Locality and Spin-Locality in Higher-Spin Theory

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Quantum Gravity Challenge

QG effects should matter at ultrahigh energies of Planck scale

$$m_P^2 = \frac{hc}{G} \qquad m_P \sim 10^{19} GeV$$

To proceed conjecture that the regime of ultra high (transPlanckian) energies exhibits some high symmetries.

Starting point: spin s free Fronsdal HS gauge fields $\quad \delta\varphi_{n_1\dots n_s} = \partial_{(n_1}\varepsilon_{n_2\dots n_s}$

In 60th it was argued (Weinberg, Coleman-Mandula) that

HS symmetries cannot be realized in a non-trivial local field theory

in Minkowski space For a review: Bekaert, Boulanger, Sundell (2010)

Green light: $(A)dS$ background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ is singular

Non-analiticity in Λ via dimensionless combination $\Lambda^{-\frac{1}{2}}\frac{\partial}{\partial x}$ Fradkin, MV 1987

Background HS gauge fields contribute to higher-derivative terms in the evolution equations: no geodesic motion in presence of HS fields

Space-Time and Spin

Space-time M is where symmetry $G = O(d-1,2)$ acts

Spin s: different G-modules V_s where fields $\phi^A(x)$ are valued.

V_s contain ground (primary) fields $\phi^A(x)$ along with their derivatives $\partial_{n_1} \ldots \partial_{n_k} \phi^A(x)$ (descendants)

HS vertices contain higher derivatives Bengtsson, Bengtsson, Brink (1983), Berends, Burgers and H. Van Dam (1984), (1985), Fradkin, MV; Metsaev,...

HS symmetries Fradkin, MV 1986 are infinite dimensional extesions of G

Infinite towers of spins $\Rightarrow$ infinite towers of derivatives.

How (non)local is HS gauge theory?

Locality and Non-Locality

Equations of motion in perturbatively local field theory $E_{A_0,s_0}(\partial,\phi)=0$

$$E_{A_0,s_0}(\partial,\phi)=\sum_{k=0,l=1}^{\infty} a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0,s_1,\dots,s_l)\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_l}_{s_l}$$

have a finite # of non-zero coefficients $a^{n_1\dots n_k}_{A_0\dots A_l}$ at any order l.

s_0 is the spin of the field on which the linearized equation is imposed

HS theory involves infinite towers of Fronsdal fields of all spins.

$a^{n_1\dots n_k}_{A_0\dots A_l}$ may take an infinite # of values.

It makes sense to distinguish between Gelfond, MV 2018

local: finite number of derivatives at any order

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,\dots s_l)=0 \quad \text{at } k>k_{max}(l)$$

spin-local: finite number of derivatives for any finite subset of fields

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,s_2,\dots s_l)=0 \quad \text{at } k>k_{max}(s_0,s_1,s_2,\dots s_l)$$

non-local: infinite number of derivatives for a finite subset of fields at some order.

Compact Spin-Locality

The simplest option: replacement of the class of local field theories with the finite # of fields by spin-local models with infinite # of fields.

Spin-local-compact vertices in addition obey

$$a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0, s_1, \dots, s_k + t_k, \dots, s_l) = 0 \quad t_k > t_k^0 \quad \forall k$$

non-compact otherwise.

Compactness is in the space of spins, not in space-time

Both types of vertices in HS theory:

Cubic HS vertices $\omega * \omega$ built from HS gauge potentials are spin-local-compact: spins s_0, s_1, s_2 obey the triangle inequalities $s_0 \le s_1 + s_2$ etc.

Vertices associated with the conserved currents built from gauge invariant field strength are spin-local non-compact. These include conserved currents of any integer s_0 built from two spin-zero fields $(s_1 = s_2 = 0)$.

Field Redefinitions

A local theory remains local under perturbatively local field redefinitions

$$\phi^B_{s_0} \to \phi^B_{s_0} + \delta\phi^B_{s_0}\,, \qquad \delta\phi^B_{s_0} = \sum_{k=0,l=1}^{\infty} b^{B\,n_1\dots n_k}{}_{A_1\dots A_l}(s_0, s_1, \dots, s_l)\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_l}_{s_l}$$

with a finite # of non-zero coefficients at any order.

Which field redefinitions leave vertices spin-local?

