Quark exchange effects in the nuclear equation of state at high densities

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1. Story: "Three-window picture of dense matter"

2. Another "Three-window picture of dense matter"

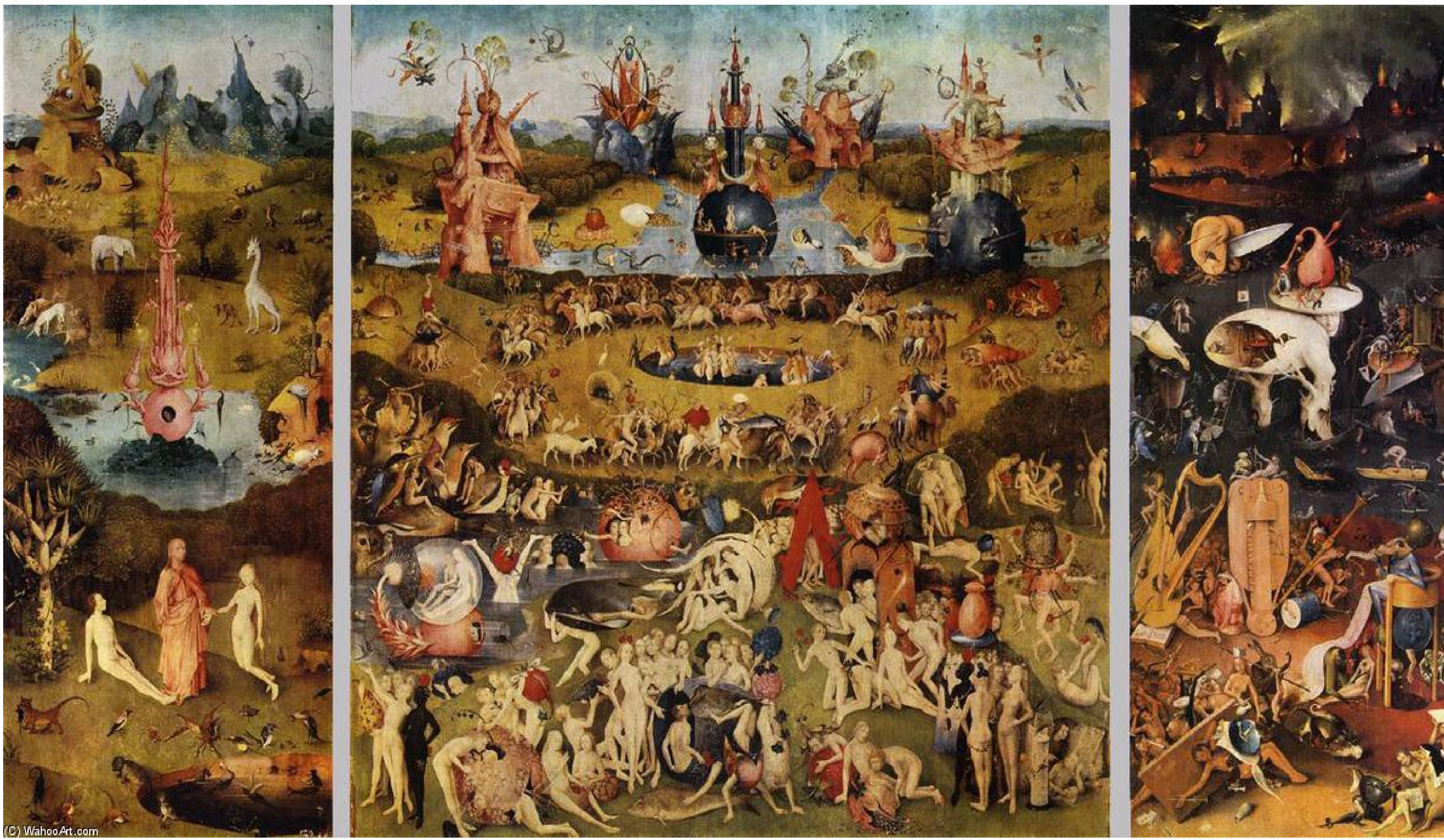
3. Pauli blocking vs. excluded volume in nuclear matter

4. Towards "measuring" the EoS in the T-μ_B-μ_I box!

Based on work with H. Grigorian, G. Roepke, D. Alvarez-Castillo, A. Ayriyan, N.-U. Bastian et al.

6th Conference "Compact Stars & QCD Phase Diagram", Dubna, 28.09.2017

1. Story “Three-window picture of dense matter”



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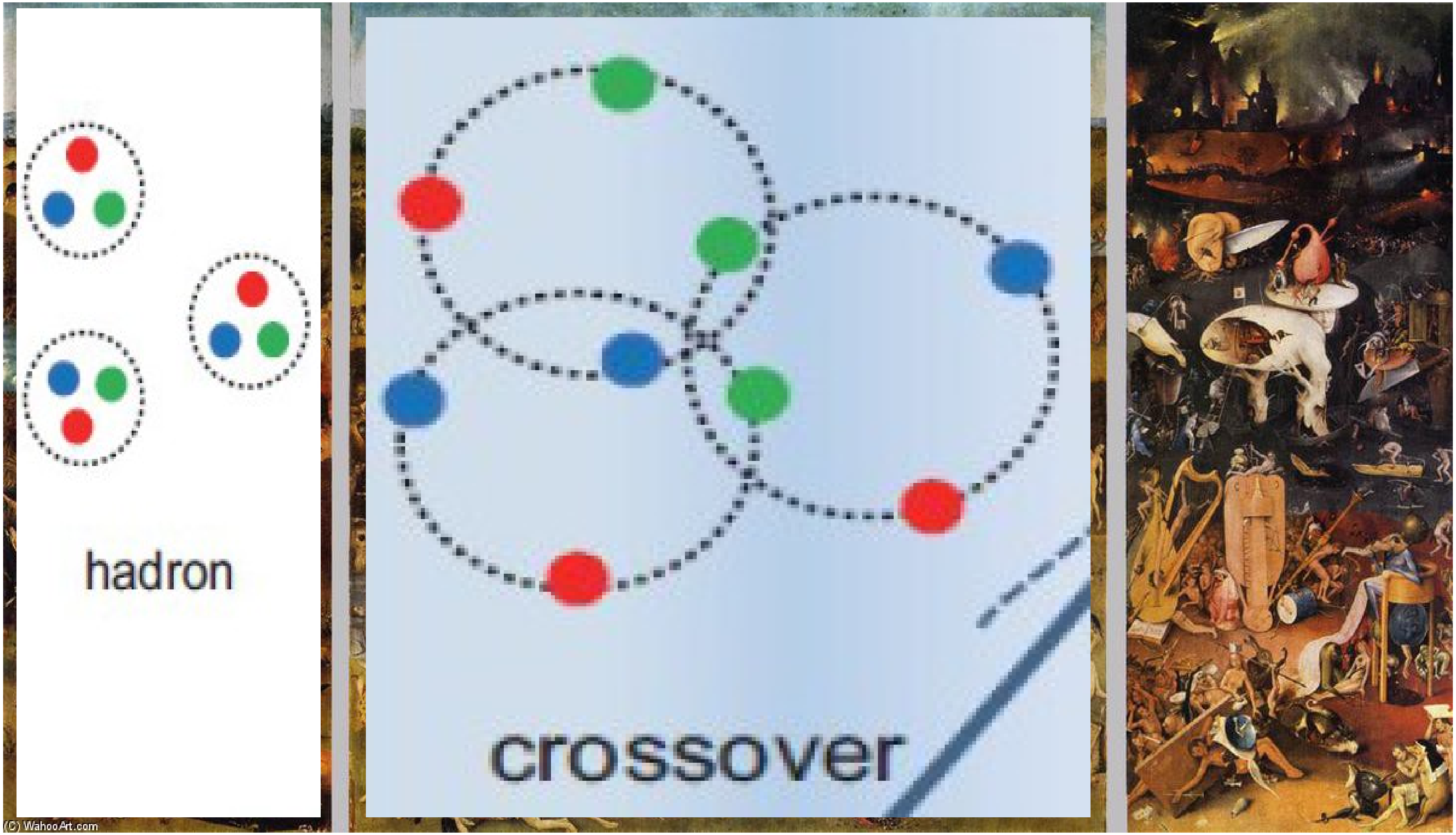

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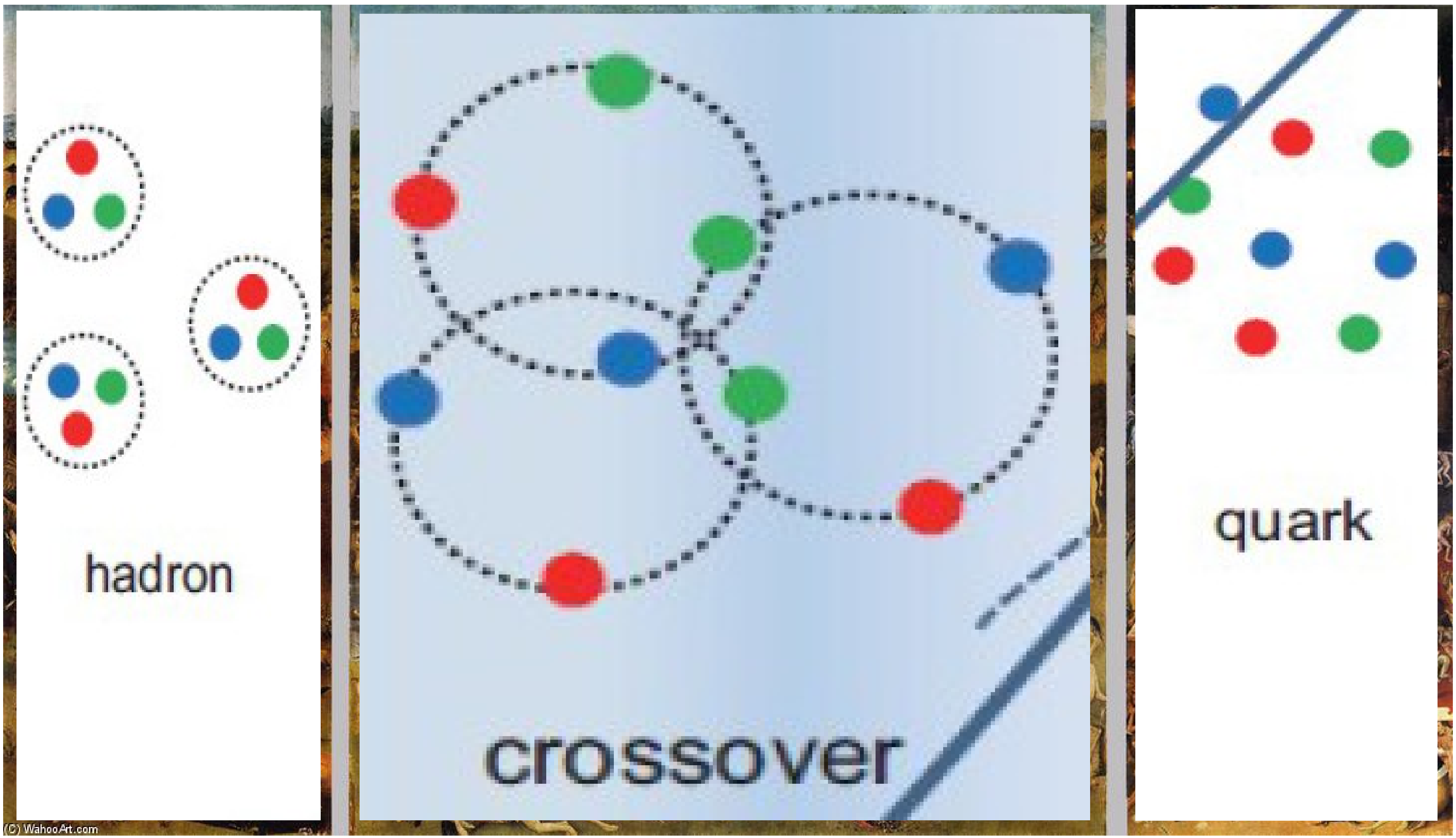

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Hadron-Quark Crossover and Massive Hybrid Stars

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On the basis of the percolation picture from the hadronic phase with hyperons to the quark phase with strangeness, we construct a new equation of state (EOS) with the pressure interpolated as a function of the baryon density. The maximum mass of neutron stars can exceed $2M_{\odot}$ if the following two conditions are satisfied; (i) the crossover from the hadronic matter to the quark matter takes place at around three times the normal nuclear matter density, and (ii) the quark matter is strongly interacting in the crossover region. This is in contrast to the conventional approach assuming the first order phase transition in which the EOS becomes always soft due to the presence of the quark matter at high density. Although the choice of the hadronic EOS does not affect the above conclusion on the maximum mass, the three-body force among nucleons and hyperons plays an essential role for the onset of the hyperon mixing and the cooling of neutron stars.

Subject Index Neutron stars, Nuclear matter aspects in nuclear astrophysics, Hadrons and quarks in nuclear matter, Quark matter

1. Introduction

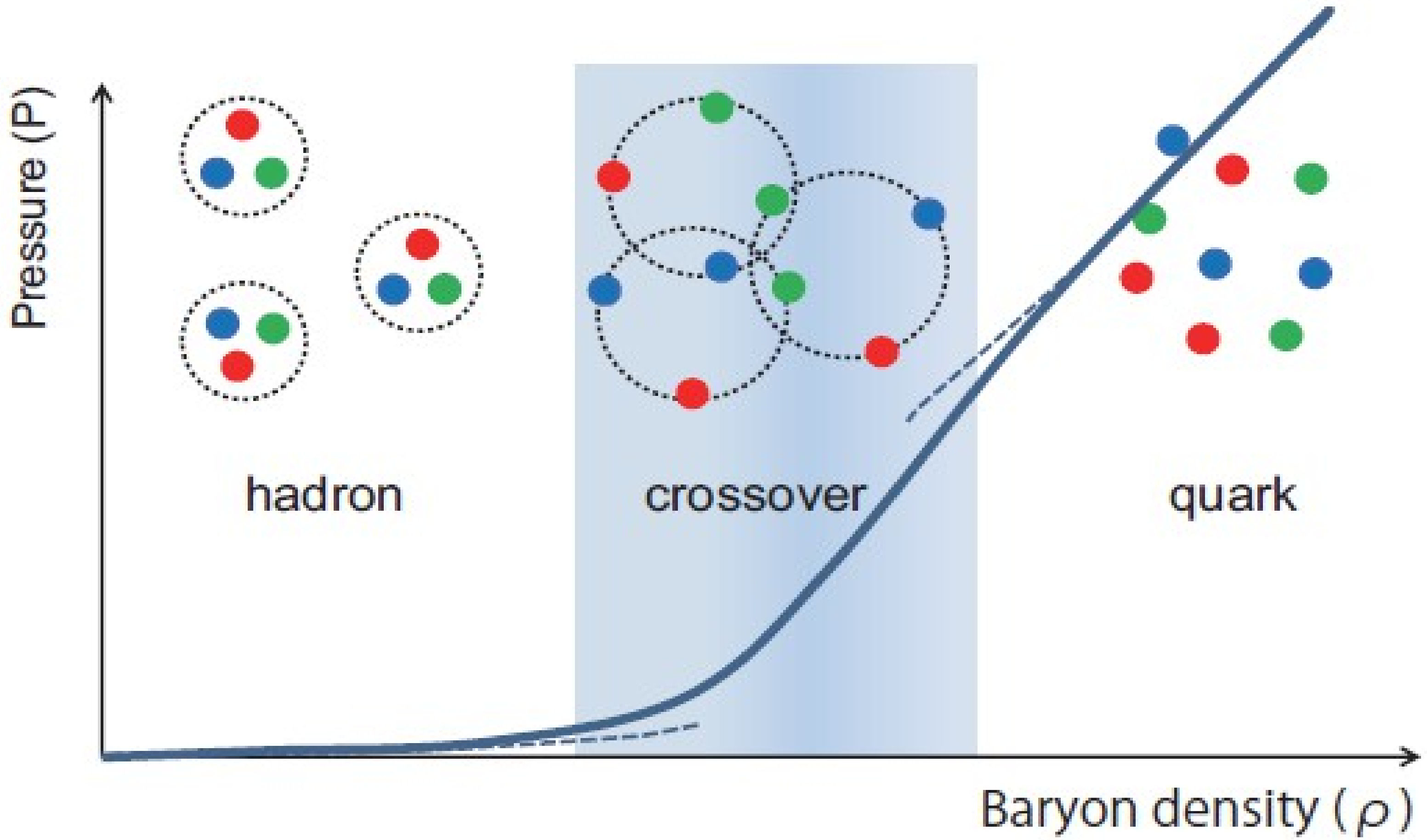
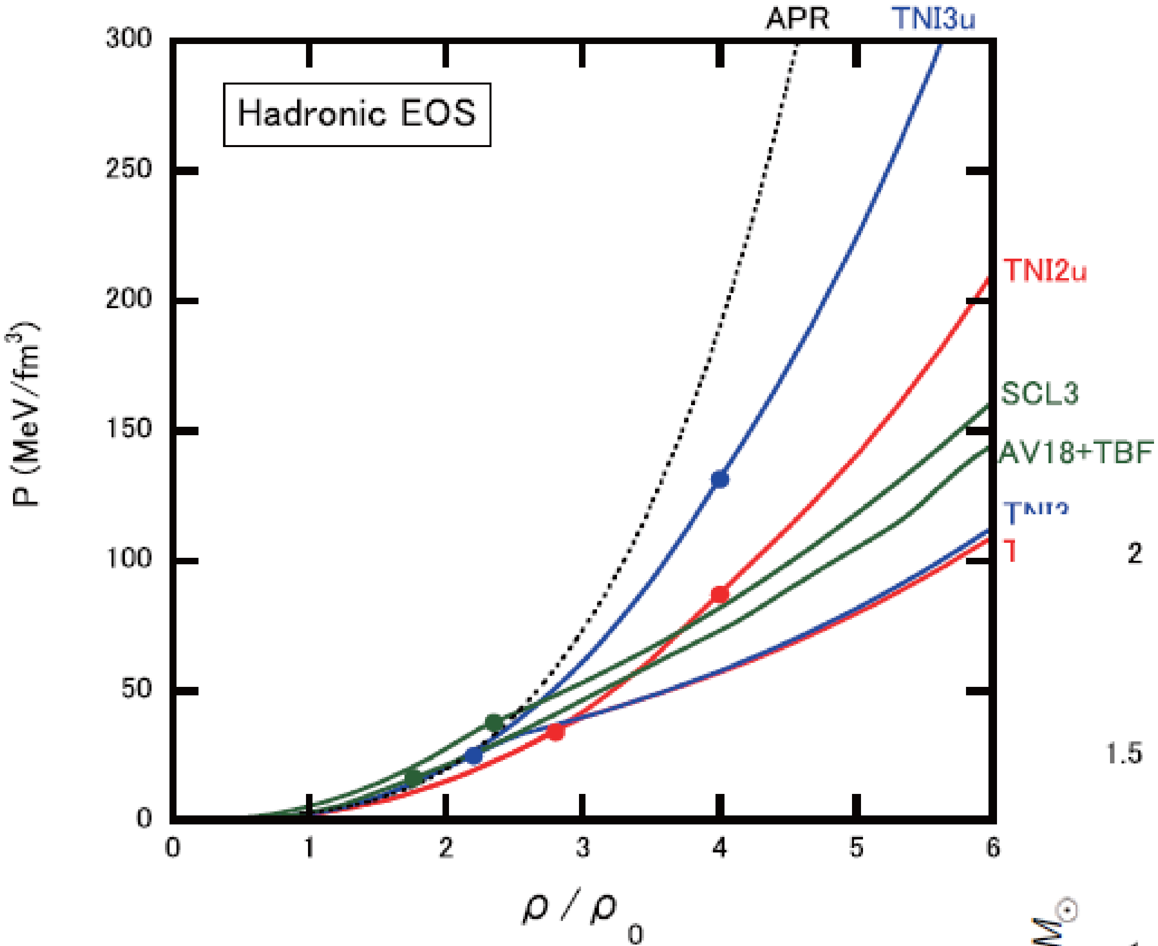


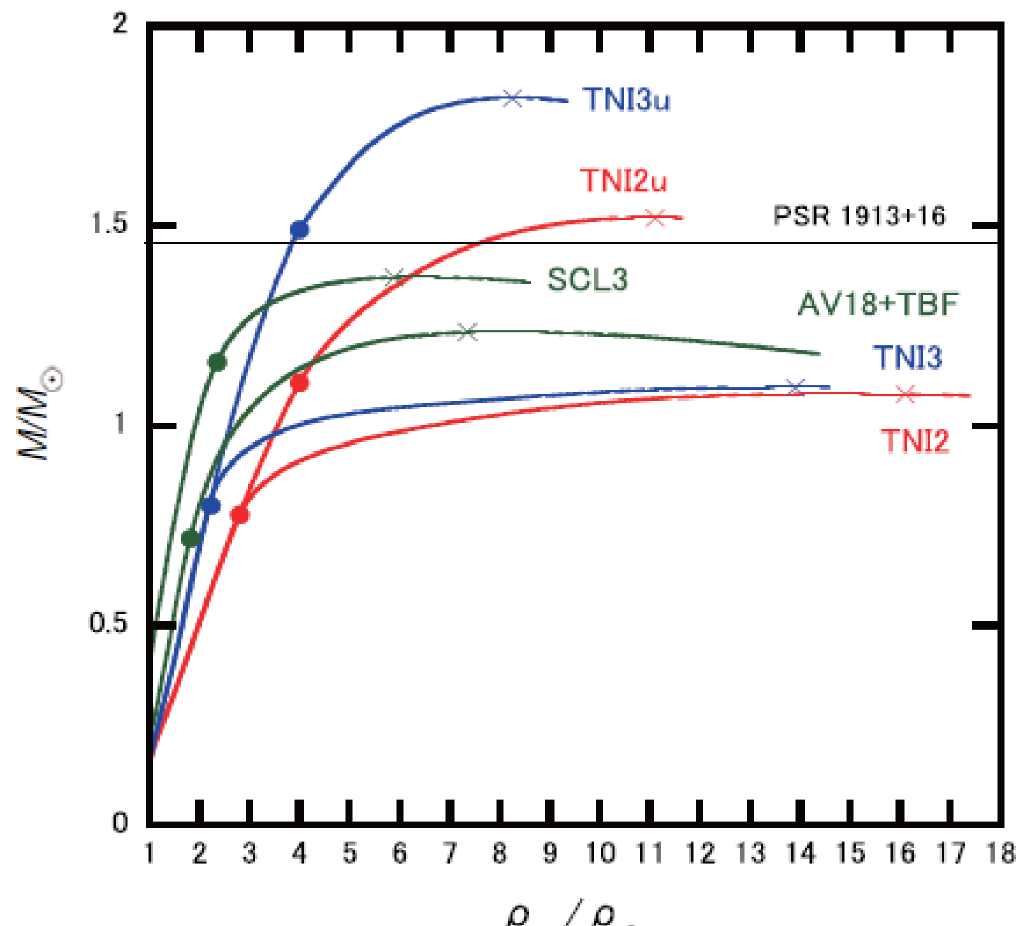
Fig. 1 Schematic picture of the QCD pressure (P) as a function of the baron density (ρ) under the assumption of the hadron-quark crossover. The crossover region where finite-size hadrons start to overlap and percolate is shown by the shaded area. The pressure calculated on the basis of the point-like hadrons (shown by the dashed line at low density) and that calculated on the basis of weakly interacting quarks (shown by the dashed line at high density) lose their validity in the crossover region, so that the naive use of the Gibbs conditions by extrapolating the dashed lines is not justified in general.

2. Hadronic EOS (H-EOS)

Table 1 Properties of various hadronic EOSs with hyperons; TNI2, TNI3, TNI2u, TNI3u [33, 34], Paris+TBF, AV18+TBF [36–38] and SCL3ΛΣ [39]. κ is the nuclear incompressibility and $\rho_{\rm th}$ is the threshold density of hyperon-mixing with ρ_0 (=0.17/fm^3) being the normal nuclear density. R and ρ_c denote the radius and central density for the maximum mass ($M_{\rm max}$) NS, respectively. The numbers in the parentheses are those without hyperons. $*$s indicate that the numbers are read from the figures in [36].

