Valentin Kuzmin

MPD TPC alignment

- TPC alignment (theoretical basis for the MPD case)
- Mini Monte-Carlo for TPC alignment task
- Alignment study on cosmic rays & TPC laser system
- Conclusion&Outlook

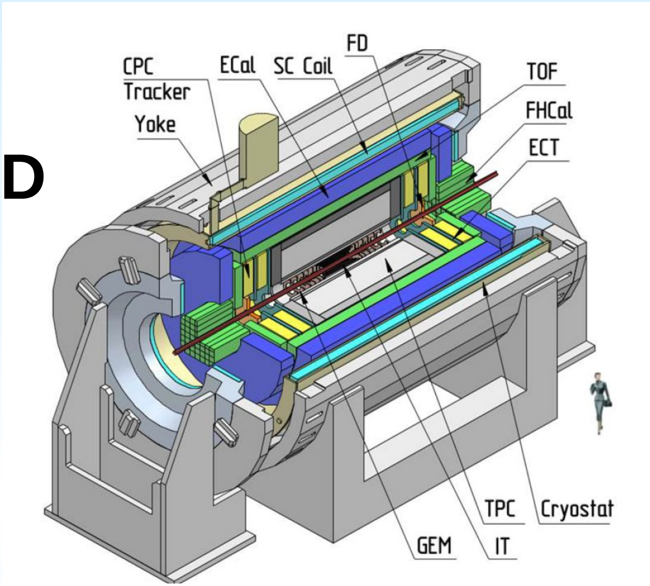
Many physical conclusions in high energy physics experiments are made by comparing the simulated data of a theoretical model with the experimental distributions. Tracking provides basic information about charged particles.

In order to have high resolution and unbiased tracking, the detector elements must be correctly aligned. Partly this can be achieved by optical survey, and for example laser alignment systems, to track short-term movements. TPC laser system is created mainly promptly to update the parameters of the gas system. But we will test a possibility of using it to align the detector.

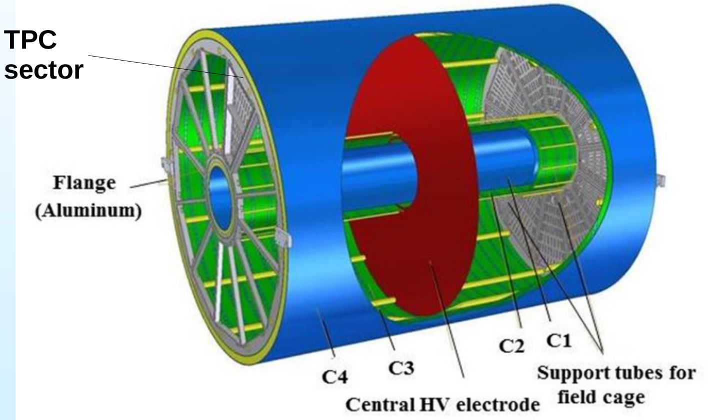
The ultimate alignment precision, however, is achieved by using the fitted tracks themselves.

TPC alignment

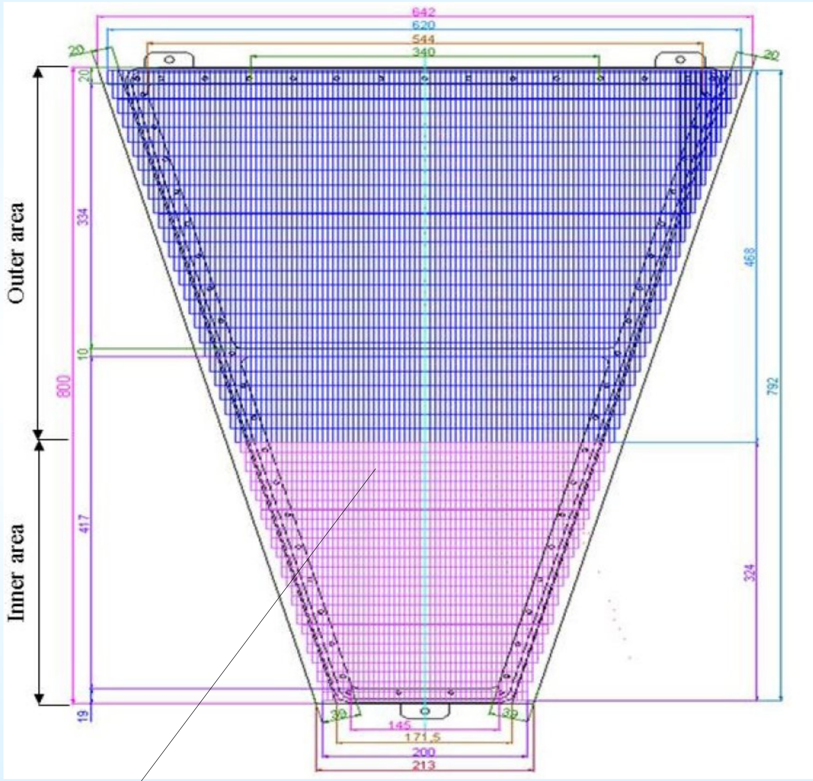
MPD



TPC



TPC sector



4074 sensitive elements that fix the projection of the track on the sector

TPC alignment

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The TPC sensing elements, pads, are located in sectors. The sector position as a solid body relative to the Global Coordinate System (GCS) of the detector is determined by 6 parameters.
They are
3 parameters of the position of the starting point of the sector local Coordinate System (LCS)

p_1, p_2, p_3

and 3 Euler angles determine the rotation of the axes of the LCS relative to the GCS *roll(alpha)* - p_4, *yaw(beta)* - p_5, *pitch(gamma)* - p_6**:**

$\boldsymbol{p}(p_1, p_2, p_3, p_4, p_5, p_6)$

These parameters are called global parameters in the alignment task.

In the case of a TPC where all pads are hard-wired, the total number of global parameters to be defined is equal to

6x12x2=144

TPC alignment

To reconstruct a track, its model is used, usually it is helix (cylinder detector geometry) or a straight line (lack of magnetic field).

Each model is defined by a set of parameters called **local parameters**

helix

$\boldsymbol{q}(q_1, q_2, q_3, q_4, q_5, q_6)$

$$\left(\begin{array}{c} x = q_1 + q_4 \cos(q_5 + t) \\ y = q_2 + q_4 \sin(q_5 + t) \\ z = q_3 + q_6 t \end{array}\right.$$

line

$\boldsymbol{q}(q_1, q_2, q_3, q_4, q_5)$

$$\left(\begin{array}{c} x = q_1 + \sin(q_4)\cos(q_5) t \\ y = q_2 + \sin(q_4)\sin(q_5) t \\ z = q_3 + \cos(q_4) t \end{array}\right.$$

TPC alignment

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How to find all 144 parameters p which determine the positions of 24 sectors from experimental data?

