Dynamical breaking of symmetries beyond the standard model and supergeometry

Andrej B. Arbuzov and Diego J. Cirilo-Lombardo

Group theoretical realizations containing the electroweak sector of the Standard Model are discussed from the supersymmetry point of view. Dynamical breaking of the symmetry is performed and the corresponding quadratic (super Yang-Mills) Lagrangian is obtained. Supercoherent states of the Klauder-Perelomov type are defined to enlarge the symmetry taking into account the geometry of the coset based in the simplest supergroup SU (2|1) as the structural basis of the electroweak sector of the SM. The extended model is superintegrable and the superconnection in the odd part takes a dynamical character. The physical and geometrical implications of the additional degrees of freedom interpreted as a hidden sector of the representation are briefly discussed

1) *AA and DJCL, Phys.Scripta* 94 (2019) 12, 125302
2) *DJCL, Int.J.Geom.Meth.Mod.Phys.* 18 (2021) 06, 2150087

Preliminary aspects

- Phenomenological motivation:
- Unification, hierarchies and matter antimatter asymmetry
- Exotic states necessity in astrophysical/cosmological scenarios
- Mathematical structure
- Physical origin: works of Fairlie, Neeman, spin-statistics interpretation of the couplings with physical fields. Electroweak sector of SM and beyond.
- Mathematical/geometrical: Quillen (superconnection) with a fiber bundle concept
- Other developments: Todorov (Clifford), Therry Mieg and Jarvis, Kanno and Sugimoto (anomalies)
- Extended geometry: Azcarraga et al. (expansion from MC), Bergshoeff et al. (3D sugra), Gomis et al. , etc
- Coherent states appr.: SO(2,4) gauge grav., CS construction SU(2I1) and rel. with nonlinear realized symmetries, Noncommutative issues (Moyal, star product, etc).

3) AA andDJCL, *Int.J.Geom.Meth.Mod.Phys.* 15 (2017) 01, 1850005
4) DJCL, *Int.J.Geom.Meth.Mod.Phys.* 19 (2022) 01, 2250006
5) DJCL, *Int.J.Geom.Meth.Mod.Phys* Vol. 19, No. 1 (2022) 2250006

MAIN FEATURES

- (i) Introduction of a superconnection valued in the supergroup and not in the superalgebra. This fact, allows us to introduce the geometric imprint of the group manifold (or the tangent space).

- (II) The principal idea is to develop the theory beyond the SM using the group and not the vector space defined by the algebra, since physics lives in the group: the equations are similar but the geometry of the group manifold is embodied in the equations through the nonlinearity of a (super)Kahler's metric.

- (ii) The other fundamental issue in our approach is the extension of the group theoretical connection by means of super coherent states [3,4] conveniently chosen and constructed by the action of an element g of *SU(2|1).* This point is novel and very important (the explicit detailed construction of the *SU(2|1) coherent* states is developed in [2]) given that it allows a dynamical breaking of symmetry alternative to other methods.

- We analyze if a NLR relationship certainly exists, because under particular conditions the coherent states which extend the structure of the superconnection have the ability of natural projectors to reduce the general element from the full supergroup to a coset

$$\boxed{G \to G' \equiv GG_0 \to G/H,}$$

3) AA andDJCL, *Int.J.Geom.Meth.Mod.Phys.* 15 (2017) 01, 1850005
4) DJCL, *Int.J.Geom.Meth.Mod.Phys.* 19 (2022) 01, 2250006

GAUGE STRUCTURE AND SUPERCONNECTIONS: SU(2|1)

A. Supergroup structure

$$J = \begin{pmatrix} M & \phi \\ \overline{\phi} & N \end{pmatrix} . \quad (1)$$

The even generators in this structure are

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (2)$$

and the odd ones are

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} . \quad (3)$$

The diagonal matrices are related to the following important operators: $I_L^3 \equiv \frac{1}{2}\lambda_3$, the gradation operator $U_Q \equiv \sqrt{3}\lambda_8$, and the new operator that is fixed by the orthogonality in $U(2/1)$:

$$\lambda_0 \equiv \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} . \quad (4)$$

In previous studies the latter was related with the chirality of the model defined by $\sqrt{\frac{3}{2}}\lambda_0$.

B. Element of the supergroup: $\mathcal{K} \in B_1, \mathcal{H} \in B_0$:

$$\mathcal{K} = \frac{1}{\sqrt{1+\phi^\dagger \phi}} \begin{pmatrix} \mathbb{I}_{2\times 2} & \phi \\ \phi^\dagger & 1 \end{pmatrix} \tag{5}$$

with

$$\phi = \frac{\tan\sqrt{v^\dagger v}}{\sqrt{v^\dagger v}} v \quad \text{and} \quad v = \begin{pmatrix} \theta_4 - i\theta_5 \\ \theta_6 - i\theta_7 \end{pmatrix} \tag{6}$$

and

$$\mathcal{H} = \begin{pmatrix} w e^{-i\theta_8/\sqrt{3}} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & e^{-2i\theta_8/\sqrt{3}} \end{pmatrix} \tag{7}$$

$$\mathcal{U} = \mathcal{K}\mathcal{H}, \tag{8}$$

with

$$w = \frac{i}{\sqrt{1+W^2}} \left\{ \overbrace{\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}}^{-i\mathbb{I}} + \overbrace{\begin{pmatrix} W_3 & W_- \\ W_+ & -W_3 \end{pmatrix}}^{Y\cdot\sigma} \right\}, \qquad W_{3,\pm} = \frac{\tan\sqrt{\theta^2}}{\sqrt{\theta^2}} \theta_{3,\pm}, \tag{9}$$

$$\theta^2 = \theta_1^2 + \theta_2^2 + \theta_3^2, \qquad \theta_- = (\theta_1 - i\theta_2), \qquad \theta_+ = (\theta_1 + i\theta_2). \tag{10}$$

The final form of the supergroup-valuated connection being the following

$$\mathcal{K} = \exp\begin{pmatrix} 0 & \nu \\ \nu^\dagger & 0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{1+\phi^\dagger\phi}}\begin{pmatrix} \mathbb{I}_{2\times 2} & \phi \\ \phi^\dagger & 1 \end{pmatrix}$$

$$\mathcal{H} = \frac{1}{\sqrt{1+\phi^\dagger\phi}}\begin{pmatrix} \mathbb{I}_{2\times 2} & \phi \\ \phi^\dagger & 1 \end{pmatrix}\begin{pmatrix} \frac{\mathbb{I}_{2\times 2}}{\sqrt{1+W^2}} + \frac{i}{\sqrt{1+W^2}}\begin{pmatrix} W_3 & W_- \\ W_+ & -W_3 \end{pmatrix} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix}\underbrace{\begin{pmatrix} \left(\frac{1}{\sqrt{1+B'^2}} - i\frac{B'}{\sqrt{1+B'^2}}\right)\bullet\mathbb{I}_{2\times 2} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & \frac{\left(1-iB'\right)^2}{1+B'^2} \end{pmatrix}}_{\equiv\begin{pmatrix} A\bullet\mathbb{I}_{2\times 2} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & A^2 \end{pmatrix}}$$

tan(((θ₈)/(√3))) identified with B′≡B/√3 due the exponential

B. Element of the supergroup: $\mathcal{K}\in B_1, \mathcal{H}\in B_0$:

$$\mathcal{U} = \begin{pmatrix} \widetilde{w}\widetilde{A} & \widetilde{\phi}\widetilde{A}^2 \\ \widetilde{\phi}^\dagger \widetilde{w}\widetilde{A} & \widetilde{A}^2 \end{pmatrix}. \tag{13}$$