EOS	TNI2	TNI3	TNI2u	TNI3u	Paris+TBF	AV18+TBF	SCL3ΛΣ
κ (MeV)	250	300	250	300	281	192	211
$\rho_{\rm th}(\Lambda)/\rho_0$	2.95	2.45	4.01	4.01	2.9*	2.8*	2.24
$\rho_{\rm th}(\Sigma^-)/\rho_0$	2.83	2.23	4.06	4.01	1.9*	1.8*	2.24
$M_{\rm max}/M_\odot$	1.08	1.10	1.52	1.83	1.26	1.22	1.36
	(1.62)	(1.88)			(2.06)	(2.00)	(1.65)
R(km)	7.70	8.28	8.43	9.55	10.46	10.46	11.42
	(8.64)	(9.46)			(10.50)	(10.54)	(10.79)
ρ_c/ρ_0	16.10	13.90	11.06	8.26	7.35	7.35	6.09
	(9.97)	(8.29)			(6.47)	(6.53)	(6.85)


Hadronic EOS
APR
TNI3u
TNI2u
SCL3
AV18+TBF
P (MeV/fm³)
ρ / ρ0


TNI3u
TNI2u
PSR 1913+16
SCL3
AV18+TBF
TNI3
TNI2
M/M⊙
ρc / ρ0

3. Quark EOS (Q-EOS)

$$\mathcal{L}_{\rm NJL} = \overline{q}(i\partial\!\!\!/ - m)q + \frac{1}{2}G_{S}\sum_{a=0}^{8}[(\overline{q}\lambda^a q)^2 + (\overline{q}i\gamma_5\lambda^a q)^2] - G_{D}[\det\overline{q}(1+\gamma_5)q + \text{h.c.}]$$

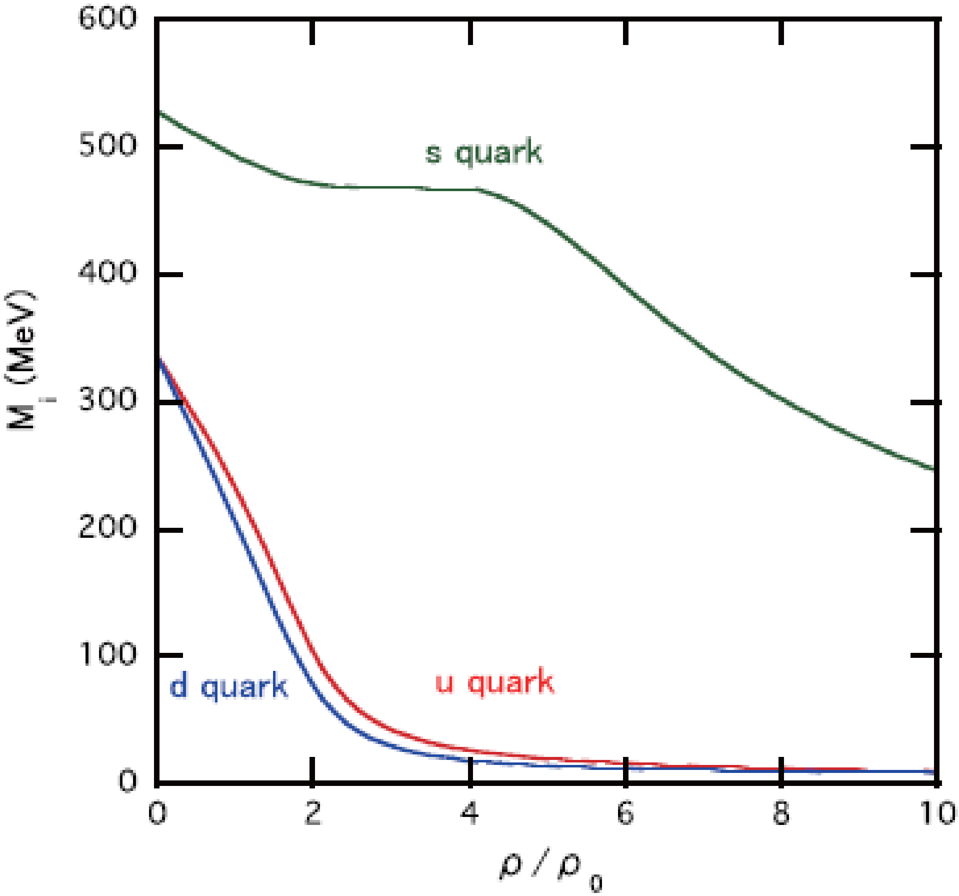
$$- \begin{cases} \frac{1}{2}g_{V}(\overline{q}\gamma^\mu q)^2 \\ \frac{1}{2}G_{V}\sum_{a=0}^{8}\left[(\overline{q}\gamma^\mu\lambda^a q)^2 + (\overline{q}i\gamma^\mu\gamma_5\lambda^a q)^2\right] \end{cases}$$

$$P(T,\mu_{u,d,s}) = T\sum_i\sum_\ell\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\ln\left(\frac{S_i^{-1}(i\omega_\ell,\mathbf{p})}{T}\right)$$

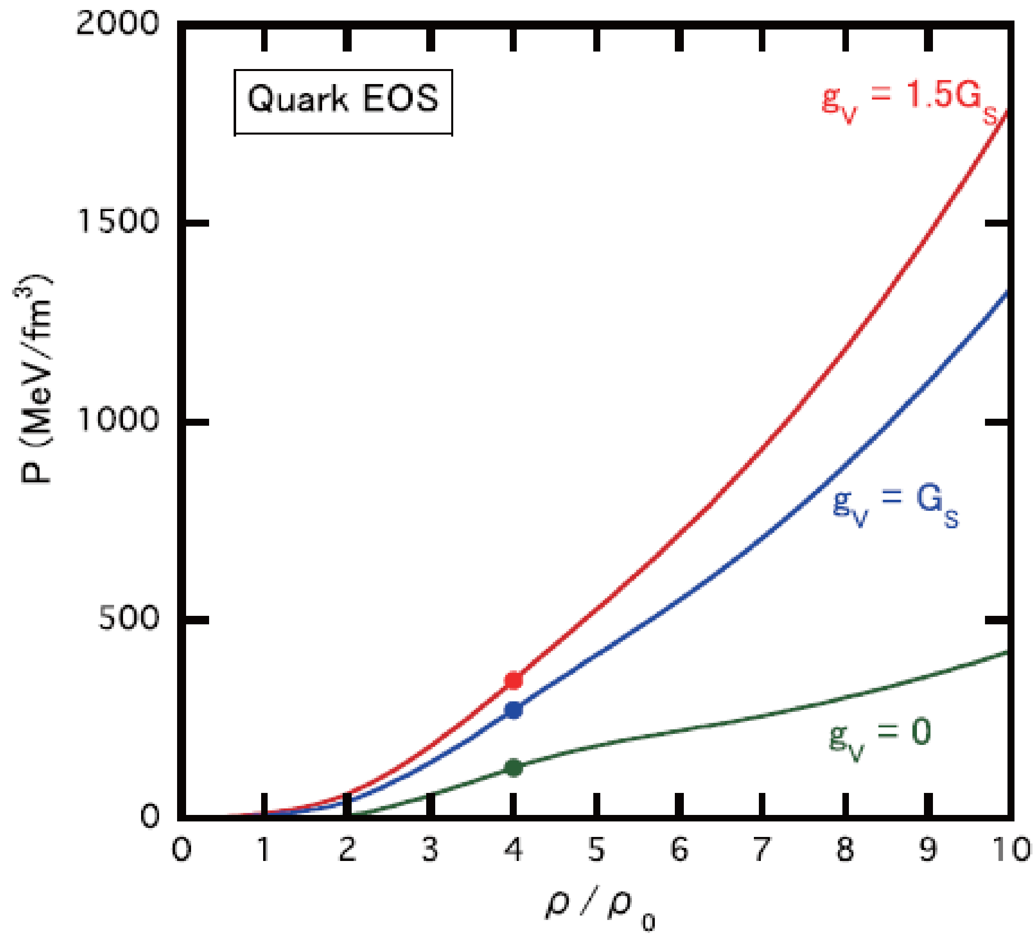
$$-G_{S}\sum_i\sigma_i^2 - 4G_{D}\sigma_u\sigma_d\sigma_s + \begin{cases} \frac{1}{2}g_{V}\left(\sum_i n_i\right)^2 \\ \frac{1}{2}G_{V}\sum_i n_i^2 \end{cases}$$

$$S_i^{-1} = p\!\!\!/ - M_i - \gamma^0\mu_i^{\rm eff}, \quad \mu_i^{\rm eff} \equiv \begin{cases} \mu_i - g_{V}\sum_j n_j \\ \mu_i - G_{V} n_i \end{cases}$$

	Λ(MeV)	$G_{S}\Lambda^2$	$G_{D}\Lambda^5$	$m_{u,d}$(MeV)	m_s(MeV)
HK	631.4	3.67	9.29	5.5	135.7
RKH	602.3	3.67	12.36	5.5	140.7
LKW	750	3.64	8.9	3.6	87



600
500
400
300
200
100
0
M_i (MeV)
s quark
d quark
u quark
0
2
4
6
8
10
ρ / ρ_0



Quark EOS
2000
1500
1000
500
0
P (MeV/fm^3)
$g_V = 1.5G_S$
$g_V = G_S$
$g_V = 0$
0
1
2
3
4
5
6
7
8
9
10
ρ / ρ_0

4. Hadron-Quark crossover

As discussed in §1, treating the point-like hadron as an independent degree of freedom loses its validity as the baryon density approaches to the percolation region. In other words, the system cannot be described neither by an extrapolation of the hadronic EOS from the low-density side nor by an extrapolation of the quark EOS from the high-density side. Under such situation, it does not make much sense to apply the Gibbs criterion of two phases I and II, $P_{\mathrm{I}}(T_c, \mu_c) = P_{\mathrm{II}}(T_c, \mu_c)$ since P_{I} and P_{II} are not reliable in the transition region.

we will consider a phenomenological "interpolation" between the H-EOS and Q-EOS as a first step. Such an interpolation is certainly not unique, but we adopt a simplest

$$\begin{aligned} P(\rho) &= P_H(\rho) f_-(\rho) + P_Q(\rho) f_+(\rho), \\ f_\pm(\rho) &= \frac{1}{2}\left(1 \pm \tanh\left(\frac{\rho - \bar{\rho}}{\Gamma}\right)\right), \end{aligned}$$

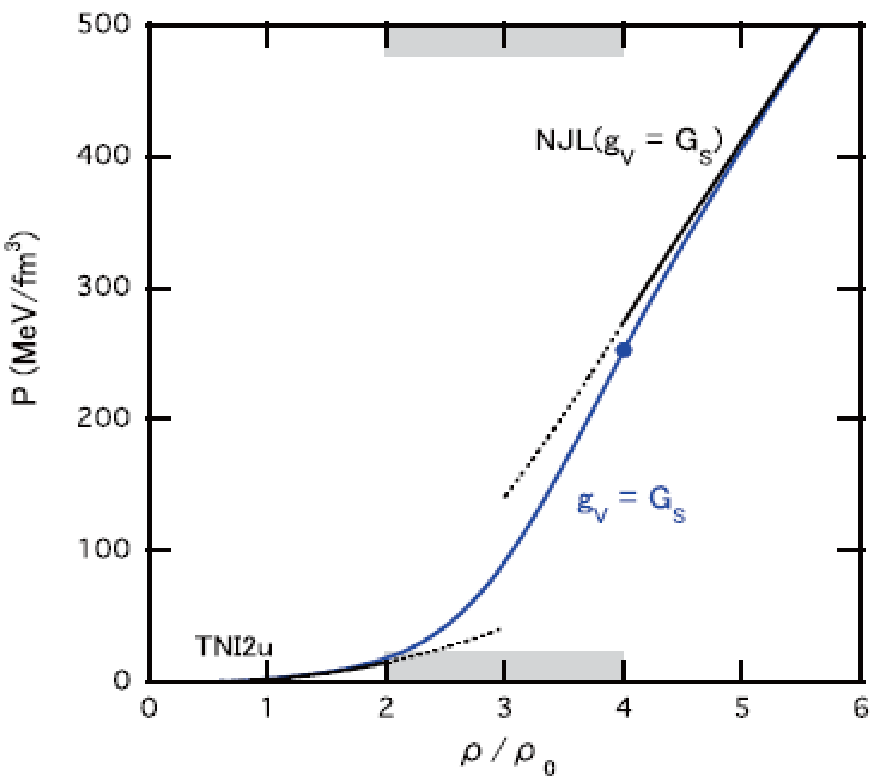
where P_H and P_Q are the pressure in the hadronic matter and that in the quark matter.

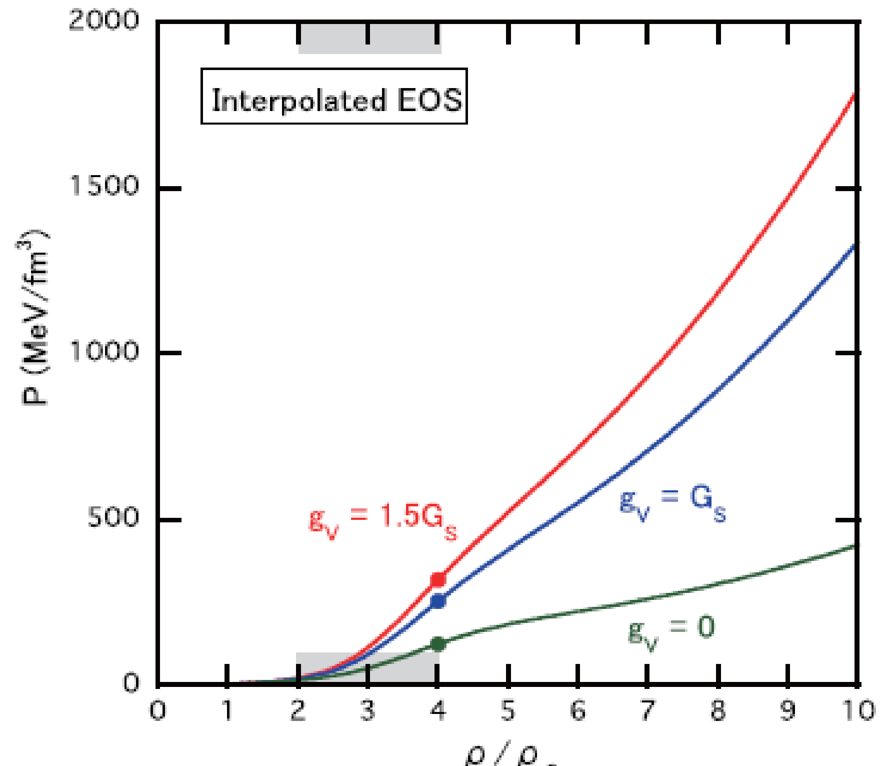
One should not confuse Eq.(7) with the pressure in the mixed phase associated with the first-order phase transition in which $f_\pm$ is considered to the volume fraction of each phase. In our crossover picture, the system is always uniform and f_- (f_+) should be interpreted as the degree of reliability of H-EOS (Q-EOS) at given baryon density.

To calculate the energy density ε as a function of ρ in thermodynamically consistent way, we integrate the thermodynamical relation, $P = \rho^2 \partial(\varepsilon/\rho)/\partial\rho$ and obtain

$$\begin{aligned} \varepsilon(\rho) &= \varepsilon_H(\rho) f_-(\rho) + \varepsilon_Q(\rho) f_+(\rho) + \Delta\varepsilon \\ \Delta\varepsilon &= \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_H(\rho') - \varepsilon_Q(\rho')) \frac{g(\rho')}{\rho'} d\rho' \end{aligned}$$

with $g(\rho) = \frac{2}{\Gamma}(e^X + e^{-X})^{-2}$ and $X = (\rho - \bar{\rho})/\Gamma$. Here ε_H (ε_Q) is the energy density obtained from H-EOS (Q-EOS). $\Delta\varepsilon$ is an extra term which guarantees the thermodynamic consistency.





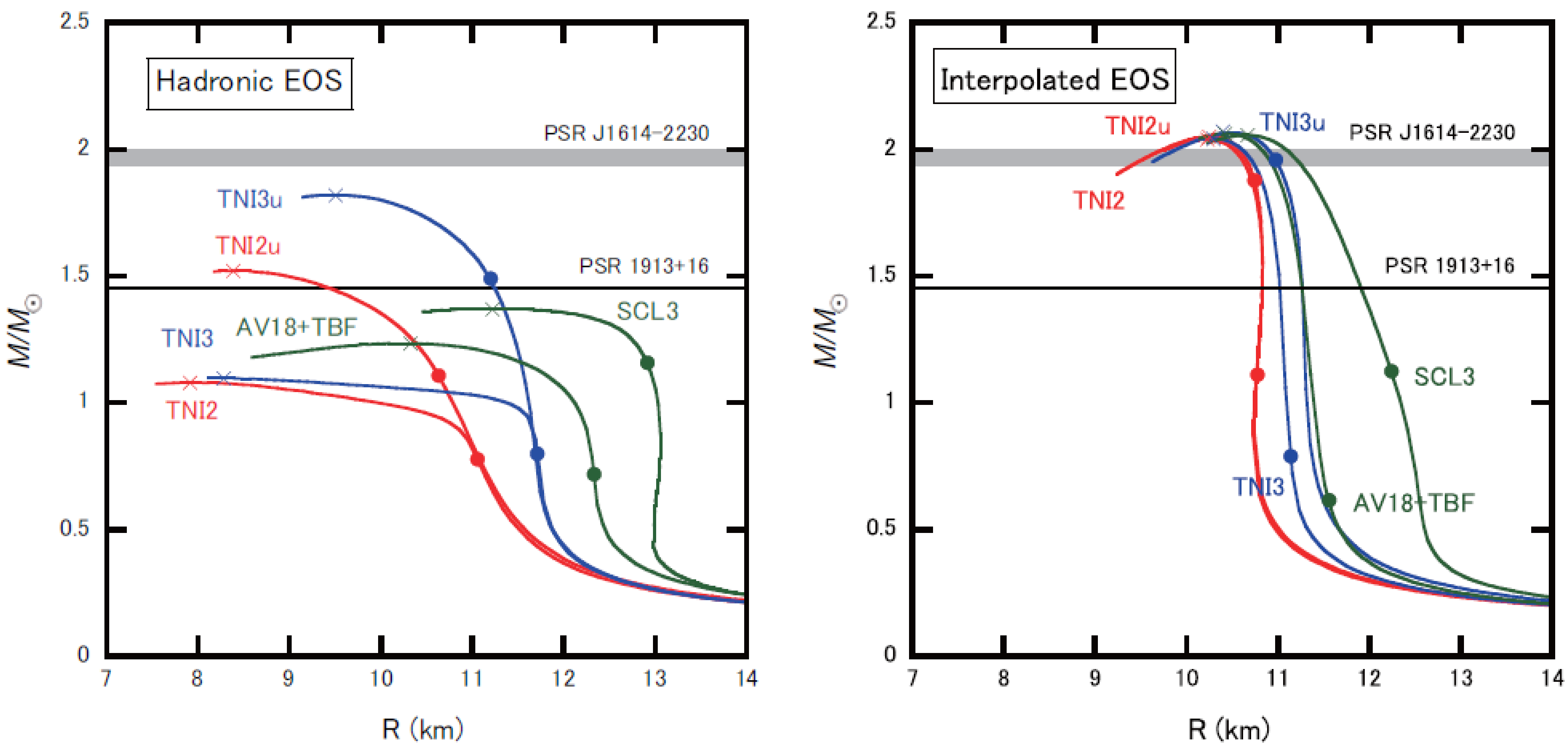
5. Numerical results and discussions

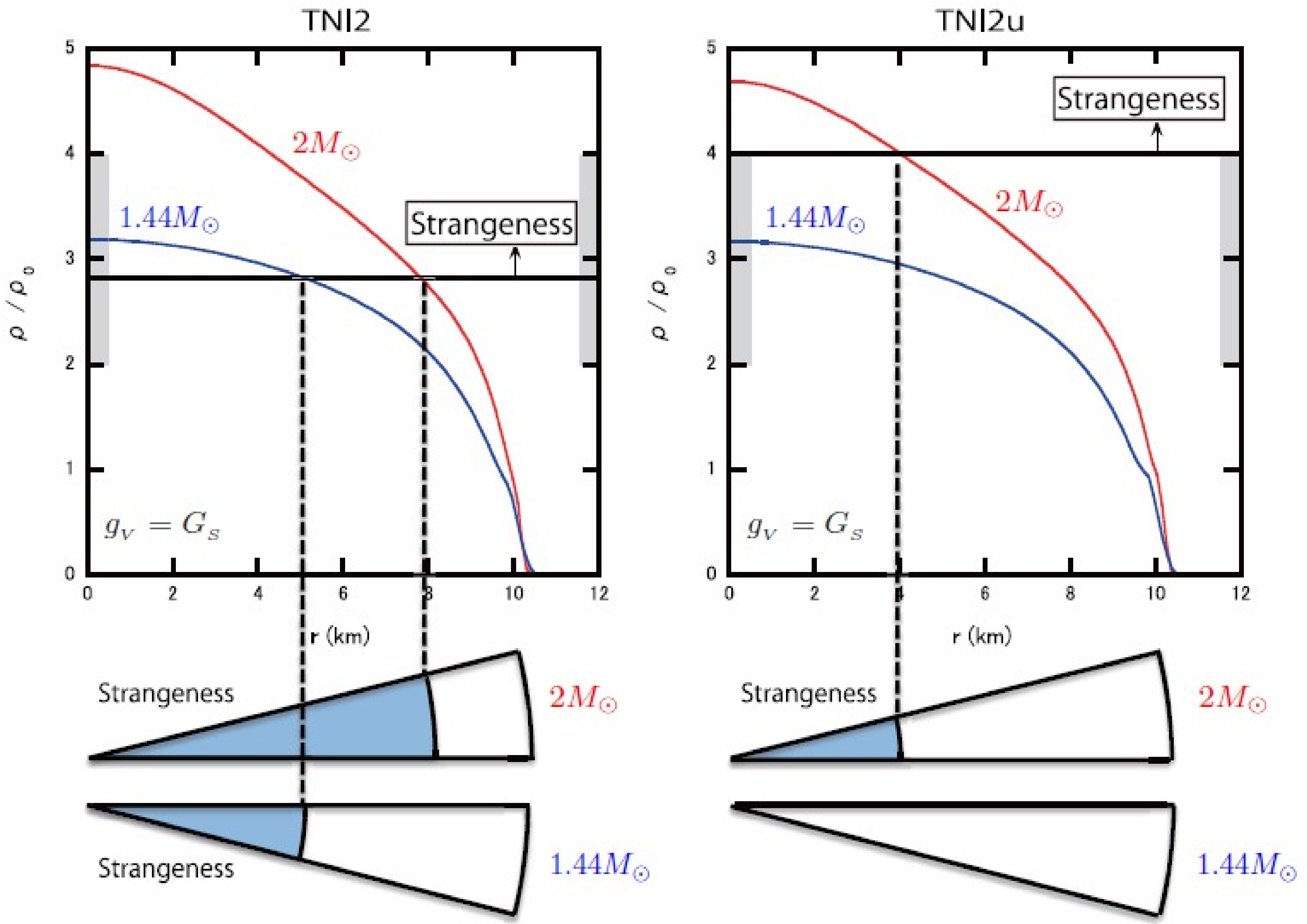
5.1. *Massive hybrid star with strangeness*

We now solve the following Tolman-Oppenheimer-Volkov (TOV) equation to obtain M-R relationship by using the EOSs with and without the hadron-quark crossover:

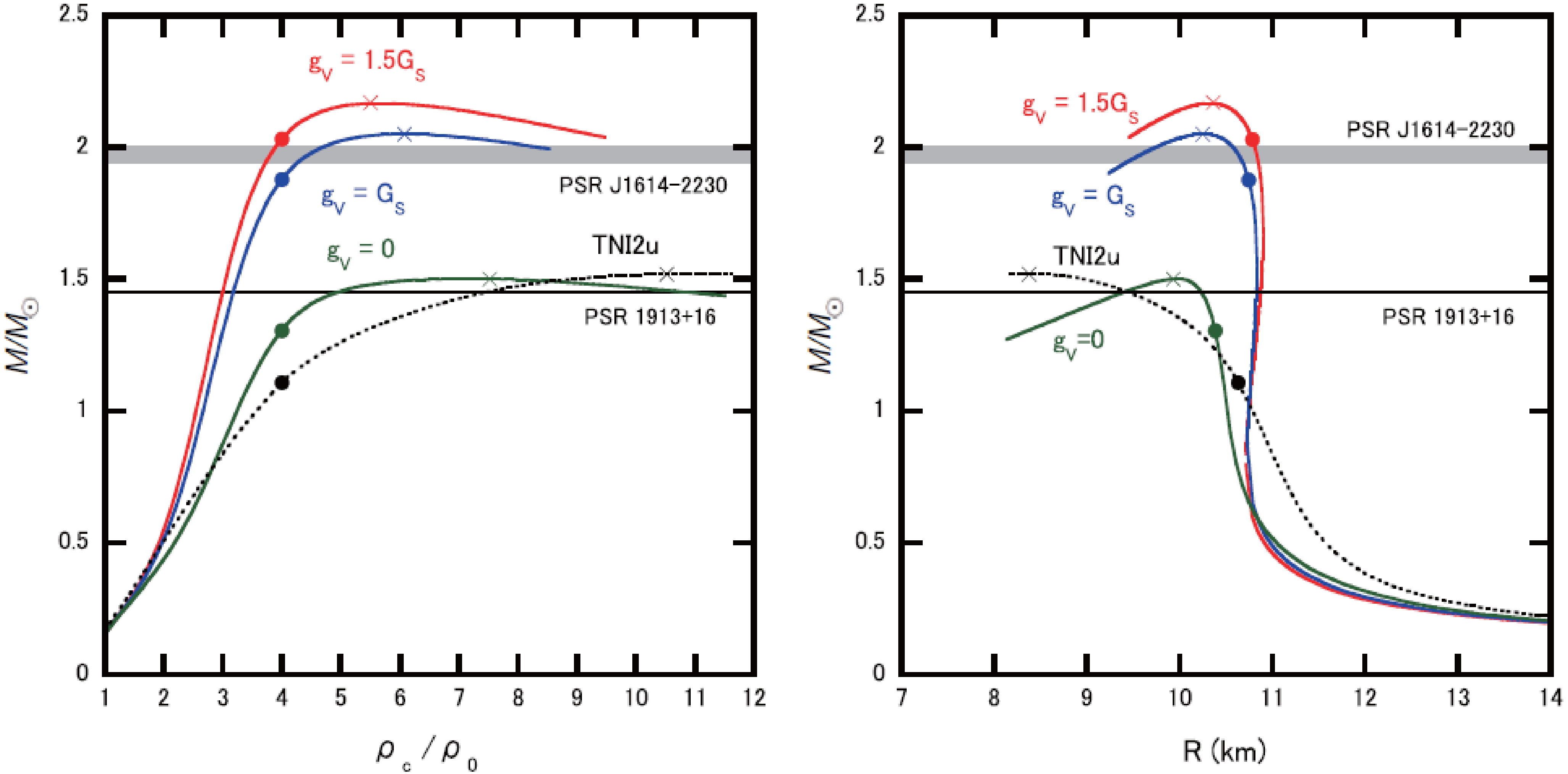
$$\frac{dP}{dr} = -\frac{G}{r^2}\left(M(r) + 4\pi P r^3\right)(\varepsilon + P)\left(1 - 2GM(r)/r\right)^{-1},$$

$$M(r) = \int_0^r 4\pi r'^2 \varepsilon(r') dr',$$





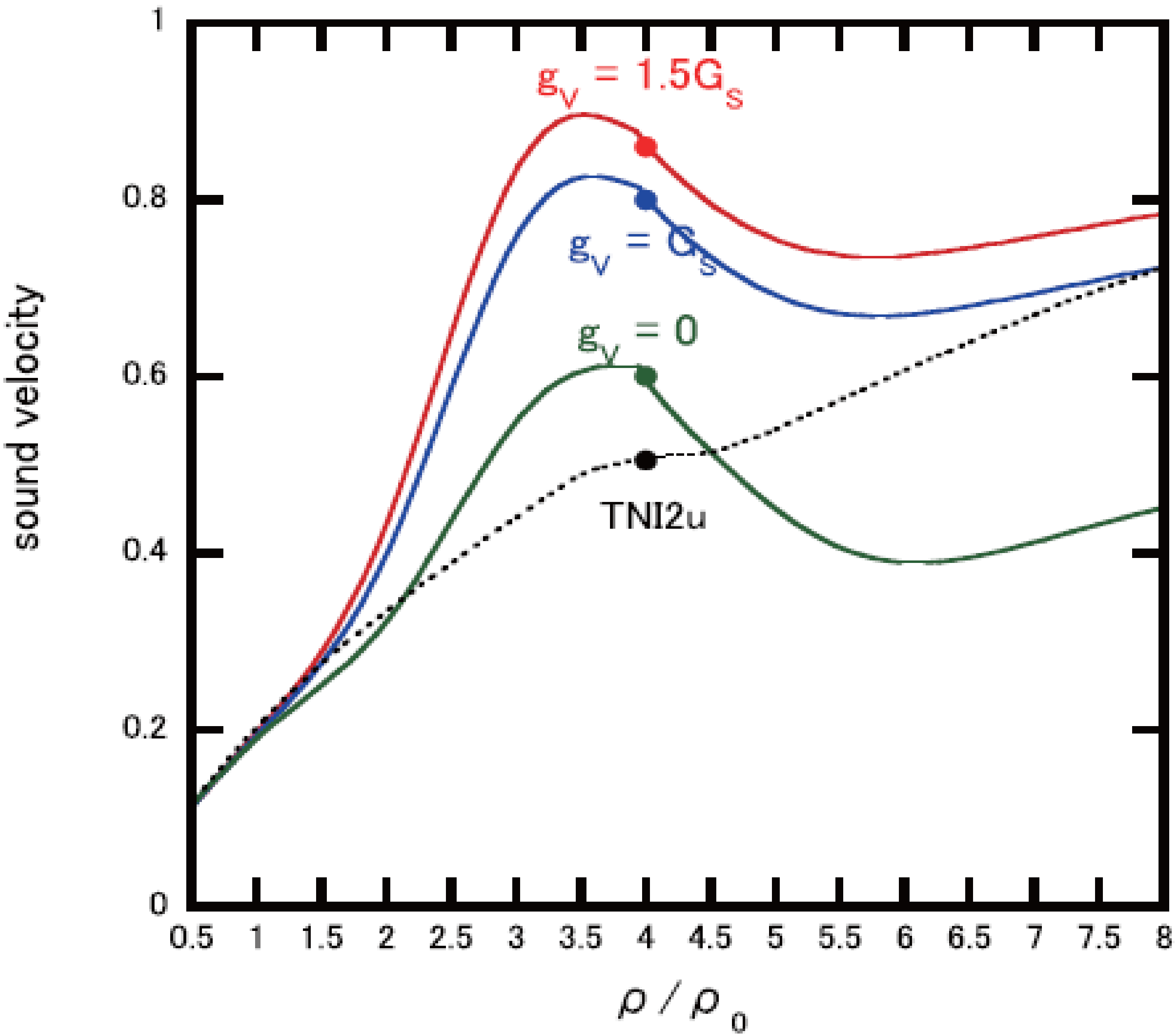
TNI2
TNI2u
ρ / ρ_0
r (km)
$2M_\odot$
$1.44M_\odot$
Strangeness
$g_V = G_S$



g_V = 1.5G_S
g_V = G_S
g_V = 0
PSR J1614−2230
TNI2u
PSR 1913+16
$M/M_\odot$
ρ_c / ρ_0
g_V = 1.5G_S
PSR J1614−2230
g_V = G_S
TNI2u
PSR 1913+16
g_V=0
$M/M_\odot$
R (km)

5.4. *Sound velocity of interpolated EOS*

One of the measures to quantify the stiffness of EOS is the sound velocity $v_s = \sqrt{dP/d\varepsilon}$.



