Vector interaction enhanced bag model.

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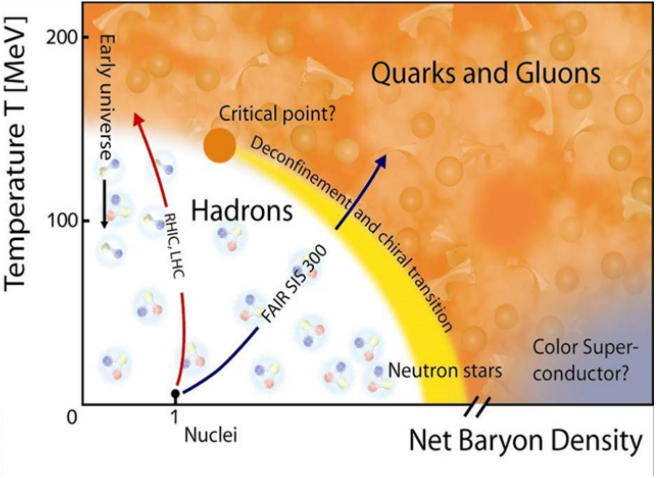
28th September, 2017

Overview

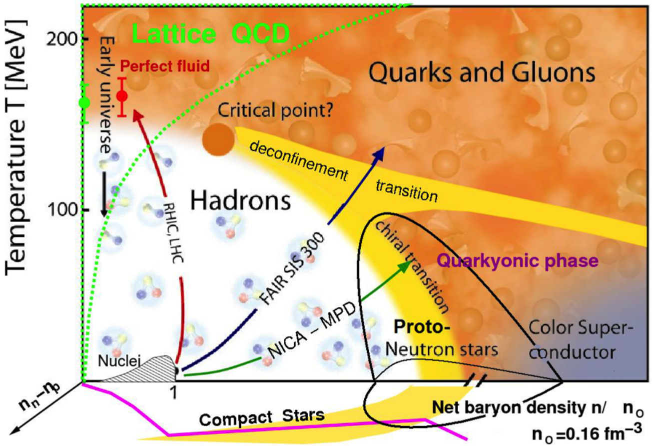
Motivation

QCD phase diagram



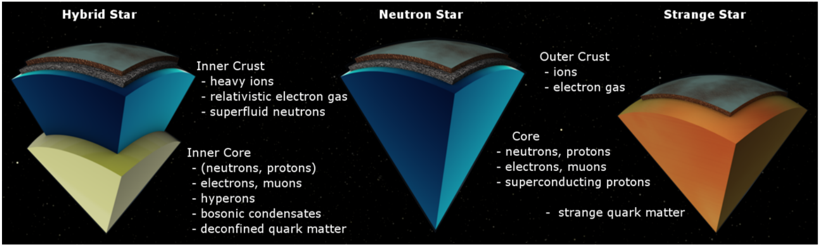
[1] Image retrieved from `http://www.gsi.de`

QCD phase diagram



[2]Image retrieved from http://theor0.jinr.ru/twiki-cgi/view/NICA.

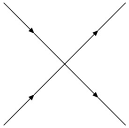
Why?



[3] Image courtesy of Thomas Klähn

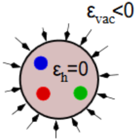
Solutions - Effective models

Nambu–Jona-Lasino model



- Assumes only contact interactions between quarks
- Exhibits DCSB but no confinement

Bag model[4]



- Designed to mimic confinement
- Assumes constant quark masses

Both models are inspired by, but not originate from QCD!

[4]Image retrieved from `https://arxiv.org/pdf/0811.2024.pdf`

The Dyson–Schwinger formalism

The basic concept $\int_a^b \frac{d}{dz} f(z) dz = 0$
This can be applied to the QCD generating functional

$$Z = \int [D\Phi] e^{iS + i \int d^4x (J_a^\mu G_\mu^a + \bar{\eta}\psi + \eta\bar{\psi})}$$

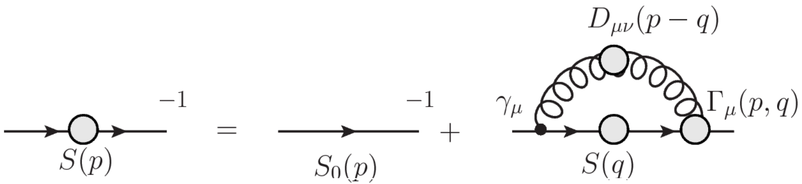
giving us

$$\frac{dZ}{d\eta} = 0$$

which is helpful because

$$G^{(N)}(x_1, ..., x_N) = \frac{(-i)^N}{Z[0]} \frac{\partial^N Z[J]}{\partial J(x_1) ... \partial J(x_N)}\bigg|_{J=0}$$