Consider the distance between a single measurement of a track point (hit)

$h_i(p^k)$ (i — number of the track point, k – number of the sector)

and the curve, set by the track model

$$r = T(q),$$

where q is the vector of track parameters and p^k are 6 global parameters that determine the position of the sector k.

let's build the sum:

$$\chi^2 = F(\overline{q}, \overline{p}) = \sum_{events}^{all} \sum_{i}^{track} \frac{\left(\overline{r}_i(\overline{p}_k) - T_i(\overline{q})\right)^2}{\sigma^2}$$

TPC alignment

It can be shown that the value of *F(**p**,**q**)* **for any incorrect set of global parameters** ***p*** **will be greater than when using the correct values.**
To find the alignment of the detector, we need to find the minimum of the function *F(**p**,**q**)* **or the point where**

$$dF(\boldsymbol{p},\boldsymbol{q}) = 0$$

If $\boldsymbol{p}_0$ **is the current alignment that needs to be improved and** $\boldsymbol{q}_0$ **are found track parameters, it is more convenient(faster) to look for the increments** $\Delta\boldsymbol{p}$**, which are the solution of the system of** $144+n_q$ **equations:**

$$\begin{cases} \dfrac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}} + \dfrac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}^2}\triangle\boldsymbol{q} + \dfrac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}\partial\boldsymbol{p}}\triangle\boldsymbol{p} = 0 \\ \dfrac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{p}} + \dfrac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}\partial\boldsymbol{p}}\triangle\boldsymbol{q} + \dfrac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{p}^2}\triangle\boldsymbol{p} = 0 \end{cases}$$

This system is approximate, with an accuracy of first-order terms in increments, so a solution with a given accuracy may require consecutive iterations.

The principal difficulty is to estimate the real accuracy of the obtained solution depending on the size and quality of the experimental data used.

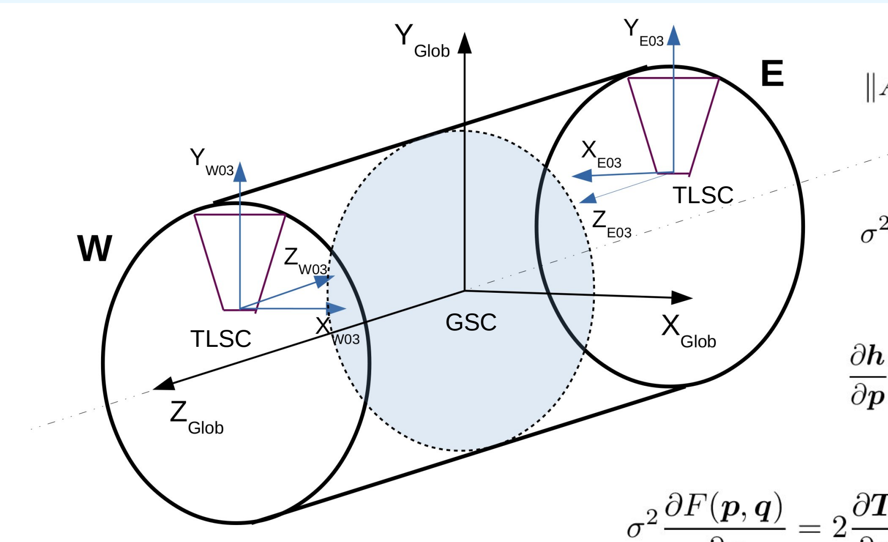
TPC alignment

MPD Global Coordinate System (GCS)
Theoretical Local Coordinate System of the sector (TLCS)
Real Local Coordinate System of the sector (LCS) (not shown)

$$\boldsymbol{X_g} = \boldsymbol{S_i^{tl}} + \left\|T_i^{-1}\right\| \boldsymbol{X_{tl}}$$

$$\boldsymbol{X_{tl}} = \boldsymbol{S_i^{A}} + \left\|A_i^{-1}\right\| \boldsymbol{X_{l}}$$

$$\left\|R_i^{-1}\right\| = \left\|T_i^{-1}\right\| \left\|A_i^{-1}\right\|$$



$$\|A_i\| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma_i) & \sin(\gamma_i) \\ 0 & -\sin(\gamma_i) & \cos(\gamma_i) \end{pmatrix} \times \begin{pmatrix} \cos(\beta_i) & 0 & -\sin(\beta_i) \\ 0 & 1 & 0 \\ \sin(\beta_i) & 0 & \cos(\beta_i) \end{pmatrix} \times \begin{pmatrix} \cos(\alpha_i) & \sin(\alpha_i) & 0 \\ -\sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sigma^2 \frac{\partial F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{p}} = \frac{\partial(\boldsymbol{h}(\boldsymbol{p}) - \boldsymbol{T}(\boldsymbol{q}))^2}{\partial \boldsymbol{p}} = 2\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}}(\boldsymbol{h} - \boldsymbol{T})$$

$$\sigma^2 \frac{\partial^2 F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{p}^2} = 2\left(\frac{\partial^2 \boldsymbol{h}}{\partial \boldsymbol{p}^2}(\boldsymbol{h} - \boldsymbol{T}) + \left(\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}}\right)^2\right)$$

$$\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}} = \left\|T^{-1}\right\| \frac{\partial}{\partial \boldsymbol{p}} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} + \frac{\partial \left\|R^{-1}\right\|}{\partial \boldsymbol{p}} \boldsymbol{h_s} = \begin{pmatrix} T_{11}^{-1} & T_{12}^{-1} & T_{13}^{-1} & \sum_j \frac{\partial R_{1j}^{-1}}{\partial p_4} h_{s_j} & \sum_j \frac{\partial R_{1j}^{-1}}{\partial p_5} h_{s_j} & \sum_j \frac{\partial R_{1j}^{-1}}{\partial p_6} h_{s_j} \\ T_{21}^{-1} & T_{22}^{-1} & T_{23}^{-1} & \sum_j \frac{\partial R_{2j}^{-1}}{\partial p_4} h_{s_j} & \sum_j \frac{\partial R_{2j}^{-1}}{\partial p_5} h_{s_j} & \sum_j \frac{\partial R_{2j}^{-1}}{\partial p_6} h_{s_j} \\ T_{31}^{-1} & T_{32}^{-1} & T_{33}^{-1} & \sum_j \frac{\partial R_{3j}^{-1}}{\partial p_4} h_{s_j} & \sum_j \frac{\partial R_{3j}^{-1}}{\partial p_5} h_{s_j} & \sum_j \frac{\partial R_{3j}^{-1}}{\partial p_6} h_{s_j} \end{pmatrix}$$

$$\sigma^2 \frac{\partial F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{q}} = 2\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{q}}(\boldsymbol{T} - \boldsymbol{h})$$

$$\frac{\partial^2 F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{q}^2} = 2\left(\frac{\partial^2 \boldsymbol{T}}{\partial \boldsymbol{q}^2}(\boldsymbol{T} - \boldsymbol{h}) + \frac{\partial \boldsymbol{T}}{\partial q_i}\frac{\partial \boldsymbol{T}}{\partial q_j}\right)$$

$$\sigma^2 \frac{\partial^2 F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{q} \partial \boldsymbol{p}} = -2\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{q}}\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}}$$

$$\frac{\partial \boldsymbol{T}(\boldsymbol{q})}{\partial \boldsymbol{q}} = \begin{pmatrix} 1 & 0 & 0 & \cos(q_4)\cos(q_5)t & -\sin(q_4)\sin(q_5)t \\ 0 & 1 & 0 & \cos(q_4)\sin(q_5)t & \sin(q_4)\cos(q_5)t \\ 0 & 0 & 1 & -\sin(q_4)t & 0 \end{pmatrix}$$

A detailed description of the theoretical basis of the adjustment task for MPD TPC is here: ***http://mpdroot-forum.jinr.ru/download/file.php?id=1***

Millipede-II and alignment equations system

MSU

There is the code Millipede-II (Software alignment for Tracking Detectors,
V. Blobel, NIM A, 566 (2006), pp. 5-13, doi:10.1016/j.nima.2006.05.157)
designed specifically for the alignment task of detectors with a large number of sensors.