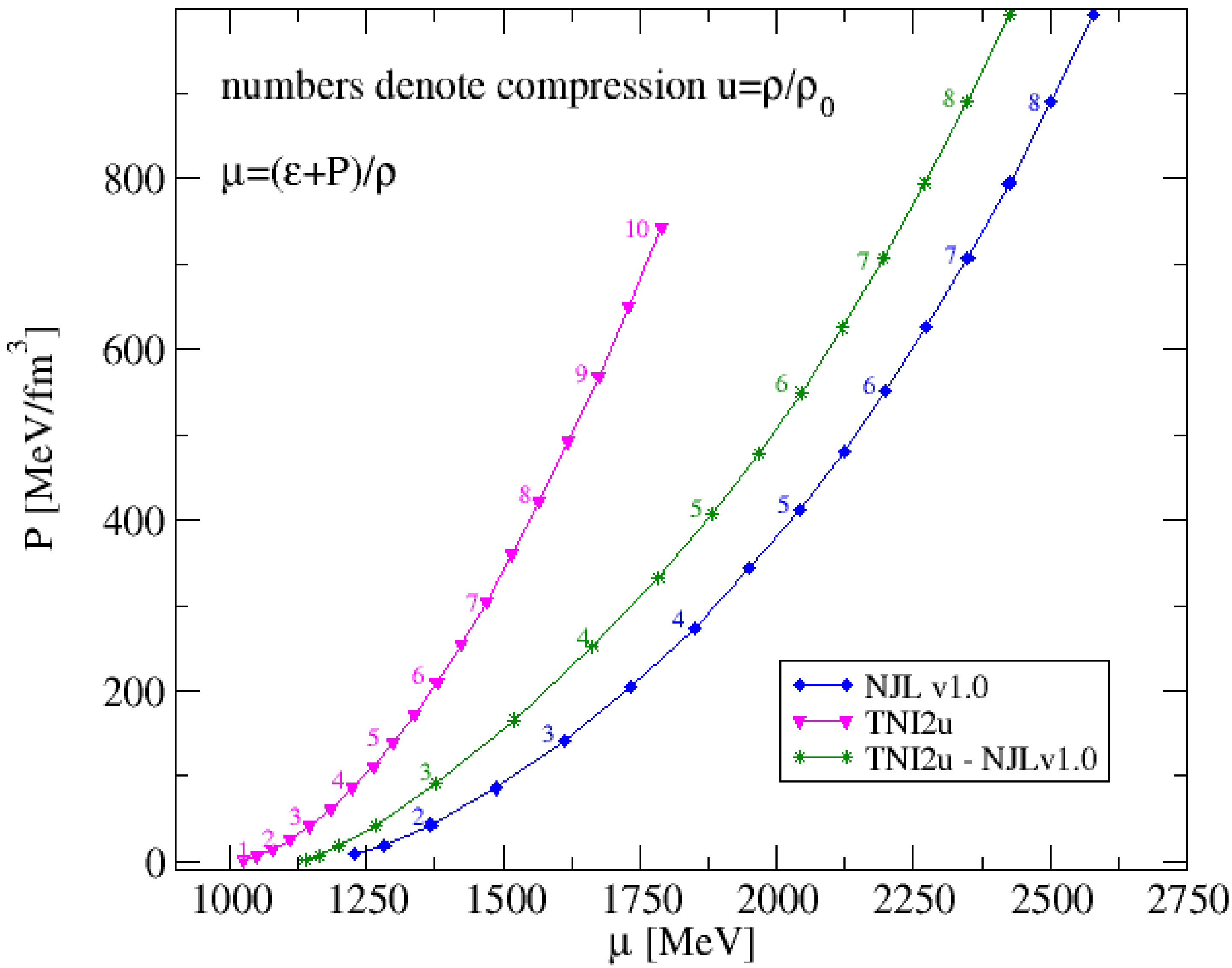
6. Summary and concluding remarks

We have constructed an EOS by the interpolation between the H-EOS at lower densities and the Q-EOS at higher densities, and found that the hybrid stars could have $M_{\max} \sim 2M_{\odot}$, compatible with the observation. This conclusion is in contrast to the conventional EOS for hybrid stars derived through the Gibbs construction in which the resultant EOS becomes always softer than hadronic EOS and thereby leads to smaller $M_{\max}$.

The idea of rapid stiffening of the EOS starting from $2\rho_0$ opens a possibility that the experimental nuclear incompressibility $\kappa = (240 \pm 20)$MeV at $\rho \sim \rho_0$ is compatible with the existence of massive neutron stars. Also, the idea may well be checked by independent laboratory experiments with medium-energy heavy-ion collisions.

Finally, we remark that the crossover region may contain richer non-perturbative phases such as color superconductivity, inhomogeneous structures and so on [1]. How these structures as well as the associated cooling processes affect the results of the present paper would be an interesting future problem to be examined.

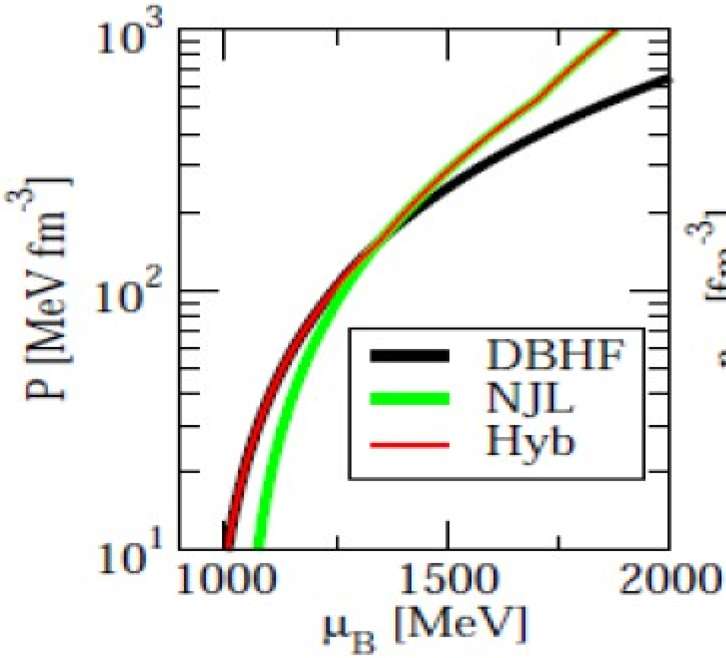
Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS with "Crossover"

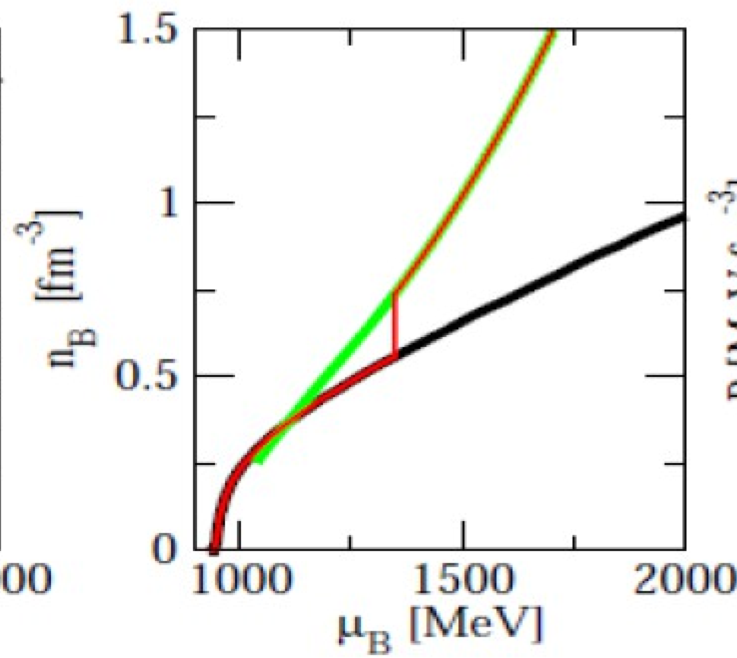


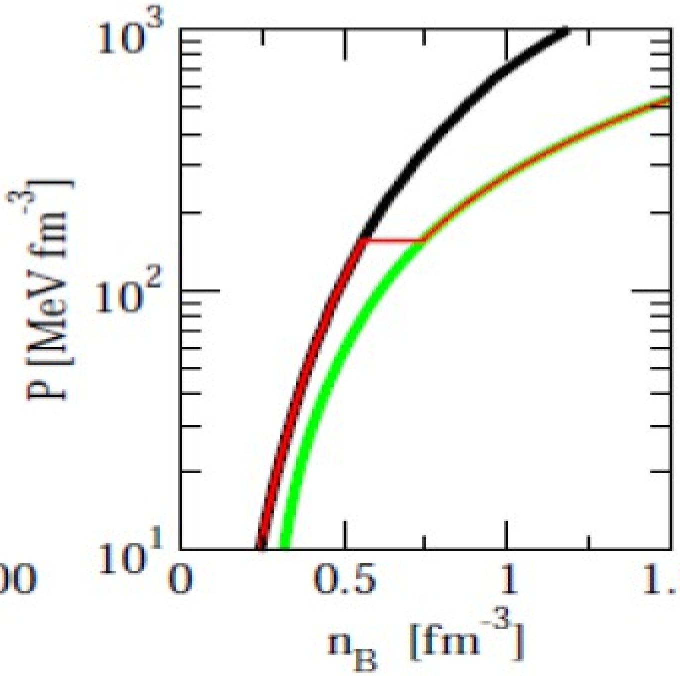
PROBLEM:
The interpolation in $P(\rho)$ is not a "crossover", since thermodynamic consistency requires a shift $\Delta\varepsilon$ which isolates the resulting hybrid EoS (green) from the hadronic (magenta) and quark (blue) asymptotes.

Figure prepared with data from arxiv:1212.6803v1

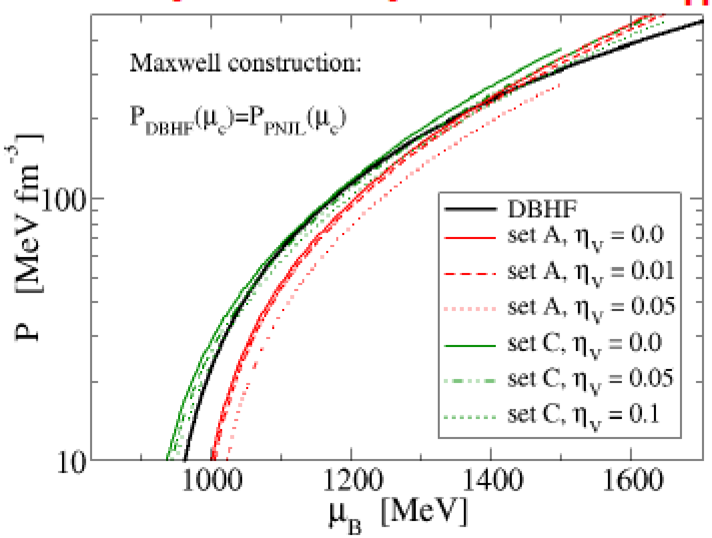
Traditional: Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS

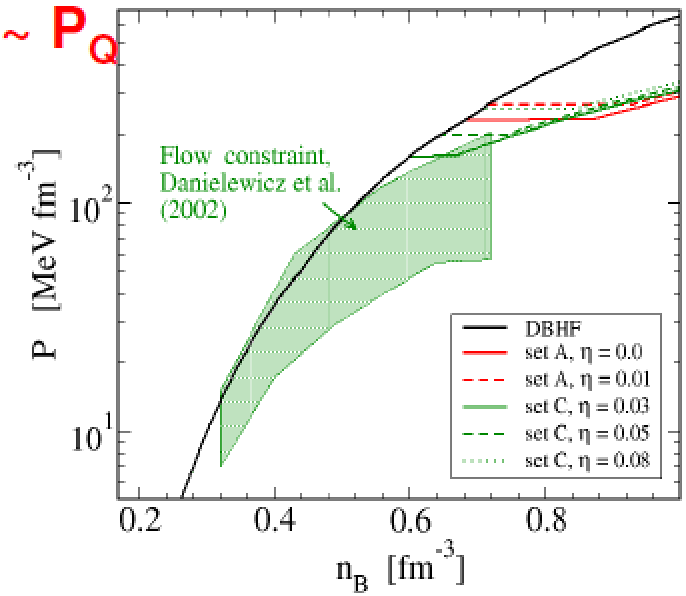


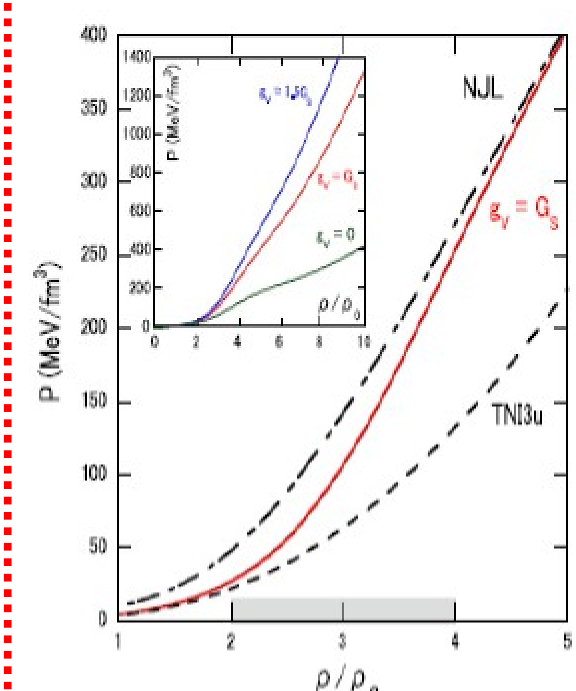




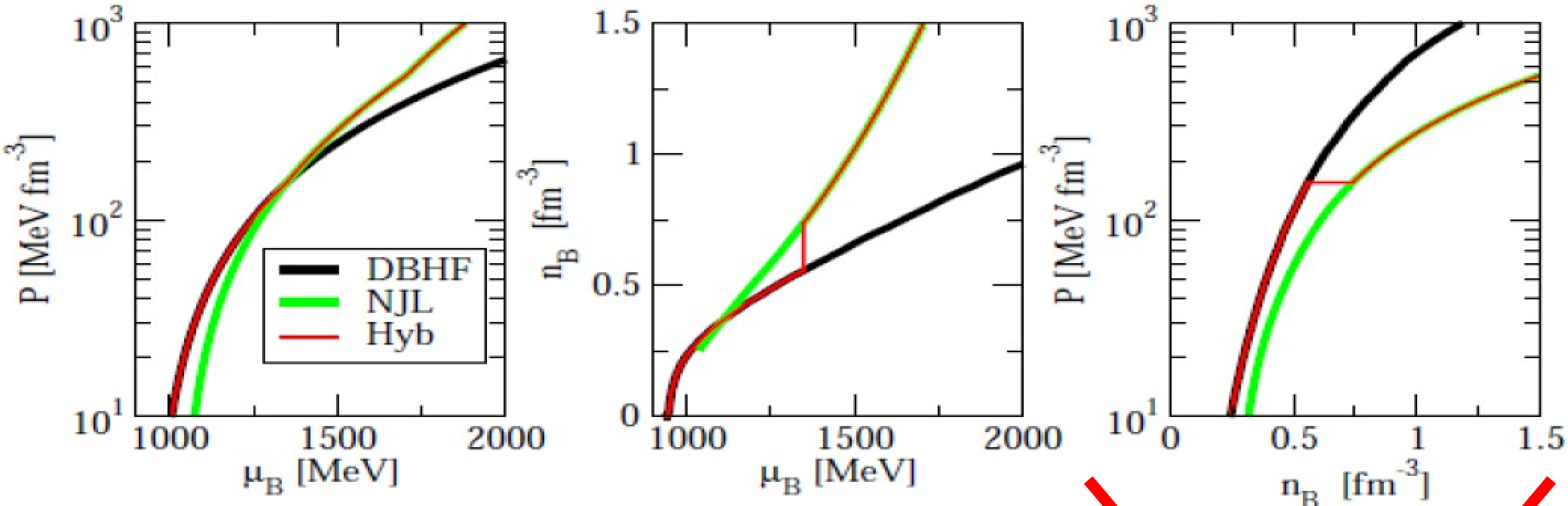
Masquerade problem: $P_H \sim P_Q$



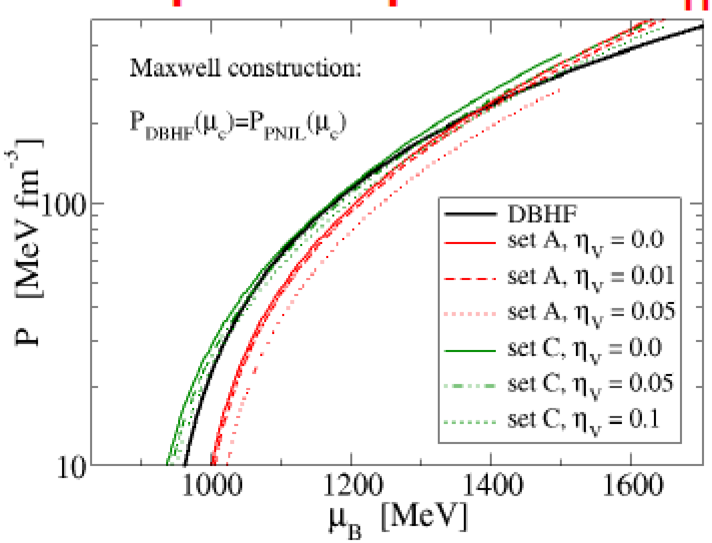


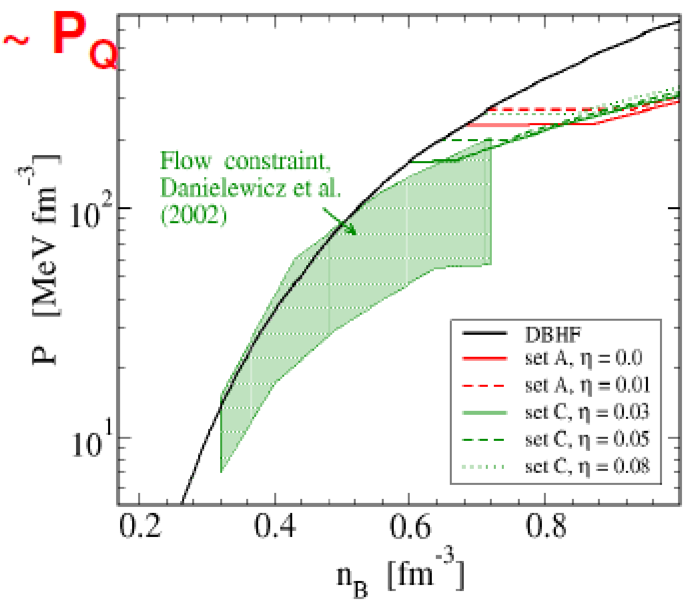


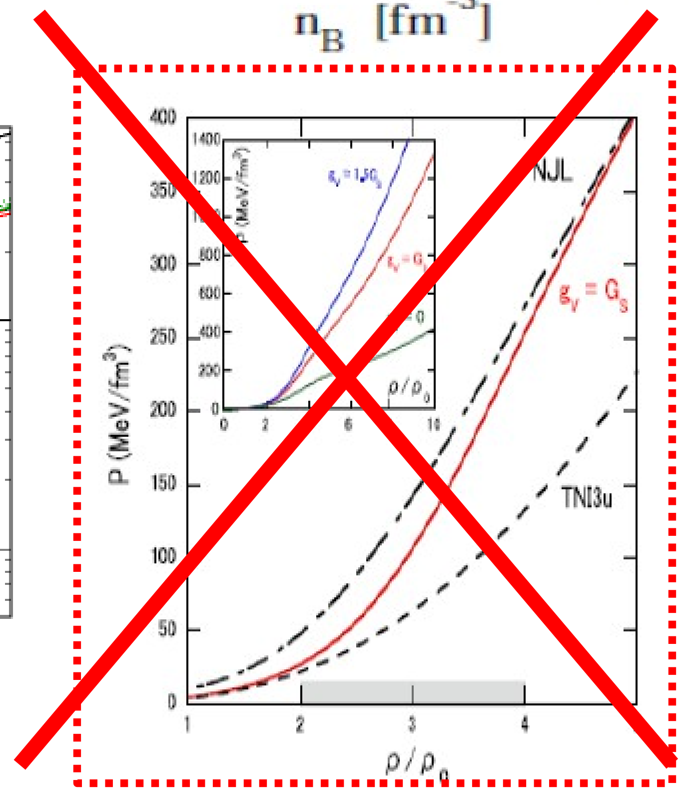
Traditional: Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS



Masquerade problem: $P_H \sim P_Q$







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Maximum mass of neutron stars with a quark core

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Hadron-quark phase transition in dense matter and neutron stars

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HADRONIC PHASE

$$\epsilon=\frac{1}{2}m_\omega^2\bar{\omega}_0^2+\frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2+\frac{1}{2}m_\sigma^2\bar{\sigma}^2+\frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3$$

$$+\frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4+\sum_i \epsilon_{\mathrm{FG}}(\bar{m}_i,\bar{\mu}_i)+\sum_l \epsilon_{\mathrm{FG}}(m_l,\mu_l),$$

$$P=\frac{1}{2}m_\omega^2\bar{\omega}_0^2+\frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2-\frac{1}{2}m_\sigma^2\bar{\sigma}^2-\frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3$$

$$-\frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4+\sum_i P_{\mathrm{FG}}(\bar{m}_i,\bar{\mu}_i)+\sum_l P_{\mathrm{FG}}(m_l,\mu_l)$$

QUARK PHASE

$$\epsilon_Q=\sum_q (\Omega_q+\mu_q\rho_q)+B, \qquad P_Q=-\sum_q \Omega_q-B,$$

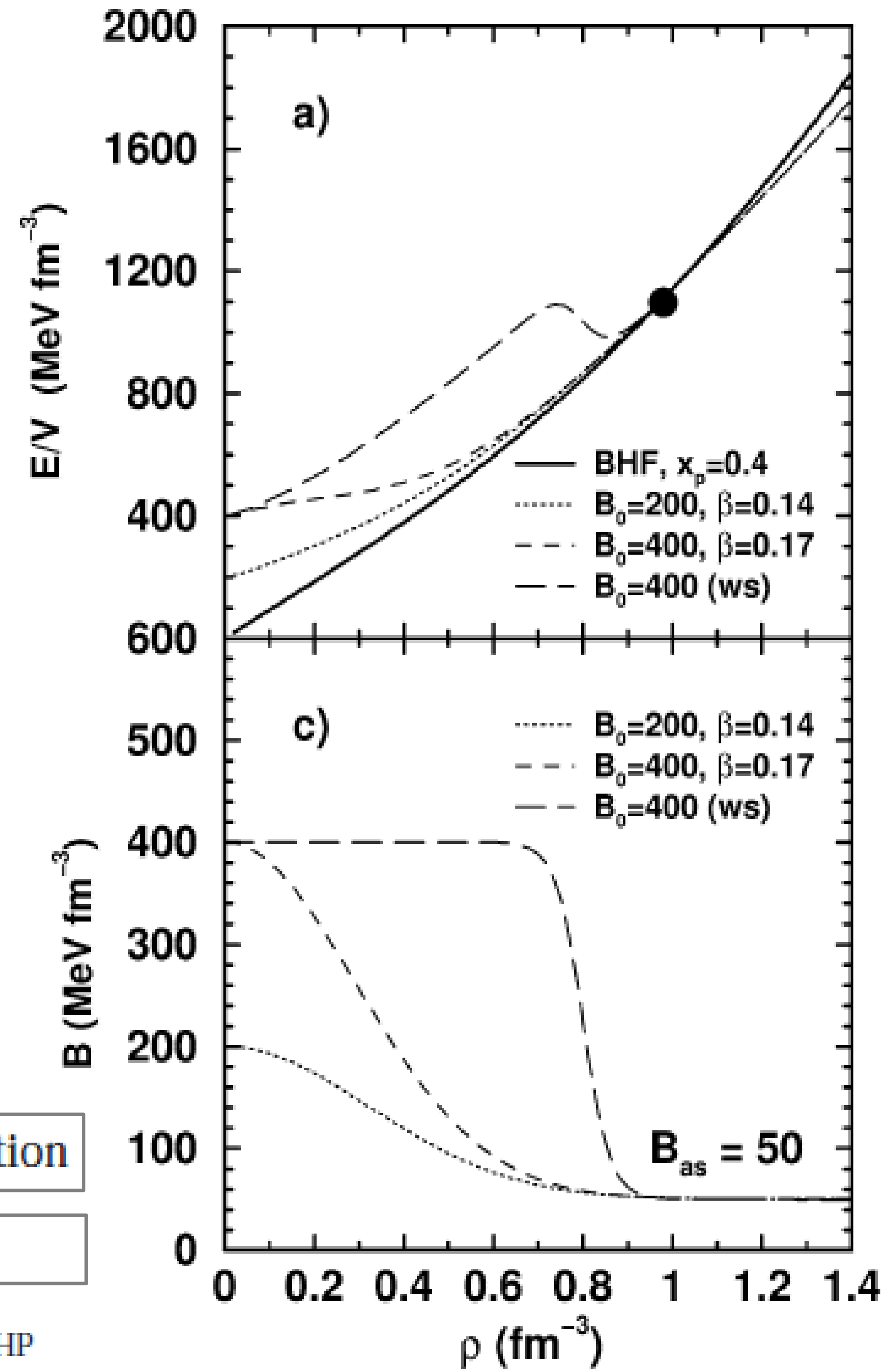
$$B(\rho)=B_{\mathrm{as}}+(B_0-B_{\mathrm{as}})\left[1+\exp\left(\frac{\rho-\bar{\rho}}{\rho_d}\right)\right]^{-1}$$

