Supporting the existence of the QCD critical point by compact star observations

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Compact Stars in the QCD Phase Diagram VI
Dubna Russia
September 28, 2017

Collaborations

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Special thanks to our supporters:

Heisenberg-Landau Program (with S. Typel)
Bogoliubov-Infeld Program (with D. Blaschke)
Ter Antonian - Smorodinsky Program (with H. Grigorian)
RSA - JINR Collaboration (visits in SA for Schools and Conference)
COST Action STSM Program (with D. Blaschke & CompStar Conferences)
Narodowe Centrum Nauki (visits in Wroclaw, Poland)

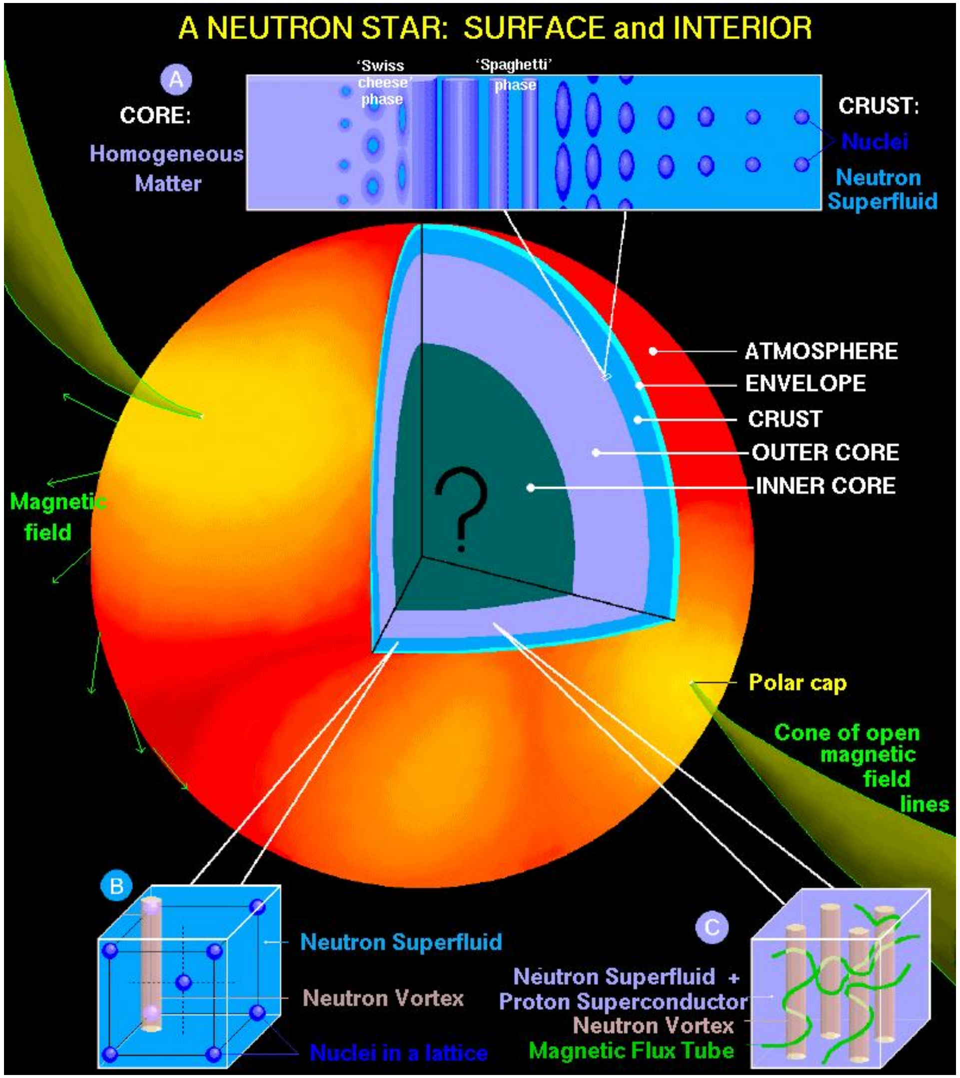
Key Questions

- Can compact star observations provide compelling evidence about a first order phase transition in QCD?
- What are the relevant observables?

Outline

- Introduction to the neutron star equation of state.
- First order phase transition and deconfinement in compact stars: neutron star twins.
- Astrophysics measurements of compact stars.
- Astrophysical implications and perspectives.