The following definitions to simplify (to hide the nonlinearity of square root factors) are introduced:

$$\widetilde{\phi} = \frac{\phi}{\sqrt{1+\phi^\dagger\phi}}, \quad \widetilde{W}_{3,\pm} \equiv \frac{W_{3,\pm}}{\sqrt{1+W^2}}, \quad \widetilde{B}' = \frac{B'}{\sqrt{1+B'^2}}, \tag{14}$$

$$\widetilde{w} = \frac{\mathbb{I}_{2\times 2}}{\sqrt{1+W^2}} + \mathrm{i}\begin{pmatrix} \widetilde{W}_3 & \widetilde{W}_- \\ \widetilde{W}_+ & -\widetilde{W}_3 \end{pmatrix} = \frac{\mathbb{I}_{2\times 2}}{\sqrt{1+W^2}} + \mathrm{i}\widetilde{W}\cdot\sigma. \tag{15}$$

- i) in U, the even part of the group structure (corresponding with the electroweak sector, namely SU(2)⊗U(1)) is exactly preserved;
- ii) the non diagonal blocks (the odd part) in U can be interpreted as new fermion-boson interaction;
- iii) $\mathcal{H}$=exp(∑$\theta_k\sigma_k$}) (k=1,2,3,8) and w=exp(∑$\theta_k\sigma_k$}) (k=1,2,3) ∈SU(2)$_L$;
- iv) there are other parameterization symmetrical (Borel type) ones involving ladder operators that we will use in the construction of the corresponding supercoherent states.

SUPERCURVATURE AND LAGRANGIAN

Due to the identification of the superconnection and $\mathcal{U}$, we can see that from

$$\Gamma = \frac{e^{-i\theta_8/\sqrt{3}}}{\sqrt{1+\phi^\dagger\phi}} \left\{ \begin{bmatrix} w & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & e^{-i\theta_8/\sqrt{3}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{2\times 2} & \phi e^{-i\theta_8/\sqrt{3}} \\ \phi^\dagger w & 0 \end{bmatrix} \right\} \tag{13}$$

$$\equiv \Gamma_{even} + \Gamma_{odd} \tag{14}$$

one can obtain the super-Riemannian curvature in the language of superforms:

$$d\Gamma_{even} + \Gamma_{even} \wedge \Gamma_{even} + \Gamma_{odd} \vee \Gamma_{odd} \rightarrow d\Gamma_{even} + [\Gamma_{even}, \Gamma_{even}] + \{\Gamma_{odd}, \Gamma_{odd}\} \tag{15}$$

$$d\Gamma_{odd} + \Gamma_{even} \wedge \Gamma_{odd} \rightarrow d\Gamma_{even} + [\Gamma_{even}, \Gamma_{odd}]\,. \tag{16}$$

To compute the above equations that define the supercurvature, it is useful to have in mind that the tensor product of a commutative superalgebra of differential forms and a Lie superalgebra is again a Lie superalgebra with the product

$$[a \otimes X, b \otimes Y] = -1^{|X||b|}\,(a \wedge b) \otimes [X, Y]\,. \tag{17}$$

A. Odd supercurvature and Weinberg angle

$$d\Gamma_{odd} + \Gamma_{even} \wedge \Gamma_{odd} \rightarrow$$
$$\rightarrow -i \begin{pmatrix} \mathbf{0}_{2\times 2} & d\widetilde{\phi} - 2i\left[g\widetilde{W}\cdot\sigma - g'\widetilde{B}\cdot\mathbb{I}_{2\times 2}\right]\widetilde{\phi} \\ d\widetilde{\phi}^{\dagger} + 2i\widetilde{\phi}^{\dagger}\left[g\widetilde{W}\cdot\sigma - \frac{g'}{\sqrt{3}}\widetilde{B}\cdot\mathbb{I}_{2\times 2}\right] & 0 \end{pmatrix}, \tag{38}$$

where we defined $g' \equiv \frac{1}{\sqrt{3}\sqrt{1+W^2}}$, $g \equiv \frac{1}{\sqrt{1+B^2}}$;

$$\sin^2\theta_W = \frac{g\prime^2}{g^2 + g\prime^2} \rightarrow \frac{1+B^2}{3\left(1+W^2\right)+1}. \tag{40}$$

If $W^2, B^2 \sim 0$ then

$$\sin^2\theta_W\big|_{W^2,B^2\rightarrow 0} \rightarrow 0.25\,. \tag{41}$$

It is important to note that, if we take $\sin^2\theta_W$ with respect to the fields without tilde of the expression (39) we see that, due to $\frac{g}{\sqrt{3}} = g'$ with $g = \frac{1}{\sqrt{1+W^2}\sqrt{1+B^2}} = f$,

$$\sin^2\theta_W = \frac{g\prime^2}{g^2 + g\prime^2} = \frac{1}{4}$$

exactly. In this case we see that the result $\sin^2\theta_W = \frac{1}{4}$ is independent of the fields W and B, depending only on the particular structure of the supergroup.

B. Even supercurvature

In the same way as before for the odd part of the supercurvature, discarding terms involving pure geometry of the group manifold (e.g., terms involving df), we get the even part in the form

$$d\Gamma_{even} + [\Gamma_{even}, \Gamma_{even}] + \{\Gamma_{odd}, \Gamma_{odd}\} \rightarrow \tag{42}$$
$$\rightarrow \begin{pmatrix} d\widetilde{W}_k + \widetilde{W}_i \wedge \widetilde{W}_j \epsilon_{ijk}\sigma_k + d\left(\frac{1}{\sqrt{3}}\widetilde{B}\right) - 4\widetilde{\phi}\widetilde{\phi}^\dagger \mathbb{M} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & d\left(\frac{2}{\sqrt{3}}\widetilde{B}\right) - 4\widetilde{\phi}^\dagger\widetilde{\phi}\,\langle \mathbb{M}\rangle_{\widetilde{\phi}} \end{pmatrix}.$$

Consequently, the full supercurvature takes the form

$$\mathcal{F} \equiv \begin{pmatrix} d\widetilde{W}_k + \widetilde{W}_i \wedge \widetilde{W}_j \epsilon_{ijk}\sigma_k + d\left(\frac{1}{\sqrt{3}}\widetilde{B}\right) - 4\widetilde{\phi}\widetilde{\phi}^\dagger \mathbb{M} & D\widetilde{\phi} \\ \left(D\widetilde{\phi}\right)^\dagger & d\left(\frac{2}{\sqrt{3}}\widetilde{B}\right) - 4\widetilde{\phi}^\dagger\widetilde{\phi}\,\langle \mathbb{M}\rangle_{\widetilde{\phi}} \end{pmatrix}. \tag{43}$$