Phase transition in β-stable neutron star matter

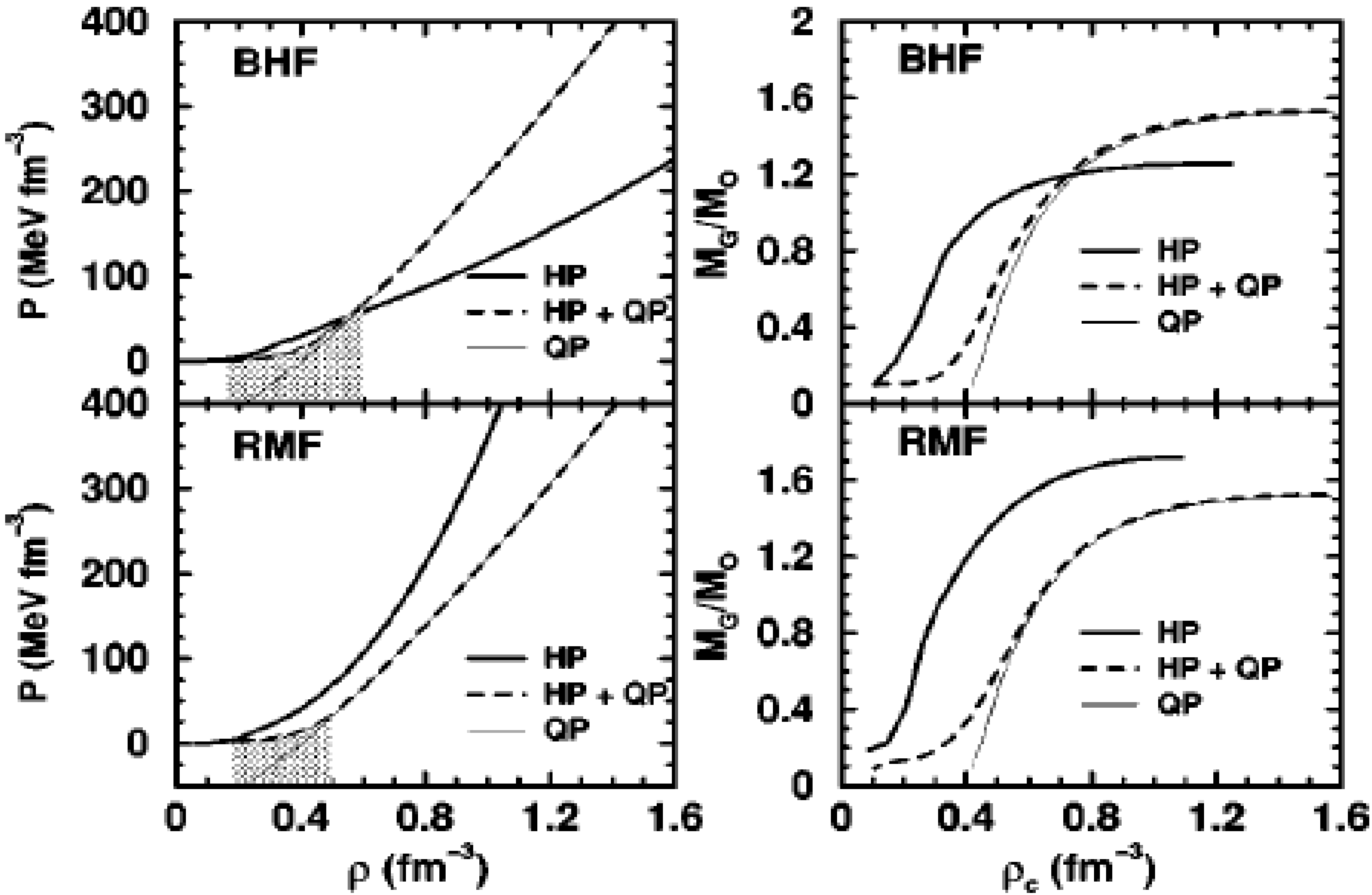
$$P_{\mathrm{HP}}(\mu_e,\mu_n)=P_{\mathrm{QP}}(\mu_e,\mu_n)=P_{\mathrm{MP}} \qquad \boxed{\text{Gibbs condition}}$$

$$\chi\rho_c^{\mathrm{QP}}+(1-\chi)\rho_c^{\mathrm{HP}}=0. \qquad \boxed{\text{Global charge conservation}}$$

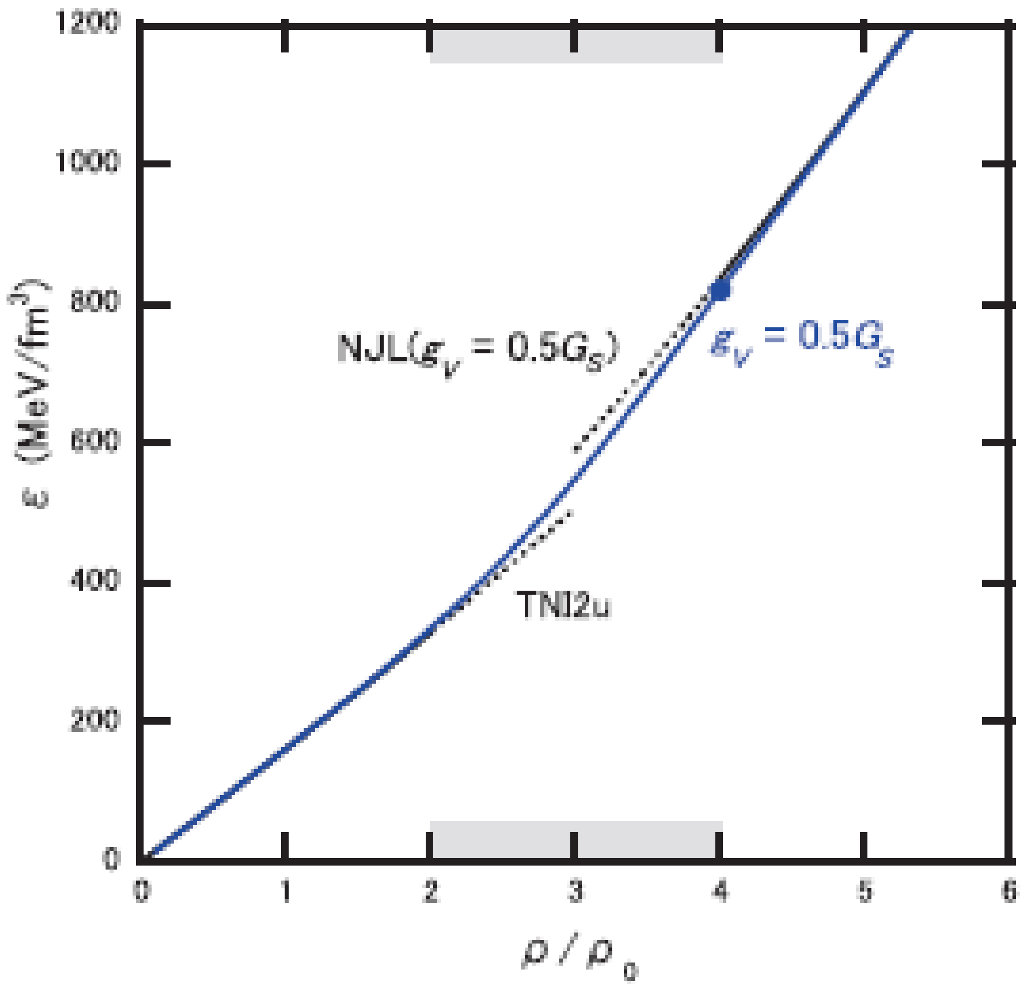
$$\epsilon_{\mathrm{MP}}=\chi\epsilon_{\mathrm{QP}}+(1-\chi)\epsilon_{\mathrm{HP}}, \qquad \rho_{\mathrm{MP}}=\chi\rho_{\mathrm{QP}}+(1-\chi)\rho_{\mathrm{HP}}$$

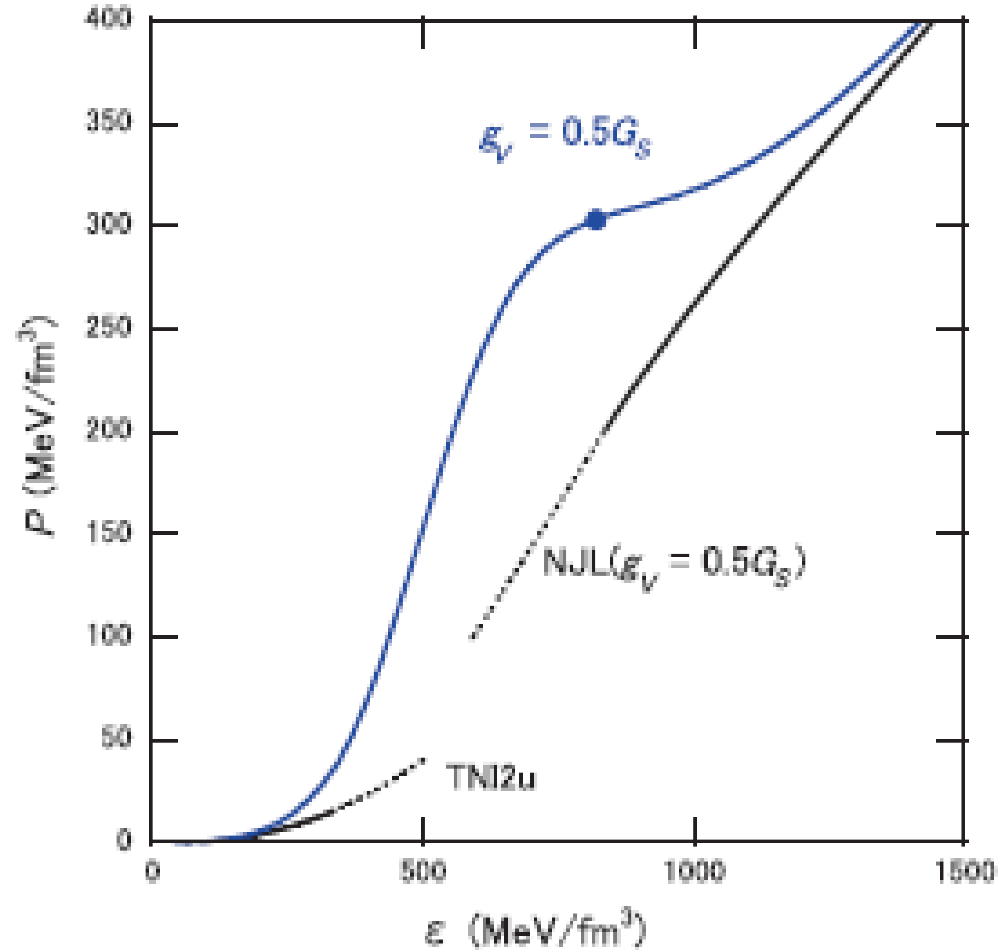


Gibbs Phase transition → Mixed phase, Softening the EoS; Quark Phase: stiff



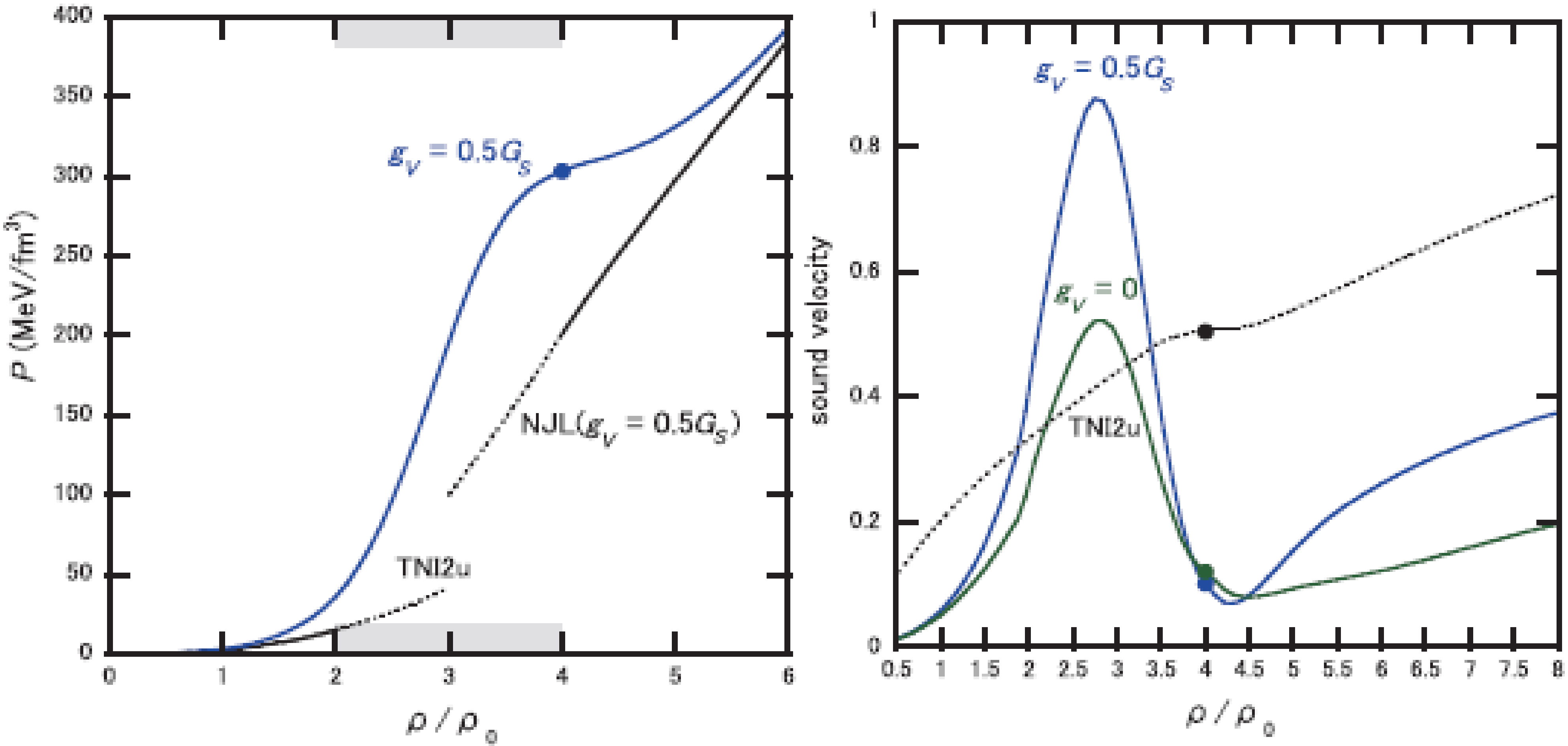
2nd attempt: interpolation between energy densities ε(ρ)





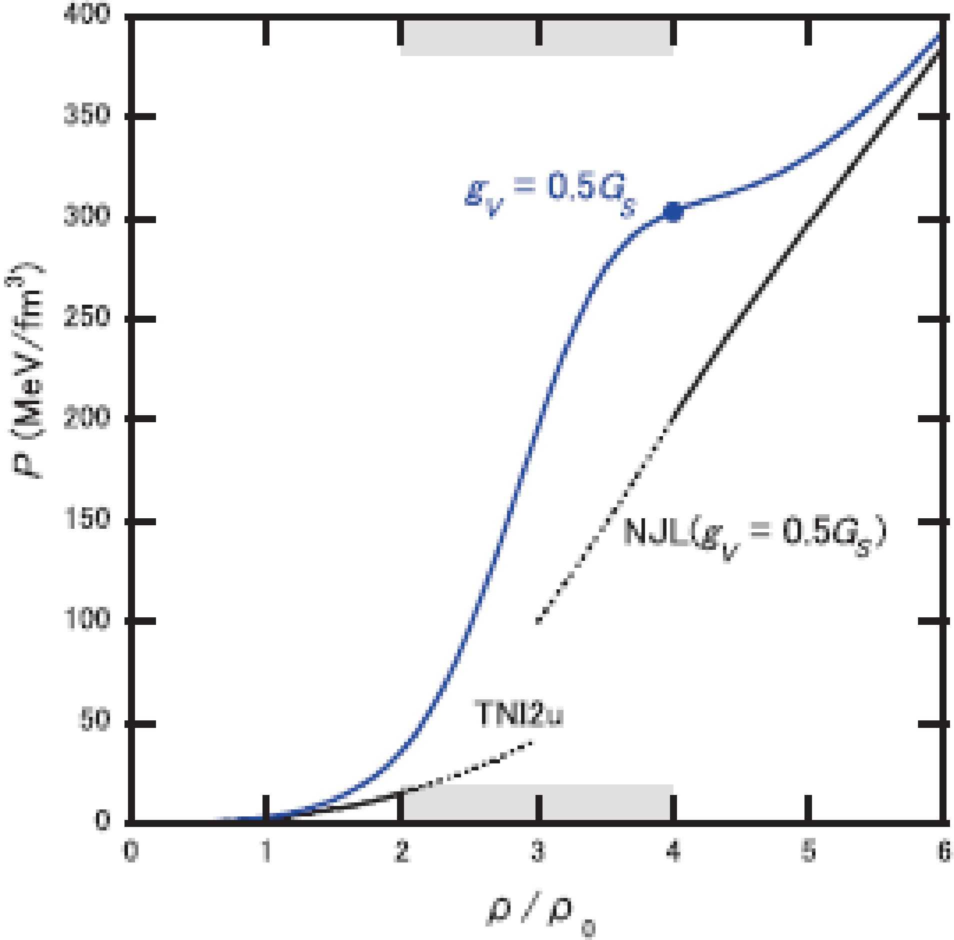
Masuda, Hatsuda, Takatsuka, PTP 073D01 (2013); [arxiv:1212.6803v2]

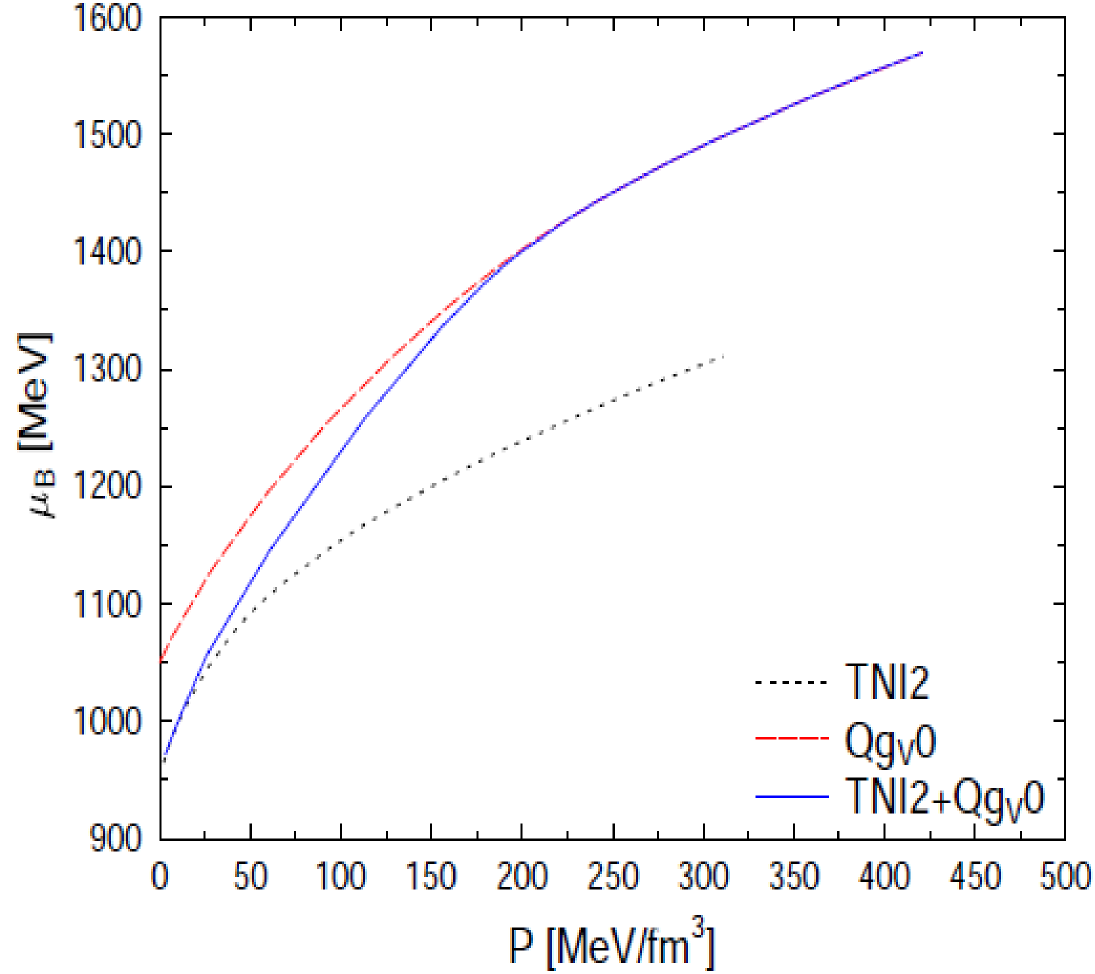
2nd attempt: interpolation between energy densities ε(ρ)



NOTE: After a strong stiffening one observes the "dip" in the speed of sound which is typical for a phase transition and corresponds to the "plateau" in P(ρ)

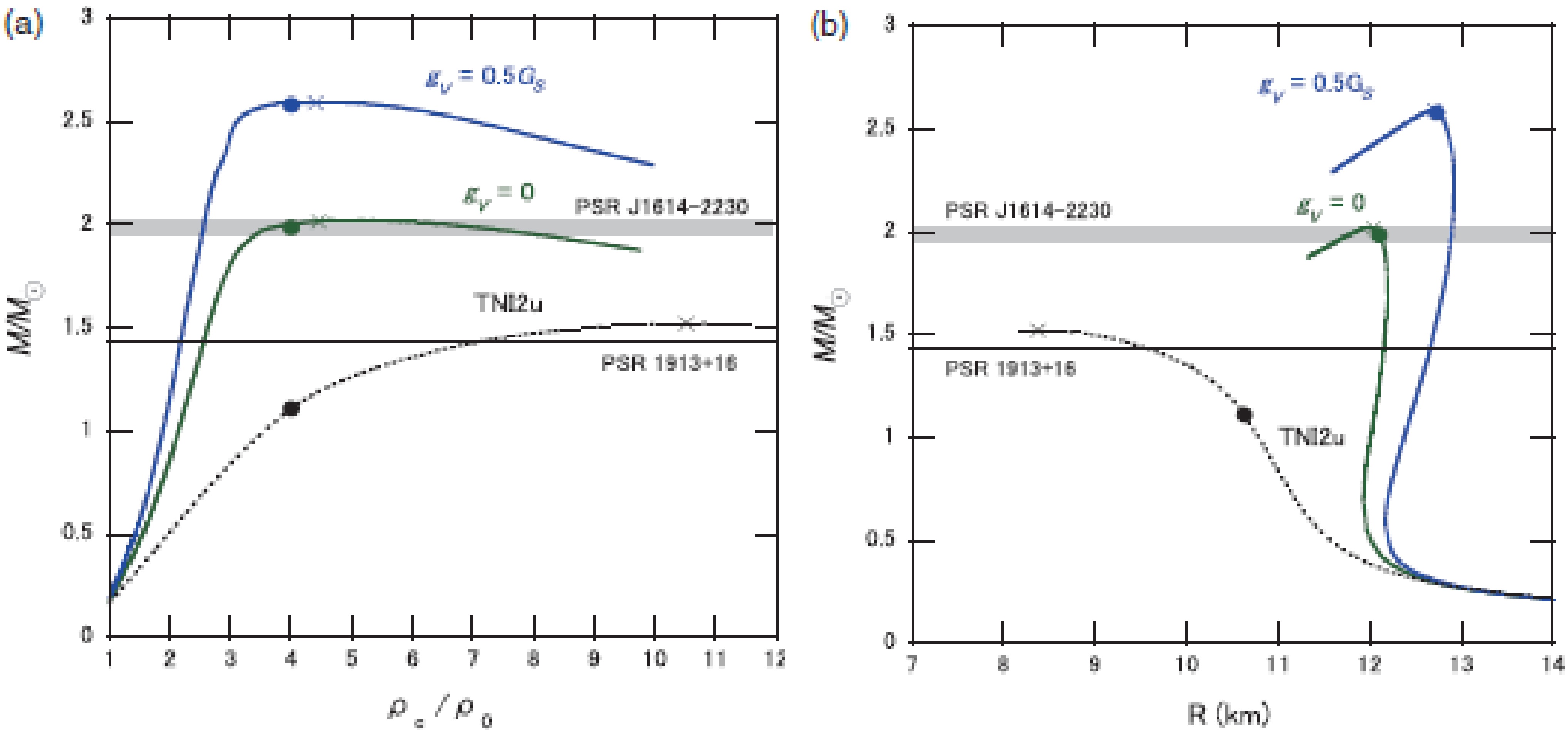
2^{nd} attempt: interpolation between energy densities ε(ρ)





NOTE: This interpolation procedure in ε(ρ) is not only thermodynamically consistent, but also a true interpolation, as can be seen from P(μ) or its inversion μ(P).
Courtesy: Matthias Hempel, using data from arxiv:1212.6803v2
For hybrid star EoS with interpolation in P(μ), see arxiv:1302.6275; arxiv:1310.3803

2^{nd} attempt: interpolation between energy densities $\varepsilon(\rho)$

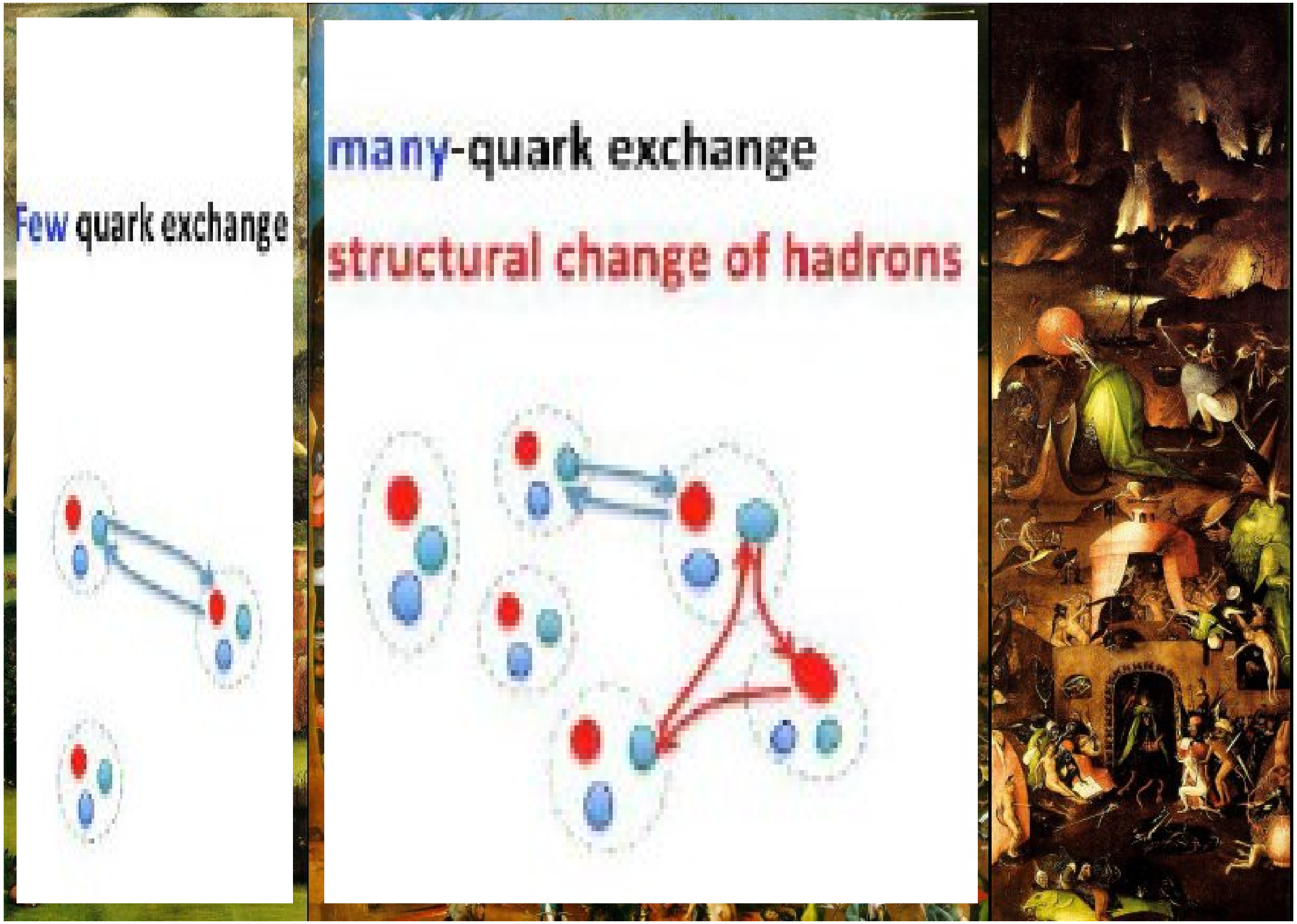


Attention:
Results with interpolation between energy densities $\varepsilon(\rho)$ are different from those with interpolation in pressures $P(\rho)$
Which one is correct? ...

