)+}_{11}L^{(2)}. \quad (37)$$

we have used momenta conversation -law and non-invariance $L^{(i)}$ when acting by trace operator: $(\widehat{L}^{(i)}_{11}(L^{(i)})^2|0\rangle \neq 0)$ and

$$L^{(ii+1)+}_{11} = a^{(i)\mu+}a^{(i+1)+}_\mu - \frac{1}{2}\mathcal{P}^{(i)+}_1\eta^{(i+1)+}_1 - \frac{1}{2}\mathcal{P}^{(i+1)+}_1\eta^{(i)+}_1. \quad (38)$$

(2), one has new 2-, 4- , ..., $[s_i/2]$ forms in powers in $\rightarrow$

$$U^{(s_i)}_{j_i}(\eta^{(i)+}_{11}, \mathcal{P}^{(i)+}_{11}) \ := \ (\widehat{L}^{(i)+}_{11})^{(j_i-2)}\{(\widehat{L}^{+(i)}_{11})^2 - j_i(j_i-1)\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\}, \quad i=1,2,3.$$

First,,we should check (36) and (38) - Q^{tot}-BRST-closed as extensions of BRST operators Q^{tot}_c in Alkalaev Grigoriev,Tipunin, 2008, 2011, R.R. Metsaev, Phys. Lett. B 720 (2013) in view of trace operators.

$$\widehat{L}^{(i)}_{11}(L^{(i+j)})^k|0\rangle \equiv 0, j=1,2, \quad \forall k\in\mathbb{N}\,,$$
$$\widehat{L}^{(i)}_{11}L^{(i)}|0\rangle = \Big(-(p^{(i+1)}_\mu - p^{(i+2)}_\mu)a^{(i)\mu} + \imath\big(\mathcal{P}^{(i+1)}_0 - \mathcal{P}^{(i+2)}_0\big)\eta^{(i)}_1 + L^{(i)}\widehat{L}^{(i)}_{11}\Big)|0\rangle = 0,$$
$$\widehat{L}^{(i)}_{11}(L^{(i)})^2|0\rangle = \Big(-(p^{(i+1)}_\mu - p^{(i+2)}_\mu)a^{(i)\mu} + \imath\big(\mathcal{P}^{(i+1)}_0 - \mathcal{P}^{(i+2)}_0\big)\eta^{(i)}_1 + L^{(i)}\widehat{L}^{(i)}_{11}\Big)L^{(i)}|0\rangle,$$
$$= \Big(\eta^{\mu\nu}(p^{(i+1)}_\mu - p^{(i+2)}_\mu)(p^{(i+1)}_\nu - p^{(i+2)}_\nu) + L^{(i)}\imath\big(\mathcal{P}^{(i+1)}_0 - \mathcal{P}^{(i+2)}_0\big)\eta^{(i)}_1 + (L^{(i)})^2\widehat{L}^{(i)}_{11}\Big)|0\rangle$$
$$= \eta^{\mu\nu}(p^{(i+1)}_\mu - p^{(i+2)}_\mu)(p^{(i+1)}_\nu - p^{(i+2)}_\nu)|0\rangle \neq 0.$$

latter for $(L^{(i)})^k$ does not vanish under sigh of inner product (35). Analogously, $\forall k$: Z^k, not BRST closed.

$\mathcal{Z}_j$ is determined from the condition of BRST closeness, e.g. for $j=1$

$$\mathcal{Z}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j} = Z\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j} - \sum_{i=1}^{3}k_i\frac{b^{(i)+}}{h^{(i)}}\big[[L^{(i)}_{11},Z\},L^{(i)}\big\}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j-\delta_{ij}} \tag{39}$$
$$+\sum_{i\neq e}^{3}k_ik_e\frac{b^{(i)+}b^{(e)+}}{h^{(i)}h^{(e)}}\big[L^{(e)}_{11},\big[[L^{(i)}_{11},Z\},L^{(i)}\big\}L^{(e)}\big\}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j-\delta_{ij}-\delta_{ej}}$$
$$-\sum_{i\neq e\neq o}^{3}k_ik_ek_o\frac{b^{(i)+}b^{(e)+}b^{(o)+}}{h^{(i)}h^{(e)}h^{(o)}}\big[L^{(o)}_{11},\big[L^{(e)}_{11},\big[[L^{(i)}_{11},Z\},L^{(i)}\big\}L^{(e)}\big\}L^{(0)}\big\}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j-1}.$$

For $j>1$ expression for $\mathcal{Z}_j$ is deduced analogously.