General spin-local field redefinitions do not work since contributions of all spin s_p redefined fields may develop non-locality

$$\delta E_{A_0,s_0}(\partial,\phi) = \sum_{s_p=0}^{\infty}\sum_{p,k,k'=0,l,l'=1}^{\infty} a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0, s_1, s_2, \dots, s_p, \dots, s_l)$$
$$\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_{p-1}}_{s_{p-1}}\phi^{A_{p+1}}_{s_{p+1}}\dots\phi^{A_l}_{s_l} b^{A_p\,m_1\dots m_{k'}}{}_{B_1\dots B_{l'}}(s_p, t_1, \dots, t_{l'})\partial_{m_1}\dots\partial_{m_k}\phi^{B_1}_{t_1}\dots\phi^{B_{l'}}_{t_{l'}}$$

Spin-local-compact field redefinitions in spin-local theories: proper substitute since summation over s_p is finite.

One of the central problems in HS theory is to find a field frame making it (spin-)local. Given non-locally looking field theory, the essential question is whether or not it is spin-local in some other variables.

HS Multiplets

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2 \ldots$

$\omega_{\alpha_1 \ldots \alpha_n\,, \dot\beta_1 \ldots \dot\beta_m}$ **and** $C_{\alpha_1 \ldots \alpha_n\,, \dot\beta_1 \ldots \dot\beta_m}$ **with all** $n \geq 0$ **and** $m \geq 0$.

Generating functions $\omega(Y|x)$ **and** $C(Y|x)$: **unrestricted functions of commuting spinor variables** $Y = (y_\alpha, \bar y_{\dot\alpha})$

$$A(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} A_{\alpha_1 \ldots \alpha_n\,, \dot\alpha_1 \ldots \dot\alpha_m} y^{\alpha_1} \ldots y^{\alpha_n} \bar y^{\dot\alpha_1} \ldots \bar y^{\dot\alpha_m}$$

Gauge one-forms $\qquad \omega_{\alpha_1 \ldots \alpha_n\,, \dot\beta_1 \ldots \dot\beta_m}\,, \qquad n + m = 2(s-1)$

$s = 1: \quad \omega(x) = dx^{\underline{n}} \omega_{\underline{n}}(x)$

$s = 2: \quad \omega_{\alpha\dot\beta}(x)\,, \qquad \omega_{\alpha\beta}(x)\,, \qquad \bar\omega_{\dot\alpha\dot\beta}(x)$

$s = 3/2: \quad \omega_\alpha(x)\,, \qquad \bar\omega_{\dot\alpha}(x)$

Frame-like fields: $|n-m| = 0$ **(bosons) or** $|n-m| = 1$ **fermions**

Auxiliary Lorentz-like fields: $|n-m| = 2$ **(bosons)**

Extra fields: $|n-m| > 2$ **and zero-forms** $C(Y|x)$: **higher derivatives**

Free Field Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is 1989

$$\star \qquad R_1(y,\overline{y}\mid x)=\frac{i}{4}\Big(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\,C(0,\overline{y}\mid x)+\bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,C(y,0\mid x)\Big)$$

$$\star\star \qquad \tilde{\mathrm{D}}_0 C(y,\overline{y}\mid x)=0$$

$$R_1(y,\bar y\mid x):=D_0^{ad}\omega(y,\bar y\mid x) \qquad D_0^{ad}:=D^L-e^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial\bar y^{\dot\beta}}+\frac{\partial}{\partial y^\alpha}\bar y_{\dot\beta}\Big)$$

$$\tilde{\mathrm{D}}_0=D^L+e^{\alpha\dot\beta}\Big(y_\alpha\bar y_{\dot\beta}+\frac{\partial^2}{\partial\mathbf{y}^\alpha\partial\bar{\mathbf{y}}^{\dot\beta}}\Big) \qquad D^L:=\mathsf{d}_x-\Big(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta}+\bar\omega^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\frac{\partial}{\partial\bar y^{\dot\beta}}\Big)$$

$$H^{\alpha\beta}:=e^{\alpha}{}_{\dot\alpha}e^{\beta\dot\alpha}\,, \qquad \overline{H}^{\dot\alpha\dot\beta}:=e_\alpha{}^{\dot\alpha}e^{\alpha\dot\beta}$$

$\star\star$ **implies that higher-order terms in y and $\bar y$ describe higher-derivative descendants of the primary HS fields**