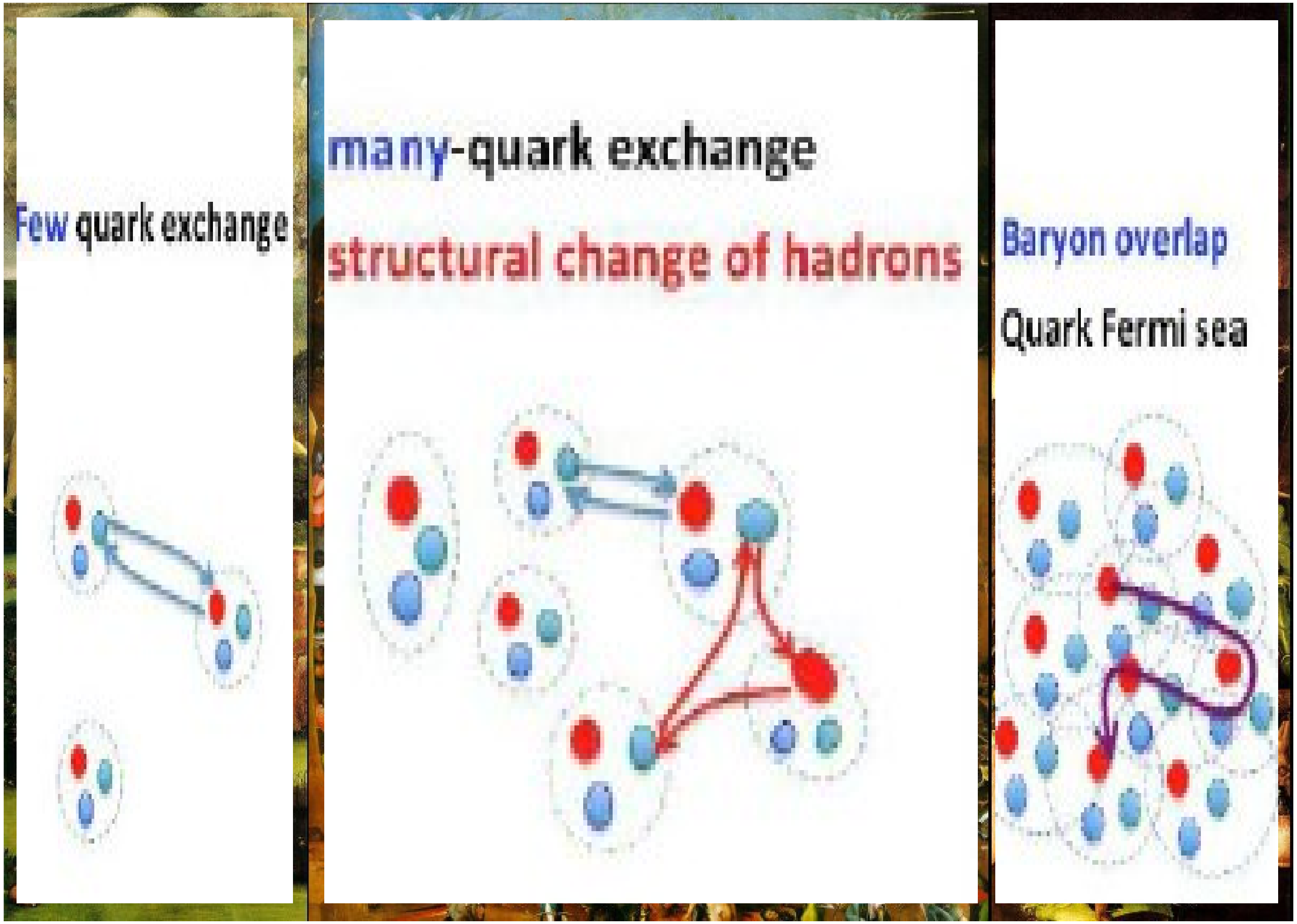
2. Another “three-window picture of dense matter”

2. Another "three-window picture of dense matter"

2. Another “three-window picture of dense matter”

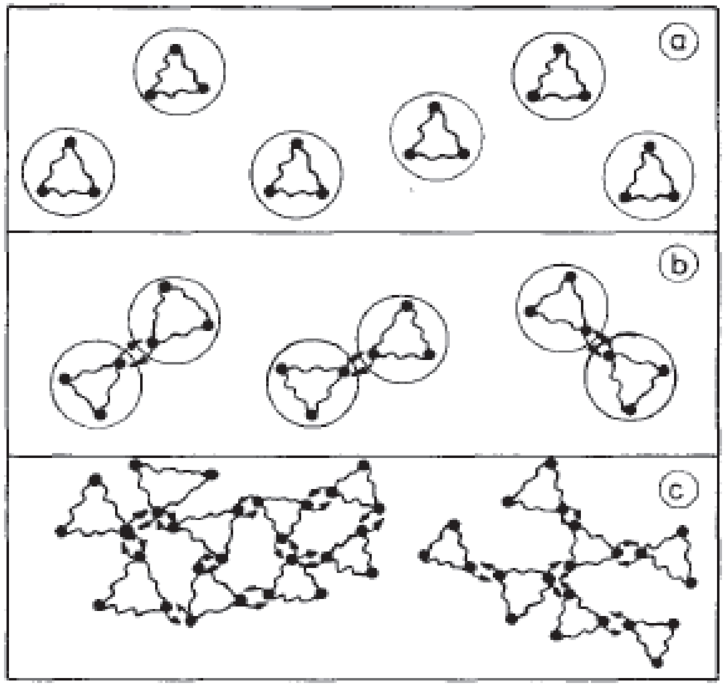


2. Another "three-window picture of dense matter"



Toru Kojo, EPJA 52, 51 (2016)

2.1. Pauli blocking among baryons

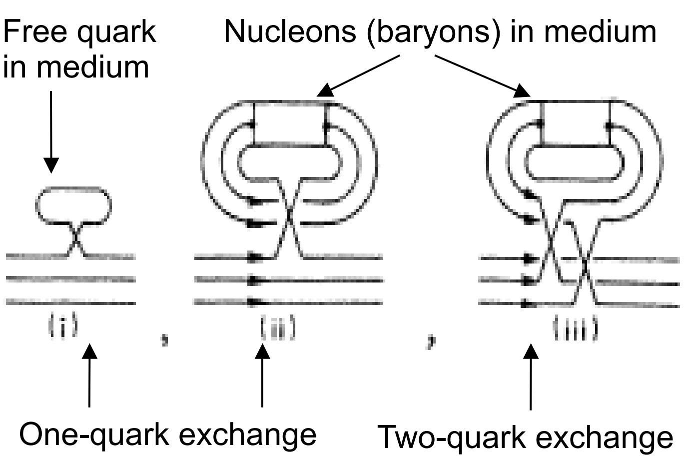


a) Low density: Fermi gas of nucleons (baryons)

b) ~ saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)

Roepke & Schulz, Z. Phys. C 35, 379 (1987); Roepke, DB, Schulz, PRD 34, 3499 (1986)



Nucleon (baryon) self-energy --> Energy shift

$$\begin{aligned}\Delta E^{\mathrm{Pauli}}_{\nu P} =& \sum_{123} |\psi_{\nu P}(123)|^2 [E(1)+E(2)+E(3)-E^0_{\nu P}][f_{\alpha_1}(1)+f_{\alpha_2}(2)+f_{\alpha_3}(3)] \\ &+ \sum_{123}\sum_{456}\sum_{\nu' P'} \psi^*_{\nu P}(123)\psi_{\nu' P'}(456) f_3(E^0_{\nu' P'})\{\delta_{36}\psi_{\nu P}(123)\psi^*_{\nu' P'}(456) - \psi_{\nu P}(453)\psi^*_{\nu' P'}(126)\} \\ &\times [E(1)+E(2)+E(3)+E(4)+E(5)+E(6)-E^0_{\nu P}-E^0_{\nu' P'}] \\ =& \Delta E^{\mathrm{Pauli,free}}_{\nu P} + \Delta E^{\mathrm{Pauli,bound}}_{\nu P} .\end{aligned}$$

PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

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(Received 16 December 1985)

2.1. Pauli blocking among baryons - details

$$\Sigma_\nu(p, p_{Fn}, p_{Fp}) = \sum_{\nu'=\{n,p\}} \sum_{\alpha=1,2} C^{(\alpha)}_{\nu\nu'} W_\alpha(p_{F\nu'}, p)$$

$$W_\alpha(p_{F\nu'}, p) = \frac{\Omega}{2\pi^2} \int_0^{p_{F\nu'}} p'^2 \bar{V}^{(\alpha)}(p, p') dp';$$

$$\bar{V}^{(\alpha)}(p, p') = \frac{1}{2} \int_{-1}^{1} V^{(\alpha)}(\vec{p}, \vec{p}') dz;$$

$$V^{(\alpha)}(\vec{p}, \vec{p}') = \frac{V_0 b}{\Omega m} \left(\frac{15}{2} - \lambda_\alpha^2 (\vec{p} - \vec{p}')^2 \right) \exp(-\lambda_\alpha^2 (\vec{p} - \vec{p}')^2).$$

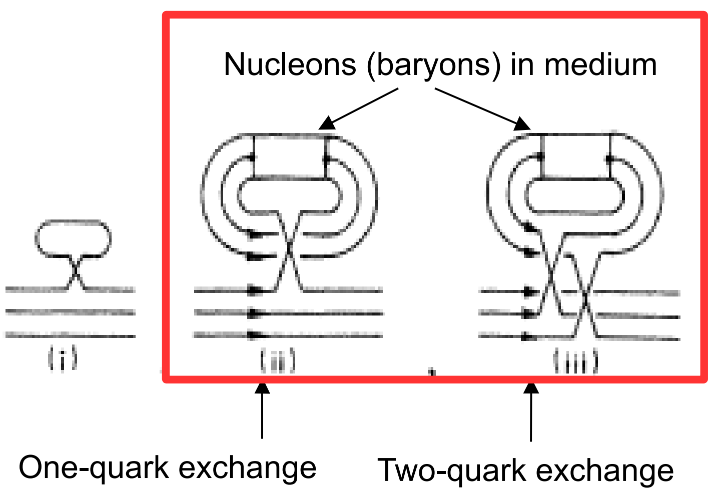
	$C^{(1)}_{n\ \nu}$	$C^{(2)}_{n\ \nu}$
neutron	$-\frac{96}{243}$	$\frac{10}{27}$
proton	$-\frac{28}{81}$	$\frac{8}{27}$

$$b^{-2} = \sqrt{3} m\omega$$

$$\omega = 178.425\ \text{MeV}$$

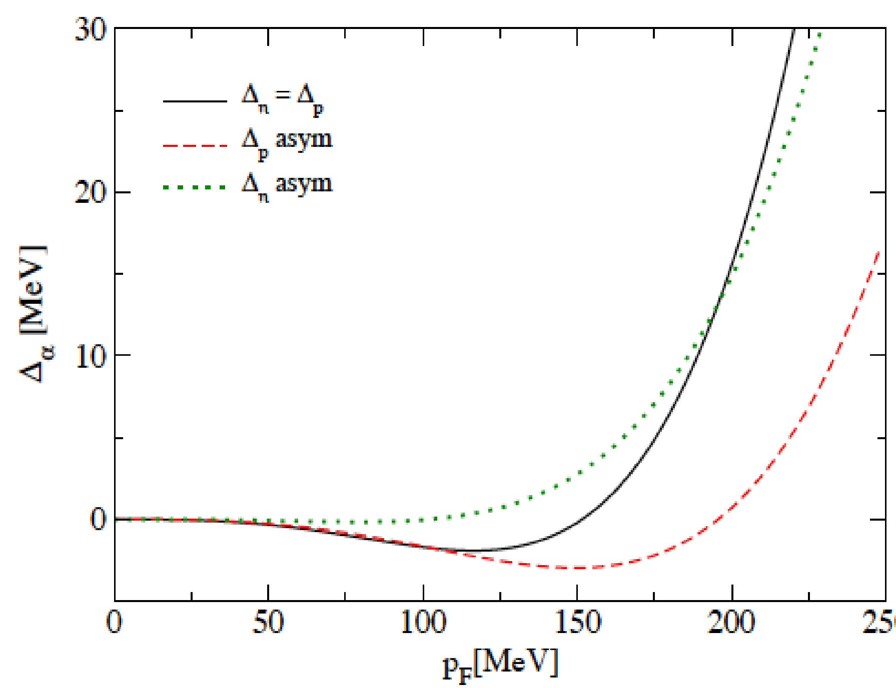
$$m = 350\ \text{MeV} \qquad b = 0.6\ \text{fm}$$

$$V_0 = \frac{9\sqrt{3}\pi^{3/2}}{2} \text{ and } \lambda_\alpha = \frac{b}{\sqrt{3}\alpha}.$$



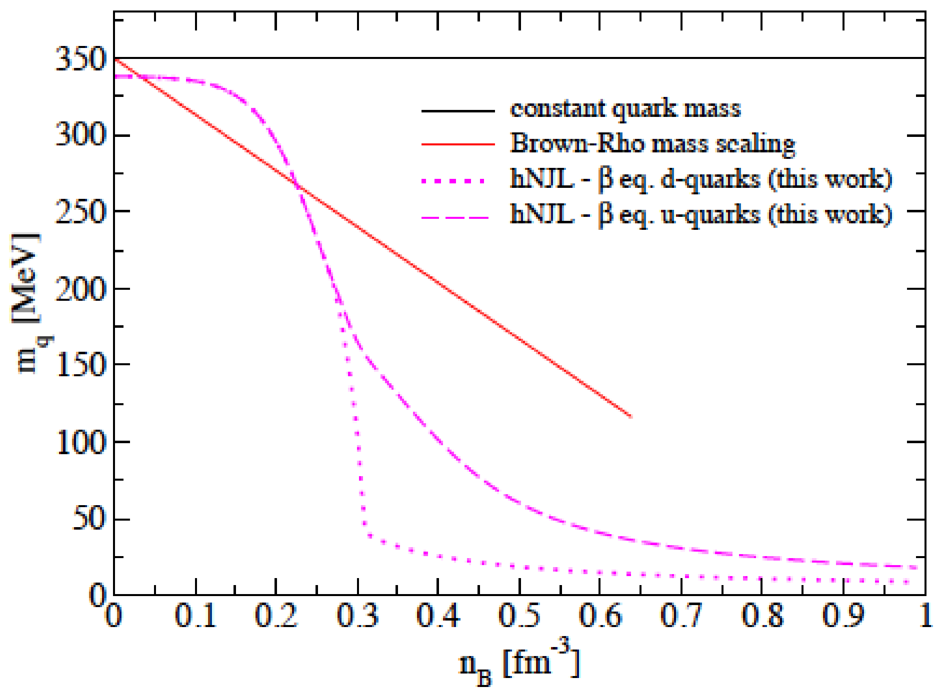
$$\begin{aligned} W_\alpha(p_{F\nu'}, p) = \frac{V_0 b}{32\pi^2 \lambda_\alpha^4 m} & \{12\lambda_\alpha \sqrt{\pi} \left[\operatorname{erf}(\lambda_\alpha(p_{F\nu'} - p)) + \operatorname{erf}(\lambda_\alpha(p_{F\nu'} + p))\right] \\ & + \frac{1}{p} \left[\left(11 - 2\lambda_\alpha^2\, p_{F\nu'}(p_{F\nu'} + p)\right) e^{-\lambda_\alpha^2 (p_{F\nu'} + p)^2} \right. \\ & + \left. \left(11 - 2\lambda_\alpha^2\, p_{F\nu'}(p_{F\nu'} - p)\right) e^{-\lambda_\alpha^2 (p_{F\nu'} - p)^2} \right] \} \end{aligned}$$

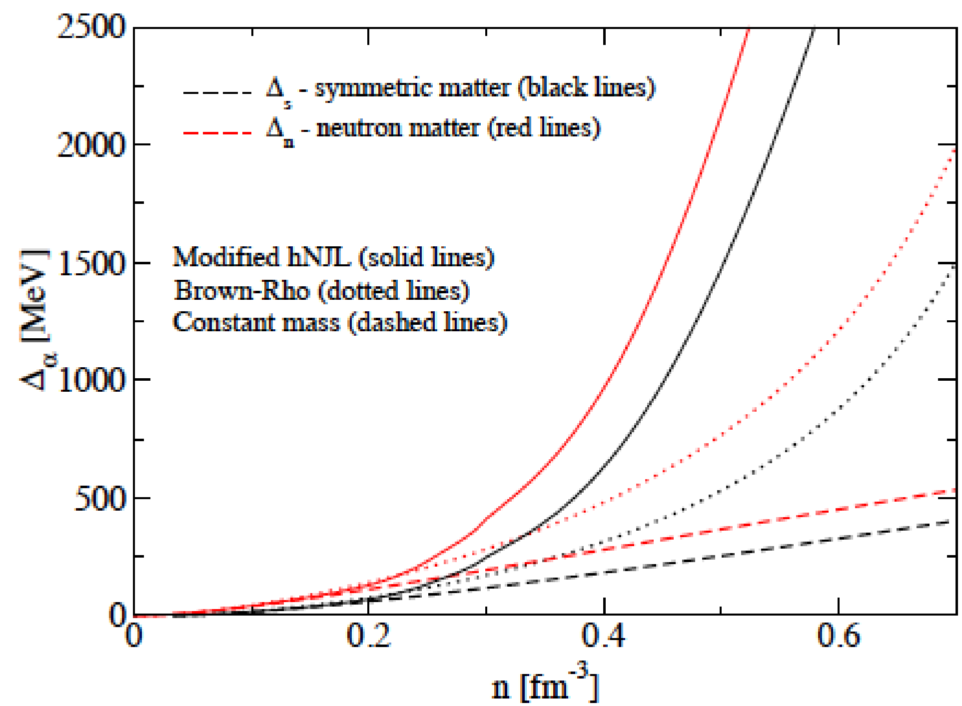
$$\Delta^{Pauli}_{\nu_A, P} = \frac{1}{24\sqrt{3\pi}} \frac{b}{m} \sum_{\nu'} [15 a_{\nu,\nu'} P_F(\nu')^3 + \frac{17}{12} b_{\nu,\nu'} b^2 (P^2 + P_F(\nu')^2) P_F(\nu')^3]$$



2.1. Pauli blocking among baryons – details

New aspect: chiral restoration --> dropping quark mass





Increased baryon swelling at supersaturation densities:
--> dramatic enhancement of the Pauli repulsion !!

D.B., H. Grigorian, G. Roepke: “Quark exchange effects in dense nuclear matter”, (in prep.)

2.2. Pauli blocking among baryons – results

New EoS: Joining RMF (Linear Walecka) for pointlike baryons with chiral Pauli blocking

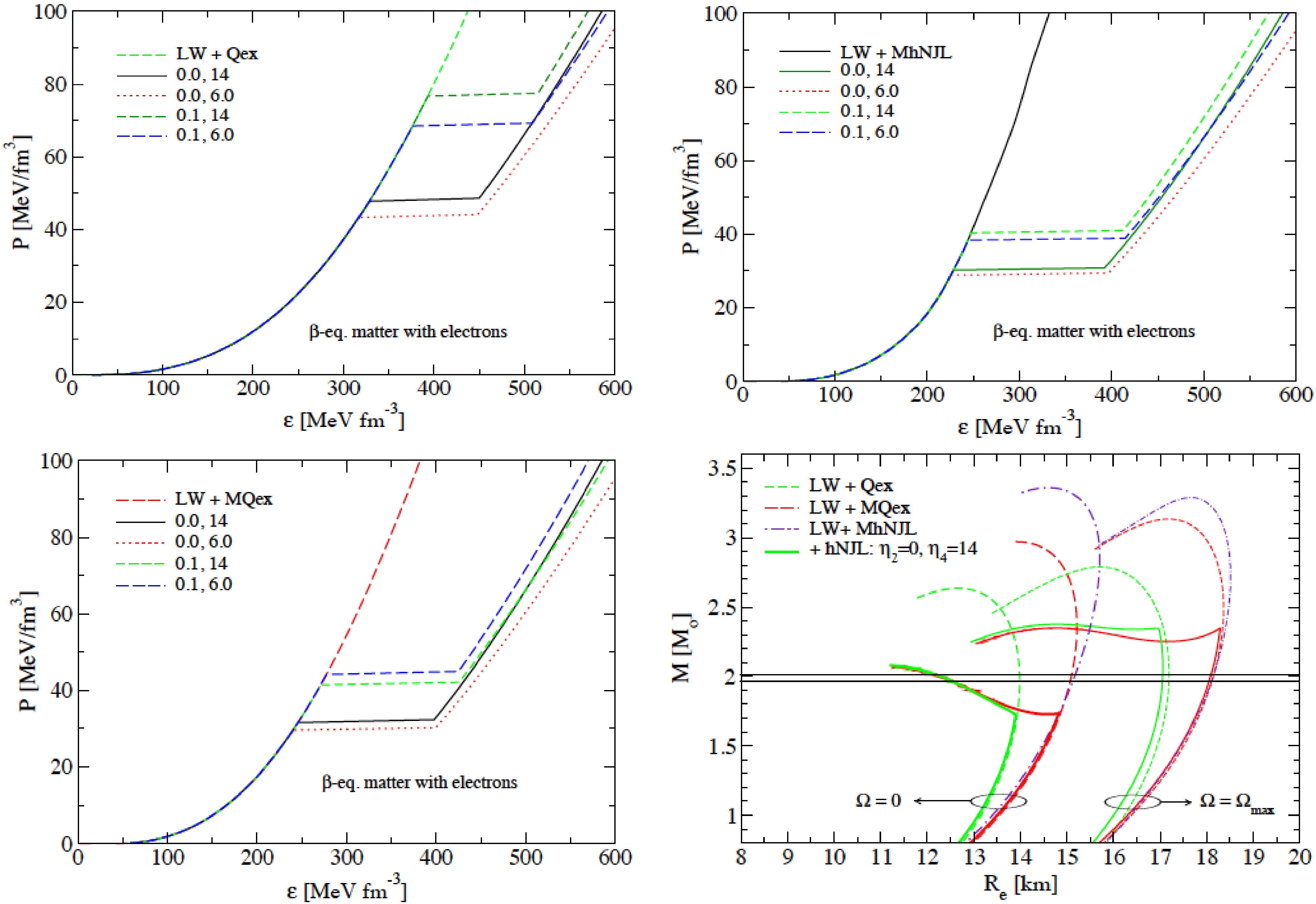
$$p = \frac{1}{4\pi^2}\sum_\nu \left[-E_\nu^* m_\nu^{*2} p_{F\nu} + \frac{2}{3}E_\nu^* p_{F\nu}^3 + m_\nu^{*4}\log\left(\frac{E_\nu^* + p_{F\nu}}{m_\nu^*}\right)\right]$$
$$+ \frac{1}{2}\left(\frac{g_\omega}{m_\omega}\right)^2 n^2 - \frac{1}{2}\left(\frac{g_\sigma}{m_\sigma}\right)^2 n_s^2 + p_{ex};$$

$$\epsilon = \frac{1}{4\pi^2}\sum_\nu \left[2\,E_\nu^{*3} p_{F\nu} - E_\nu^* m_\nu^{*2} p_{F\nu} - m_\nu^{*4}\log\left(\frac{E_I^* + p_{F\nu}}{m_\nu^*}\right)\right]$$
$$+ \frac{1}{2}\left(\frac{g_\omega}{m_\omega}\right)^2 n^2 + \frac{1}{2}\left(\frac{g_\sigma}{m_\sigma}\right)^2 n_s^2 + \epsilon_{ex},$$

$$\mu_{ex,\nu} = \Delta_\nu(n,x) = \Sigma_\nu(p_{F,\nu}; p_{Fn,}, p_{Fp}),$$
$$\epsilon_{ex} = \sum_\nu \int_0^n dn' \{x\Delta_p(n',x) + (1-x)\Delta_n(n',x)\},$$
$$p_{ex} = \sum_\nu \mu_{ex,\nu}\, n_\nu - \epsilon_{ex},$$

$$n_{s,\nu} = \frac{m_\nu^*}{\pi^2}\left[E_\nu^* p_{F\nu} - m_\nu^{*2}\log\left(\frac{E_\nu^* + p_{F\nu}}{m_\nu^*}\right)\right],$$
$$E_\nu^* = \sqrt{m_\nu^{*2} + p_{F\nu}^2}$$
$$n_\nu = \frac{p_{F\nu}^3}{3\pi^2},$$
$$m_\nu^* = m_\nu - \left(\frac{g_\sigma}{m_\sigma}\right)^2 n_{s,\nu},$$
$$\mu_\nu = E_\nu^* + \left(\frac{g_\omega}{m_\omega}\right)^2 n_\nu + \mu_{ex,\nu}.$$

2.2. Pauli blocking among baryons – results



2.3. Pauli blocking among baryons – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:
- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high n
- quark exchange among baryons -> six-quark wavefunction -> “bag melting” -> deconfinement

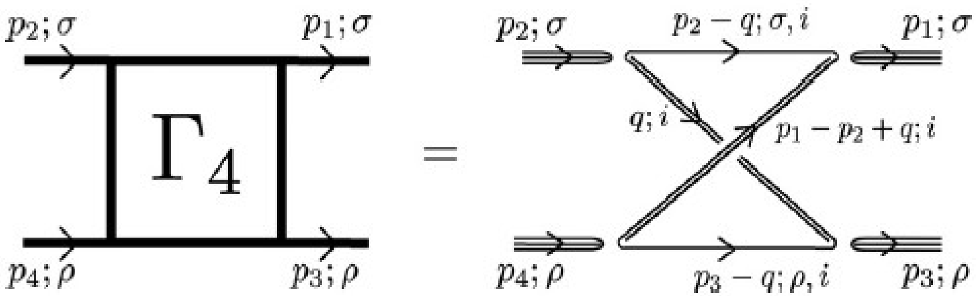
Chiral stiffening of nuclear matter --> reduces onset density for deconfinement

Hybrid EoS:
Convenient generalization of RMF models,
Take care: eventually aspects of quark exchange already in density dependent vertices!

Other baryons:
- hyperons
- deltas
Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

Relativistic generalization:
Box diagrams of quark-diquark model ...

K. Maeda, Ann. Phys. 326 (2011) 1032



2.4. Pauli blocking effect → Excluded volume

Well known from modeling dissociation of clusters in the supernova EoS:

- excluded volume: Lattimer-Swesty (1991), Shen-Toki-Oyematsu-Sumiyoshi (1996), ...
- Pauli blocking: Roepke-Grigo-Sumiyoshi-Shen (2003), Typel et al. PRC 81 (2010)
- excl. Vol. vs. Pauli blocking: Hempel, Schaffner-Bielich, Typel, Roepke PRC 84 (2011)

Here: nucleons as quark clusters with finite size --> excluded volume effect !

