Quantum Representation Theory and Manin matrices

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Supersymmetries and Quantum Symmetries

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Plan

- Introduction
- Quantum groups and quantum analogue of actions
- Quantum linear spaces: quadratic (super-)algebras
- B-Manin matrices
- Quantum representations
- Example: extended super-Yangian.

Theory and applications of Manin matrices

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Quantum (non-commutative) spaces

- Let $\mathbb{K}$ be a filed, e.g. $\mathbb{K} = \mathbb{C}$ or $\mathbb{K} = \mathbb{R}$.
- *(Affine) algebraic set* is a subset $X \subset \mathbb{K}^n$ given by a system of polynomial equations $F_\alpha(x^1, \ldots, x^n) = 0$, $\alpha = 1, \ldots, r$.
- A map $X \to Y$ between two algebraic sets $X \subset \mathbb{K}^n$ and $Y \subset \mathbb{K}^m$ is called *regular* iff it has the form $(x^1, \ldots, x^n) \mapsto \big(P_1(x), \ldots, P_m(x)\big)$ for some polynomials $P_i(x) = P_i(x^1, \ldots, x^n)$.
- The set of regular functions $f \colon X \to \mathbb{K}$ is a commutative algebra denoted by $A(X)$. Any algebraic set X is uniquely given by its algebra of functions $A(X)$ up to isomorphism.
- The regular maps $X \to Y$ are in one-to-one correspondence with the algebra homomorphisms $A(Y) \to A(X)$.
- Quantum (non-commutative) space is given by an arbitrary (non-commutative) algebra of functions.

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Quantum groups

- *Affine algebraic group* is an algebraic set G with structure of group such that the multiplication

$$G \times G \to G, \qquad (g,h) \mapsto gh \tag{1}$$

and the map $G \to G$, $g \mapsto g^{-1}$ are regular.

- Note that $A(G \times G) = A(G) \otimes A(G)$. Define the homomorphisms $\Delta \colon A(G) \to A(G) \otimes A(G)$, $\varepsilon \colon A(G) \to \mathbb{K}$ as

$$\Delta(f)(g,h) = f(gh), \qquad \varepsilon(f) = f(e), \qquad f \in A(G). \tag{2}$$

- They satisfy

$$(\Delta \otimes \mathrm{id})\Delta = (\mathrm{id} \otimes \Delta)\Delta, \qquad (\varepsilon \otimes \mathrm{id})\Delta = (\mathrm{id} \otimes \varepsilon)\Delta = \mathrm{id}\,. \tag{3}$$

- *Hopf algebra* is an algebra H with homomorphisms $\Delta \colon H \to H \otimes H$ and $\varepsilon \colon H \to \mathbb{K}$ satisfying conditions (3) and the existence of *antipode*.
- [**Drinfeld**]: *Quantum group* is given by an arbitrary Hopf algebra.

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Actions of quantum groups on quantum spaces

- Let G be an affine algebraic group and X be an algebraic set. *Action* of G on X is a regular map

$$a\colon G \times X \to X, \qquad\qquad a(g,x) = g.x, \tag{4}$$

such that $g.(h.y) = (g \cdot h).x$, $e.x = x$.

- Let H be a quantum group (Hopf algebra) and R be a quantum space (algebra). The quantum analogue of action is a homomorphism $\delta\colon R \to H \otimes R$ such that

$$(\mathrm{id} \otimes \delta)\delta = (\Delta \otimes \mathrm{id})\delta, \qquad\qquad (\varepsilon \otimes \mathrm{id})\delta = \mathrm{id}\,. \tag{5}$$

- The algebra R equipped with such δ is called *H-comodule algebra*.

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Linear representations of groups

- Let V be (super-)vector space over $\mathbb{K}$. A representation of a group G on V is a homomorphism

$$\rho\colon G \to \mathrm{End}(V), \qquad \rho(gh) = \rho(g)\rho(h), \qquad \rho(e) = \mathrm{id}_V\,. \tag{6}$$

- Let V be finite-dimensional with a basis $e_1, \ldots, e_n$, then

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- *Quadratic algebra* is a graded algebra $\mathcal{A}$ generated by $x^1,\ldots,x^n \in \mathcal{A}_1$ with quadratic commutation relations

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- **Proposition.** For any quadratic algebra the commutation relations can be written in the form

$$\sum_{i,j=1}^{n} B^{kl}_{ij} x^i x^j = 0, \qquad\qquad k,l = 1,\ldots,n, \tag{11}$$

where B^{kl}_{ij} are entries of some $n^2 \times n^2$ idempotent matrix $B = (B^{kl}_{ij})$, i.e.