Note that a tilde indicates here that the respective quantity is affected by the induced supercurvature due to the pullback from the algebra (vector space) to the group representation.

$$k = (1+f) \quad \text{and} \quad f = \frac{1}{\sqrt{1+W^2}\sqrt{1+B'^2}},$$

$$\mathbb{M} \equiv \left(f\mathbb{I}_{2\times 2} + \widetilde{W}\widetilde{B}'\right),$$

$$\langle \mathbb{M}\rangle_{\widetilde{\phi}} \equiv \frac{\widetilde{\phi}^\dagger \mathbb{M}\widetilde{\phi}}{\widetilde{\phi}^\dagger\widetilde{\phi}} = f + \left(\frac{\widetilde{\phi}^\dagger \widetilde{W}\widetilde{B}'\widetilde{\phi}}{\widetilde{\phi}^\dagger\widetilde{\phi}}\right).$$

Analog to [5,6] model but the geometry of the group manifold is embodied into the equations

5)Fairlie *Phys.Lett.B* 82 (1979) 97-100; 6)Aydemir et al, *Phys.Rev.D* 91 (2015) 045020

Gauge transformation properties

QCD

$dG + \frac{1}{2}[G,G]$ transform as $F_G \to U F_G U^\dagger$ under $SU(3)$ gauge transformations

Our case

$$\widetilde{W} \to u_e \widetilde{W} u_e^\dagger + \mathrm{i} \mathrm{d} u_e u_e^\dagger, \quad \widetilde{B} \to \widetilde{B} - \mathrm{d}\theta_8, \quad \widetilde{\phi} \to u_e \mathrm{e}^{\theta_8 \lambda_8} \widetilde{\phi}.$$

Consequently the curvatures remain with the required invariance, namely

$$F_{\widetilde{W}} \to u_e F_{\widetilde{W}} u_e^\dagger, \qquad F_{\widetilde{B}} \to F_{\widetilde{B}}, \qquad D\widetilde{\phi} \to u_e \mathrm{e}^{\mathrm{i}\theta_8/\sqrt{3}} D\widetilde{\phi}.$$

even part of the $SU(2|1)$ superalgebra: $U_e = \exp\left(\sum_{k=1,2,3,8} \theta_k \lambda_k\right)$ with $u_e = \exp\left(\sum_{k=1,2,3} \theta_k \sigma_k\right)$

in[6] explicitly broken g.inv

$$F_W + \frac{1}{\sqrt{3}} F_B - 2\phi\phi^\dagger + \boxed{\xi\xi^\dagger} \qquad \xi^\dagger \xi = v^2. \quad \hat{\phi} = \phi + \frac{\xi}{\sqrt{2}},$$

SUPERCONNECTIONS AND SUPERGEOMETRY

- The strategy to extend the symmetry without breaking the group theoretical features of the model is realized as follows.

i) If we have two diffeomorphic (or gauge) non-equivalent $SU(2|1)$ valuated superconnections, namely Γ^{AB} and $\widetilde{\Gamma}^{AB}$. Their difference transforms as a second rank three-supertensor under the action of $SU(2|1)$:

$$\kappa^{AB} = G^A_{\ C} G^B_{\ D} \kappa^{CD}, \tag{37}$$

$$\kappa^{AB} \equiv \widetilde{\Gamma}^{AB} - \Gamma^{AB}. \tag{38}$$

ii) If we calculate now the curvature from $\widetilde{\Gamma}^{AB}$, we obtain

$$\widetilde{R}^{AB} = R^{AB} + \mathcal{D}\kappa^{AB}, \tag{39}$$

where the $SU(2|1)$ supercovariant derivative is defined in the usual way (see the previous Section)

$$\mathcal{D}\kappa^{AB} = d\kappa^{AB} + \Gamma^A_{\ C} \wedge \kappa^{CB} + \Gamma^B_{\ D} \wedge \kappa^{AD}. \tag{40}$$

iii) Redefining the $SU(2|1)$ three vectors as $V_2^A \equiv \psi^A$ and $V_1^B \equiv \varphi^B$ (in order to put all in the standard notation), the 2-form κ^{AB} can be constructed as

$$\kappa^{AB} \rightarrow \psi^{[A}\varphi^{B\}}dU, \tag{41}$$

where U is a super-scalar function. Then we introduce all into the $\widetilde{R}^{AB}$ and get

$$\begin{aligned}\widetilde{R}^{AB} &= R^{AB} + \mathcal{D}\left(\psi^{[A}\varphi^{B\}}dU\right) \\ &= R^{AB} + \left(\psi^{[A}\mathcal{D}\varphi^{B\}} - \varphi^{[A}\mathcal{D}\psi^{B\}}\right) \wedge dU.\end{aligned} \tag{42}$$

Note that the supercurvature $\widetilde{R}^{AB}$ splits into even and odd parts as indicated in the previous Section, being the capital letters the multi-index A, B, C etc. corresponding to the supercoordinates of the $su(2|1)$ superspace.

iv) Let us define

$$\widetilde{\theta}^A = \widetilde{\mathcal{D}}\varphi^A \tag{43}$$

with the extended superconnection $\widetilde{\Gamma}^{AB} = \Gamma^{AB} + \kappa^{AB}$, then

$$\begin{aligned} \widetilde{\theta}^A &= \underbrace{\mathcal{D}\varphi^A}_{\theta^A} + \kappa^A{}_B\varphi^B, \\ \widetilde{\theta}^A &= \theta^A + \left[\psi^A\left(\varphi^B\right)^2 - \varphi^A\left(\psi\cdot\varphi\right)\right]\wedge dU, \end{aligned} \tag{44}$$

where $\left(\varphi^B\right)^2 = \left(\varphi_{\ B}\varphi^B\right)$ and $\left(\psi\cdot\varphi\right) = \psi_B\varphi^B$ etc.

In the same manner we also define

$$\begin{aligned} \widetilde{\eta}^A &= \widetilde{\mathcal{D}}\psi^A, \\ \widetilde{\eta}^A &= \eta^A + \left[\psi_2^A\left(\psi\cdot\varphi\right) - \varphi^A\left(\psi^B\right)^2\right]\wedge dU. \end{aligned} \tag{45}$$

Some aspects of coset coherent states

$$H_0 = \{g \in G \mid \mathcal{U}(g) V_0 = V_0\} \subset G. \tag{53}$$

Consequently the orbit of the V_0 is isomorphic to the coset, i.e.

$$\mathcal{O}(V_0) \simeq G/H_0. \tag{54}$$

Analogously, if we remit to the operators

$$|V_0\rangle \langle V_0| \equiv \rho_0. \tag{55}$$

Then the orbit

$$\mathcal{O}(V_0) \simeq G/H \tag{56}$$

with

$$\begin{aligned} H &= \{g \in G \mid \mathcal{U}(g) V_0 = \theta V_0\} \\ &= \left\{g \in G \mid \mathcal{U}(g) \rho_0 \mathcal{U}^{\dagger}(g) = \rho_0\right\} \subset G. \end{aligned} \tag{57}$$

Some aspects of coset coherent states

$$\begin{pmatrix} \widetilde{w}\widetilde{A} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix} V_0 = V_0, \tag{59}$$