Available volume fraction: $\Phi = V_{\rm av}/V = 1 - v \sum_{i=n,p} n_i, \quad v = \frac{1}{2}\frac{4\pi}{3}(2r_{\rm nuc})^3 = 4V_{\rm nuc}$

Equations of state for T=0 nuclear matter:

$$p_{\rm tot}(\mu_{\rm n},\mu_{\rm p}) = \frac{1}{\Phi}\sum_{i=n,p} p_i + p_{\rm mes},$$

$$\varepsilon_{\rm tot}(\mu_{\rm n},\mu_{\rm p}) = -p_{\rm tot} + \sum_{i=n,p} \mu_i n_i,$$

$$p_i = \frac{1}{4}\left(E_i n_i - m_i^* n_i^{(s)}\right),$$

$$n_i = \frac{\Phi}{3\pi^3} k_i^3,$$

$$n_i^{(s)} = \frac{\Phi m_i^*}{2\pi^2}\left[E_i k_i - (m_i^*)^2 \ln\frac{k_i + E_i}{m_i^*}\right],$$

$$E_i = \sqrt{k_i^2 + (m_i^*)^2} = \mu_i - V_i - \frac{v}{\Phi}\sum_{j=p,n} p_j,$$

Effective mass: $m_i^* = m_i - S_i.$

Scalar meanfield: $S_i \sim n_i^{(s)}$

Vector meanfield: $V_i \sim n_i$

2.5. Stiff quark matter at high densities

S. Benic, Eur. Phys. J. A 50, 111 (2014)

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8\ ,\ \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2\ ,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\mathrm{MF}} = \bar{q}(i\partial\!\!\!/ - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U\ ,$

$$M = m + 2\frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle + 4\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle\langle q^\dagger q\rangle^2\ ,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle - 4\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle\ ,$$

$$U = \frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle^2 + 3\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^4 - 3\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle^2 - \frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle^2 - 3\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^4\ .$$

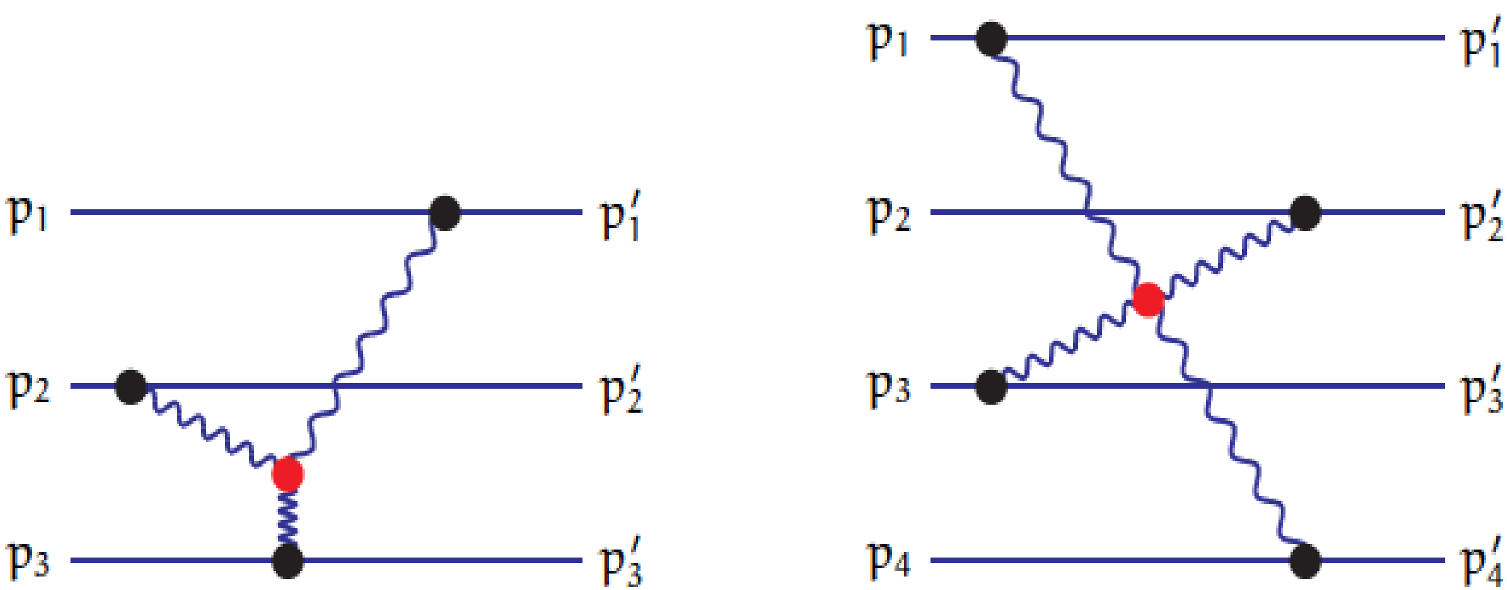
Thermodynamic Potential:

$$\Omega = U - 2N_f N_c \int \frac{d^3p}{(2\pi)^3}\left\{E + T\log[1 + e^{-\beta(E-\tilde{\mu}_q)}] + T\log[1 + e^{-\beta(E+\tilde{\mu}_q)}]\right\} + \Omega_0$$

2.5 Support for universal stiffening of hadron and quark matter at high densities

Universal repulsion from multi-pomeron exchanges in baryon-baryon scattering.
(Th. A. Rijken, private communication)

Y. Yamamoto, T. Furumoto, N. Yasutake, Th.A. Rijken; EPJA 52 (3) (2016)

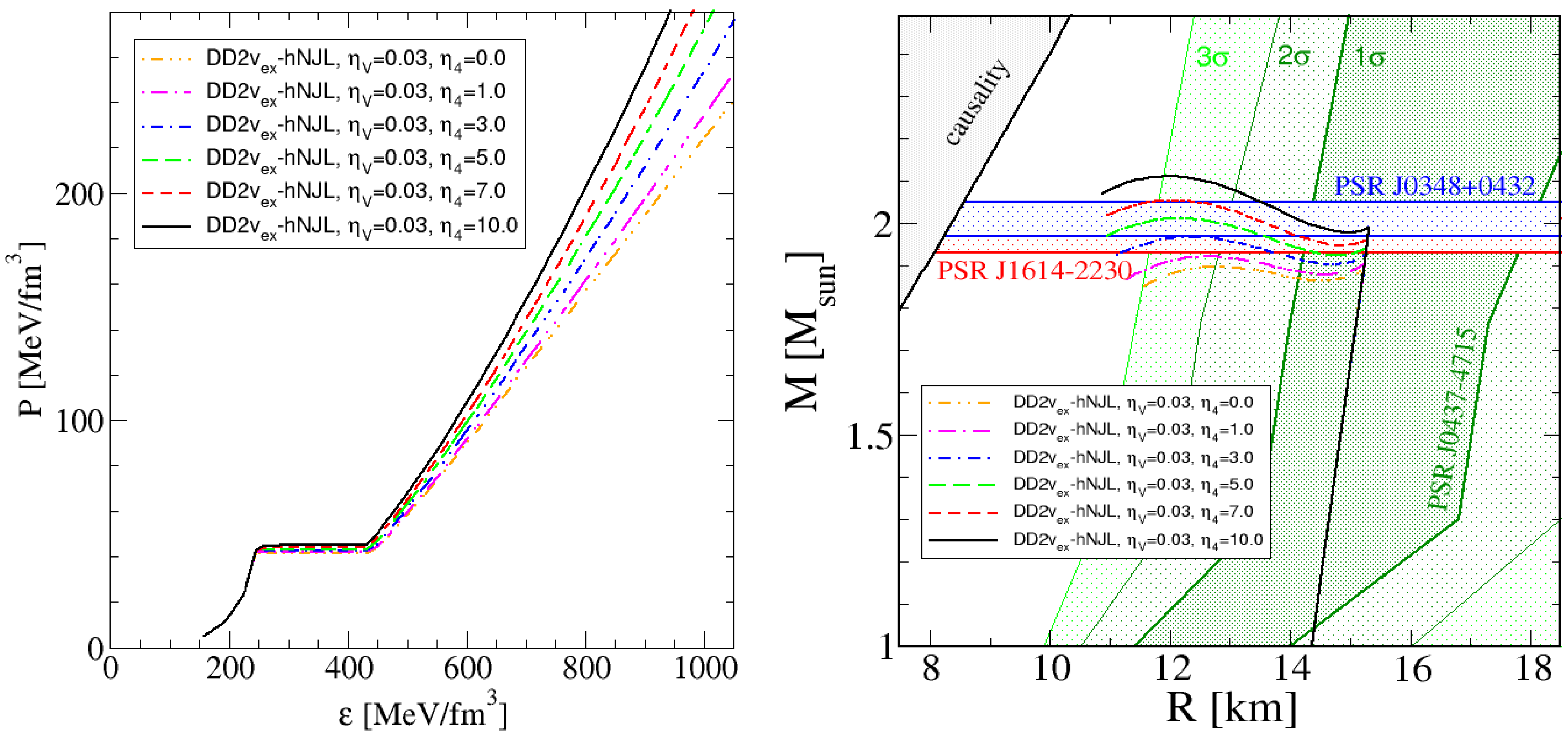


(a) Triple-pomeron vertex (b) Quartic-pomeron vertex

The pomerons (wavy lines) couple to different quarks (solid lines) in quark matter (as in the hNJL Lagrangian) or to quarks in different baryons in nuclear matter (giving rise to repulsive 3- and 4- body interactions).

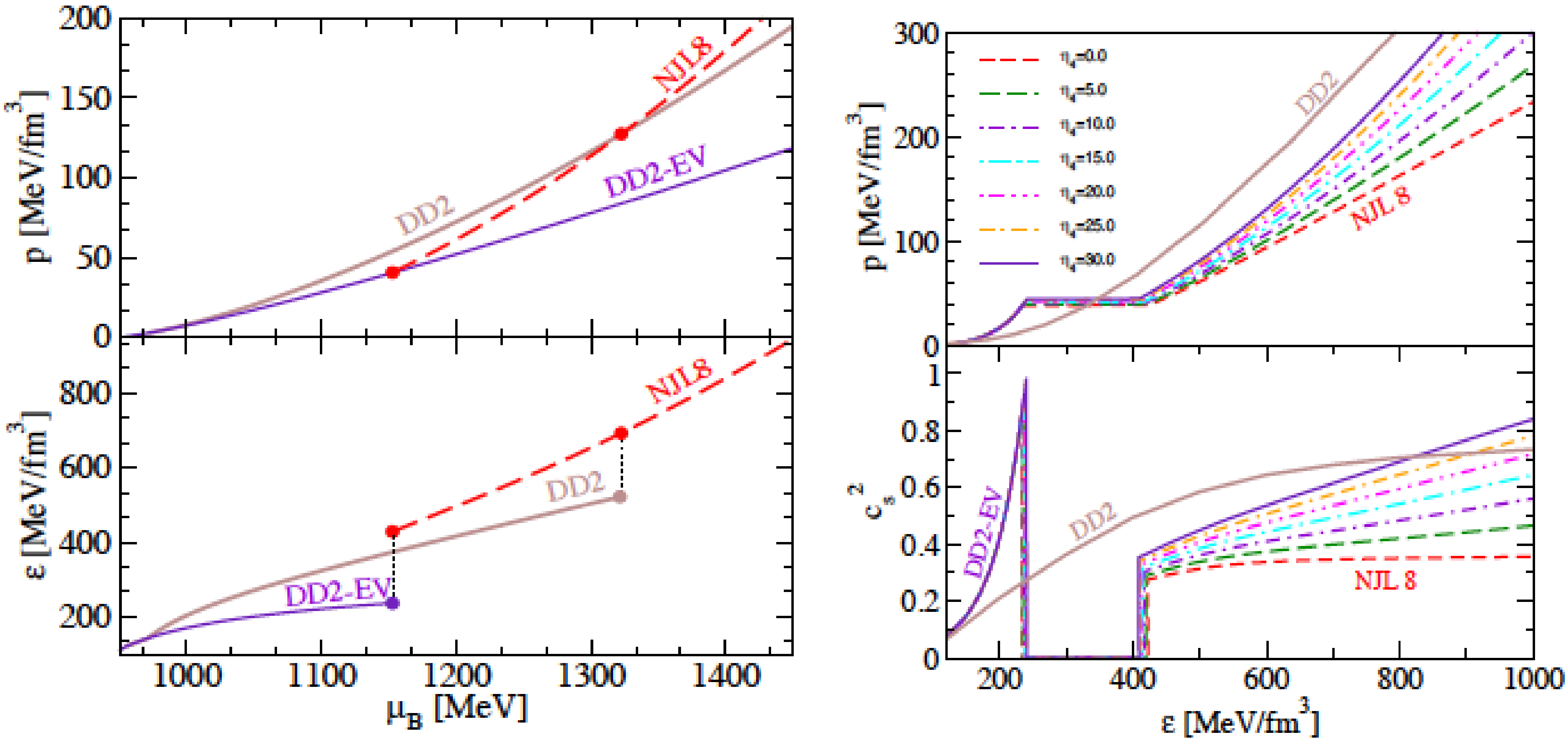
Result: high-mass twins ↔ 1st order PT

S. Benic, D. Blaschke, D. Alvarez-Castillo, T. Fischer, S. Typel, arxiv:1411.2856



Hybrid EoS supports M-R sequences with high-mass twin compact stars

2.5. Stiff quark matter at high densities

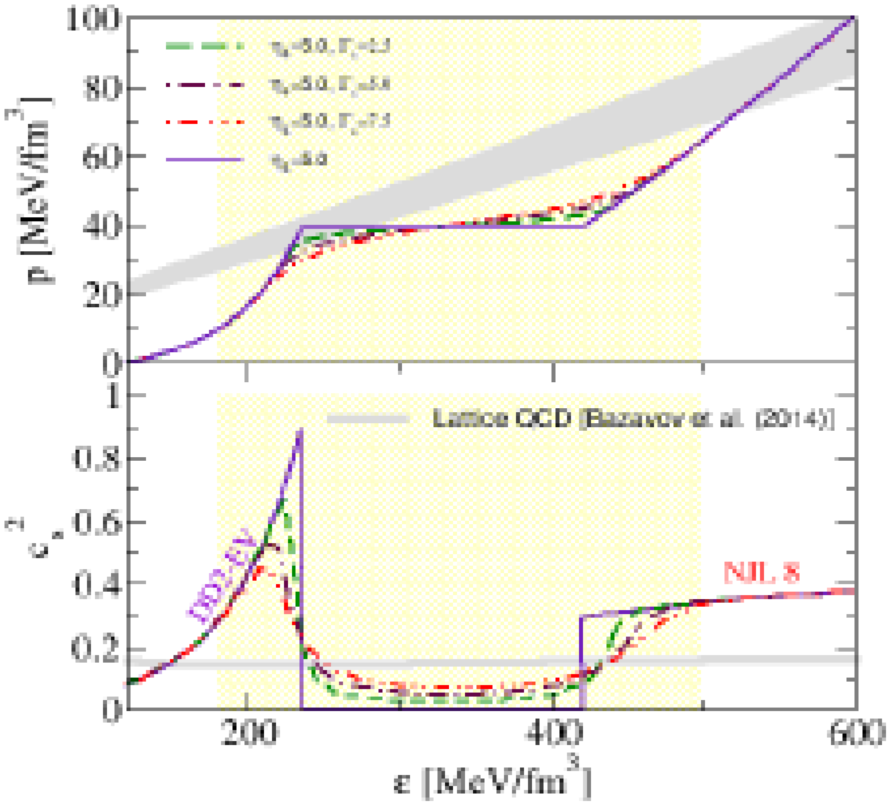


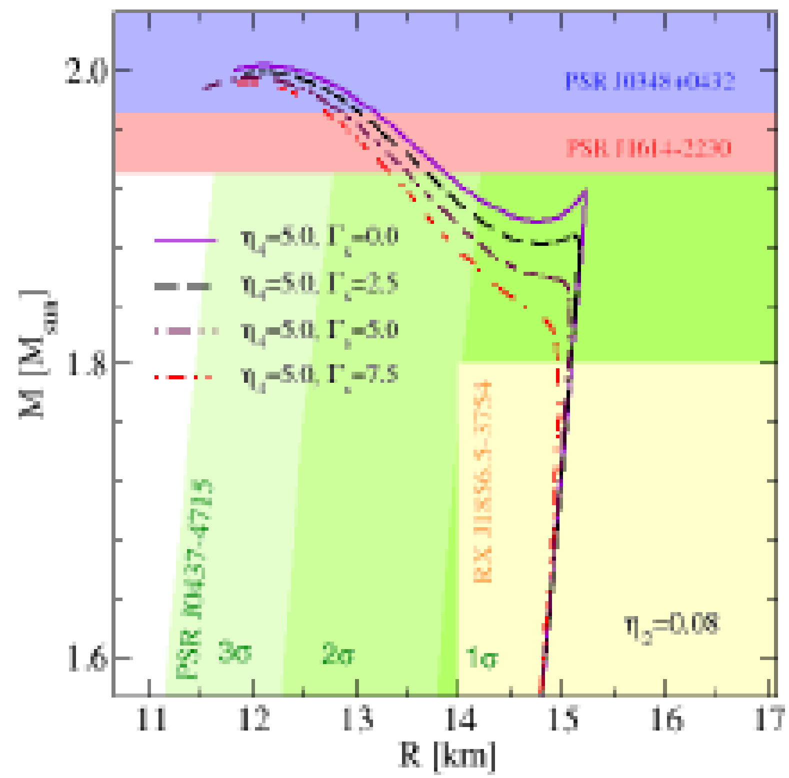
Here: Stiffening of dense hadronic matter by excluded volume in density-dependent RMF

S. Benic, D.B., D. Alvarez-Castillo, T. Fischer, S. Typel, A&A 577, A40 (2015)

2.5. Stiff quark matter at high densities

Estimate effects of structures in the phase transition region (“pasta”)



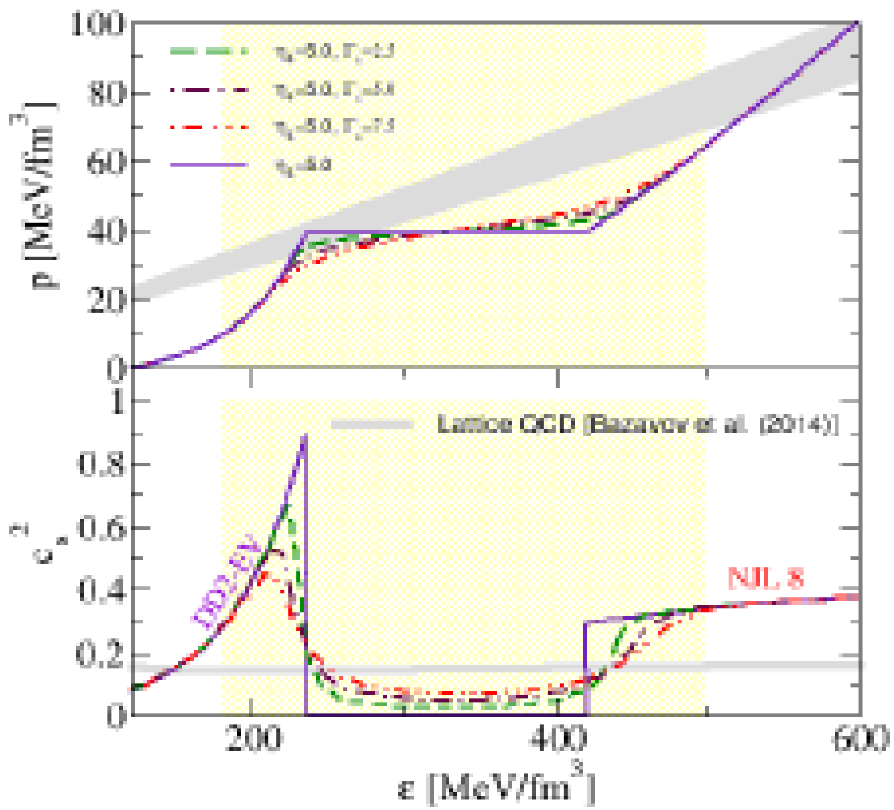


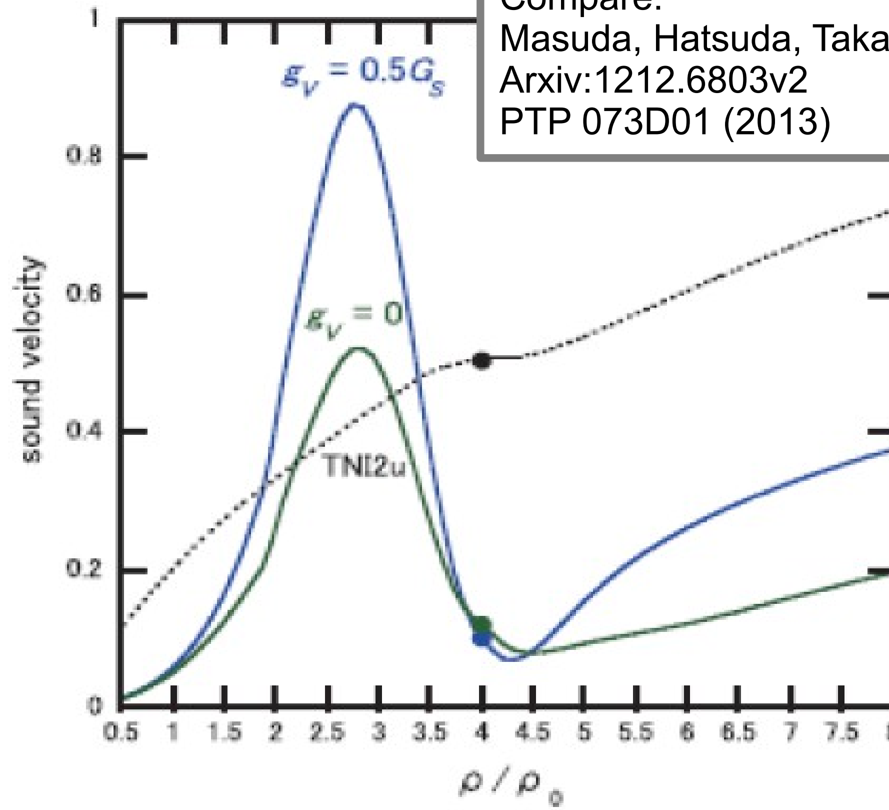
High-mass Twins relatively robust against “smoothing” the Maxwell transition construction

D. Alvarez-Castillo, D.B., arxiv:1412.8463; Phys. Part. Nucl. 46 (2015) 846

2.5. Stiff quark matter at high densities

Estimate effects of structures in the phase transition region (“pasta”)





Compare:
Masuda, Hatsuda, Takatsuka
Arxiv:1212.6803v2
PTP 073D01 (2013)

High-mass Twins relatively robust against “smoothing” the Maxwell transition construction

D. Alvarez-Castillo, D.B., arxiv:1412.8463; Phys. Part. Nucl. 46 (2015) 846

2.6. Rotation

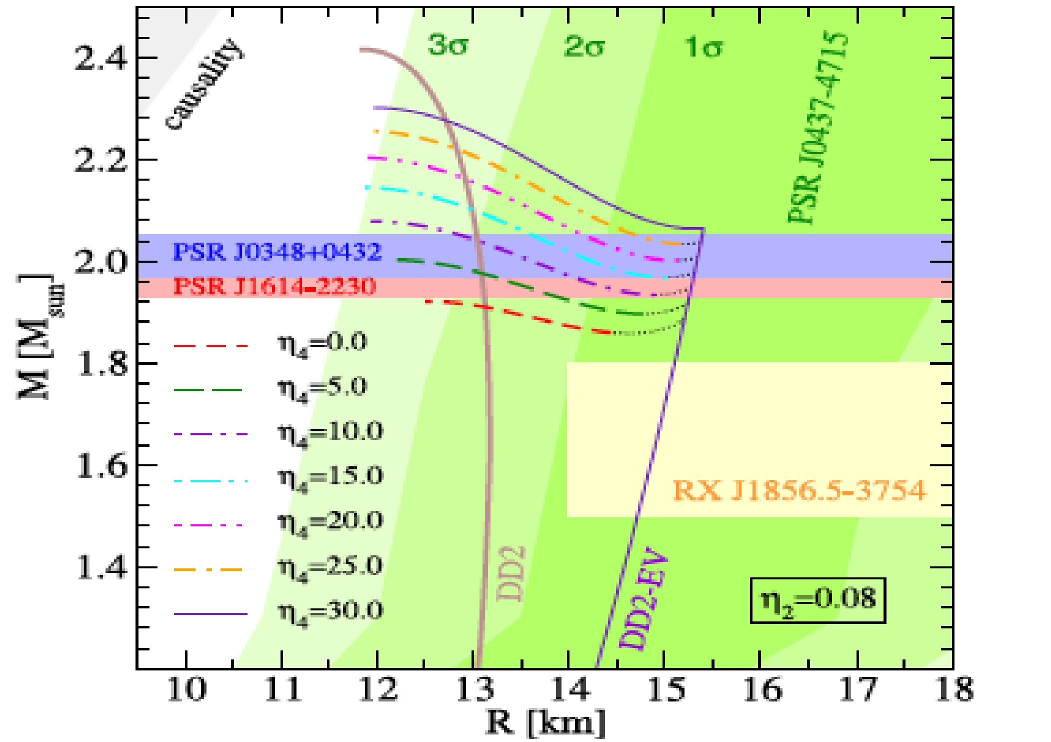
- existence of 2 M_sun pulsars and possibility of high-mass twins raises question for their inner structure: (Q)uark or (N)ucleon core ??
  --> degenerate solutions
  --> transition from N to Q branch

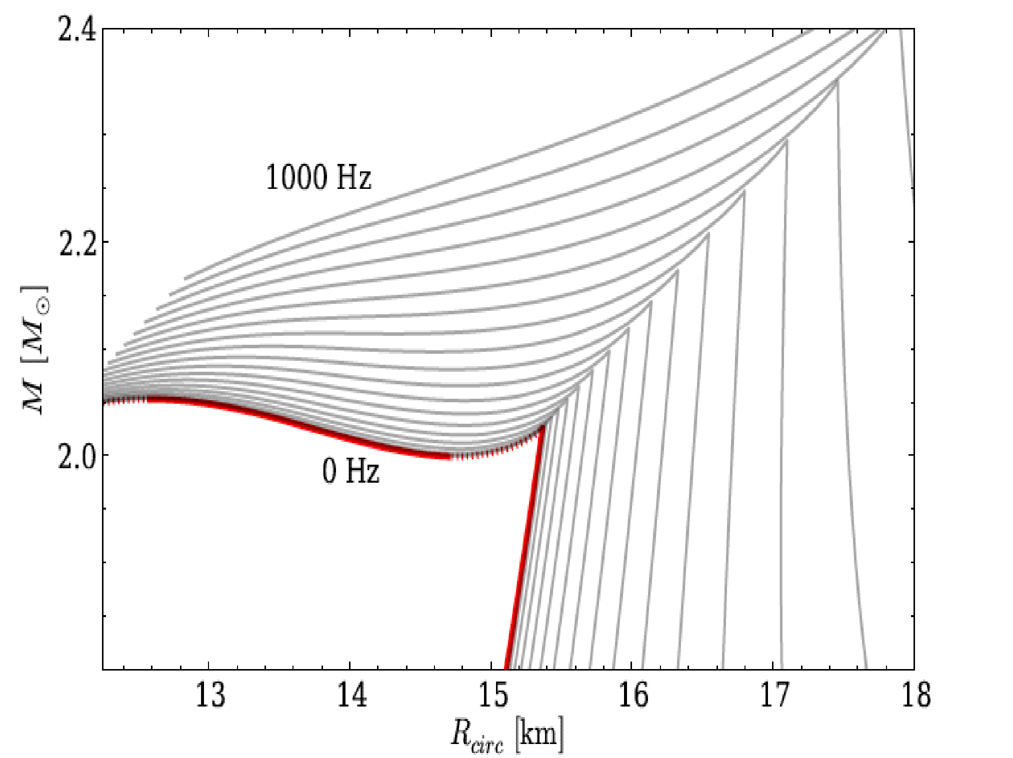
- PSR J1614-2230 is millisecond pulsar, period P = 3.41 ms, consider rotation !

- transitions N --> Q must be considered for rotating configurations:
  --> how fast can they be?