Neutron Stars



Credit:
Dany Page, UNAM

Neutron Star EoS

- Nuclear interaction:

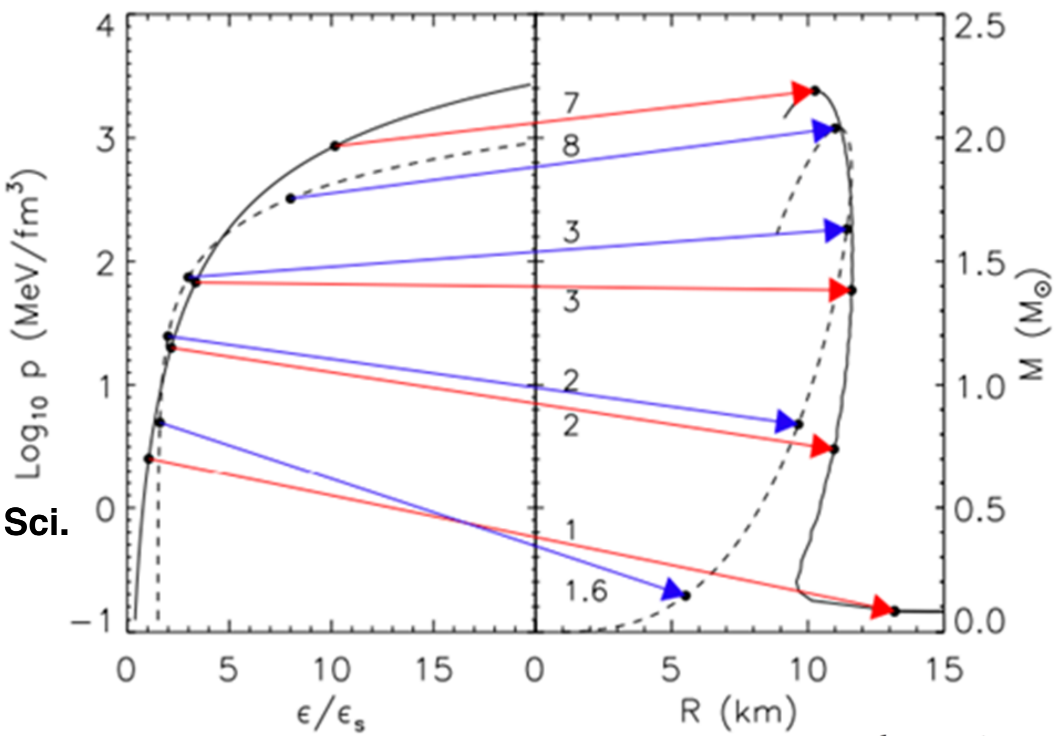
$$E(n,x) = E(n,x=1/2) + E_s(n) * \alpha^2(x),$$

- Beta equilibrium: $\mu_n - \mu_p = \mu_e = \mu_\mu$
- 2 phase construction under Gibbs conditions: $p^I = p^{II}$ $\mu_n^I = \mu_n^{II}$ $\mu_e^I = \mu_e^{II}$
- Charge neutrality: $x_p = x_e$
- TOV equations + Equation of State

$$\frac{dp}{dr} = -\frac{(\rho + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)} \qquad p(\rho)$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

Compact Star Sequences
(M-R ↔ EoS)



Lattimer,
Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)
arXiv: 1305.3510