The description of this code says that it was created specifically for solving systems of linear equations of the type:

$$\begin{cases} \frac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}} + \frac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}^2}\triangle \boldsymbol{q} + \frac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}\partial \boldsymbol{p}}\triangle \boldsymbol{p} = 0 \\ \frac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{p}} + \frac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}\partial \boldsymbol{p}}\triangle \boldsymbol{q} + \frac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{p}^2}\triangle \boldsymbol{p} = 0 \end{cases}$$

Its main feature is working with the number of equations up to tens of thousands.
However:
1. The program is written in Fortran 90 and works in single precision mode with so many equations.
2. The code uses only first-order derivatives *F*(***p***,***q***) as input parameters. Based on such initial data, we do not know where does the code get needed second order derivatives from?!
3. Millipede-II does not want to work with reconstructed MPD TPC tracks, not accepting 95% of the tracks, and declares "matrix rank insufficient".

Therefore, the above system of linear equations for MPD TPC was solved by the usual Gauss method, which confirms the non-degeneracy of the system by its non-zero determinant.

Mini Monte-Carlo for TPC

To study the accuracy of measuring the alignment of the device, we need artificial events with the simulation of the reaction of sensitive elements (pads) in the TPC sectors. Currently, such modeling for MPD TPC is not ready in full (with the final reconstruction of the tracks).

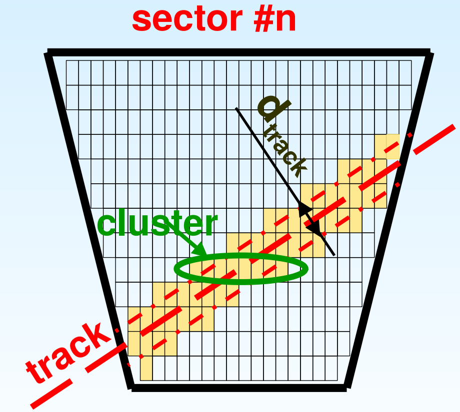
For the problem of alignment, a simplified Monte Carlo simulation of cluster formation in the sectors was developed and the hits from the tracks of charged particles were reconstructed in the MPD space.

According to the projection of the track on the plane of the sector, the pads which are covered by it are determined, and the signal values in them are played. In each row of pads there are clusters of neighboring pads with signals, the time of arrival of the signal is played. The "hit" of the track is restored in the GCS of MPD.

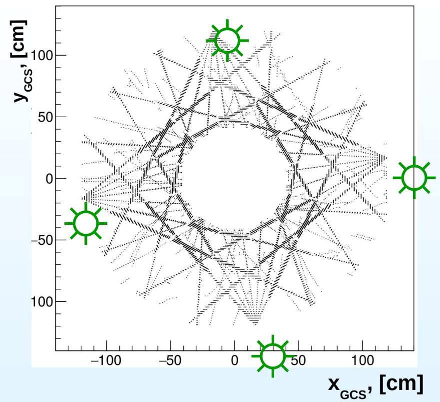
Each set of hits was reconstructed, i.e., the local parameters ***q*** were found and they were used to calculate χ^2 and the function $F(\mathbf{p},\mathbf{q})$ with its derivatives.

Simulated variants:

1. Cosmic rays without a magnetic field in the detector.
2. The beams of the TPC laser system. These rays in each chamber lie in 4 planes perpendicular to the Z-axis of the camera and are arranged so that they pass through all sectors.



Sources of 7 laser rays



On 4 planes perpendicular to Z inside the TPC camera from 4 points, there are 7 rays, the projections of which intersect all sectors.

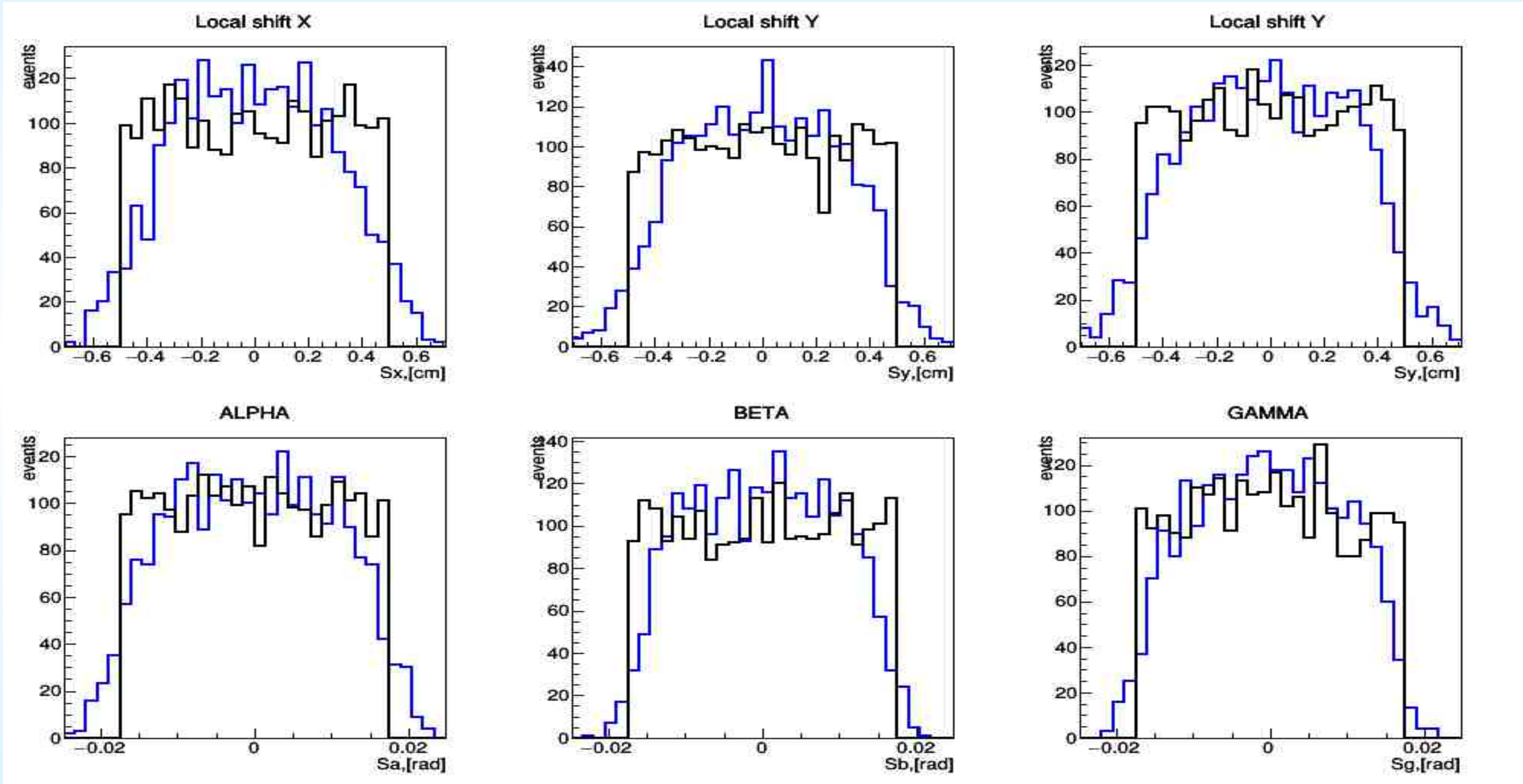
χ^2 track simulation in TPC

The model experiment consists of a random simulating of distortions of the position & Euler angles of sectors.