(V_0 is extreme in the sense of the previous paragraph) and it also belongs to the vacuum, that is, it is annihilated by all the lower generators of the complete supergroup, that is to say

$$U_- \begin{pmatrix} A \\ B \\ C \end{pmatrix} = 0 \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ -\omega_l'^* & 0 & 0 \\ -\overline{\Phi_l}' & -\overline{\rho}_l' & 0 \end{pmatrix} V_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow V_0 = \begin{pmatrix} 0 \\ 0 \\ \overline{C} \end{pmatrix}, \tag{60}$$

where $-\omega_l'^*$: even , $-\overline{\Phi_l}', -\overline{\rho}_l'$: odd (e.g. Grassmannian), are the generic parameters associated with the lower generators corresponding to $SU(2|1)$.Consequently with $g \in SU(2|1)$

$$V = gV_0 = \begin{pmatrix} \Phi\left(1 - \overline{\rho}\rho + \omega^*\omega - \overline{\Phi}\Phi\right)^{-1/4} \overline{C} \\ \rho\left(1 - \overline{\rho}\rho + \omega^*\omega - \overline{\Phi}\Phi\right)^{1/4} \overline{C} \\ \overline{C} \end{pmatrix}, \text{ that is: } gV_0 \rightarrow V \tag{61}$$

e.g.: V is in the orbit of V_0.

Some aspects of coset coherent states

$$\psi = \pm \begin{pmatrix} \Omega^{-1/4}\Phi^{\alpha}_{\beta}\psi^0_a \\ \Omega^{-1/4}\rho^{\alpha}_{\beta}\psi^0_{\alpha} \\ \pm\psi^0_{\beta} \end{pmatrix}, \varphi = \pm \begin{pmatrix} \Omega'^{-1/4}\Phi^{\alpha'}_{\beta}\varphi^0_a \\ \Omega'^{-1/4}\rho^{\alpha'}_{\beta}\varphi^0_a \\ \pm\varphi^0_{\beta} \end{pmatrix}$$

where

$$\Omega = \left(1 - \overline{\rho}\rho + \omega^{*}\omega - \overline{\Phi}\Phi\right)$$

$$SU(2|1) \ni \psi \wedge \varphi = \Omega^{-1/2} \begin{pmatrix} \Phi^{\alpha\beta}\psi^0_a \wedge \Phi^{\gamma}_{\beta}\varphi^0_{\gamma} & \Phi^{\alpha}_{\delta}\psi^0_a \wedge \rho^{\gamma}_{\beta}\varphi^0_{\gamma} & \pm\Omega'^{1/4}\Phi^{\alpha}_{\delta}\psi^0_a \wedge \varphi^0_{\beta} \\ \rho^{\alpha}_{\delta}\psi^0_{\alpha} \wedge \Phi^{\gamma}_{\beta}\varphi^0_{\gamma} & \rho^{\alpha}_{\delta}\psi^0_{\alpha} \wedge \rho^{\gamma}_{\beta}\varphi^0_{\gamma} & \pm\Omega'^{1/4}\rho^{\alpha}_{\delta}\psi^0_{\alpha} \wedge \varphi^0_{\beta} \\ \pm\Omega^{1/4}\psi^0_{\delta} \wedge \Phi^{\gamma}_{\beta}\varphi^0_{\gamma} & \pm\Omega^{1/4}\psi^0_{\delta} \wedge \rho^{\gamma}_{\beta}\varphi^0_{\gamma} & \psi^0_{\delta}\varphi^0_{\beta} - \psi^0_{\beta}\varphi^0_{\delta} \end{pmatrix}$$

$$\Downarrow$$

$$SU(2|1) \ni \kappa^{AB} \propto \Omega^{-1/2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \psi^0_{\delta}\varphi^0_{\beta} - \psi^0_{\beta}\varphi^0_{\delta} \end{pmatrix}$$

EXTENDED SUPERCURVATURE

From the original superconnection
the extended supercurvature is

$$\mathcal{F}^{AB}_{ext} = \mathcal{F}^{AB} + \mathcal{D}_{\epsilon}\left(\psi^{[A}\varphi^{B\}}dU\right) \tag{46}$$

$$= \mathcal{F}^{AB} + \left(\psi^{[A}\mathcal{D}\varphi^{B\}} - \varphi^{[A}\mathcal{D}\psi^{B\}}\right)\wedge dU$$

$$+ \left(\psi^{[A}\kappa^{B}_{C}\varphi^{C\}} - \varphi^{[A}\kappa^{B}_{C}\psi^{C\}}\right)\wedge dU \tag{47}$$

$$= \begin{pmatrix} D\widetilde{W}_k + \frac{1}{\sqrt{3}}D\widetilde{B} - 4\widetilde{\phi}\widetilde{\phi}^{\dagger} & D\widetilde{\phi} - 2\widetilde{\psi}\widetilde{\phi}\widetilde{\varphi} \\ \left(D\widetilde{\phi}\right)^{\dagger} + 2\widetilde{\varphi}\widetilde{\phi}^{\dagger}\widetilde{\psi} & \frac{2}{\sqrt{3}}D\widetilde{B} - 4\widetilde{\phi}^{\dagger}\widetilde{\phi} + \left(\widetilde{\psi}d\widetilde{\varphi} - \widetilde{\varphi}d\widetilde{\psi}\right) + 2\widetilde{\psi}^2\widetilde{\varphi}^2 \end{pmatrix}, \tag{48}$$

$$\mathrm{d}\widetilde{W}_k + \widetilde{W}_i \wedge \widetilde{W}_j \epsilon_{ijk}\sigma_k$$

$$\mathrm{d}\left(\frac{1}{\sqrt{3}}\widetilde{B}\right)$$

Note that, at the supercurvature level, a Dirac-type term, namely $(\widetilde{\psi}\mathrm{d}\widetilde{\varphi} - \widetilde{\varphi}\mathrm{d}\widetilde{\psi}) + 2\widetilde{\psi}^2\widetilde{\varphi}^2$ plus the couplings with $\widetilde{\phi}^{\dagger}$ and $\widetilde{\phi}$ are geometrically induced by the extension of the original superalgebra.

Superlagrangian

Dynamically introduce through the coherent states the ϕ VEV , resulting symmetry breaking in the proposed BSM model.