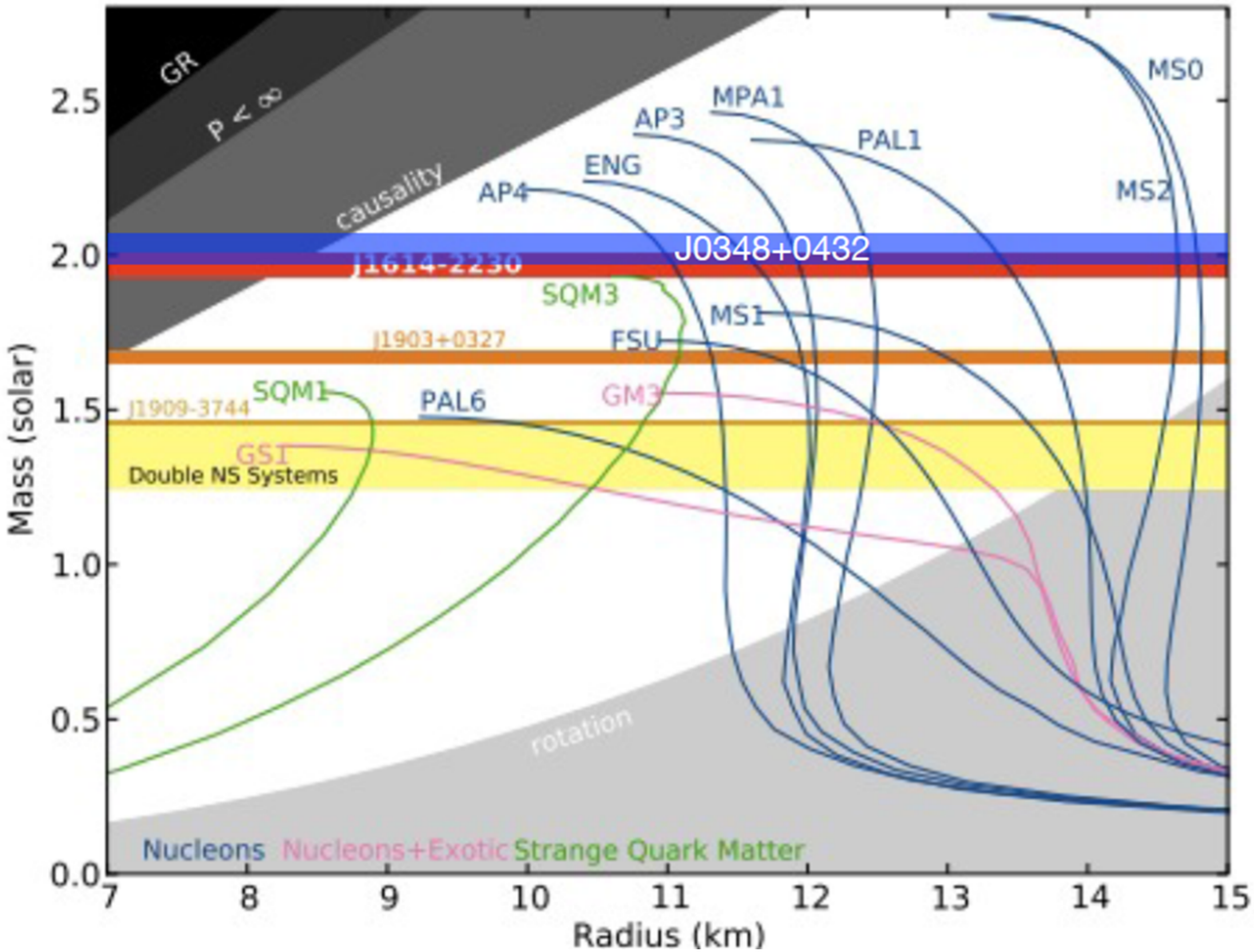
- TOV Equations
- Equation of State (EoS)

$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