The uniform distribution was used as the input (BLACK histograms).
The BLUE color shows the distribution of the recovered values for cosmic rays samples.
The difference at the ends of the intervals is due to the accuracy of parameter recovery.

200 such experiments were conducted for sets of 10,000 tracks of cosmic rays and the TPC laser system.

Alignment global variables



χ^2 track simulation in TPC

MSU

Function *F(p,q)* (χ^2)

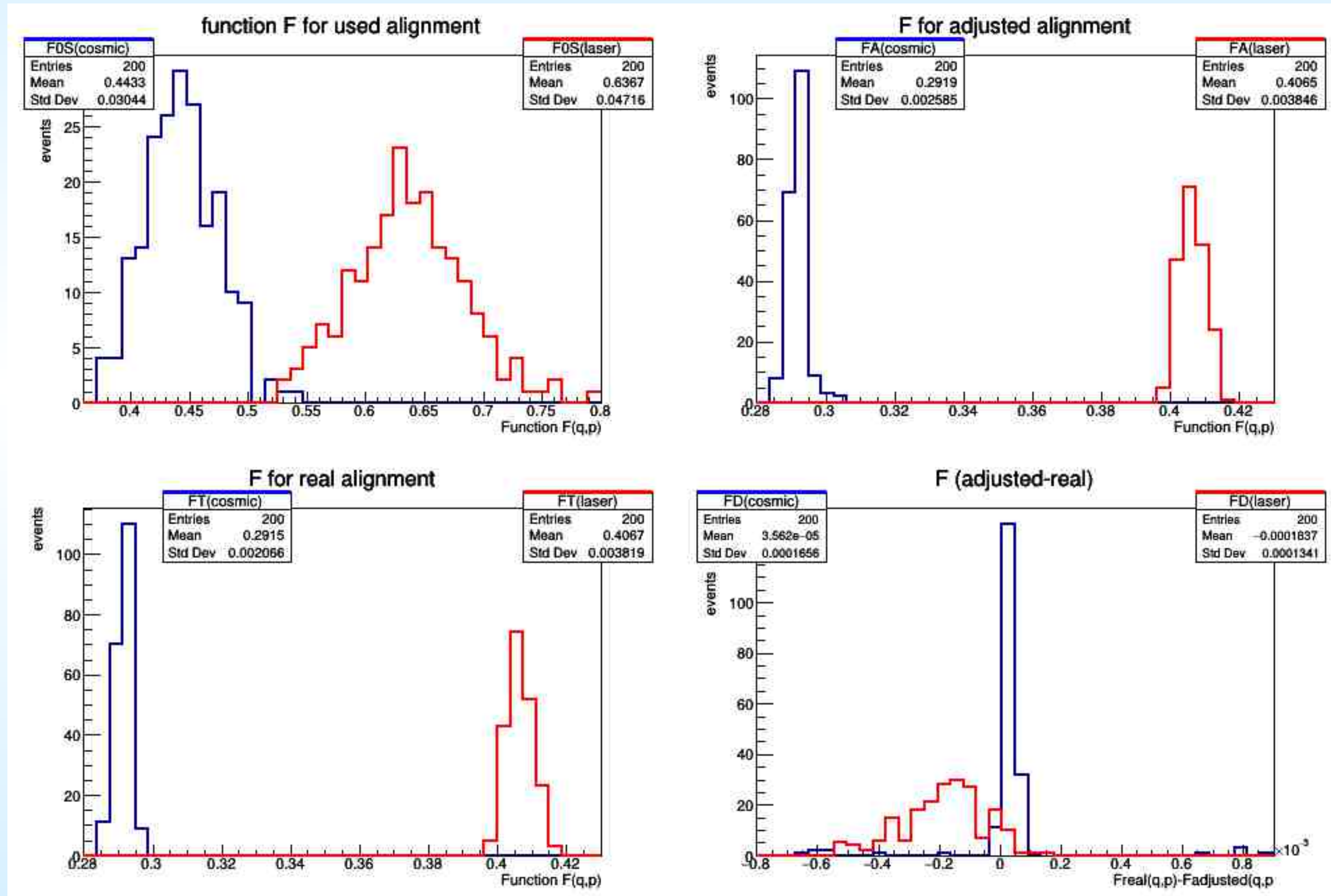
BLUE – cosmic rays
RED – laser system rays

"F for used alignment" are results of random alignment when for tracks reconstruction theoretical “zero” alignment was used.

"F for adjusted alignment" are final results of finding the alignment, i.e. after minimizing the function F.

"F for real alignment" are simulated (not recovered) values of the function F.

"F(adjucted-real)" is the difference between F-values of found and simulated alignments.