For $j = 2$

$$\mathcal{Z}_2 \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)} = Z\mathcal{Z} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)} - \sum_{i_1=1}^{3} \frac{b^{(i_1)+}}{h^{(i_1)}} \Big[\Big[\widehat{L}_{11}^{(i_1)}, Z\Big\}, \mathcal{Z} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)}\Big\} \tag{40}$$
$$+ \sum_{i_1 \neq e_1}^{3} \frac{b^{(i_1)+} b^{(e_1)+}}{h^{(i_1)} h^{(e_1)}} \Big[\widehat{L}_{11}^{(e_1)}, \Big[\Big[\widehat{L}_{11}^{(i_1)}, Z\Big\}, \mathcal{Z} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)}\Big\}\Big\}$$
$$- \sum_{i_1 \neq e_1 \neq o_1}^{3} \frac{b^{(i_1)+} b^{(e_1)+} b^{(o_1)+}}{h^{(i_1)} h^{(e_1)} h^{(o_1)}} \Big[\widehat{L}_{11}^{(o_1)}, \Big[\widehat{L}_{11}^{(e_1)}, \Big[\Big[\widehat{L}_{11}^{(i_1)}, Z\Big\}, \mathcal{Z} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)}\Big\}\Big\}\Big\}.$$

for $j \geq 1$ we have by induction

$$\mathcal{Z}_j \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)} = Z\mathcal{Z}_{j-1} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)} - \sum_{i_{j-1}=1}^{3} \frac{b^{(i_{j-1})+}}{h^{(i_{j-1})}} \Big[\Big[\widehat{L}_{11}^{(i_{j-1})}, Z\Big\}, \mathcal{Z}_{j-1} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)}\Big\} \quad (4$$
$$+ \sum_{i_{j-1} \neq e_{j-1}}^{3} \frac{b^{(i_{j-1})+} b^{(e_{j-1})+}}{h^{(i_{j-1})} h^{(e_{j-1})}} \Big[\widehat{L}_{11}^{(e_{j-1})}, \Big[\Big[\widehat{L}_{11}^{(i_{j-1})}, Z\Big\}, \mathcal{Z}_{j-1} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)}\Big\}\Big\} -$$
$$\sum_{i_{j-1} \neq e_{j-1} \neq o_{j-1}}^{3} \frac{b^{(i_{j-1})+} b^{(e_{j-1})+} b^{(o_{j-1})+}}{h^{(i_{j-1})} h^{(e_{j-1})} h^{(o_{j-1})}} \Big[\widehat{L}_{11}^{(o_{j-1})}, \Big[\widehat{L}_{11}^{(e_{j-1})}, \Big[\Big[\widehat{L}_{11}^{(i_{j-1})}, Z\Big\}, \mathcal{Z}_{j-1} \prod_{p=1}^{3} \mathcal{L}_{k_p}^{(p)}\Big\}$$

it is sufficiently present in the form $\big|V^{(3)}\rangle_{(s)_3} = U^{(s_i)}_{j_i}\big|X^{(3)}\rangle_{(s)_3-2j_i}$ with some $\big|X^{(3)}\rangle_{(s)_3}$: $(\varepsilon, gh)\big|X^{(3)}\rangle = (1,3)$ & check BRST-closeness for $i\neq k$ and $j=1,2$

$$\eta^{(i)+}_{11}\widehat{L}^{(i)}_{11}U^{(s_k)}_{j_k}|0\rangle \equiv 0, k\neq i\,,$$

$$\eta^{(i)+}_{11}\widehat{L}^{(i)}_{11}U^{(s_i)}_{1_i}\big|X^{(3)}\rangle_{(s)_3-2_i} = \big(\sigma^{(i)}\eta^{(i)+}_{11} - 2\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}\big)\big|X^{(3)}\rangle_{(s)_3-2_i} = 0, \tag{42}$$

$$\eta^{(i)+}_{11}\widehat{L}^{(i)}_{11}U^{(s_i)}_{2_i}\big|X^{(3)}\rangle_{(s)_3-4_i} = \big(\{\sigma^{(i)}\eta^{(i)+}_{11}, \widehat{L}^{(i)+}_{11}\} - 2\eta^{(i)+}_{11}\widehat{L}^{(i)+}_{11} + 2\eta^{(i)+}_{11}\widehat{L}^{(i)+}_{11}$$
$$-4\widehat{L}^{(i)+}_{11}\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}\big)\big|X^{(3)}\rangle_{(s)_3-4_i} = 2\sigma^{(i)}\eta^{(i)+}_{11}\widehat{L}^{(i)+}_{11}\big|X^{(3)}\rangle_{(s)_3-4_i} = 0.$$