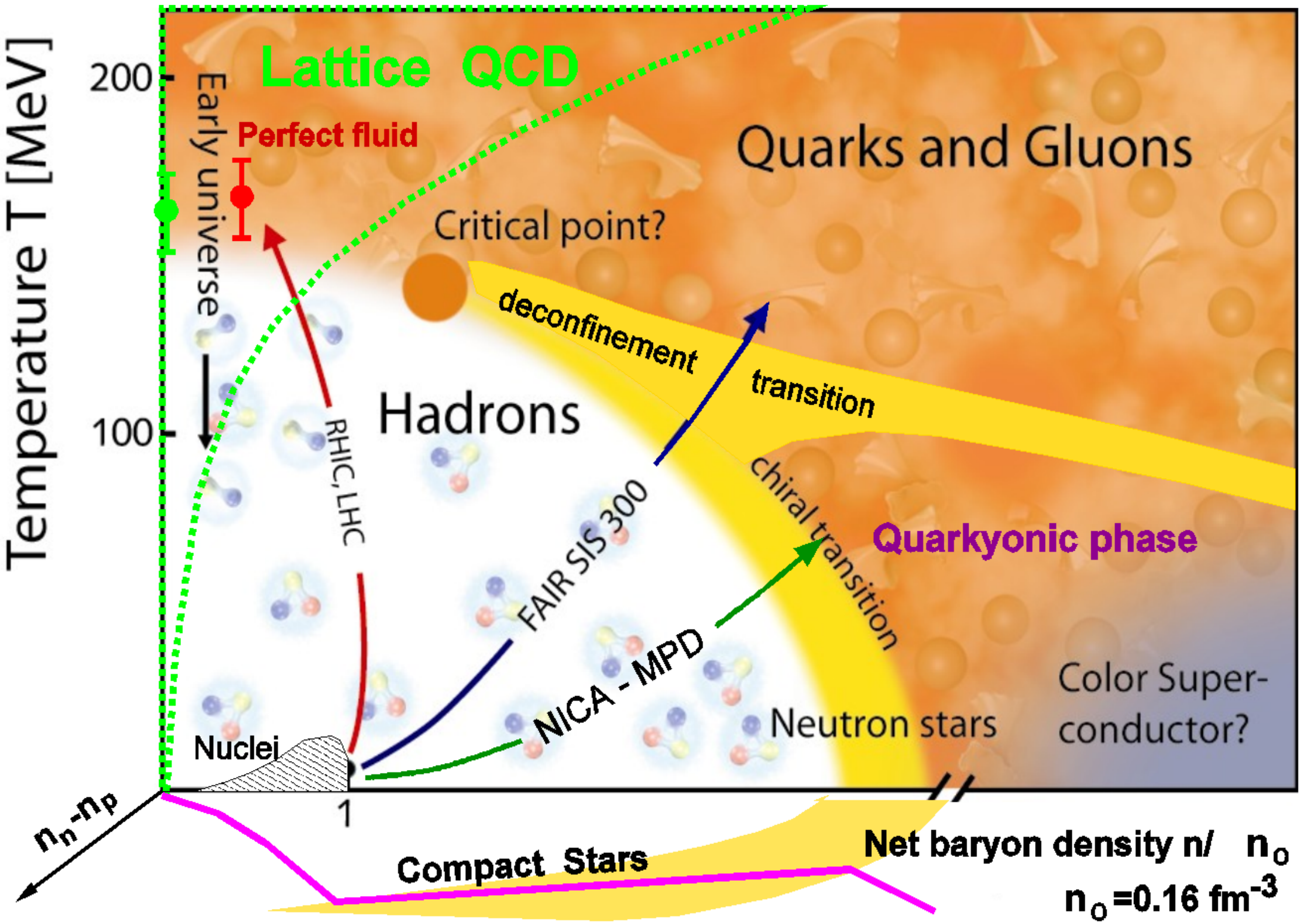
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