Alignment accuracy

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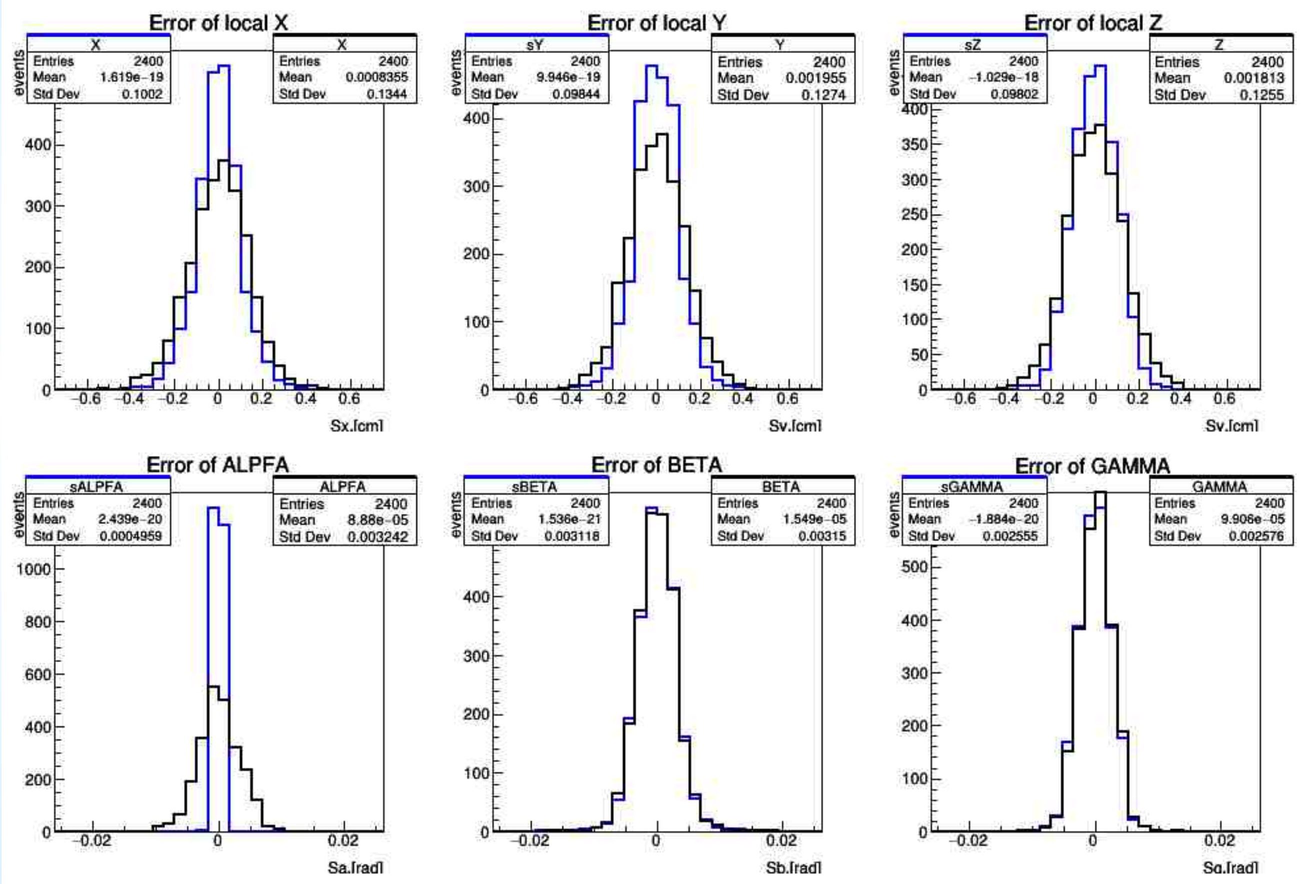
The reconstruction task is defined with accuracy up to the simultaneous transfer of all local coordinate systems of sectors to the same vector or simultaneous rotation of all sectors as a whole around the global Z axis.
In both cases, the minimum value of the function F does not change. We need a fulcrum.

To fix the shift, which can be accumulated during minimization, next condition was imposed when minimizing F:

the average value of the sum of each local shift and each average Euler angle over all sectors is constant.
It is our Archimedes' fulcrum.

The nature of this condition lies in the equations for the function F, which do not change when the variables are shifted to a constant value.
The results with applying this condition are shown in blue, without it in black.

(simulated - adjusted) alignment parameters



cosmic rays case

Alignment accuracy

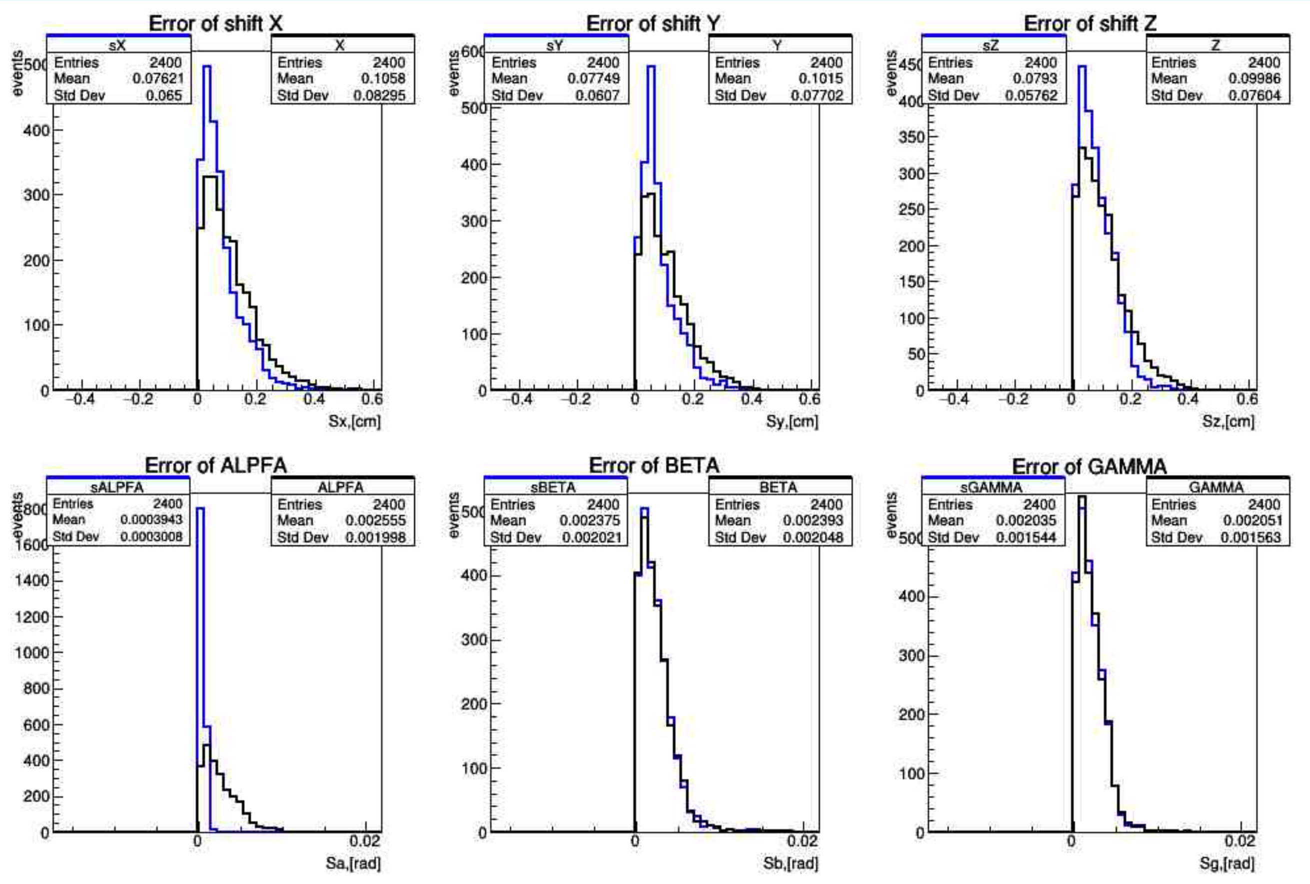
The results with applying the condition on average shifts of global parameters are shown in BLUE, without it in BLACK.

BLUE histograms for local coordinates shifts along their axises show 200-300 microns smaller error. It’s in average over all 200 M-C experiments.

In a separate experiment the shift can be as positive as negative and its value will be directly add to the “black” values.