for $j>2$

$$\eta^{(i)+}_{11}\widehat{L}^{(i)}_{11}U^{(s_i)}_{j_i}\big|X^{(3)}\rangle_{(s)_3-2j_i} = \Big(\sum_{e=0}^{j_i-1}(\widehat{L}^{(i)+}_{11})^e\{\sigma^{(i)}\eta^{(i)+}_{11}, \widehat{L}^{(i)+}_{11}\}(\widehat{L}^{(i)+}_{11})^{j_i-e-2} \tag{43}$$
$$+ j_i\eta^{(i)+}_{11}\big[(j_i-1) - 2\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}\big](\widehat{L}^{(i)+}_{11})^{j_i-1}\Big)\big|X^{(3)}\rangle_{(s)_3-2j_i}$$
$$= \Big((j_i-1)\sigma^{(i)}\eta^{(i)+}_{11} - 2j_i\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}\Big)\big(\widehat{L}^{(i)+}_{11}\big)^{j_i-1}\big|X^{(3)}\rangle_{(s)_3-2j_i} = 0.$$

In (42)–(43) we have used s/commutators linear in $\widehat{L}^{(i)+}_{11}$

$$\eta^{(i)+}_{11}\Big[\widehat{L}^{(i)}_{11},\, \widehat{L}^{(i)+}_{11}\Big\} = \eta^{(i)+}_{11}\big\{(\sigma^{(i)}+2) - 2\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}\big\} = \sigma^{(i)}\eta^{(i)+}_{11} - 2\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11},$$
$$\Big[\widehat{L}^{(i)+}_{11},\, \sigma^{(j)}\Big\} = -2\delta^{ij}\widehat{L}^{(i)+}_{11}, \tag{44}$$

and polynomial in $\widehat{L}^{(i)+}_{11}$

$$\eta^{(i)+}_{11}\Big[\widehat{L}^{(i)}_{11},\, \big(\widehat{L}^{(i)+}_{11}\big)^{j_i}\Big\} = \sum_{e=0}^{j_i-1}\big(\widehat{L}^{(i)+}_{11}\big)^{e}\eta^{(i)+}_{11}\Big[\widehat{L}^{(i)}_{11},\, \widehat{L}^{(i)+}_{11}\Big\}\big(\widehat{L}^{(i)+}_{11}\big)^{j_i-e-1}$$

$$= \sum_{e=0}^{j_i-1}\big(\widehat{L}^{(i)+}_{11}\big)^{e}\sigma^{(i)}\eta^{(i)+}_{11}\big(\widehat{L}^{(i)+}_{11}\big)^{j_i-e-1} - 2j_i\big(\widehat{L}^{(i)+}_{11}\big)^{j_i-1}\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}.$$

$$= (j_i-1)\Big[\sigma^{(i)} - j_i\Big]\eta^{(i)+}_{11}\big(\widehat{L}^{(i)+}_{11}\big)^{j_i-1} - 2j_i\big(\widehat{L}^{(i)+}_{11}\big)^{j_i-1}\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\eta^{(i)}_{11}$$

$$\partial\big|X^{(3)}\rangle_{(s)_3-2j_i})/\partial\mathcal{P}^{(i)+}_{11} = 0.$$

BRST-closeness of all introduced operators is guaranteed.

general solution for covariant cubic vertex preserving irreps of $ISO(1,d-1)$ for HS fields (s_1,s_2,s_3) (thus correct degrees of freedom) when passing to interacting theory

$$|V^{(3)}\rangle_{(s)_3} = |V^{M(3)}\rangle_{(s)_3} + \sum_{(j_1,j_2,j_3)>0}^{([s_1/2],[s_2/2],[s_3/2])} U^{(s_1)}_{j_1} U^{(s_2)}_{j_2} U^{(s_3)}_{j_3} |V^{M(3)}\rangle_{(s)_3-2(j)_3}, \quad (45)$$