$$p(\varepsilon)$$

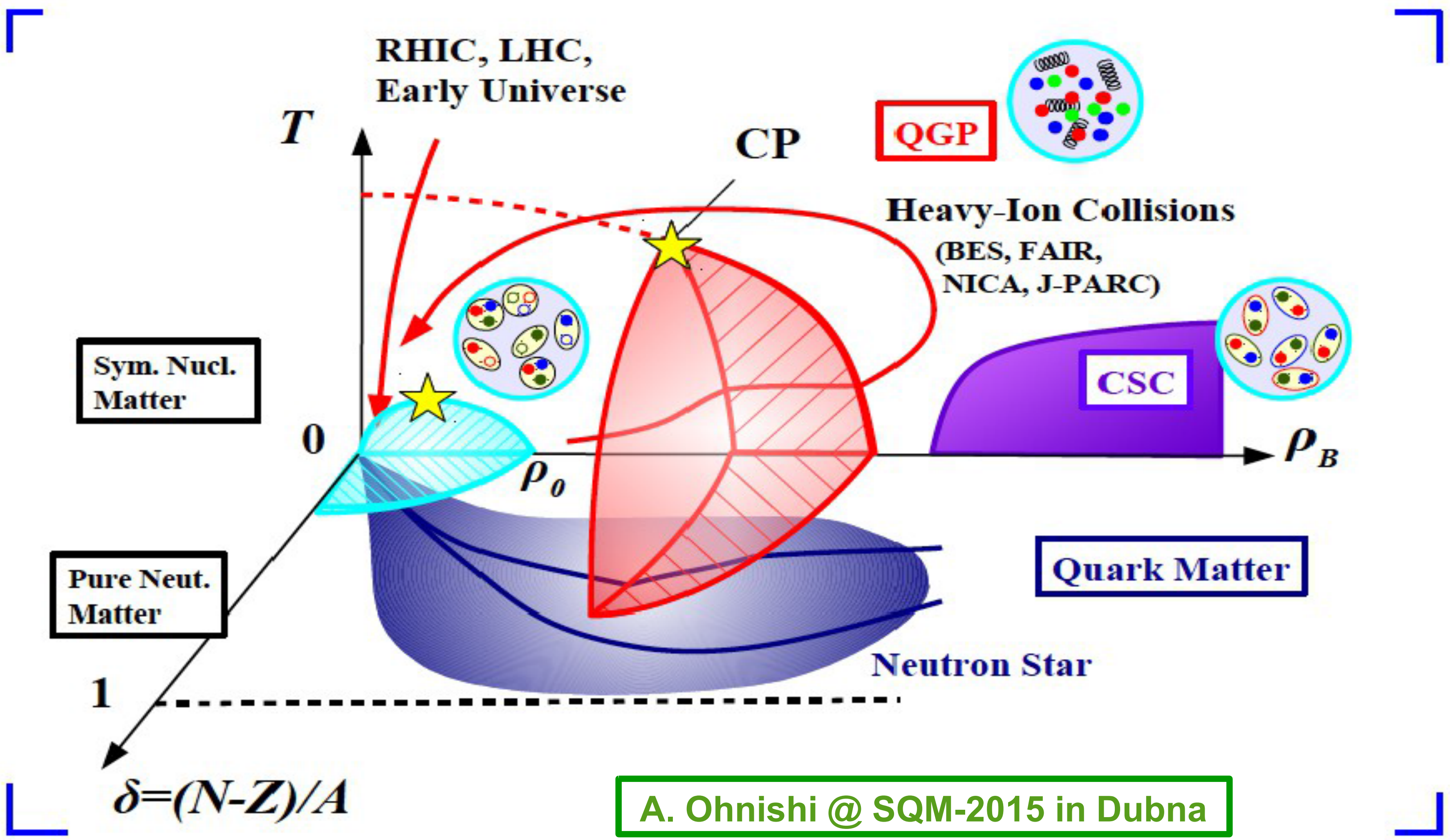
Massive neutron stars



Critical Endpoint in QCD



Support a CEP in QCD phase diagram with Astrophysics?

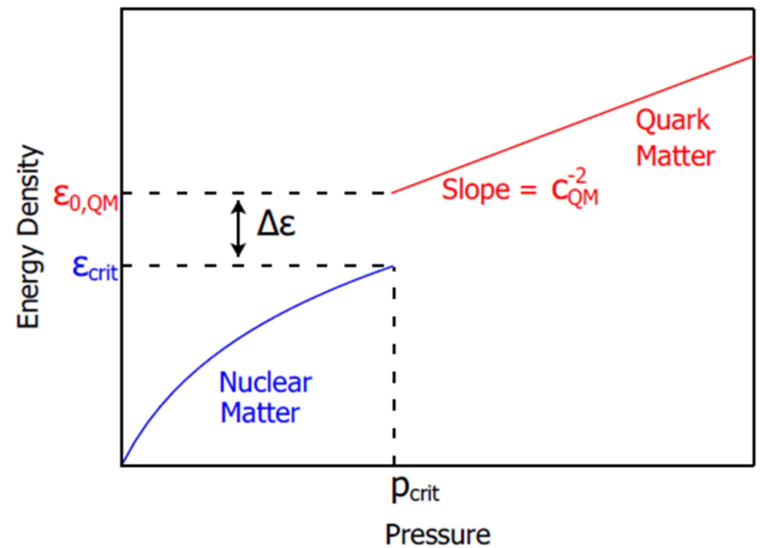


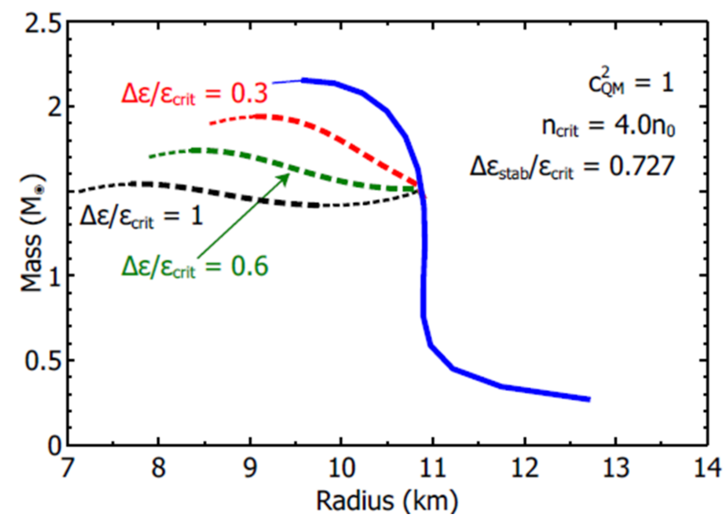
Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics)
=> Critical endpoint exists!

Neutron Star Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the "latent heat" (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → "**third family of CS**".