For the best final alignment we need to know the position at list one sector. If we don’t have it we should subtract from the found adjusted global parameters their average values to minimize absolute errors of the global alignment parameters.

abs(simulated - adjusted) alignment parameters



cosmic rays case

Alignment accuracy

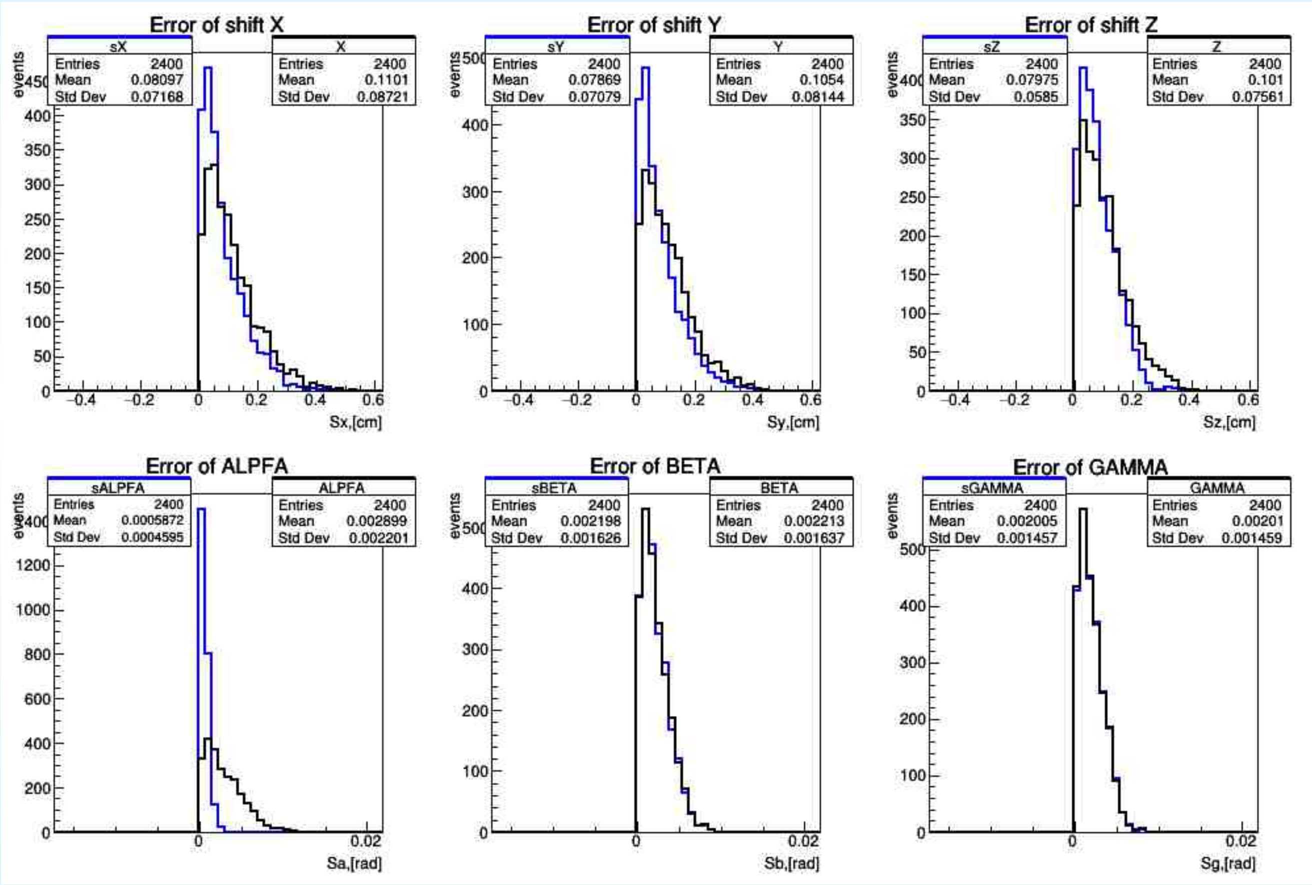
MSU

abs(simulated - adjusted) alignment parameters

The accuracy of recovered alignment parameters in the case of laser system rays is practically the same as for cosmic muons.

In both cases we observe big difference between blue and black histograms for the ALPHA angle. It is due to fine sensitivity of the minimized function F to the value of the average rotation around local Z-axises of sectors because we turn all sectors simultaneously by the same angle around different rotation centers.

Runs with laser system will be taken much more often then cosmic rays runs. It allows to monitor the MPD TPC alignment permanently.



MPD TPC laser system rays case

Alignment accuracy

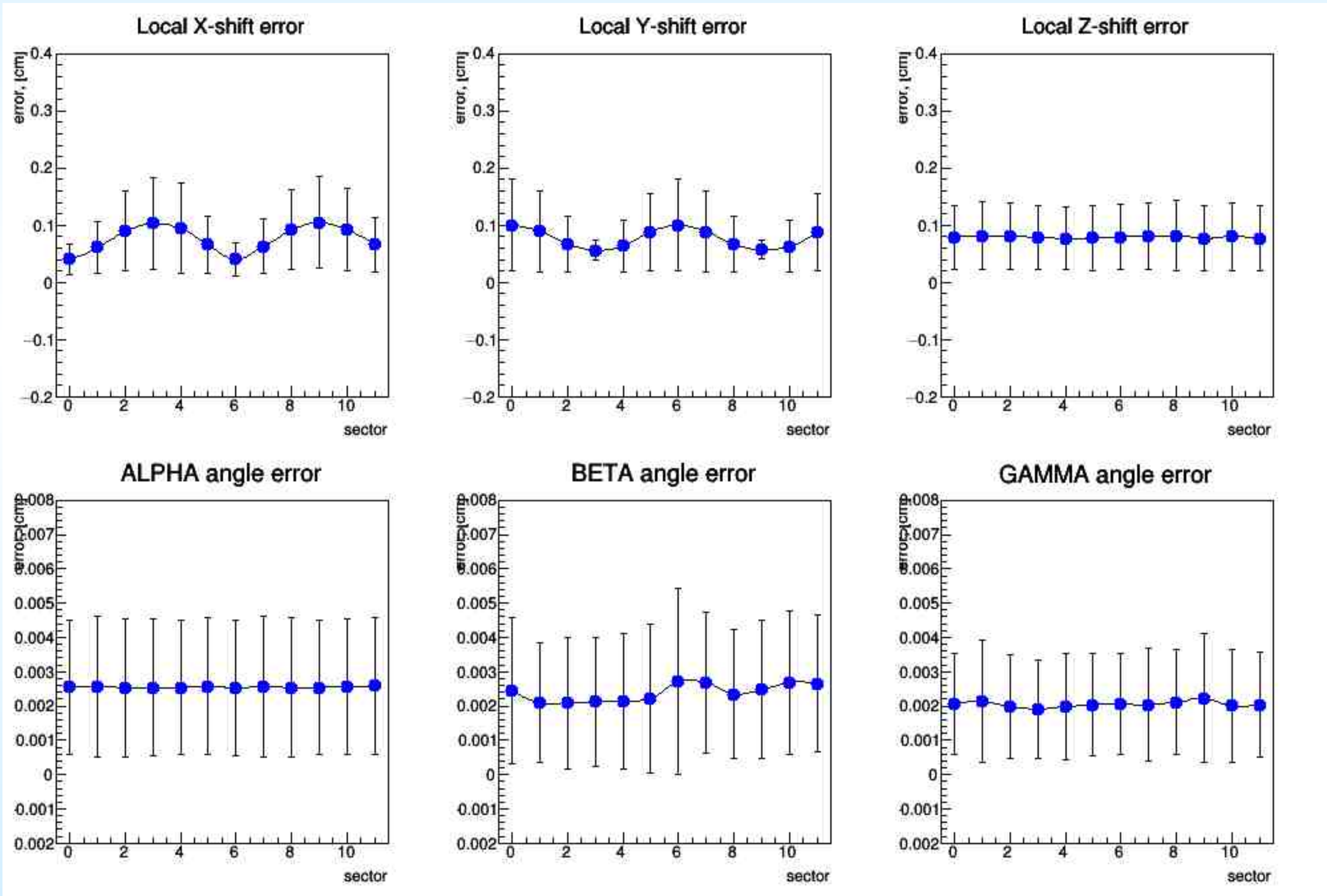
(simulated - adjusted) alignment parameters

cosmic rays case

The dependence of the accuracy of recovered alignment parameters on the sector number.