$V^{M(3)}\rangle_{(s)_3-2(j)_3}$ determined in R.R. Metsaev, PLB 720 (2013) with modified forms respecting trace $\mathcal{L}^{(i)}_{k_i}$, (35) и $\mathcal{Z}_j$

$$\begin{aligned} |V^{M(3)}\rangle_{(s)_3-2(j)_3} &= \textstyle\sum_k \mathcal{Z}_{1/2\{(s-2J)-k\}} \prod_{i=1}^3 \mathcal{L}^{(i)}_{s_i-2j_i-1/2(s-2J-k)}|0\rangle, && (46)\\ (s,J) &= \left(\textstyle\sum_i s_i,\ \sum_i j_i\right). && (47)\end{aligned}$$

and enumerated by naturals (k,j_1,j_2,j_3) satisfying to the equations

$$\boxed{s-2J-2s_{\min} \leq k \leq s-2J, \quad k = s-2J-2p,\ p\in\mathbb{N}_0,\ 0\leq j_i\leq [s_i/2].} \quad (48)$$

General vertex (45) besides modified terms (46) contains new ones. These are linear in trace $U^{(s_i)}_{1_i} = \widehat{L}^{(i)+}_{11}$ for each field copy

$$\textstyle\sum_{(j_1,j_2,j_3)>0}^{(1,1,1)} (\widehat{L}^{(1)+}_{11})^{j_1} (\widehat{L}^{(2)+}_{11})^{j_2} (\widehat{L}^{(3)+}_{11})^{j_3} |V^{M(3)}\rangle_{(s)_3-2(j)_3}. \quad (49)$$

as differed from $V^{M(3)}\rangle_{(s)_3-2(j)_3}$ in vertex: $b^{(i)+}$, $\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}$, $i=1,2,3$.

Thus, for even s_i, $i = 1,2,3$ vertex for $j_i = [s_i/2]$ be with $s_i/2$ traces for initial fields $|\phi\rangle_{s_i}$, $\prod_i Tr^{s_i/2}\phi^{(i)}$, without derivatives (!)

$$\big|\overline{V}^{(3)}\rangle_{(s)_3} \;=\; \prod_{i=1}^{3} U^{(s_i)}_{[s_i/2]}|0\rangle = \prod_{i=1}^{3}(\widehat{L}^{(i)+}_{11})^{(j_i-2)}\big\{(\widehat{L}^{+(i)}_{11})^2 - j_i(j_i-1)\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\big\}|0\rangle. \tag{50}$$

For 1, 2 or for all odd s_1, s_2, s_3, the vertices with minimal number of derivatives will contain 1, 2 or 3 derivatives:

$$\big|V^{(3)}_1\rangle_{(s)_3} \;=\; U^{(s_1)}_{[s_1-1/2]}U^{(s_2)}_{[s_2/2]}U^{(s_3)}_{[s_3/2]}L^{(1)}|0\rangle, \tag{51}$$

$$\big|V^{(3)}_2\rangle_{(s)_3} \;=\; U^{(s_1)}_{[s_1-1/2]}U^{(s_2)}_{[s_2-1/2]}U^{(s_3)}_{[s_3/2]}L^{(1)}L^{(2)}|0\rangle, \tag{52}$$

$$\big|V^{(3)}_3\rangle_{(s)_3} \;=\; \prod_{i=1}^{3} U^{(s_i)}_{[s_i-1/2]}\big\{\prod_{i=1}^{3} L^{(i)} + Z\big\}|0\rangle. \tag{53}$$

for $d > 4$ the number of independent (even parity invariant) vertices in $\forall|V^{M(3)}\rangle_{(s)_3-2(j)_3}$ enumerated by k & equal $(s_{\min} - 2j_{\min} + 1)$, but for $d = 4$ it reduced to 2, i.e. $k = s - 2J$, $s - 2J - 2s_{min}$ due to proportionality for k: $s - 2J - 2s_{min} < k < s - 2J$, to terms with ∂^2 (R. Metsaev, PLB 720 (2013)). For $s_1 = s_2 = s_3$ we have selfinteracting vertex (45) $\phi_{\mu(s)}$.

$|V^{(3)}\rangle \neq |V^{M(3)}\rangle$ not identical: $L_{11}^{(i)}\ |V^{M(3)}\rangle \neq 0.$

Correspondence

1) $|V_{irrep}^{M(3)}\rangle = |V^{M(3)}\rangle / L_{11}^{(i)} |V^{M(3)}\rangle,$

2) reducing $\mathcal{M}_{un} \to \mathcal{M}_c$: $|V^{(3)}\rangle_{\mathcal{M}_{un}\ \to\ \mathcal{M}_c} = |\breve{V}^{(3)}\rangle$

Then $|V_{irrep}^{M(3)}\rangle = |\breve{V}^{(3)}\rangle$!