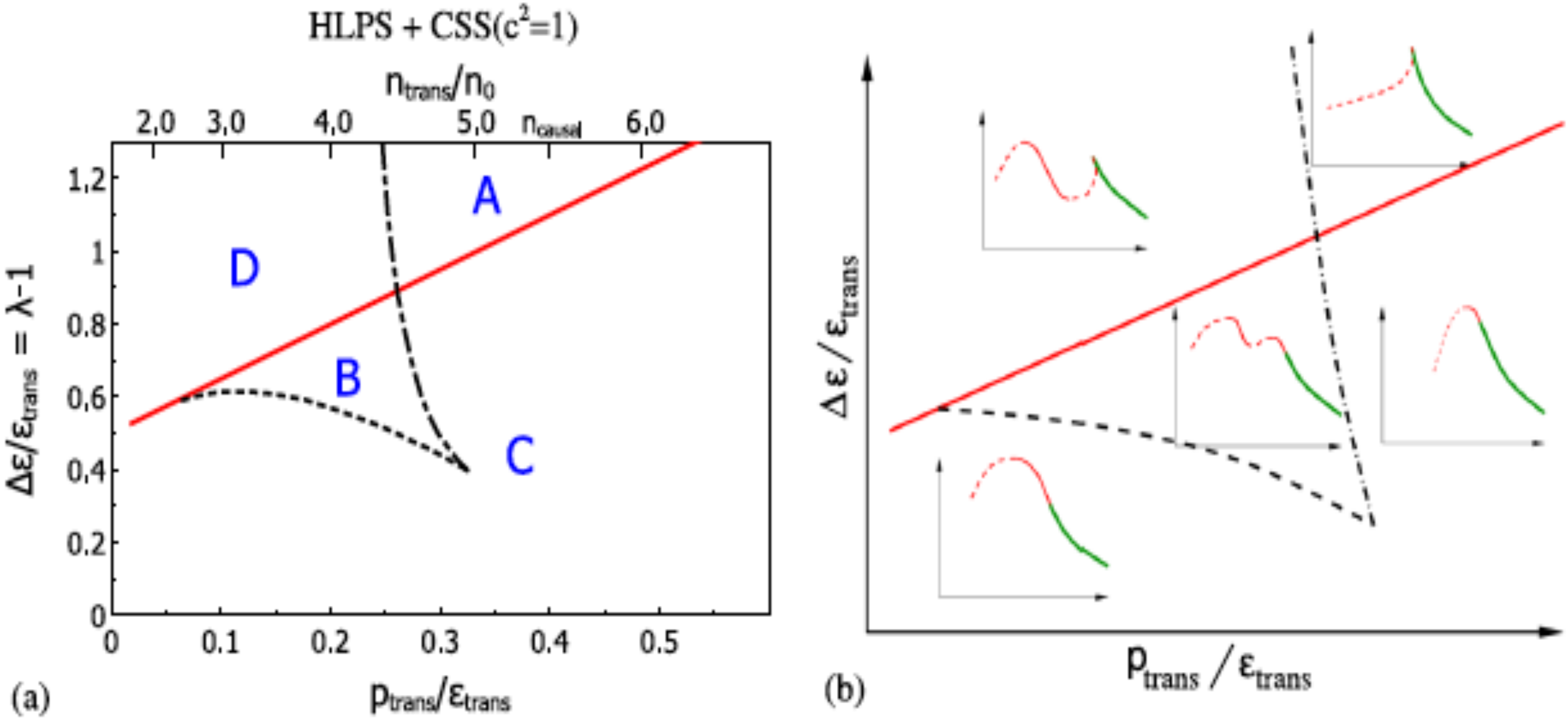
- Measuring two **disconnected populations** of compact stars in the M-R diagram would represent **the detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!





Alford, Han, Prakash,
Phys. Rev. D 88, 083013 (2013)

Key fact: Mass "twins" ↔ 1st order PT



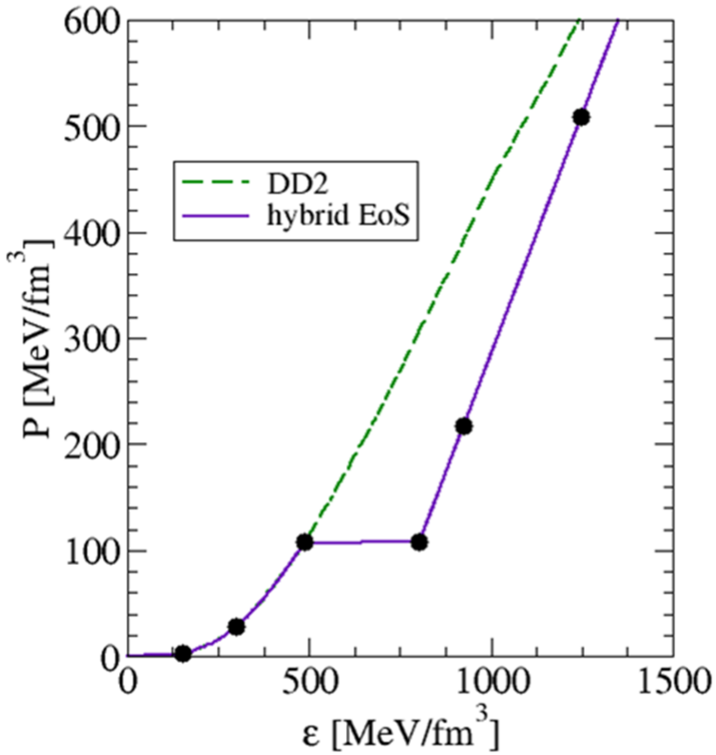
Systematic Classification [Alford, Han, Prakash: PRD88, 083013 (2013)]