Case 1: $\tilde{\psi}, \tilde{\varphi} = \text{const.}$

$$d\tilde{\varphi} = d\tilde{\psi} = 0,$$

$$S = \frac{1}{4}\langle \mathcal{F}_{\text{ext}}, \mathcal{F}_{\text{ext}}\rangle = -\frac{1}{4} \times \int d^4x [((F_W^{\text{ext}})_{\mu\nu}(F_W^{\text{ext}})^{\mu\nu} + (F_B^{\text{ext}})_{\mu\nu}(F_B^{\text{ext}})^{\mu\nu}) + (D\tilde{\phi})^\dagger D\tilde{\phi} - V(\tilde{\phi}^\dagger, \tilde{\phi}, \tilde{\psi}, \tilde{\varphi}) + Z(\tilde{\phi}^\dagger, \tilde{\phi}, \tilde{B}, \tilde{W})], \tag{69}$$

$$F_{Wk}^{\text{ext}} = d\tilde{W}_k + \tilde{W}_i \wedge \tilde{W}_j \epsilon_{ijk}, \tag{70}$$

$$F_{B\mu\nu}^{\text{ext}} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{71}$$

$$V(\tilde{\phi}^\dagger, \tilde{\phi}, \tilde{\psi}, \tilde{\varphi}) = 16\left[\left(\tilde{\phi}^\dagger\tilde{\phi} - \frac{v^2}{8}\right)\left(\tilde{\phi}^\dagger\tilde{\phi} + \frac{v^2}{8}\right)\right], \tag{72}$$

$$Z(\tilde{\phi}^\dagger, \tilde{\phi}, \tilde{B}, \tilde{W}) = 2(\tilde{\phi}^\dagger \tilde{W}\tilde{B}'\tilde{\phi})\tilde{\phi}^\dagger\tilde{\phi} + (\tilde{\phi}^\dagger\tilde{W}\tilde{B}'\tilde{\phi})^2, \tag{73}$$

where we defined

$$v^2 = 2(\tilde{\psi}^2\tilde{\varphi}^2 + (\tilde{\psi}\cdot\tilde{\varphi})^2) = \tilde{\psi}^2\tilde{\varphi}^2. \tag{74}$$

- In [6], the symmetry breaking is imposed by introducing 'by hand' the Higgs VEV v defining the zero form ϕ with a shift. Here however, VEV is dynamically introduced through the components of the coherent states (valued in the coset) taking a dynamic character. Gauge couplings g and *g' are not modified in the covariant derivative.*

$\widetilde{W}$ and $\widetilde{Z} = \frac{\sqrt{3}\,\widetilde{W}^3 - \widetilde{B}}{2}$ gauge bosons become

$$M_{\widetilde{W}} = g\frac{v}{2}, \qquad M_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2}. \tag{75}$$

For g=1 the expressions are the simplest ones (e.g. as in the SM)

$\boxed{\text{Case 2: } \tilde{\psi} = \tilde{\psi}(z), \tilde{\varphi} = \tilde{\varphi}(z)}$

$$(\widetilde{\psi}\mathrm{d}\widetilde{\varphi} - \widetilde{\varphi}\mathrm{d}\widetilde{\psi}) + \widetilde{\psi}^2\widetilde{\varphi}^2 \equiv v^2. \qquad (76)$$

$$(\widetilde{\psi}\widetilde{\varphi}) = \underbrace{(\widetilde{\psi_0\varphi_0})}_{\varrho}\frac{v}{\sqrt{2}}\tanh[(z - z_0)\sqrt{2}v]. \qquad (77)$$

•Note that the equation is on the third components of the respective fiducial vectors , z is a super coordinate of the manifold *SU(2|1) -valuated.*

•*Here z = (z E) is a* general coordinate of the supergeometry (induced via the pullback with $\{E_A\}$ *the corresponding vector basis) from su(2|1)).*

•The extended supercurvature becomes to

$$\boxed{\begin{pmatrix} D\widetilde{W} + D\left(\frac{1}{\sqrt{3}}\widetilde{B}\right) - 4\widetilde{\phi}\,\widetilde{\phi}^{\dagger}\mathrm{M} & D\widetilde{\phi} - \sqrt{2}\,\varrho v\tanh[(z - z_0)\sqrt{2}v]\widetilde{\phi} \\ (D\widetilde{\phi})^{\dagger} + \sqrt{2}\,\varrho v\tanh[(z - z_0)\sqrt{2}v]\widetilde{\phi}^{\dagger} & D\left(\frac{2}{\sqrt{3}}\widetilde{B}\right) - 4\widetilde{\phi}^{\dagger}\widetilde{\phi}\langle\mathrm{M}\rangle_{\widetilde{\phi}} + v^2 \end{pmatrix}. \qquad (78)}$$

with the definitions

$$\widetilde{W} \wedge \widetilde{W} \equiv \widetilde{W}_i \wedge \widetilde{W}_j \epsilon_{ijk} \sigma_k, \tag{30}$$

$$\widetilde{W} \equiv \widetilde{W} \cdot \sigma, \widetilde{B} \equiv \widetilde{B} \cdot \mathbb{I}_{2\times 2}, \tag{31}$$

$$k = (1 + f) \quad \text{and} \quad f = \frac{1}{\sqrt{1 + W^2}\sqrt{1 + B'^2}}, \tag{32}$$

$$\mathbb{M} \equiv (f\mathbb{I}_{2\times 2} + \widetilde{W}\widetilde{B}'), \tag{33}$$

$$\langle \mathbb{M} \rangle_{\tilde{\phi}} \equiv \frac{\widetilde{\phi}^\dagger \mathbb{M} \widetilde{\phi}}{\widetilde{\phi}^\dagger \widetilde{\phi}} = f + \left(\frac{\widetilde{\phi}^\dagger \widetilde{W}\widetilde{B}' \widetilde{\phi}}{\widetilde{\phi}^\dagger \widetilde{\phi}} \right). \tag{34}$$

As in the standard case, we can take ϕ adimensionalized as appear in the exponential representation of the $SU(2|1)$ (from the group to the physical scenario). To this end we construct the ϕ field as

$$\phi = \frac{1}{\phi_0}\begin{pmatrix} \pi^+ \\ \frac{v+h+i\pi^0}{2} \end{pmatrix} \frac{1}{\sqrt{1+\frac{1}{\phi_0^2}\left(\pi^{+2}+\frac{(v+h)^2+(\pi^0)^2}{4}\right)}} \tag{79}$$

including the values v, h for the Higgs field and π and some bare quantity ϕ_0 to be determined. Extracting from the geometrically induced extended superpotential $V(\widetilde{\phi^\dagger,\phi}, \widetilde{\psi}, \widetilde{\varphi})$ (72) the (adimensionalized) mass term for h

$$\sim \cdots + \frac{1}{\phi_0^4} \frac{16(2vh)^2}{4\left[1+\frac{1}{\phi_0^2}\left(\pi^{+2}+\frac{(v+h)^2+(\pi^0)^2}{4}\right)\right]^2} + \cdots \tag{80}$$

Then, at the tree level we obtain the *adimensionalized* Higgs mass as

$$M_H|_{\mathrm{treeAD}} \sim \frac{4\sqrt{2}\,v}{\phi_0\left[1+\frac{1}{\phi_0^2}\left(\pi^{+2}+\frac{(v+h)^2+(\pi^0)^2}{4}\right)\right]} \tag{81}$$

consequently the *physical* mass is given by

$$M_H|_{\text{tree}} = \phi_0 M_H|_{\text{treeAD}} \sim \frac{4\sqrt{2}\,v}{\left[1 + \frac{1}{\phi_0^2}\left(\pi^{+2} + \frac{(v+h)^2 + (\pi^0)^2}{4}\right)\right]}. \tag{82}$$

The normalizing field ϕ_0 can be determined from the expression (77) by the constant of integration that would indicate that $\rho^{-1} \sim \phi_0$.