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- The tensor product of two super-algebras R, S is the super-vector space $R \otimes S$ with the multiplication
  $(r \otimes s)(r' \otimes s') = (-1)^{[s][r']} rr' \otimes ss'$,
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Super-Manin matrices

- The tensor product of a super-algebra R and a graded super-algebra $\mathcal{A} = \bigoplus_{k\geqslant 0} \mathcal{A}_k$ is a graded super-algebra: $R \otimes \mathcal{A} = \bigoplus_{k\geqslant 0} R \otimes \mathcal{A}_k$. In particular, $R \otimes \mathfrak{X}_B(\mathbb{K})$ is a graded super-algebra.
- **Proposition.** Any graded homomorphism $\phi\colon \mathfrak{X}_B(\mathbb{K}) \to R \otimes \mathfrak{X}_B(\mathbb{K})$ has the form $\phi(x^i) = \sum_{j=1}^n M^i_j \otimes x^j$, where $M^i_j \in R$ have the parity $[M^i_j] = [i] + [j] \mod 2$ and satisfy

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  Any such matrix $M = (M^i_j)$ over the super-algebra R define a homomorphism $\phi(x^i) = \sum_{j=1}^n M^i_j \otimes x^j$.
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- A quantum representation of a quantum super-group (Hopf super-algebra) H on a quantum linear space (quadratic super-algebra) $\mathcal{B} = \mathfrak{X}_B(\mathbb{K})$ is defined by a graded homomorphism

$$\delta \colon \mathcal{B} \to H \otimes \mathcal{B}, \qquad \delta_k \colon \mathcal{B}_k \to H \otimes \mathcal{B}_k \tag{16}$$

satisfying $(\mathrm{id} \otimes \delta)\delta = (\Delta \otimes \mathrm{id})\delta$, $(\varepsilon \otimes \mathrm{id})\delta = \mathrm{id}$.

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Example: super-Yangian

- Let $V = V_0 \oplus V_1$ be a super-vector space over $\mathbb{C}$, $m = \dim V_0$, $p = \dim V_1$.
- Consider the *rational* R-matrix $R(z) = z - P_V$, where $P_V : V \otimes V \to V \otimes V$ is the super-permutation operator.
- *Super-Yangian* $Y(\mathfrak{gl}_{m|p})$ is the algebra over $\mathbb{C}$ generated by t^r_{ij}, $i, j = 1, \ldots, n$, $r \in \mathbb{Z}_{\geqslant 1}$, of the parity $[t^r_{ij}] = [i] + [j] \mod 2$ with the commutation relations

$$R(z-w)T^{(1)}(z)T^{(2)}(w) = T^{(2)}(w)T^{(1)}(z)R(z-w), \qquad (18)$$

where $T(z)$ is the $n \times n$ matrix over $Y(\mathfrak{gl}_{m|p})[[z^{-1}]]$ with the entries $T(z)^i_j = \delta^i_j + \sum\limits_{r \geqslant 1} t^r_{ij} z^{-r}$. We use the notations

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Quantum representation of an extended super-Yangian

- Extend the super-Yangian $Y(\mathfrak{gl}_{m|p})$ by a new invertible generator τ as

$$\tau \cdot T(z) = T(z-1) \cdot \tau, \qquad \Delta(\tau) = \tau \otimes \tau, \quad \varepsilon(\tau) = 1. \qquad (19)$$

  We obtain the cocentral extension $Y(\mathfrak{gl}_{m|p})[\tau^{\pm 1}]$.

- Let $\mathbb{K} = \mathbb{C}((z^{-1})) = \left\{ \sum_{k=-\infty}^{N} \alpha_k z^k \mid N \in \mathbb{Z}, \alpha_k \in \mathbb{C} \right\}$.
- The field extension gives the Hopf super-algebra $H = Y(\mathfrak{gl}_{m|p})[\tau^{\pm 1}]((z^{-1}))$ over the field $\mathbb{K}$.
- The product $M = T(z) \cdot \tau$ is a multiplicative A_V-Manin matrix over the Hopf super-algebra H, where $A_V = \frac{1-P_V}{2}$.
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$$\delta \colon \mathcal{B} \to H \otimes_{\mathbb{K}} \mathcal{B}. \qquad (20)$$

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