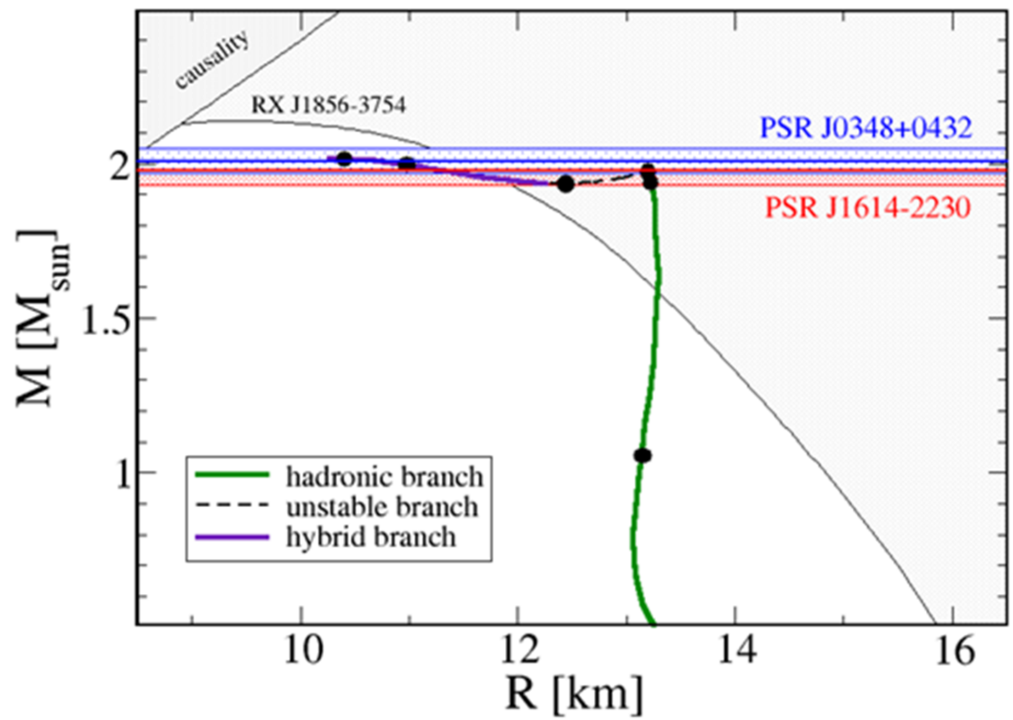
EoS P(ε) <--> Compact star phenomenology M(R)

Most interesting and clear-cut cases: (D)isconnected and (B)oth – high-mass twins!

Compact Star Twins

Third family (disconnected branch)





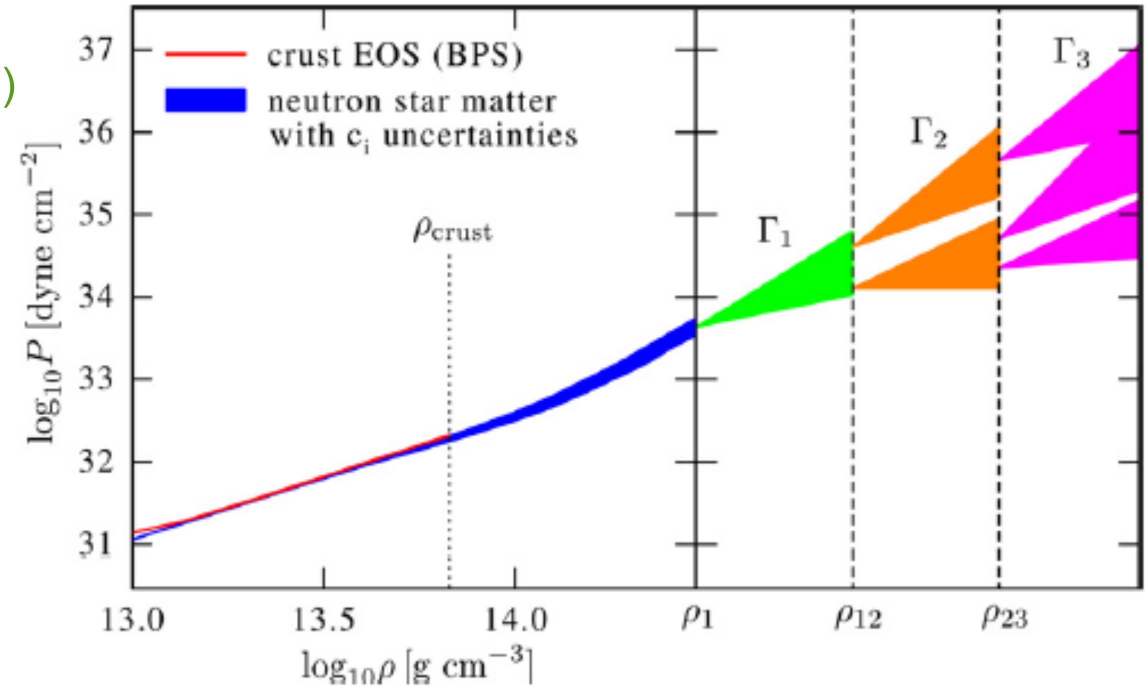
Alvarez-Castillo, Blaschke (2013),
lings of the 17th Conference of Young Scientists and Specialists,
arXiv: 1304.7758