Zero-Form Sector

Equations on the gauge invariant zero-forms C

$$C(Y;K|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x) y^{\alpha_1}\ldots y^{\alpha_n}\bar y^{\dot\alpha_1}\ldots\bar y^{\dot\alpha_m}$$

decompose into independent subsystems associated with different spins

Spin-s zero-forms are $C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x)$ with

$$n-m=\pm 2s$$

Perturbative unfolded equations

$$\mathrm{d}_x C = \sigma_- C + \text{lower-derivative and nonlinear terms}$$

$$\sigma_- := \mathbf{e}^{\alpha\dot\beta}\frac{\partial^2}{\partial \mathbf{y}^\alpha \partial \bar{\mathbf{y}}^{\dot\beta}}\,, \qquad \sigma_-^2 = 0$$

imply that higher-order terms in y and $\bar y$ in $C(y,\bar y|x)$ describe higher-derivative descendants of the primaries $C(y,0|x)$ and $C(0,\bar y|x)$. Generally, $C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x)$ contain $\frac{n+m}{2}-\{s\}$ space-time derivatives of the spin-s dynamical fields. Presence of zero-forms C in the HS vertices may induce infinite towers of derivatives and, hence, non-locality.

HS Vertices

The problem: consistent non-linear corrections 1988 in the local frame

$$\mathsf{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \ldots\,,$$

$$\mathsf{d}_x C = -[\omega, C]_* + \Upsilon(\omega, C, C) + \ldots$$

The vertices can be put into the form

$$\Upsilon(\Phi,\Phi,\ldots) = F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})\Phi(Y_1)\ldots\Phi(Y_n)|_{Y_i=0}$$

with $\Phi = \omega$, C and some non-polynomial functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ of the Lorentz-covariant combinations

$$Q^i := y^\alpha \frac{\partial}{\partial {y_i}^\alpha}\,, \qquad P^{ij} := \frac{\partial}{\partial y_i^\alpha}\frac{\partial}{\partial y_{j\alpha}}\,, \qquad \bar{Q}^i := \bar{y}^{\dot\alpha}\frac{\partial}{\partial {\bar{y}_i}^{\dot\alpha}}\,, \qquad \bar{P}^{ij} := \frac{\partial}{\partial {\bar{y}_i}^{\dot\alpha}}\frac{\partial}{\partial \bar{y}_{j\dot\alpha}}$$

The fundamental problem: find a proper class of functions $F(Q^i, P^{nm}; \bar{Q}^j,$ guaranteeing spin-locality (minimal non-locality) of the HS theory

Spinor Spin-Locality

Polynomiality of $F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$ in either P^{ij} or $\bar{P}^{ij}$ $\forall i, j$ associated with C

Restriction to the fixed spin relates the degrees in P^{ij} and $\bar{P}^{kl}$ since

$$n - m = \pm 2s$$

Non-linear corrections can affect the relation between spinor and space-time spin-locality making obscure the space-time interpretation of the locality analysis in the spinor space.

This does not happen for projectively-compact spin-local vertices with

$$F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) = Q_\omega G(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) + \bar{Q}_\omega \bar{G}(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$$

Q_ω **and** $\bar{Q}_\omega$ **being associated with the one-forms** ω **among** Φ.

Projectiely-Compact Spin-Local Vertices

Using background frame $e^{\alpha\dot\beta}$ HS equations can be represented as

$$D^L C(y,\bar y) = e^{\alpha\dot\alpha}\Big(\partial_\alpha\bar\partial_{\dot\alpha}F^{++}(y,\bar y)+y_\alpha\bar\partial_{\dot\alpha}F^{-+}(y,\bar y)+\bar y_{\dot\alpha}\partial_\alpha F^{+-}(y,\bar y)+y_\alpha\bar y_{\dot\alpha}F^{--}(y,\bar y)\Big)$$

Generally, nonlinear corrections can contribute to any of $F^{\nu\mu}$.

The contribution to F^{++} can be singled out by the projector

$$\Pi^{des} := N_y^{-1}\bar N_{\bar y}^{-1} y^\alpha \bar y^{\dot\alpha}\frac{\partial}{\partial e^{\alpha\dot\alpha}}\,,\qquad N_y := y^\alpha\partial_\alpha\,,\quad N_{\bar y} := \bar y^{\dot\alpha}\bar\partial_{\dot\alpha}$$

A spin-local vertex Υ is called projectively compact if $\Pi^{des}\Upsilon$ is spin-local-compact. In particular, if $\Pi^{des}\Upsilon = 0$.