(angular momentum J and baryon mass should be conserved simultaneously)

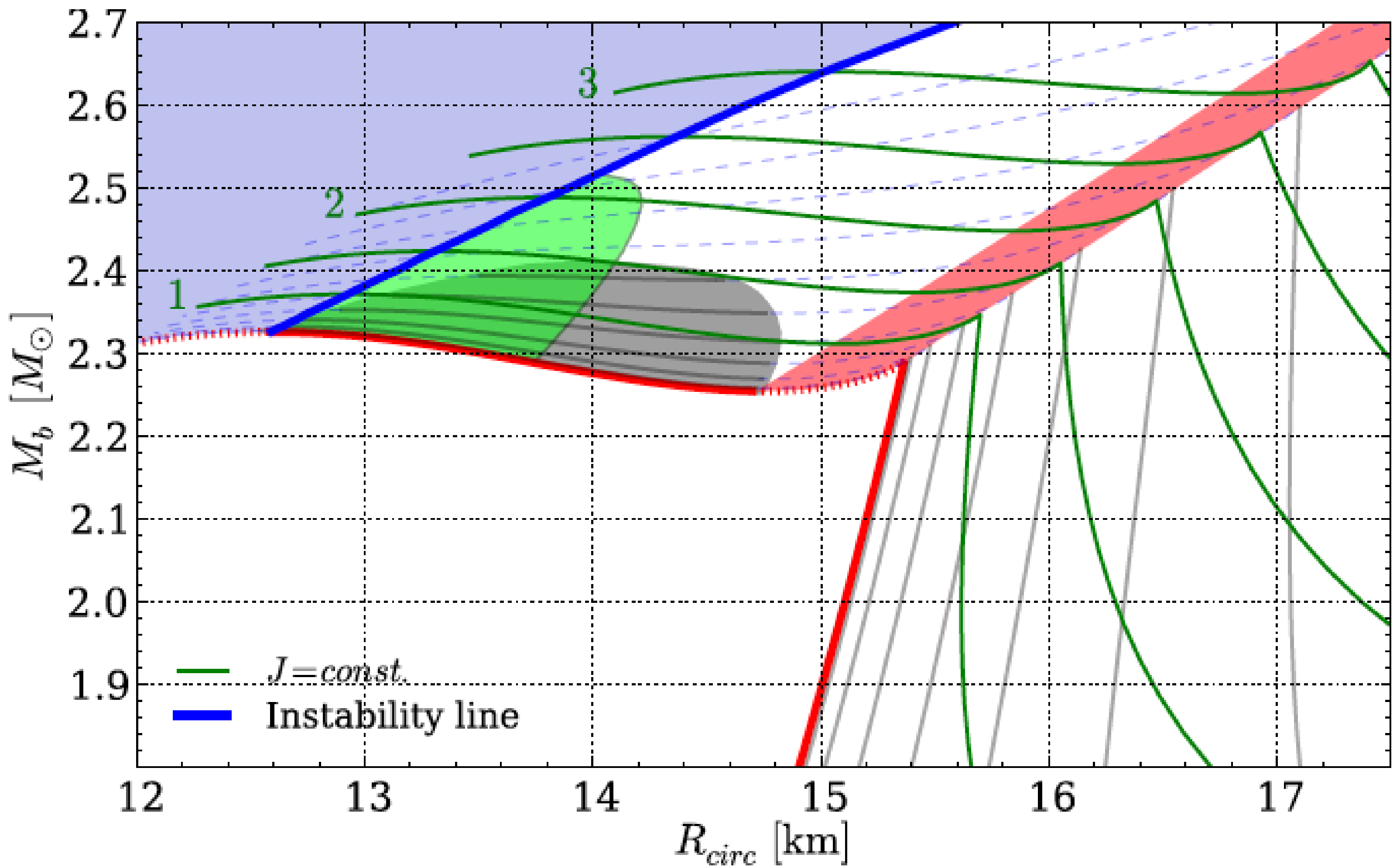
- similar scenario as fast radio bursts (Falcke-Rezzolla, 2013) or braking index (Glendenning-Pei-Weber, 1997)





M. Bejger, D.B. et al., arxiv:1608.07049
Astronomy & Astrophysics 600 (2017) A39

2.6. Rotation and stability



Red region - strong phase-transition instability,
Blue region - unstable w.r.t axisymmetric oscillations,
Grey region - no back-bending,
Green region - stable twin branch reached after the mini-collapse from the tip of $J = const.$ curve, along $M_b = const.$

2.6. Rotation - summary

This type of instability EOS provides a "natural" explanation for:

- ⋆ Lack of back-bending in radiopulsar timing,
- ⋆ Spin frequency cut-off at some moderate (but >716 Hz) frequency,
- ⋆ Falcke & Rezzolla Fast Radio Burst (FRB) engine
  - ⋆ catastrophic mini-collapse to the second branch (or to a black hole),
  - ⋆ massive rearrangement of the magnetic field $\rightarrow$ energy emission.

Astrophysical predictions:

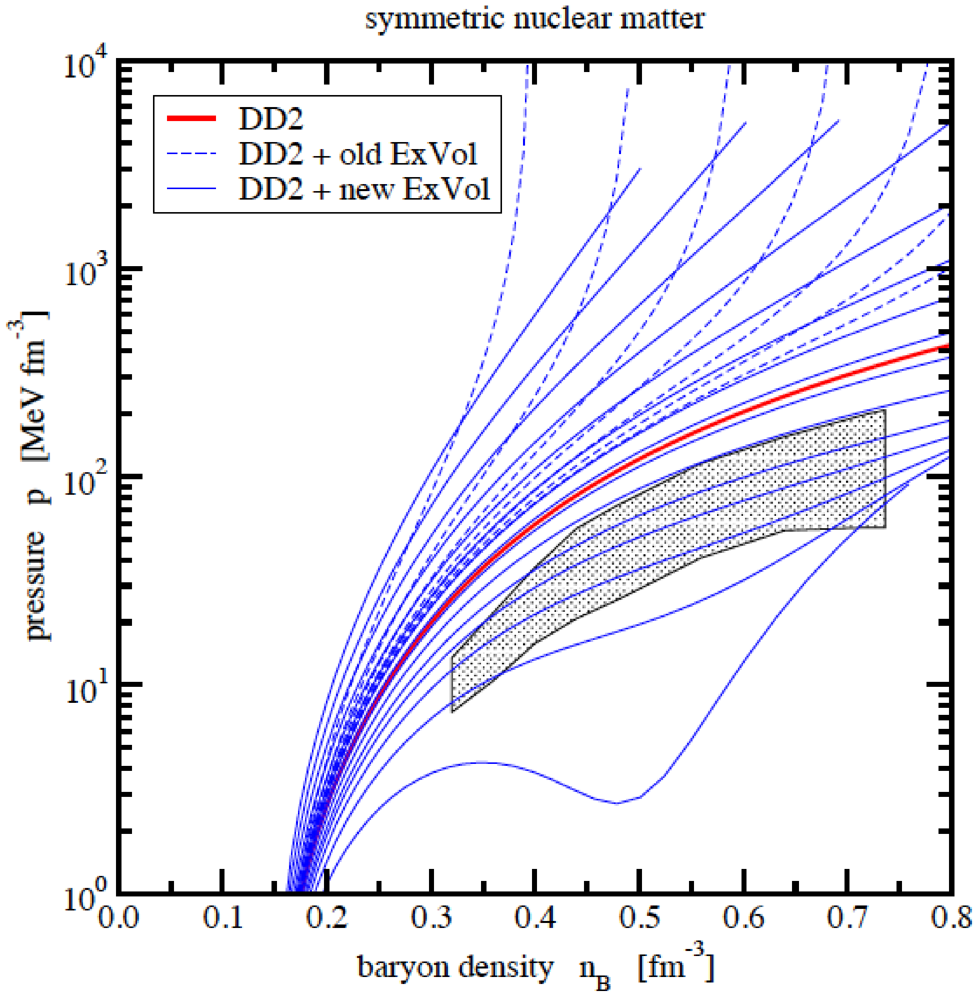
- ⋆ Way to constraint on M_b, J, I, core EOS etc.,
- ⋆ Specific shape of NS-BH mass function (no mass gap?)
- $\rightarrow$ population of massive, low B-field NSs (radio-dead?),
- $\rightarrow$ population of massive, high B-field NSs (collapse enhances the field?),
- ⋆ Characteristic burst-like signature in GW emission during the mini-collapse.

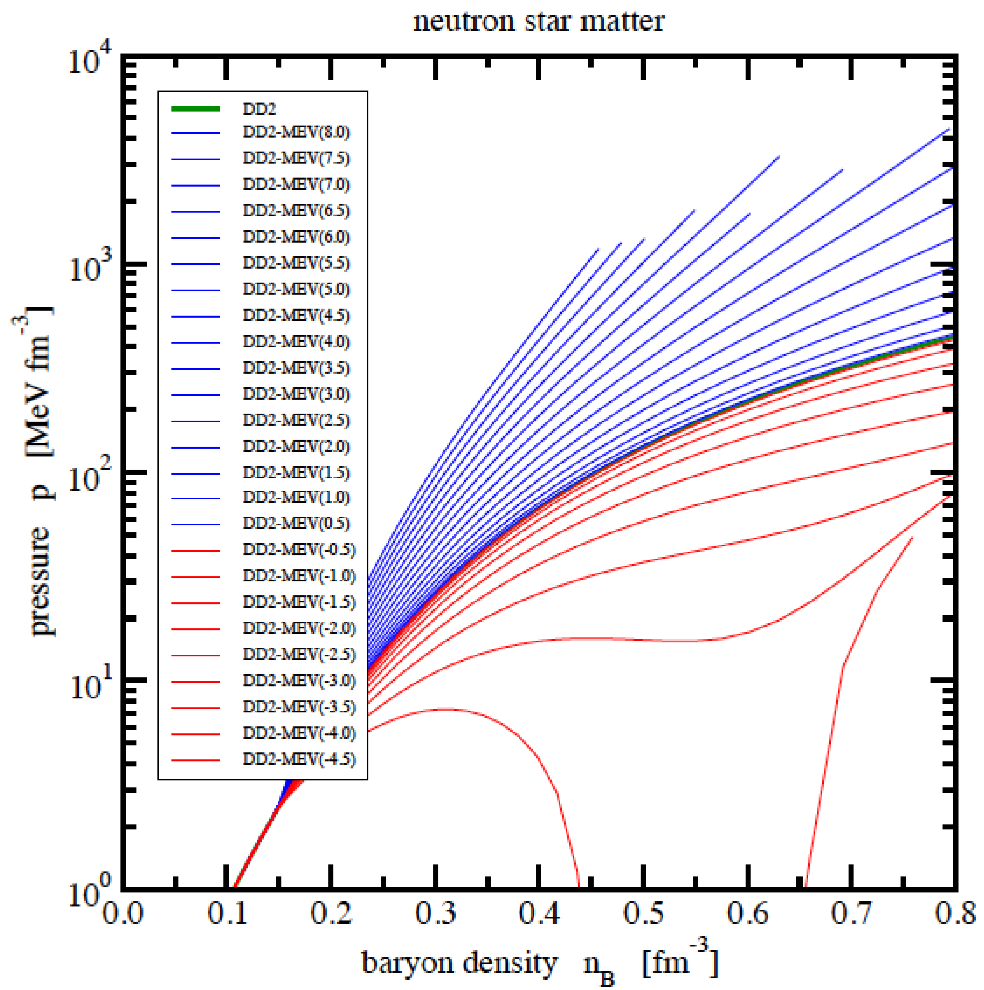
3. Pauli blocking vs. excluded volume

3.1. Equation of state

- excluded volume corrections in the hadronic EoS,

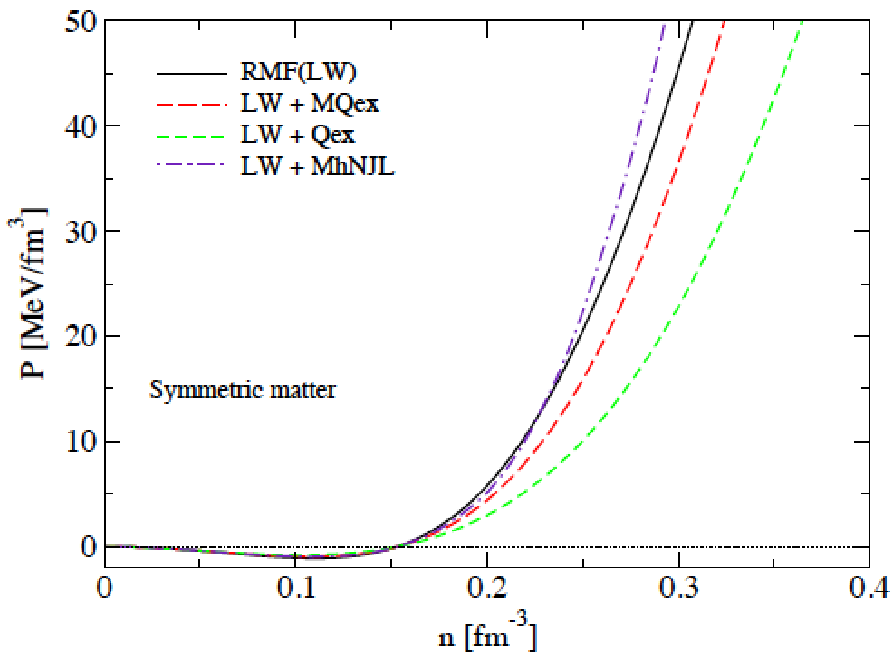
$$\Phi_N = \begin{cases} 1\,, & \text{if} \quad n \le n_{\text{sat}} \\ \exp[-v|v|(n - n_{\text{sat}})^2/2]\,, & \text{if} \quad n > n_{\text{sat}}\,, \end{cases}$$

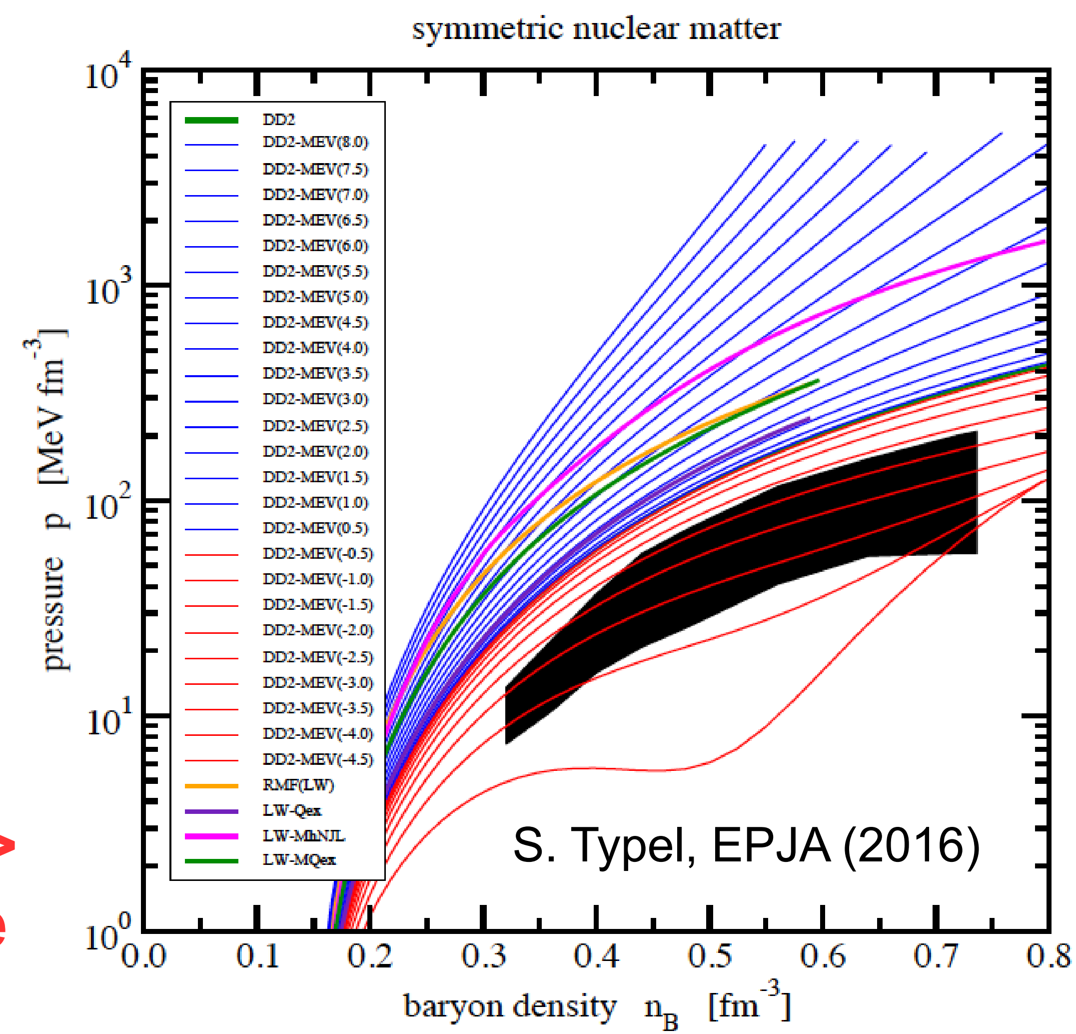




2. Pauli blocking in NM – nucleon excluded V.

New aspect: chiral restoration --> dropping quark mass

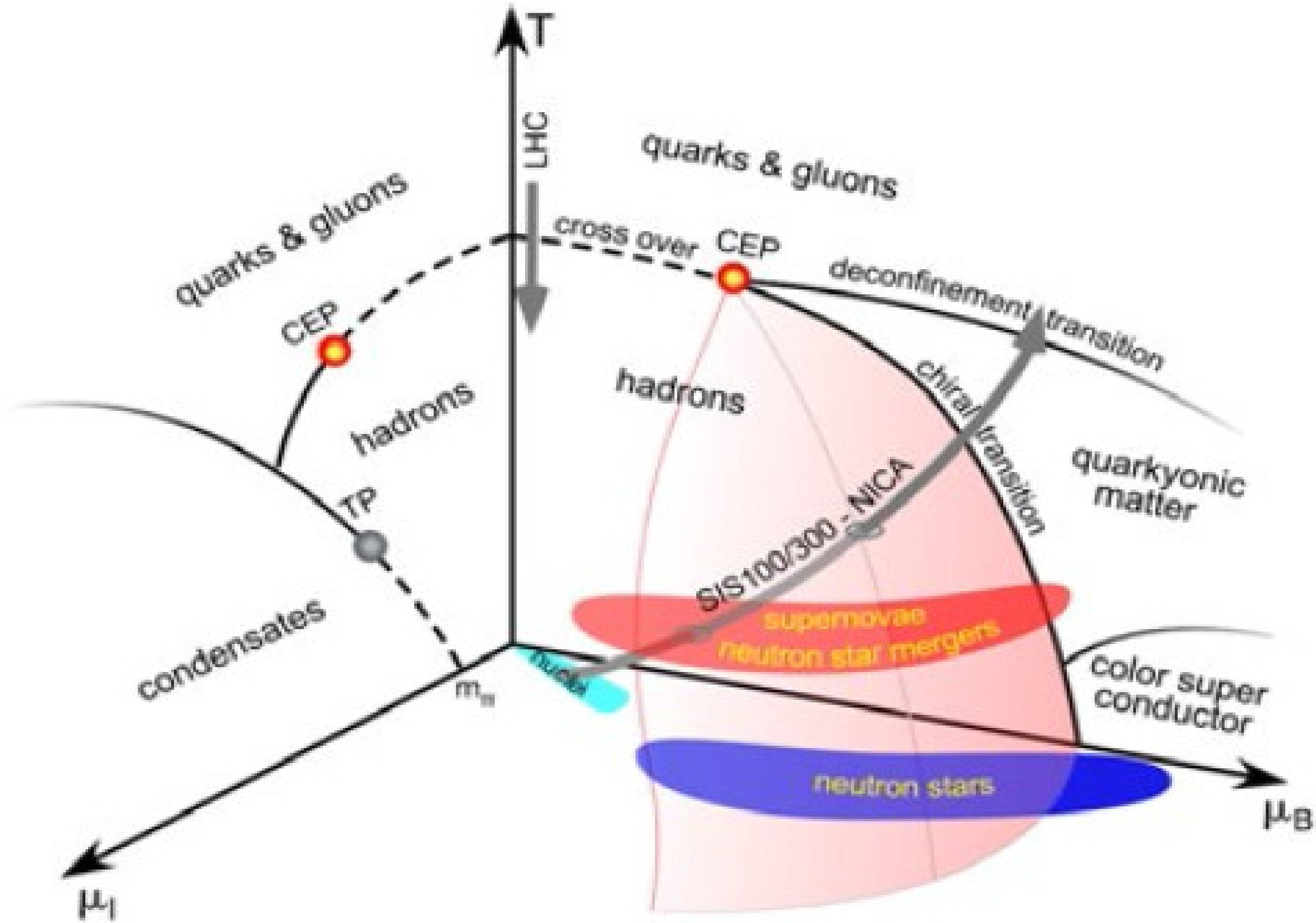




Increased baryon swelling at supersaturation densities: --> dramatic enhancement of the Pauli repulsion !!

D.B., H. Grigorian, G. Roepke: “Quark exchange effects in dense nuclear matter”, in prep. (2017)

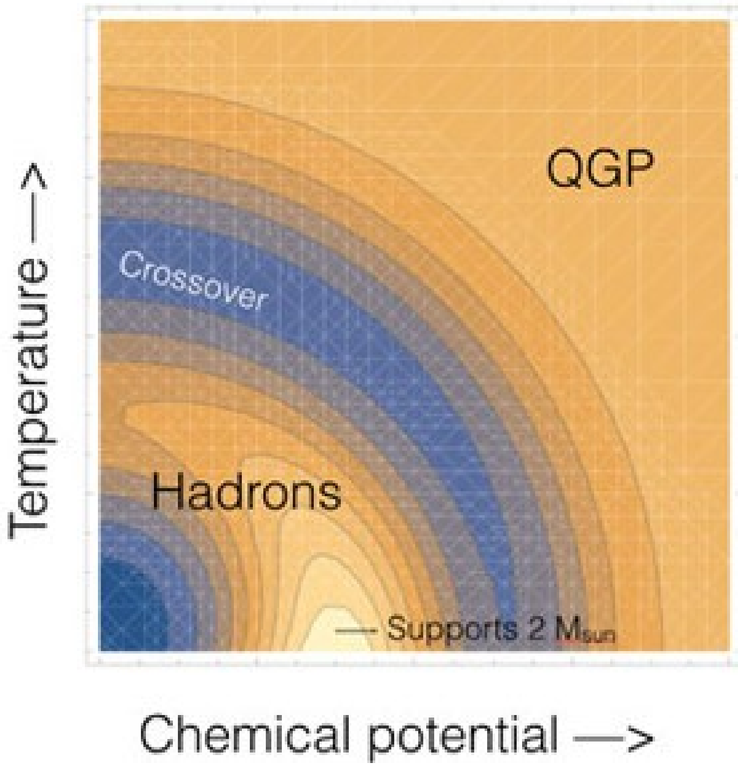
Towards measuring the EoS in the T-μ_B-μ_I phase diagram

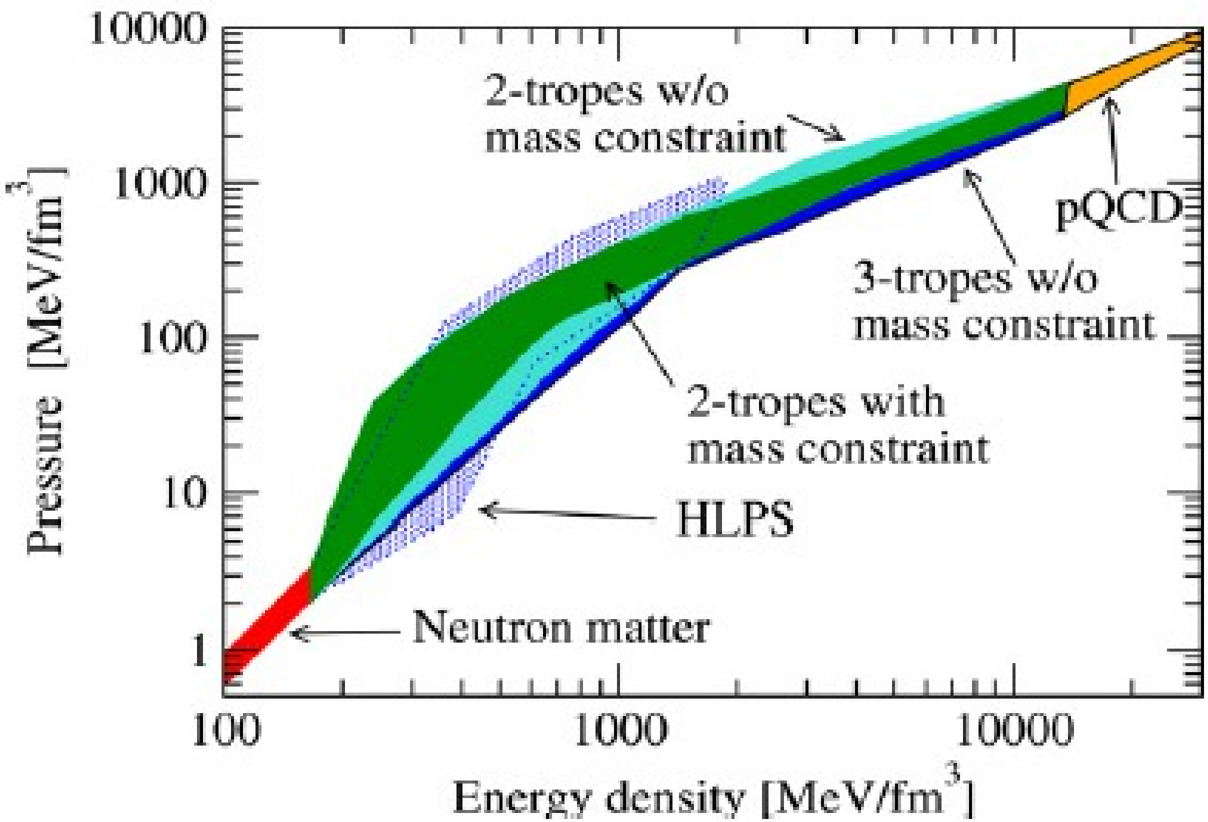


A. Andronic, D. Blaschke, et al., “Hadron production ...”, Nucl. Phys. A 837 (2010) 65 - 86

NuPECC Long Range Plan 2017; http://www.nupecc.org

Towards "measuring" the EoS in the T – mu plane (QCD phase diagram)



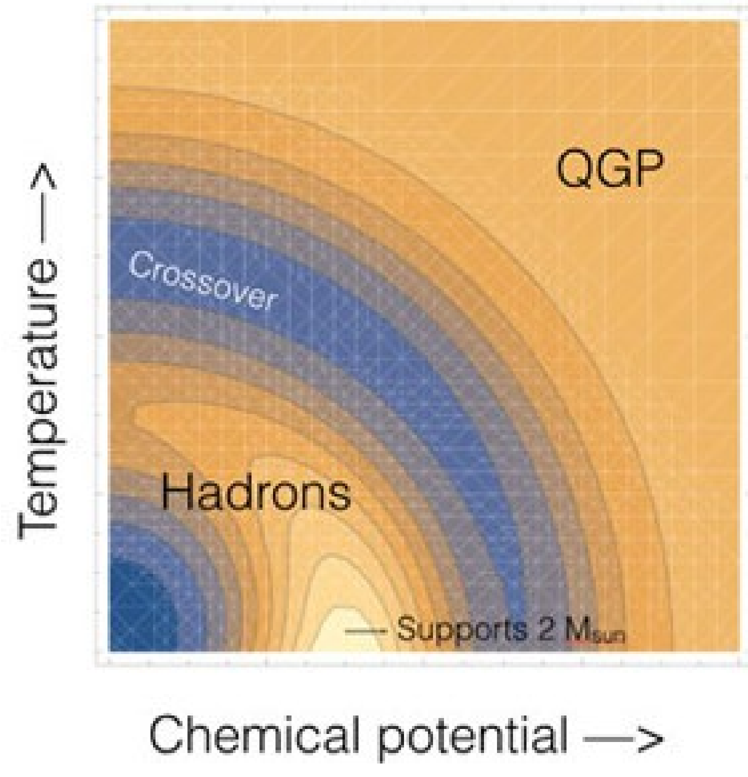


Speed-of-sound diagram from the INT program in Seattle, Summer 2016

Interpolation between lattice QCD and Compact star physics (2 M_sun)