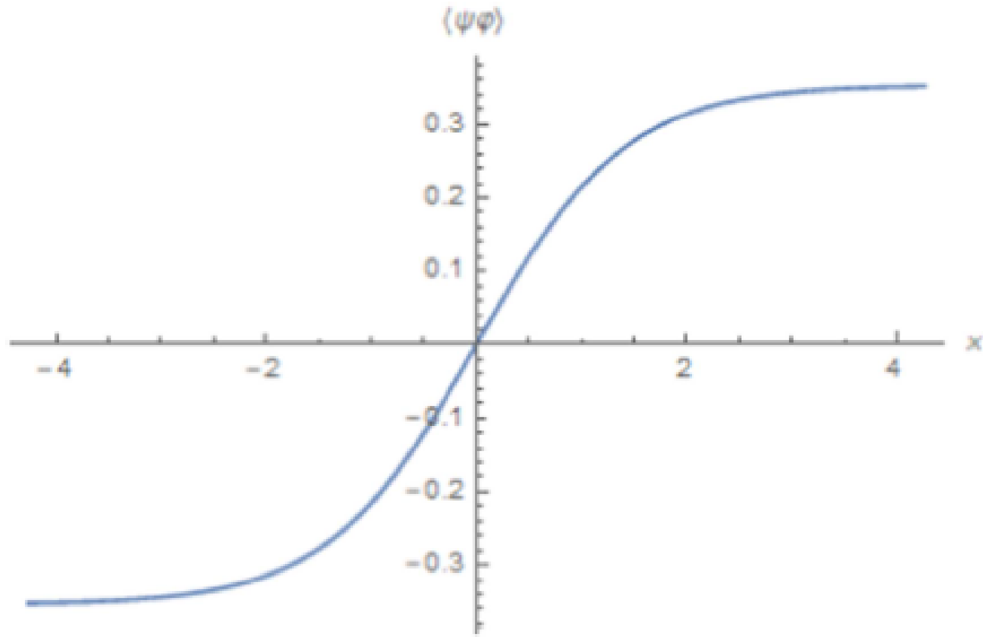

Figure 1. $(\widetilde{\psi}\widetilde{\varphi})$ for $v = 0.5$.

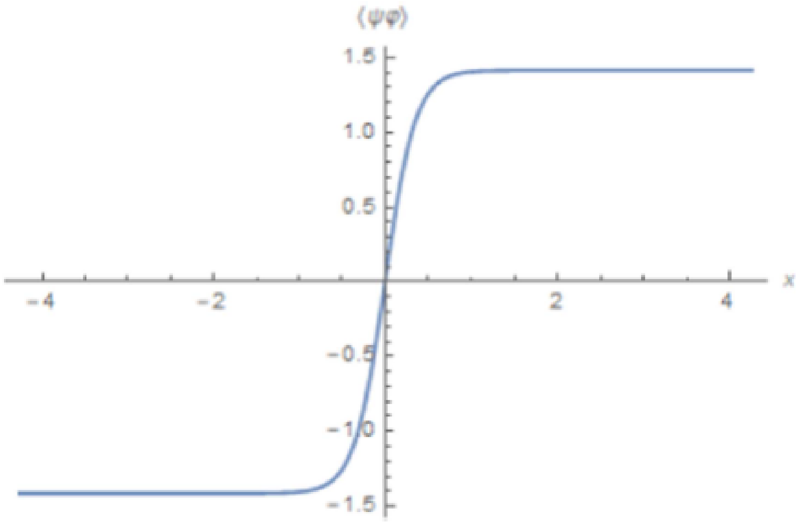

Figure 2. $(\widetilde{\psi}\widetilde{\varphi})$ for $v = 2$.

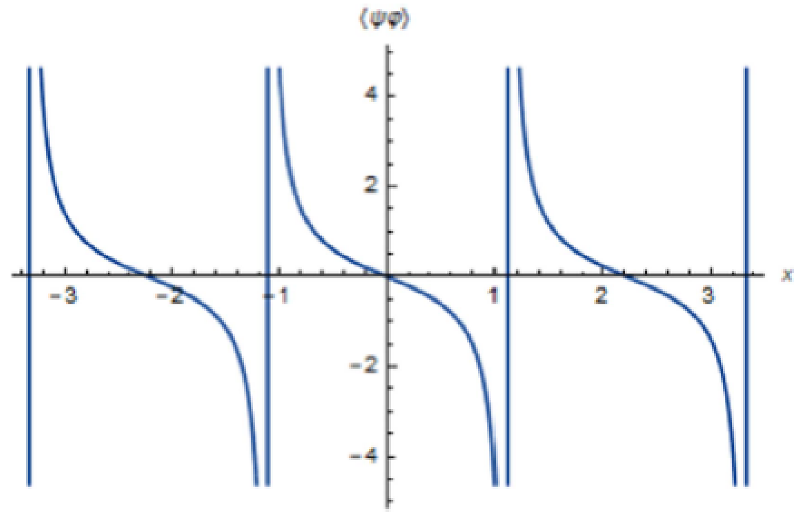

Figure 3. $v = -i$

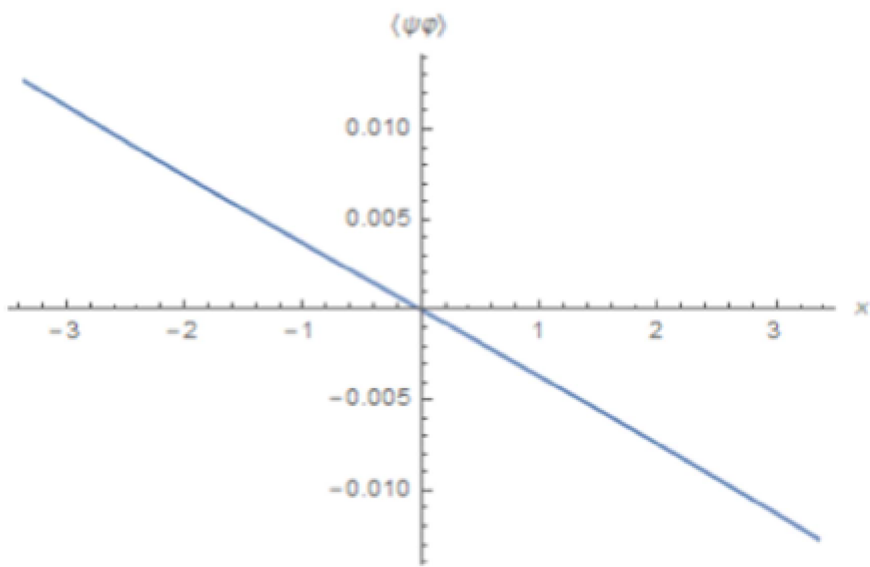

Figure 4. $(\widetilde{\psi}\widetilde{\varphi})$ for $v = 0.6i$.