Two sinusoids are clearly visible on plots for X and Y. Their origin is due to the proximity of the direction of cosmic rays to the vertical.
The bases of sectors 3 and 9 are horizontal, and the Y axes are vertical, i.e. cosmic muons intersect the sectors in a narrow interval X, that explains the maximum error in this coordinate here.
For sectors 0 and 6 rotated 90 degrees relative to the previous ones, the picture is reversed.

Errors on Y in the opposite phase to errors on X.



Conclusions & Outlook

MSU

- A theoretical basis has been created for determining the MPD TPC alignment using its experimental data.
- In the framework of the mpdroot computing system, to study the accuracy characteristics of the TPC alignment, simplified MC simulations of the response of the TPC sensitive elements from tracks of various thicknesses and the reconstruction of tracks are created.
- Within the framework of mpdroot, methods have been created to find the alignment of the TPC, which have been applied to the tracks of cosmic rays and the rays of the TPC laser system.
- It is shown that it is possible to use the TPC laser system for permanent TPC alignment monitoring which was originally designed only to monitor the properties of the gas.
- The accuracy of finding global alignment parameters by cosmic rays and laser is approximately the same. For the values of shifts in the position of the local coordinates of the sectors, this is a value of the order of 700 microns, and for the corresponding Euler angles, the accuracy is 0.002 rad or 7’.
- The alignment method can one in one be applied to other tracking detectors at the condition: the detector consists of separate solid-state parts with hard-attached sensitive elements, for example, to a silicon vertex detector.

- Simulation of TPC alignment by muons in a magnetic field and estimation of the sectors alignment accuracy.
- Integration of the final official reconstruction of tracks into the algorithm for finding the TPC alignment.
- Cooperation with other detectors to solve the problem of global MPD alignment.
- Estimation of the amount of distortions introduced by incorrect alignment in the distribution of physical quantities (e.g. the shape and width of the J/ψ mass).

Acknowledgments

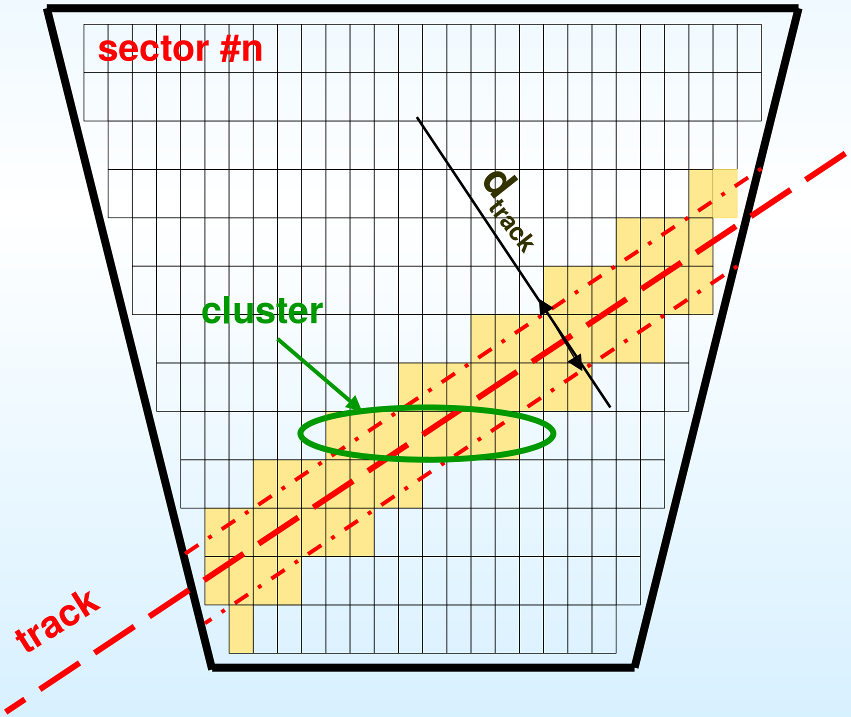
We thank Prof. E.Boos (the director of the IINR of the Moscow State University) for an informal support of this interesting job.
We thank Dr. O.Rogachevskiy (JINR Dubna) for creating conditions for a discussion of the problem both inside and outside the MPD community (http://mpdroot-forum.jinr.ru)

Thank you!

BackUp

Mini Monte-Carlo for TPC

Conditions of simulation



- A charged particle leaves a trace in the TPC in the form of a tube of ionized gas
- This trace along the electric field is projected onto the plane of the sector pads, leaving a trail with a width d_{track}
- The d_{track} value corresponds to the width of the electron avalanche near the pad surface
- Along each i-th row of the sector for each k-th pad is the area s_{ik} in i-th pad covered by the d_{track} band.
- s_{ik} defines the pad signal that is played by Gaussian

Adjustment algorithm

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A DST is created for a set of reconstructed tracks, which contains the hits of the tracks and and their restored parameters q. The reconstruction was made using the initial alignment P. For all tracks, the value Fo of the function F(p,q) is calculated and a small number of ε is selected.

⇩

B

The coefficients of the system of equations are calculated. The solution of the system is found. Corrections are made to the alignment: P=+ΔP.

⇩

All track hits are recalculated to the current alignment P.