Conclusion

- It is found general cubic interacting vertex (off-shell) for irreducible interacting HS fields with integer helicities s_1, s_2, s_3 on Minkowski $\mathbb{R}^{1,d-1}$ space;
- BRST approach is developed for constructing cubic vertices by deformation of 3 copies of free 1-st stage reducible gauge theories without any constraints for massless TS HS fields up to interacting 1-st stage reducible gauge theory with accuracy up to the 1-st order in g. Entangled system of equations are derived and solved on cubic vertex from deformed free actions and reducible gauge transformations with preservation of number of physical degrees of freedom;
- BRST-closed cubic vertex generalizes CV from [arXiv:1205.3131 [hep-th]] for massless HS fields and appears by covariant analog of even-parity vertex suggested in the light-cone formalism [hep-th/0512342] and reproduces new inputs into the vertex with traces and less numbers of space-time derivatives, including the terms without derivatives.

Outlook

- Construction (off-shell) covariant cubic interaction vertex with massive irreducible HS fields with integer spins s_1, s_2, s_3 within BRST approach (in progress);
- Construction (off-shell) covariant cubic vertex for irreducible HS fields with integer and half-integer spins $s_1, n_2 + \frac{1}{2}, n_3 + \frac{1}{2}$ on a base of BRST approach;
- Construction (off-shell) covariant cubic vertex for irreducible mixed-symmetric HS fields subject to $Ys_1^1, ..., s_1^k$, $Ys_2^1, ..., s_2^l$, $Ys_3^1, ..., s_3^m$ integer spins $\vec{s}_1^k, \vec{s}_2^l, \vec{s}_3^m$;
- Construction (off-shell) covariant cubic supersymmetric vertex within BRST approach for irreducible irreps with (half)integer superspins;
- Derivation system of equations for quartic vertex and its solution for irreducible $ISO(1, d-1)$ massless representations with integer helicities s_1, s_2, s_3, s_4 and study its (non)local realization .

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- Construction (off-shell) covariant cubic interaction vertex with massive irreducible HS fields with integer spins s_1, s_2, s_3 within BRST approach (in progress);
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Thank you very much

Quantum BRST-BV action for constrained massless HS fields

Мы предлагаем расширение БРСТ-БВ подхода с неполным БРСТ оператором гамильтоновскими БФВ осцилляторами неминимального сектора, и антигостовскими тензорными полями и тензорными поля Наканиши-Лаутрупа совместно с их антиполями для неприводимой калибровочной теории взаимодействующих ограниченных безмассовых полей высших спинов на пространстве Минковского. В результате все полевые и антиполевые компонетные поля БВ алгоритма вкладываются аддитивно в фоковский грассмановско-четный обобщенный "поле-антиполе" вектор с нулевым гостовским числом. Мы формулируем процедуру фиксации калибровки, строим БРСТ-БВ квантовое действие и производящий функционал функций Грина. Мы выводим преобразования БРСТ симметрии и получаем тождество Уорда.

C. Burdik, A.A.R, BRST-BV Quantum Actions for Constrained Totally-Symmetric Integer HS Fields, Nuclear Physics B 965 (2021) 115357, [arXiv:2010.15741]

Quantum BRST-BV action for constrained massless HS

$$Z^{[3]}\left[J^{0[3]},\phi^{[3]*}\right] = \int\prod_i d\chi^fields0i_{\mathrm{f}|c}\exp\left\{\frac{\imath}{\hbar}\left[S^{\Psi[3]}_{s_1,s_2,s_3}(\chi^{0[i]}_{\mathrm{tot}|c})+\int d\eta_0\sum_{i=1}^{3}\Big({}_{s_i}\langle J^{0i}_{\mathrm{f}|c}|\chi^{0i}_{\mathrm{f}|c}\rangle_{s_i}+\right.\right. \tag{54}$$

where

$$S^{\Psi[3]}_{s_1,s_2,s_3}(\chi^{0[3]}_{\mathrm{tot}|c}) = \sum_{i=1}^{3} S^{\Psi^i}_{s_i}(\chi^{0i}_{\mathrm{tot}|c}) + S_{\mathrm{int}}(\phi^{[3]},\phi^{\Psi[3][3]*},C^{[3]},C^{[3]*}), \tag{55}$$

$$S_{\mathrm{int}} = g\int\prod_i d\eta_0^i\Big(\otimes_{j=1}^{3}\,{}_{s_j}\langle\chi^{0j}_{\mathrm{g}|c}|V_g\rangle_{s_1,s_2,s_3} + {}_{s_1,s_2,s_3}\langle V_g^{+}|\otimes_{j=1}^{3}|\chi^{0j}_{\mathrm{g}|c}\rangle_{s_j}\Big) \tag{56}$$