The contribution of the projectively-compact spin-local vertices can affect the expressions of the descendants in terms of derivatives of the ground fields only by spin-local-compact terms that preserve space-time locality of the vertex associated with the spin-local spinor vertex.

Projectiely-Compact Spin-Local Vertices in $d_x C$

The $d_x C$ vertex is 2017

$$\Upsilon = \Upsilon_\eta(e,C) + \Upsilon_{\bar\eta}(e,C)$$

$$\Upsilon_\eta(e,C) = \frac{1}{2}\eta \exp\,(i\bar P^{1,2}) \int_0^1 d\tau e(y,(1-\tau)\bar\partial_1 - \tau\bar\partial_2) C(\tau y,\bar y;K) C(-(1-\tau)y,\bar y;K)\,,$$

$$\Upsilon_{\bar\eta}(e,C) = \frac{1}{2}\bar\eta \exp i(P^{1,2}) \int_0^1 d\tau e((1-\tau)p_1 - \tau p_2,\bar y) C(y,\tau\bar y;K) C(y,-(1-\tau)\bar y;K)\,,$$

where $e(a,\bar a) := e^{\alpha\dot\alpha} a_\alpha \bar a_{\dot\alpha}\,.$

Being non-polynomial either in P^{12} or in $\bar P^{12}$, Υ is spin-local

Since Υ contains either $e^{\alpha\dot\alpha} y_\alpha$ or $e^{\alpha\dot\alpha}\bar y_{\dot\alpha}$,

$$\Pi^{des}\Upsilon = 0 \quad \Rightarrow \Upsilon \quad \text{is projectively-compact spin-local}$$

PCSL vertices contain the minimal possible number of derivatives.

One-Form Sector

In the sector of one-forms

$$\omega(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} \omega^{A}_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x) y^{\alpha_1}\ldots y^{\alpha_n}\bar y^{\dot\alpha_1}\ldots\bar y^{\dot\alpha_m}$$

spin-s fields are the degree $s-1$ homogeneous monomials in Y:

$\omega_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x)$ with $n+m=2(s-1)$.

Dynamical HS fields, that contain Fronsdal fields, are those with $n=m$ for bosons and $|n-m|=1$ for fermions. Other components contain

$$\#(\partial_x) = \frac{1}{2}(|n-m| - 2\{s\}) \tag{1}$$

Important consequence: spin-s components of $\omega(Y)$ contain at most $s-1$ derivatives of the spin-s Fronsdal field.

Decendants

Interpretation of the components $\omega_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}$ depends on whether $n>m$ or $n<m$. At $n>m$ every next component with $n>m$ is expressed via the space-time derivatives of the previous one

$$D^L\omega(y,\bar y) - e^{\alpha\dot\beta}\partial_\alpha\bar y_{\dot\beta}\omega(y,\bar y) + \ldots = 0\,, \qquad n\geq m$$

Ellipses denotes the lower-derivative terms as well as the lhs of the Fronsdal equations or Bianchi identities. Analogously, at $m\geq n$

$$D^L\omega(y,\bar y) - e^{\alpha\dot\beta}y_\alpha\bar\partial_{\dot\beta}\omega(y,\bar y) + \ldots = 0\,, \qquad m\geq n$$

These equations can be put into the form

$$D^L\omega(y,\bar y) - e^{\alpha\dot\beta}\Big(P_+\partial_\alpha\bar y_{\dot\beta} + P_- y_\alpha\bar\partial_{\dot\beta}\Big)\omega(y,\bar y) + \ldots = 0$$

with projectors

$$P_+\omega(y,\bar y) = \omega(y,\bar y) \quad n\geq m\,, \qquad\qquad P_+\omega(y,\bar y) = 0 \quad n<m$$

$$P_-\omega(y,\bar y) = \omega(y,\bar y) \quad m\geq n\,, \qquad\qquad P_-\omega(y,\bar y) = 0 \quad m<n$$