⇩

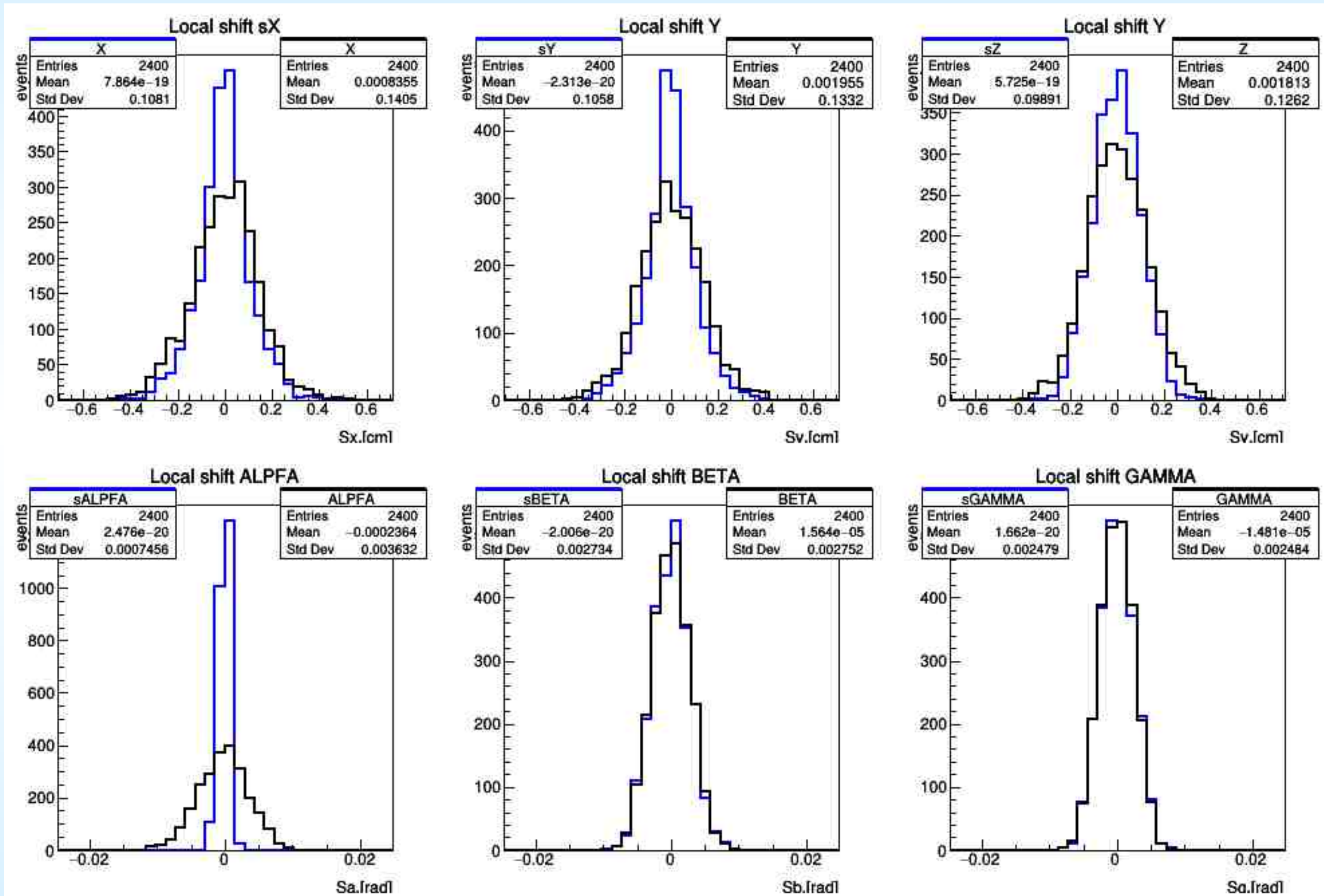
The value F of the function F(p,q) is calculated.

⇩

If |F-Fo|>ε, then F0=F and perform step B, otherwise stop

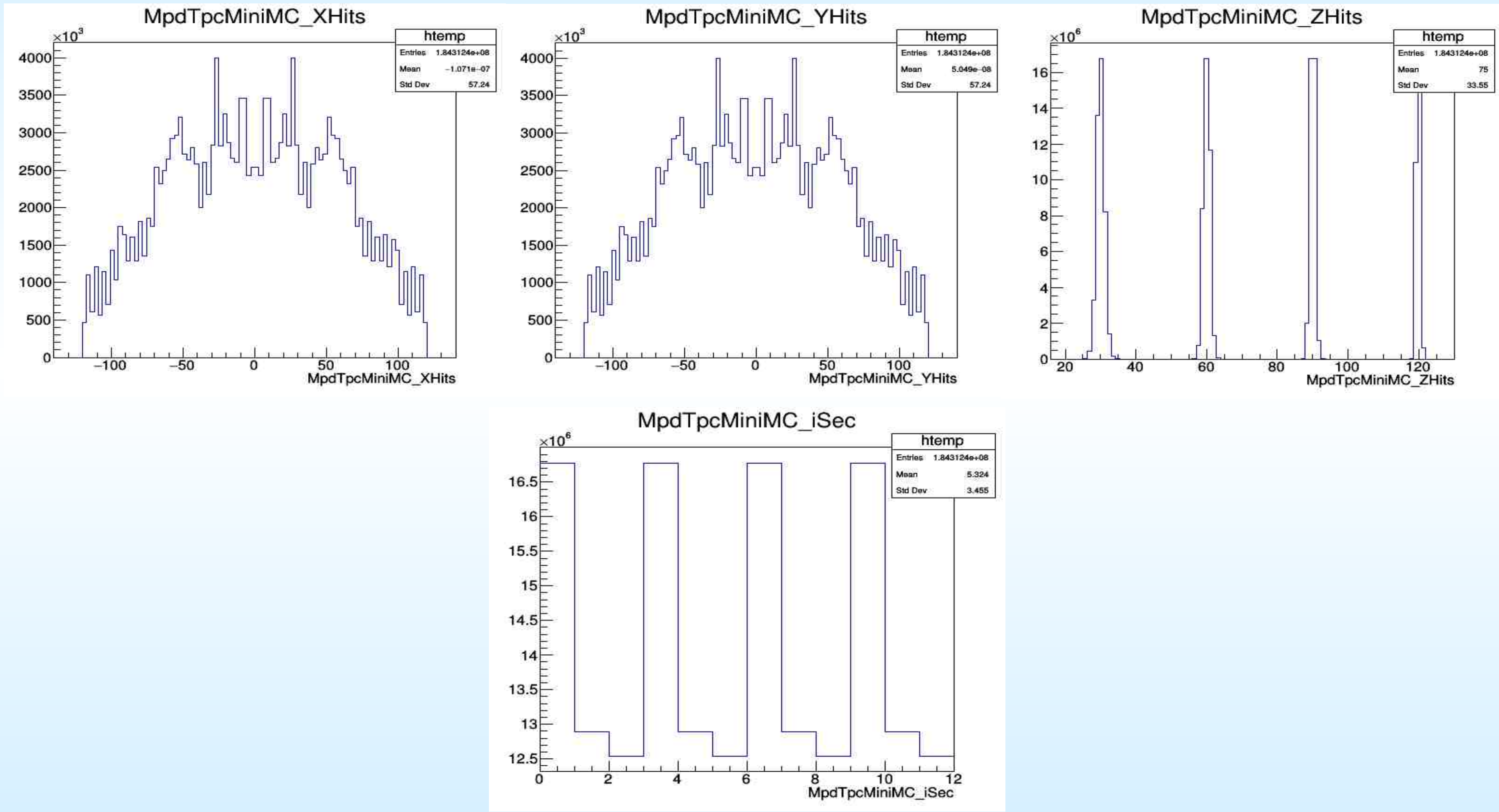
Alignment accuracy

(simulated - adjucted) alignment parameters



TPC laser system rays case

Simulation of TPC laser beams



class MpdTpcSectorGeo

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The class MpdTpcSectorGeo describes the geometry of the pads and their location relative to the local coordinate system.
The methods of the class provide the transformation of coordinates LCS->GCS and vice versa. This is the main tool when reconstructing tracks.

By default, the LCS is the same as the TLCS. The class methods allow you to set for a sector any position of its LCS relative to the TLCS.