The Quark Dyson–Schwinger equation



One particle gap equation in–medium

$$S^{-1}(p,\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p,\mu)$$

with self–energy term

$$\Sigma(p,\mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma^\rho \frac{\lambda^\alpha}{2} S(q)\Gamma^\sigma_\alpha(p,q)$$

General form of the propagator:

$$S^{-1}(p,\mu) = i\bar{\gamma}\bar{p}A(p,\mu) + i\gamma_4\tilde{p}_4 C(p,\mu) + B(p,\mu)$$

DSE results:

$$\begin{cases} A(p,\mu) = 1 \\ B(p,\mu) = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B(q,\mu)}{\vec{q}^2A^2(q,\mu)+\tilde{q}_4^2C^2(q,\mu)+B^2(q,\mu)} \\ \tilde{p}_4^2 C(p,\mu) = \tilde{p}_4 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\tilde{p}_4\tilde{q}_4 C(q,\mu)}{\vec{q}^2A^2(q,\mu)+\tilde{q}_4^2C^2(q,\mu)+B^2(q,\mu)} \end{cases}$$

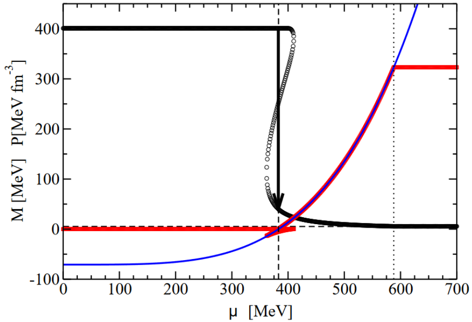
Condensates:

$$\phi = B - m = \frac{4N_c}{9m_G^2} n_s(\mu^*, B)$$

$$\tilde{p}_4 C = p_4 + i\omega = p_4 + i\mu^* => \omega = \mu^* - \mu = \frac{2N_c}{9m_G^2} n_v(\mu^*, B)$$

Chiral and deconfinement bag constants

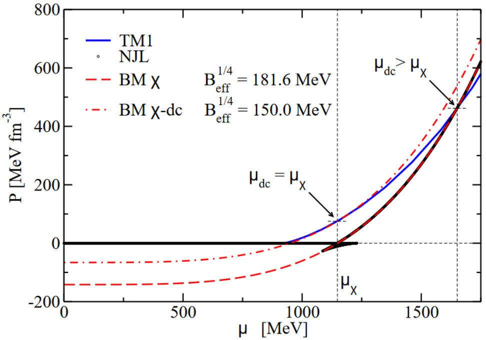
The chiral bag



$$\mu = \mu^* + K_v n_v(\mu^*)$$
$$P(\mu) = P_{FG}^{kin}(\mu^*) + \frac{K_v}{2} n_v^2(\mu^*) - B_\chi$$
$$\epsilon(\mu) = \epsilon_{FG}^{kin}(\mu^*) + \frac{K_v}{2} n_v^2(\mu^*) + B_\chi$$
$$n_v(\mu^*) = n_v(\mu)$$

[5] Klähn, Fischer, "Vector interaction enhanced bag model for astrophysical applications", ApJ 810 (2015)

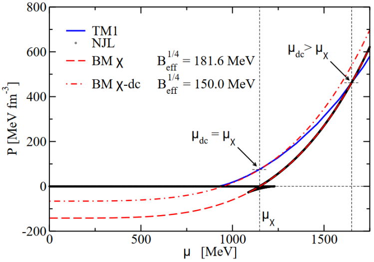
The (de)confinement bag



$$\mu = \mu^* + K_v n_v(\mu^*)$$
$$P(\mu) = P_{FG}^{kin}(\mu^*) + \frac{K_v}{2} n_v^2(\mu^*) - B_{eff}$$
$$\epsilon(\mu) = \epsilon_{FG}^{kin}(\mu^*) + \frac{K_v}{2} n_v^2(\mu^*) + B_{eff}$$
$$n_v(\mu^*) = n_v(\mu)$$
$$B_{eff} = B_\chi - B_{dc}$$