A. Kurkela, E. Fraga, J. Schaffner-Bielich, A. Vuorinen, Astrophys. J. 789 (2014) 127

Towards "measuring" the EoS in the T – mu plane (QCD phase diagram)



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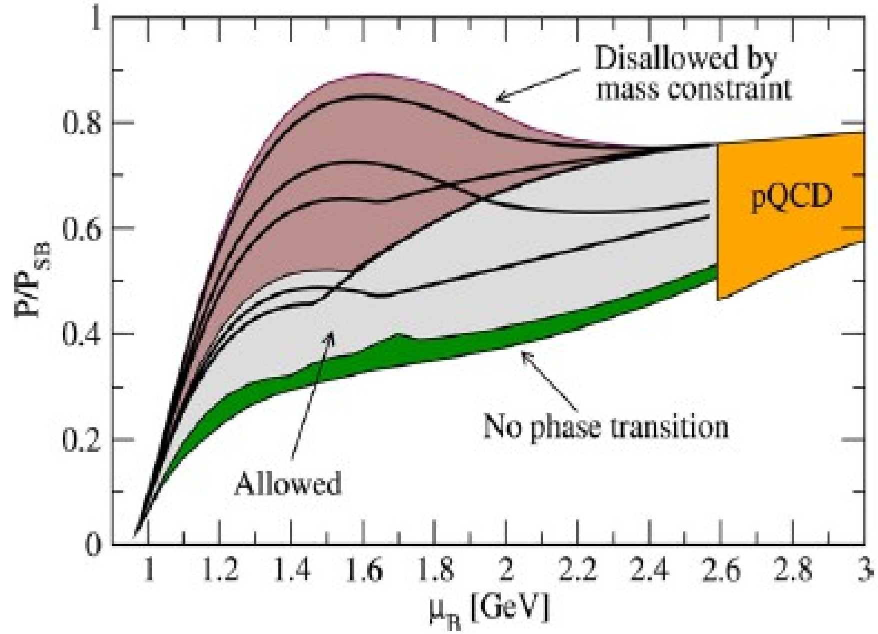
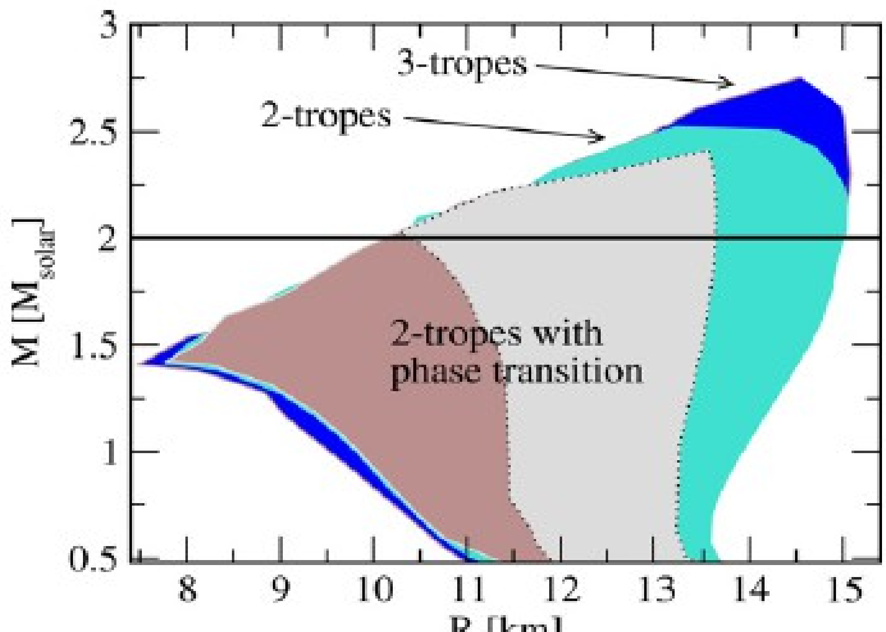
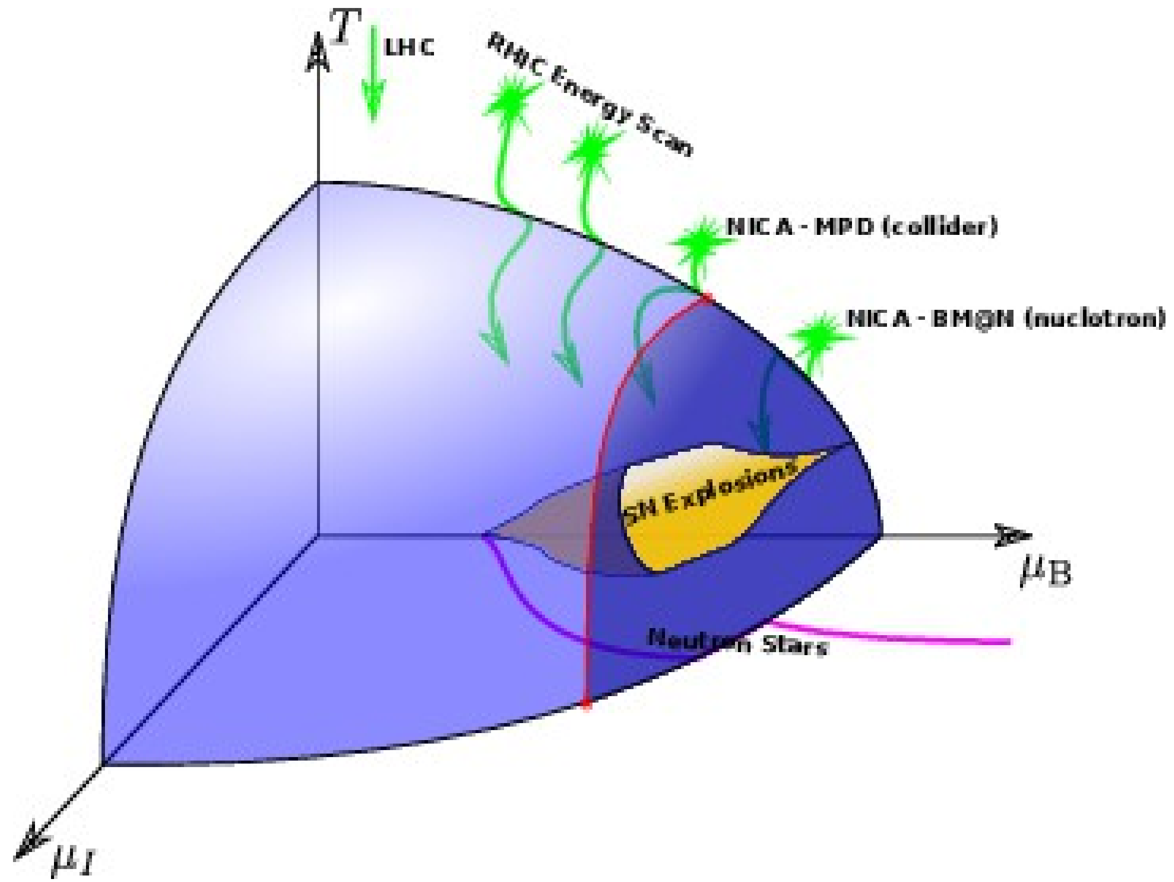




A. Kurkela, E. Fraga, J. Schaffner-Bielich, A. Vuorinen,
Astrophys. J. 789 (2014) 127

Conclusion:

High-mass twins (HMTs) with quark matter cores can be obtained within different hybrid star EoS models, e.g.,
- constant speed of sound
- higher order NJL
- piecewise polytrope
- density functional

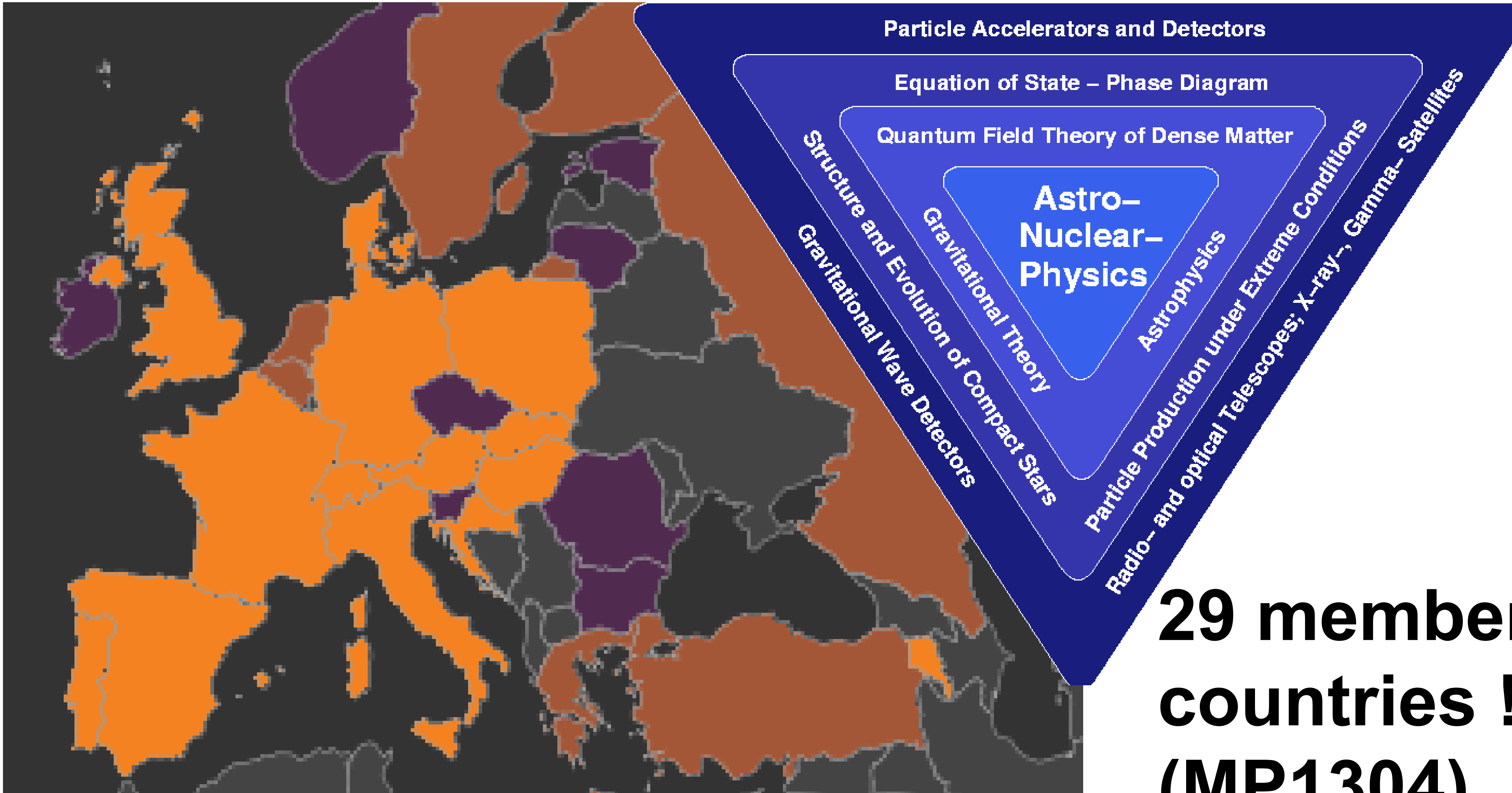


HMTs require stiff hadronic and quark matter EoS with a strong phase transition (PT)

Existence of HMTs can be verified, e.g., by precise compact star mass and radius observations (and a bit of good luck) → Indicator for strong PT !!

Extremely interesting scenarios possible for dynamical evolution of isolated (spin-down and accretion) and binary (NS-NS merger) compact stars

Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics

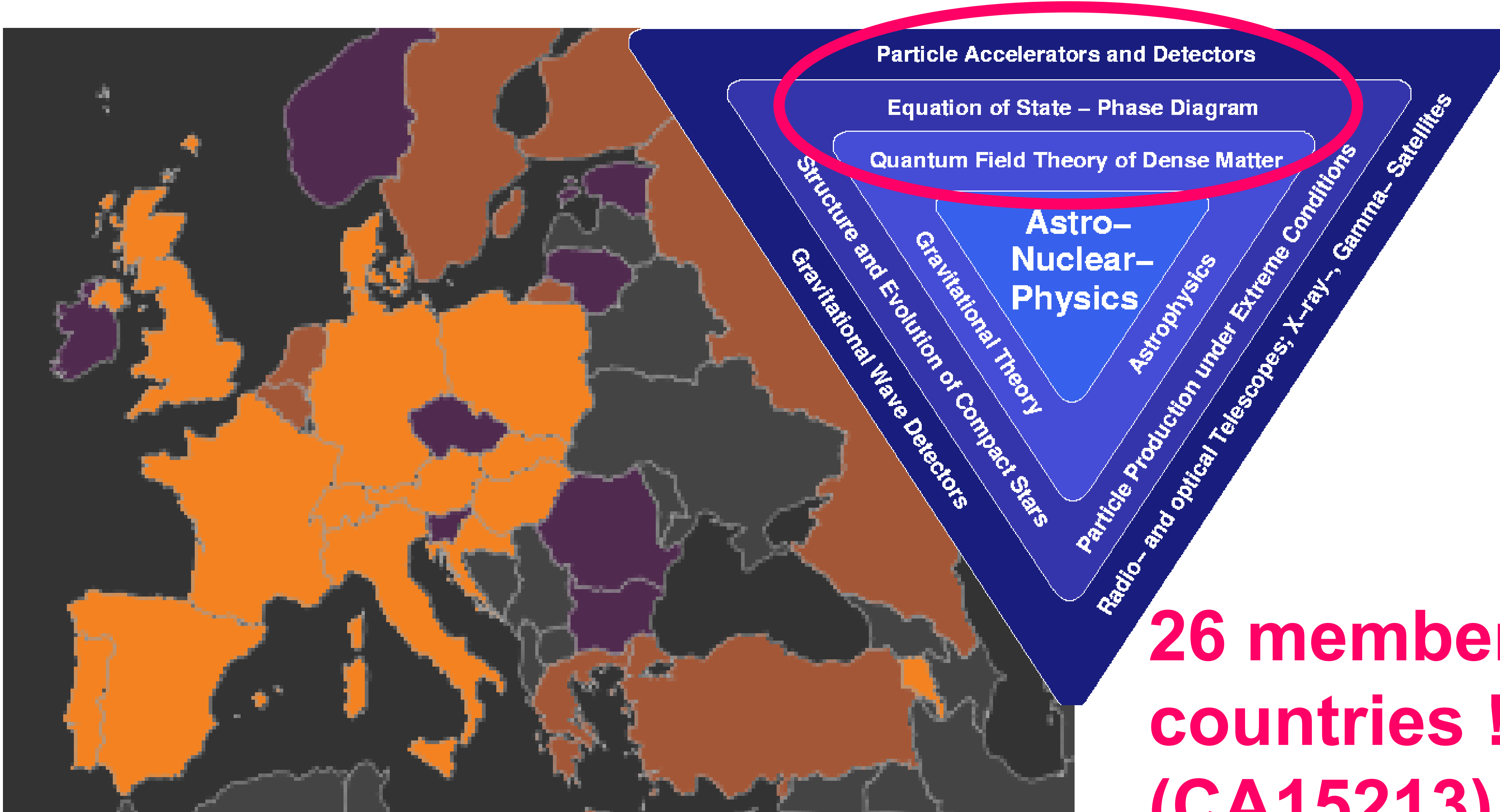


29 member countries !!
(MP1304)

New compstar **!**

cost
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

Kick-off: Brussels, November 25, 2013



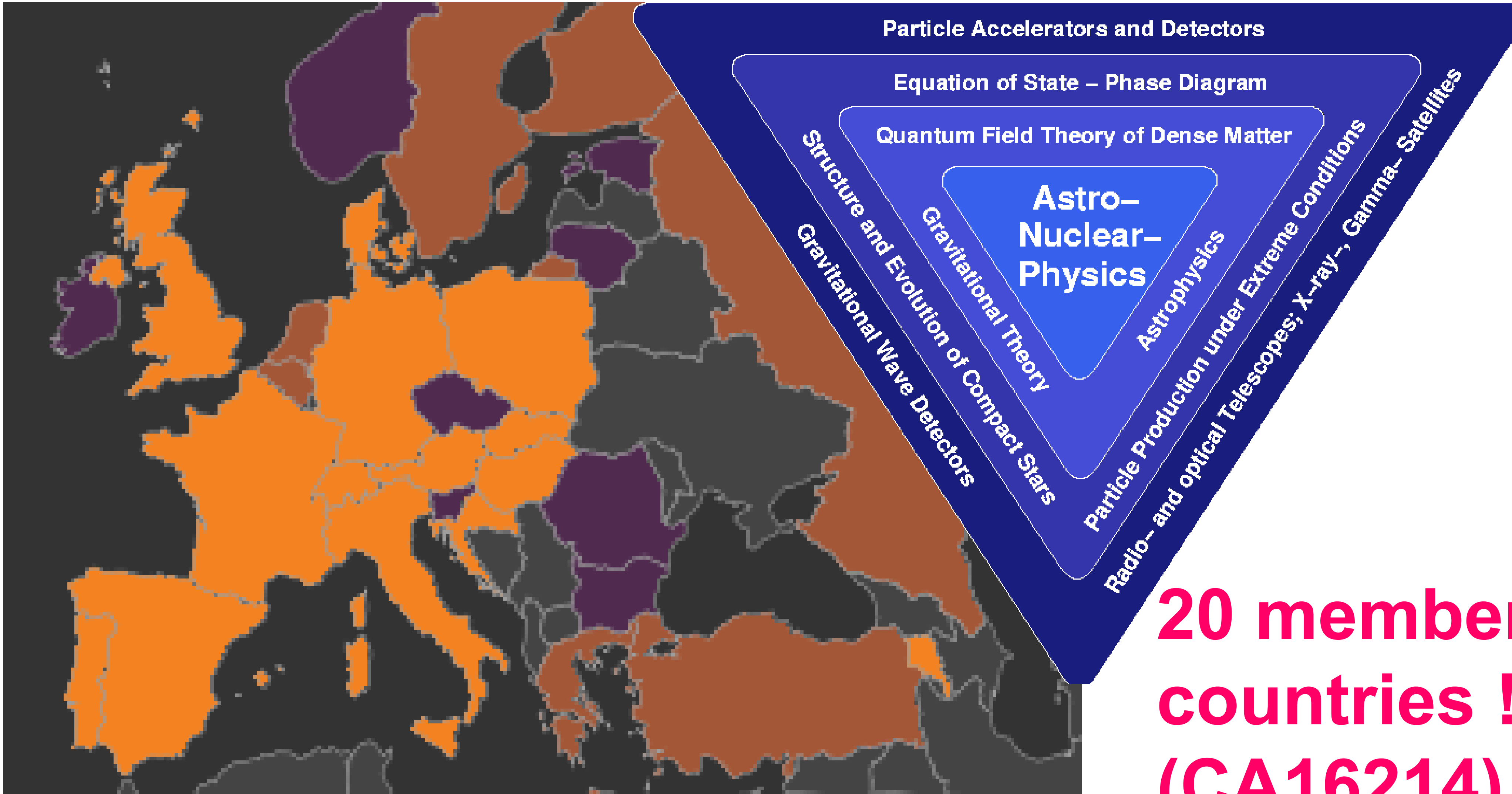
26 member countries ! (CA15213)

"Theory of HOt Matter in Relativistic Heavy-Ion Collisions"

New: THOR !

COST
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

Kick-off: Brussels, October 17, 2016



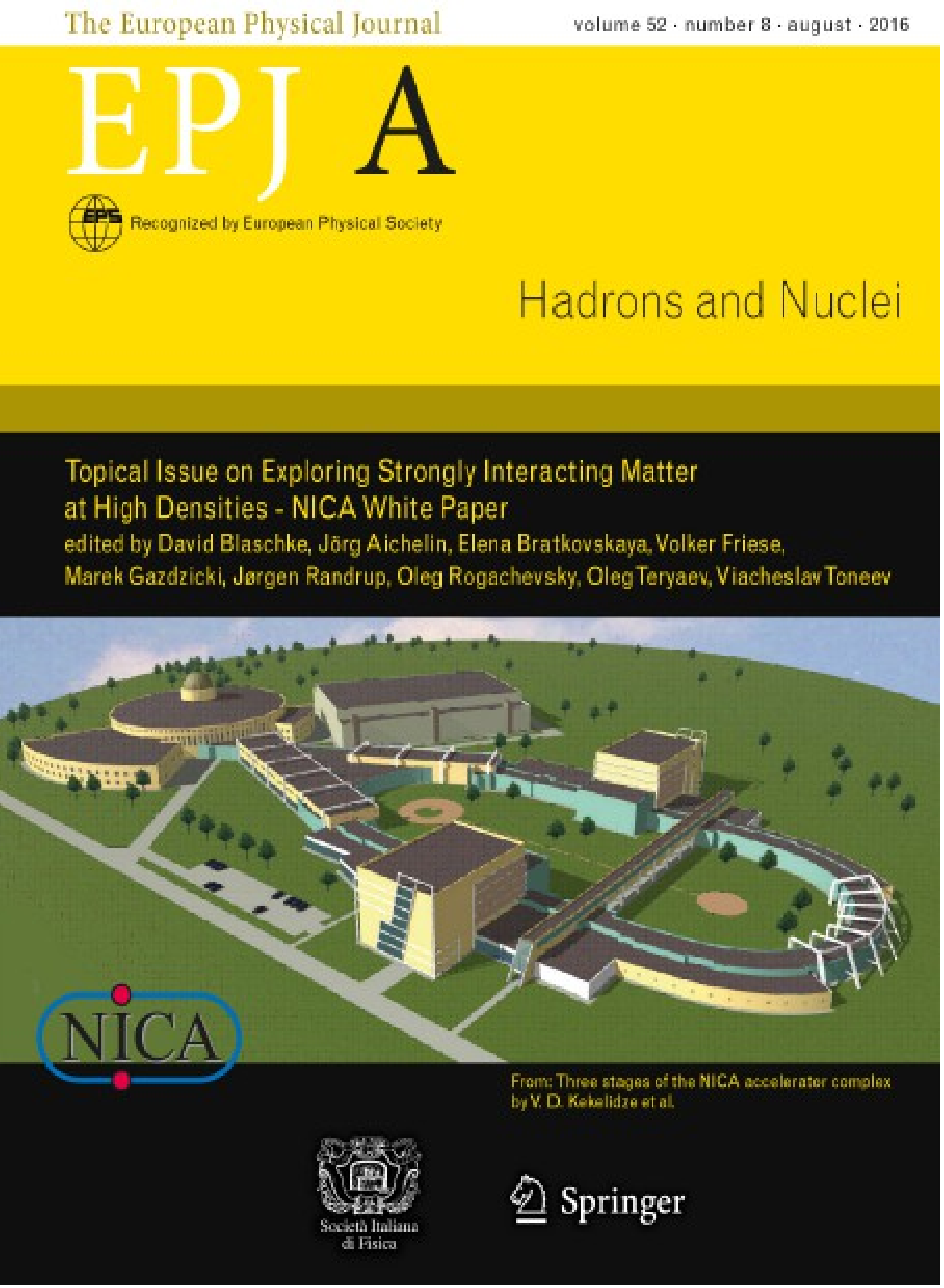
20 member countries !
(CA16214)

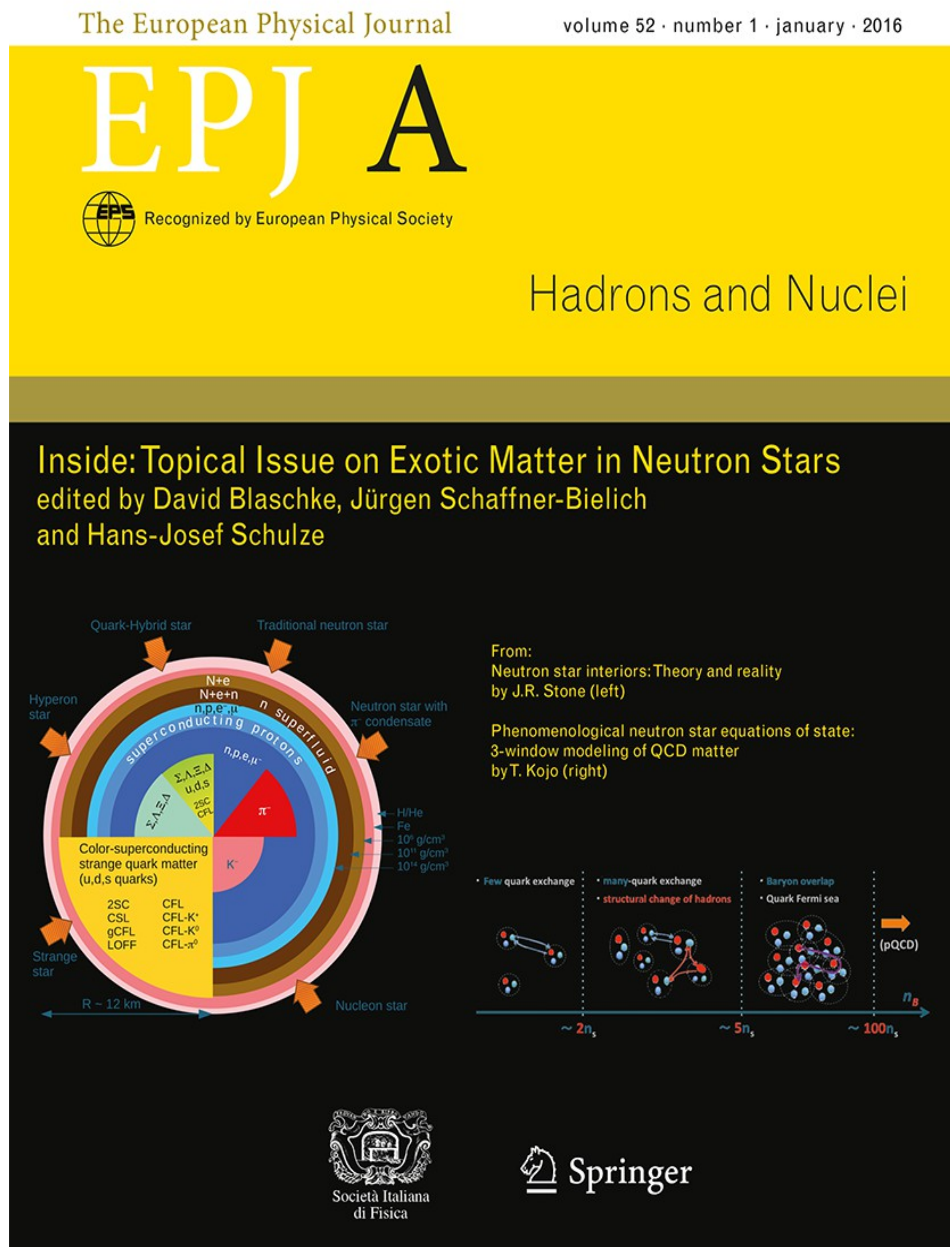
"The multi-messenger **PH**ysics and **A**strophysics of neut**RO**n **S**tars"

Newest: PHAROS!

COST
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Kick-off: Brussels, November 22, 2017





EPJA Topical Issues can be found at → http://epja.epj.org/component/list/?task=topic