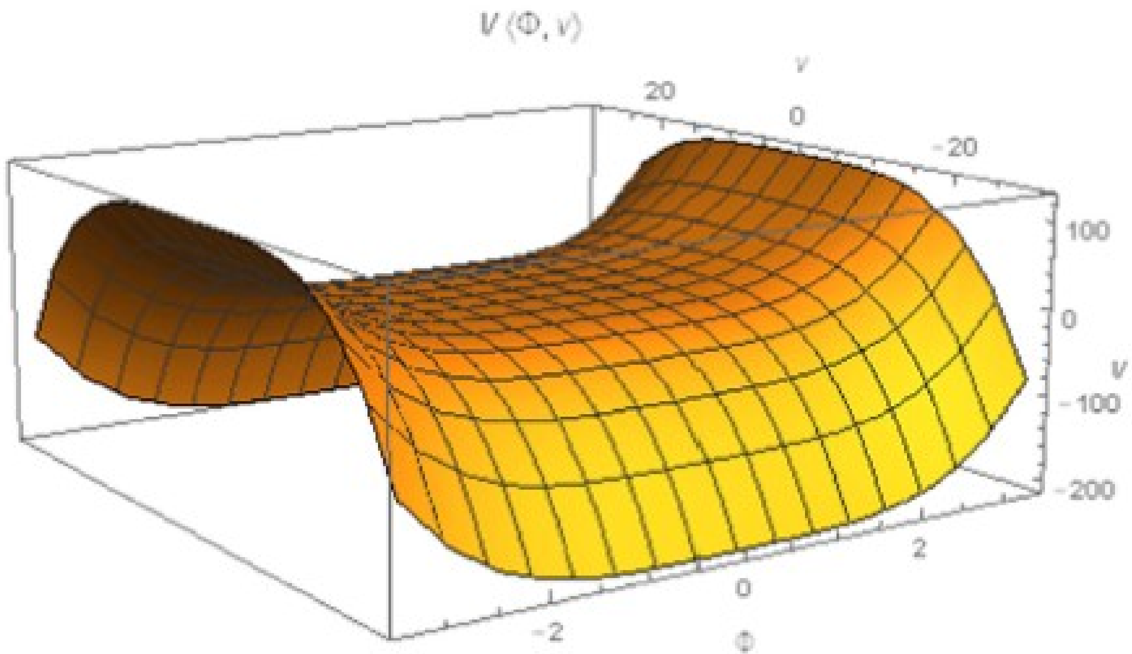

Figure 6. Superpotential as function of ϕ and ν.

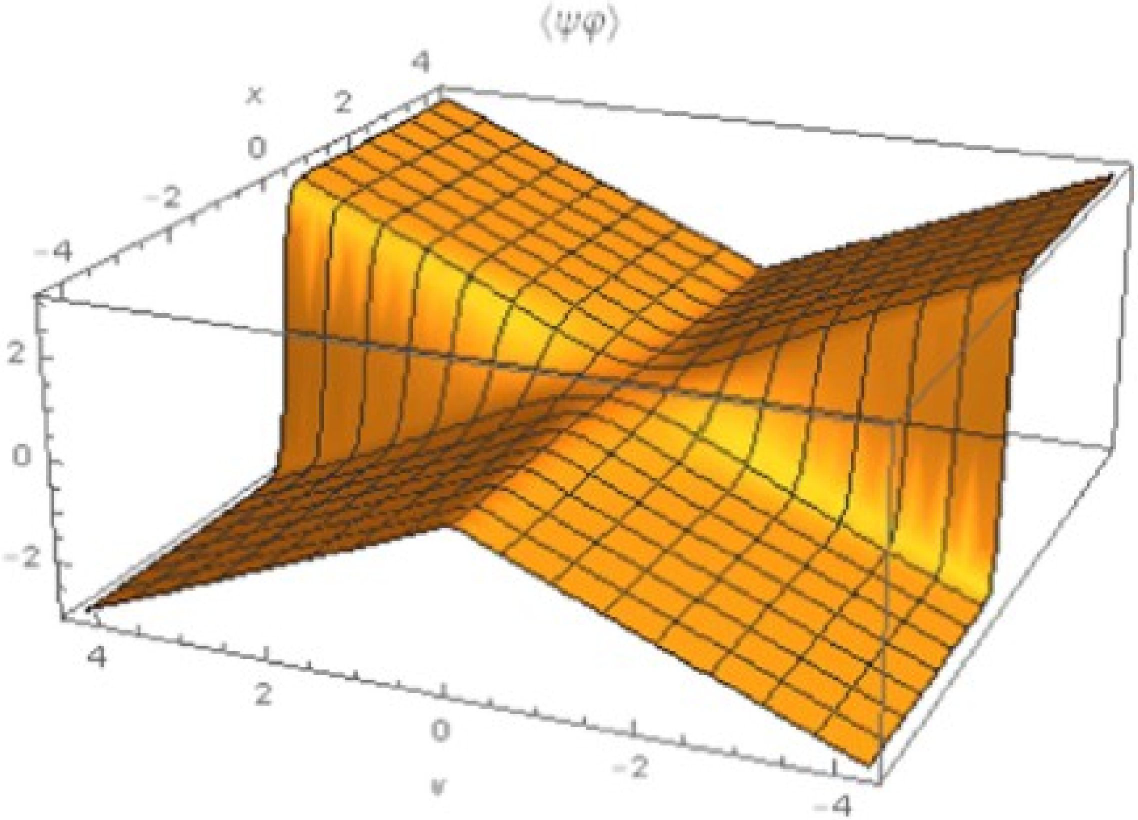

Figure 5. $\langle\tilde{\psi}\tilde{\varphi}\rangle$ as a function of spacetime coordinate and v.

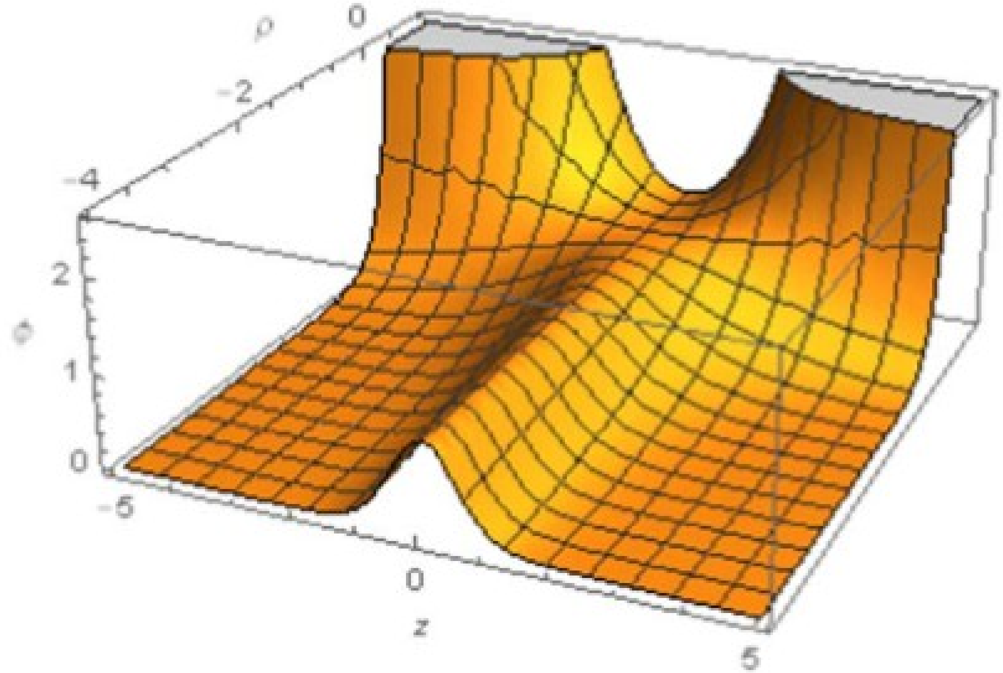

Figure 7. Shape of the Higgs doublet $\widetilde{\phi}$ as a function of the $su(2|1)$ valuated coordinates and of the fermion condensate ρ.