[6] Klähn, Fischer, "Vector interaction enhanced bag model for astrophysical applications", ApJ 810 (2015)

vBag at $T \neq 0$



$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu^*)$$

$$P_f(T,\mu_f) = P^{kin}_{FG,f}(T,\mu_f^*) + \frac{K_v}{2} n^2_{FG,f}(\mu_f^*) - B_{\chi,f}$$

$$P^Q = \sum P_f(T,\mu_f) + B_{dc}(T)$$

$$\epsilon_f(T,\mu_f) = \epsilon^{kin}_{FG,f}(T,\mu_f^*) + \frac{K_v}{2} n^2_{FG,f}(\mu_f^*) + B_{\chi,f}$$

$$\epsilon^Q = \sum \epsilon_f(T,\mu_f^*) - B_{dc}(T) + T\frac{\partial B_{dc}(T)}{\partial T}$$

$$n_f(\mu_f) = n_{FG,f}(\mu_f^*)$$

$$s_f(T,\mu_f) = \left.\frac{\partial P_f(T,\mu_f)}{\partial T}\right|_{\mu_f}$$

$$s(T,\mu_f) = \sum s_f(T,\mu_f) + \frac{\partial B_{dc}(T)}{\partial T}$$

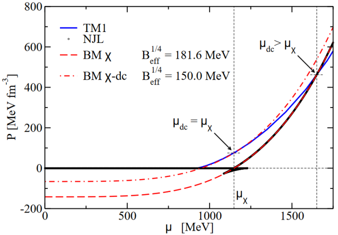
$$\mu_B = \mu_u + 2\mu_d$$

$$n_B = \frac{\partial P}{\partial \mu_B}$$

[7] Klähn, Fischer, "Vector interaction enhanced bag model for astrophysical applications", ApJ 810 (2015)

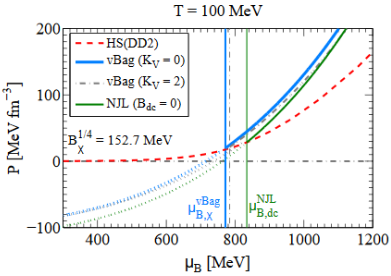
[8] Fischer, Klähn, Hempel, "Consequences of simultaneous chiral symmetry breaking and deconfinement for the isospin symmetric phase diagram", Eur.Phys.J. A (2016)

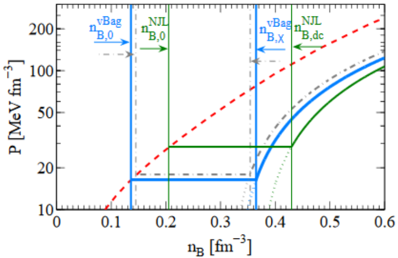
vBag at $T \neq 0$ and $\mu_C \neq 0$



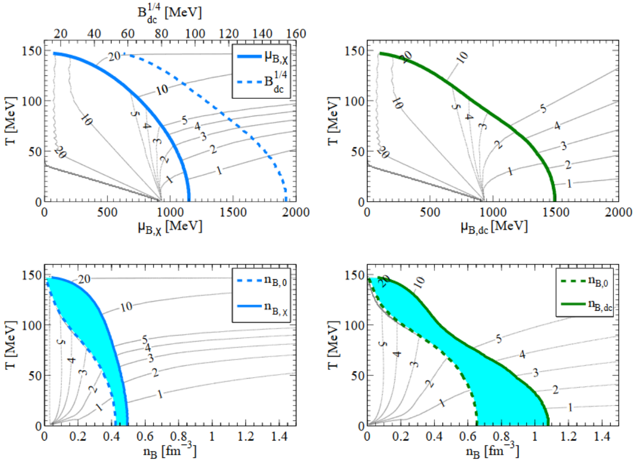
$$\mu_f = \mu_f^* + K_v n_{FG,f}(\mu^*)$$
$$P_f(T,\mu_f) = P_{FG,f}^{kin}(T,\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$$
$$P^Q = \sum P_f(T,\mu_f) + B_{dc}(T)$$
$$\epsilon_f(T,\mu_f) = \epsilon_{FG,f}^{kin}(T,\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$$
$$\epsilon^Q = \sum \epsilon_f(T,\mu_f^*) - B_{dc}(T) + T\frac{\partial B_{dc}(T)}{\partial T} + \mu_C \frac{\partial B_{dc}(T,\mu_C)}{\partial \mu_c}$$
$$n_f(\mu_f) = n_{FG,f}(\mu_f^*)$$
$$s_f(T,\mu_f) = \left.\frac{\partial P_f(T,\mu_f)}{\partial T}\right|_{\mu_f}$$
$$s(T,\mu_f) = \sum s_f(T,\mu_f) + \frac{\partial B_{dc}(T)}{\partial T}$$
$$\mu_B = \mu_u + 2\mu_d$$
$$n_B = \frac{\partial P}{\partial \mu_B}$$
$$\mu_c = \mu_u - \mu_d$$
$$n_C = \frac{\partial P}{\partial \mu_C}$$