Equation Decomposition

Representing $\omega(y,\bar y)$ in the form

$$\omega(y,\bar y) = e^{\alpha\dot\alpha}\Big(\partial_\alpha\bar\partial_{\dot\alpha}\Omega^{++}(y,\bar y)+y_\alpha\bar\partial_{\dot\alpha}\Omega^{-+}(y,\bar y)+\partial_\alpha\bar y_{\dot\alpha}\Omega^{+-}(y,\bar y)+y_\alpha\bar y_{\dot\alpha}\Omega^{--}(y,\bar y)\Big)$$

one can check that

$$e^{\alpha\dot\beta}\bar\partial_{\dot\beta}y_\alpha\omega(y,\bar y) = \frac{1}{2}\Big((N_{\bar y}+2)H^{\alpha\beta}y_\alpha\Big(\partial_\beta\Omega^{-+}(y,\bar y)+y_\beta\Omega^{--}(y,\bar y)\Big)-N_y\bar H^{\dot\alpha\dot\beta}\bar\partial_{\dot\alpha}\bar\partial_{\dot\beta}\Omega^{++}$$

$$e^{\alpha\dot\beta}\partial_\alpha\bar y_{\dot\beta}\omega(y,\bar y) = \frac{1}{2}\Big((N_y+2)\bar H^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\Big(\bar\partial_{\dot\beta}\bar\Omega^{+-}(y,\bar y)+\bar y_{\dot\beta}\bar\Omega^{--}(y,\bar y)\Big)-N_{\bar y}H^{\alpha\beta}\partial_\alpha\partial_\beta\bar\Omega^{++}$$

Suppose that the vertex is PCSL i.e., restriction of the non-linear corrections to the HS equations to the projected terms $e^{\alpha\dot\beta}\Big(P_+\partial_\alpha\bar y_{\dot\beta}+P_-y_\alpha\bar\partial_{\dot\beta}\Big)\omega(y,\bar y)$ is spin-local-compact. Then expressions for the components of $\omega(y,\bar y)$ associated with higher space-time derivatives of the Fronsdal fields will differ from those in the free theory by spin-local-compact terms that do not spoil space-time spin-locality.

Projectively-Compact Spin-Local Vertices in $\mathrm{d}_x\omega$

The vertices of the form

$$P_+\Big((\bar N+2)H^{\alpha\beta}(y_\alpha\partial_\beta\Omega^{+-}(y,\bar y)+y_\alpha y_\beta\Omega^{++}(y,\bar y))-N\bar H^{\dot\alpha\dot\beta}\bar\partial_{\dot\alpha}\bar\partial_{\dot\beta}\Omega^{--}(y,\bar y)\Big)$$

$$P_-\Big((N+2)\bar H^{\dot\alpha\dot\beta}(\bar y_{\dot\alpha}\bar\partial_{\dot\beta}\bar\Omega^{-+}(y,\bar y)+\bar y_{\dot\alpha}\bar y_{\dot\beta}\bar\Omega^{++}(y,\bar y))-\bar N H^{\alpha\beta}\partial_\alpha\partial_\beta\bar\Omega^{--}(y,\bar y)\Big)$$

do not affect the expressions for the one-form descendant fields via derivatives of the primaries hence being projectively-compact. In that case, spinor spin-locality of the next-order vertex implies its space-time spin-locality.

Remarkably, the cubic vertices found in Gelfond, MV 2017 do indeed have such a form. Moreover, they only contain the $y,\bar y$-independent terms with non-zero Ω^{--} or $\bar\Omega^{--}$.

This implies that they have the minimal number of derivatives.

Holographic Higher Spins

Klebanov-Polyakov conjecture: HS theory in AdS_4 is holographically dual to $3d$ vector model of scalar fields ϕ^i $(i = 1 \dots N)$.

Sleight and Taronna argued 2017 that a HS theory resulting from holographic analysis based on the is essentially non-local

Since HS holography is a weak-weak duality, it should be possible to test it.

No locality analysis of the full HS theory in AdS_4 has been done except for that of the Lebedev group Didenko, Gelfond, Korybut, MV 2017-2022

What has been shown so far indicates that HS theory is spin-local?!

Suggests gauged version of the KP conjecture with conformal HS boundary theory MV 2012

Conclusion

Concepts of compact and projectively compact vertices are introduced. These apply to various versions of HS theories.

For projectively-compact vertices spin-locality in the spinor space and space-time are equivalent.

PCSL vertices are conjectured to form a proper class of solutions of the non-linear HS equations that guarantee spin-locality of the HS theory at higher orders.

The new approach is designed to figure out the actual level of (potential) non-locality of the HS theory.

The analysis of HS gauge theory has a potential to affect the paradigm of the holographic corresondence replacing the gauge-gravity correspondence by the conformal gravity - gravity correspondence.