CONCLUDING REMARKS AND OUTLOOK

- We developed a possible description of the electroweak sector of the SM using the methods described in our paper [2], using naturally a coherent superstate based on the simplest supergroup *SU(2|1) which is the group* of dynamic symmetry of the supersphere, keeping invariant in the natural factorization, the even part SU(2)⊗U(1).
- Possible physical interpretation of the odd sector a potentially hidden counterpart of the SM.
- In one of the cases discussed here the diagonal part corresponding to the even sector that defines a geometrically induced differential equation for the components of a superfield.
- This differential equation can be considered non-homogeneous, specifically equal to the constant of the mean value of the condensate v^2 or superscalar product between such coherent superfields that extend the original superalgebra
- The resolution of the differential equation results in a solution that can be compact (solitonic) or not (depending on v^2 value)
- the superintegrability of the super-extended model is assured by the type of the obtained solutions [7] beside of the geometrical properties.
- the superpotential is an exact difference of squares in contrast to the standard case where the quadratic part appears.
- With respect to the statistics and other issues corresponding to the structure of the group, here the field corresponding to the Higgs appears as odd in the representation but as SU(2) scalar (doublet) from the Lagrangian point of view.

[7] Shifman M and Yung A 2009 Supersymmetric Solitons (Cambridge Monographs in Mathematical Physics) (Cambridge: Cambridge University Press)

- Relationship with nonlinear realizations, considering that the field which plays the role of Higgs could be clearly eliminated at the expense of the fields of the hidden super-sector and the constant v (playing the final role of expectation value): the antidiagonal part of the supercurvature →Maurer–Cartan superform associated with the breaking of some (super) symmetry in a standard way

(e.g. $D\widetilde{\phi} \equiv \omega_{\widetilde{\phi}}$)

$$\omega_{\widetilde{\phi}} = 0 \to \mathrm{d}\widetilde{\phi} = \sqrt{2}\,\varrho v \tanh[(z - z_0)\sqrt{2}\,v]\widetilde{\phi}$$
$$\to \mathrm{d}(\ln \widetilde{\phi}) = \varrho \tanh X \quad \mathrm{d}X, \tag{83}$$

with $X \equiv (z - z_0)\sqrt{2}\,v$ and without taking into account an ignorable phase factor (dependent on W, B) irrelevant in this stage of analysis. The result is easily obtained as

$$\widetilde{\phi} = \widetilde{\phi}_0(\cosh[(z - z_0)\sqrt{2}\,v])^{\rho}, \qquad \widetilde{\phi}_0 = \text{const} \tag{84}$$

being $z \equiv z^A E_A$ the supercoordinate $SU(2|1)$ valuated (with the superbasis carrying the superalgebra symmetry). If we define again $\tanh(X/2) = \Lambda$ then (84) can be written as

$$\widetilde{\phi} = \widetilde{\phi}_0\left(\frac{1 + \Lambda^2}{1 - \Lambda^2}\right)^{\rho}, \qquad \widetilde{\phi}_0 = \text{const.} \tag{85}$$

Compact

$$|z,\varphi,\rho\rangle = \mathcal{N}_+\left[\sum_{m=-\tau}^{m=\tau}\binom{2\tau}{m+\tau}^{1/2}\frac{z^{\tau-m}}{(1+|z|^2)^{\tau}}|b,\tau,m\rangle + \right.$$

$$+\sqrt{\tau-b}\,\overline{\varphi}\sum_{m=-\tau}^{m=\tau}\binom{2\tau}{m+\tau}^{1/2}\frac{z^{\tau-m}}{(1+|z|^2)^{\tau}}|b+1/2,\tau-1/2,m-1/2\rangle +$$

$$+\sqrt{\tau+b}\,\rho\sum_{m=-\tau}^{m=\tau}\binom{2\tau}{m+\tau}^{1/2}\frac{z^{\tau-m}}{(1+|z|^2)^{\tau}}|b-1/2,\tau-1/2,m-1/2\rangle +$$

$$\left.+\sqrt{\frac{(\tau^2-b^2)}{2\tau}}\,\overline{\varphi}\rho\sum_{m=-\tau}^{m=\tau}\binom{2\tau}{m+\tau}^{1/2}\frac{z^{\tau-m}}{(1+|z|^2)^{\tau}}|b,\tau-1,m-1\rangle\right]$$

$$\mathcal{N}_{\pm}=\frac{(1-\overline{\rho}\rho)}{\sqrt{(1-\overline{\rho}\rho\pm\omega^*\omega-\overline{\Phi}\Phi)}}S\det N$$

$$N=\begin{pmatrix}1 & 0 & \Phi\\ 0 & 1 & \rho\\ \overline{\Phi}' & \overline{\rho}' & 1\pm\bar{z}z'\end{pmatrix}$$

$$M=\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ \mathcal{C} & \mathcal{D}\end{pmatrix}\rightarrow S\det M=\frac{\det(\mathcal{A}-\mathcal{B}\mathcal{D}^{-1}\mathcal{C})}{\det\mathcal{D}}$$

Non-compact

$$
\begin{aligned}
|z,\varphi,\rho\rangle = \mathcal{N}_- \Bigg[& \sum_{m=0}^{\infty} \left(\frac{\Gamma(2\tau+m)}{m!\Gamma(2\tau)} \right)^{1/2} \frac{z^m}{(1-|z|^2)^{\tau}} |b,\tau,\tau+m\rangle + \\
& + \sqrt{\tau-b}\,\overline{\varphi} \sum_{m=0}^{\infty} \left(\frac{\Gamma(2\tau+m+1)}{m!\Gamma(2\tau+1)} \right)^{1/2} \frac{z^m}{(1-|z|^2)^{\tau}} |b+1/2,\tau-1/2,\tau+m-1/2\rangle + \\
& + \sqrt{\tau+b}\,\rho \sum_{m=0}^{\infty} \left(\frac{\Gamma(2\tau+m+1)}{m!\Gamma(2\tau+1)} \right)^{1/2} \frac{z^m}{(1-|z|^2)^{\tau}} |b-1/2,\tau-1/2,\tau+m-1/2\rangle + \\
& + \sqrt{\frac{(\tau^2-b^2)(2\tau+1)}{2\tau}}\,\overline{\varphi}\rho \sum_{m=0}^{m=\infty} \left(\frac{\Gamma(2\tau+m+2)}{m!\Gamma(2\tau+2)} \right)^{1/2} \frac{z^m}{(1-|z|^2)^{\tau}} |b,\tau-1,\tau+m-1\rangle \Bigg]
\end{aligned}
$$