[9] Klähn, Fischer, Hempel, "Simultaneous chiral symmetry restoration and deconfinement", ApJ 836 (2017)



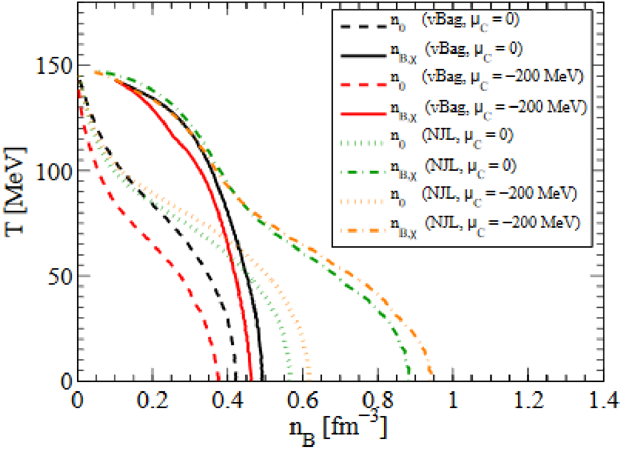


[10] Fischer, Klähn, Hempel, "Consequences of simultaneous chiral symmetry breaking and deconfinement for the isospin symmetric phase diagram", Eur.Phys.J. A (2016)



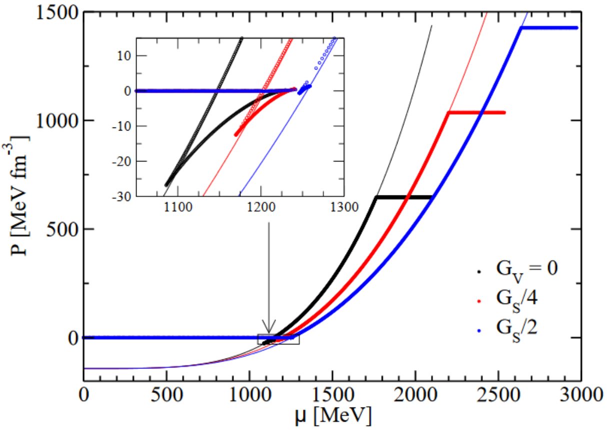
(a) vBag (b) 'standard' NJL approach

[11]Fischer, Klähn, Hempel, "Consequences of simultaneous chiral symmetry breaking and deconfinement for the isospin symmetric phase diagram", Eur.Phys.J. A (2016)

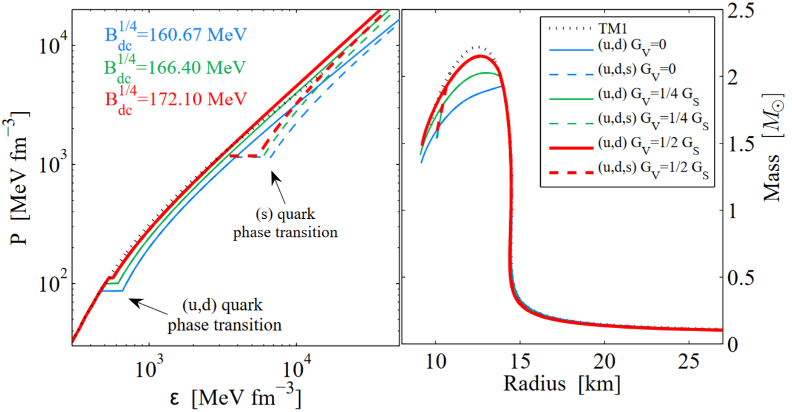


[12] Klähn, Fischer, Hempel, "Simultaneous chiral symmetry restoration and deconfinement", ApJ 836 (2017)

Vector interaction



[13] Klähn, Fischer, "Vector interaction enhanced bag model for astrophysical applications", ApJ 810 (2015)



[14] Klähn, Fischer, "Vector interaction enhanced bag model for astrophysical applications", ApJ 810 (2015)

Conclusions

- Standard NJL and BAG models result from specific approximations of the quark DSE.
- vBag is a model that introduces $D\chi SB$ and repulsive vector interactions into a standard Bag model.
- The temperature limit for the model is dictated by the hadronic EoS and the assumption of 1st order phase transition.
- B_{dc} can be understood as a quark binding energy. As expected, this energy decreases with temperature and causes the density coexistence region to increase for higher T, unlike in standard NJL approach.
- Vector interactions stiffen the quark EoS and help to achieve the 2 solar mass constraint for neutron stars.