High Mass Star Twins Based on multipolytrope EoS

Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v1

Piecewise polytrope EoS – high mass twins?

Hebeler et al., ApJ 773, 11 (2013)



$$P_i(n) = \kappa_i n^{\Gamma_i}$$

$$i = 1 \ : \ n_1 \leq n \leq n_{12}$$
$$i = 2 \ : \ n_{12} \leq n \leq n_{23}$$
$$i = 3 \ : \ n \geq n_{23} \ ,$$

Here, 1st order PT in region 2:

$$\Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\rm crit}$$

$$P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn},$$
$$\varepsilon(n)/n = \int dn \frac{P(n)}{n^2} = \int dn\, \kappa n^{\Gamma-2} = \frac{\kappa\, n^{\Gamma-1}}{\Gamma-1} + C,$$
$$\mu(n) = \frac{P(n)+\varepsilon(n)}{n} = \frac{\kappa\Gamma}{\Gamma-1} n^{\Gamma-1} + m_0,$$

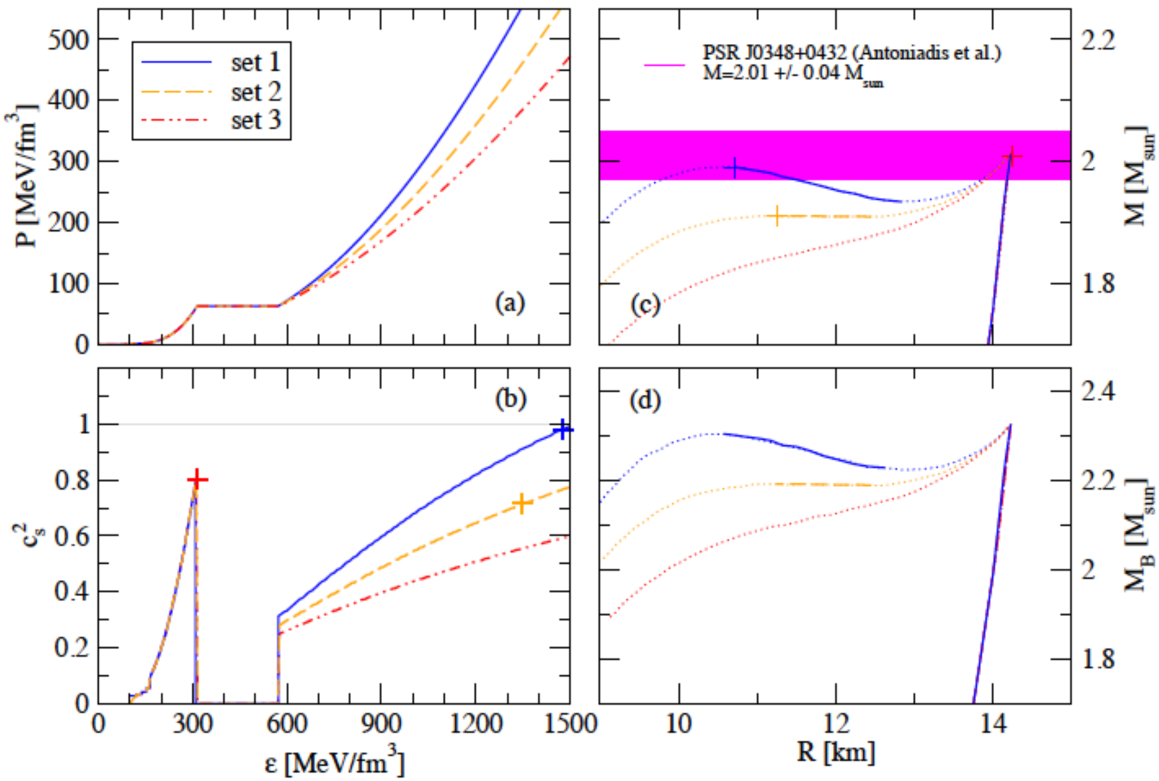
$$n(\mu) = \left[(\mu - m_0)\frac{\Gamma-1}{\kappa\Gamma}\right]^{1/(\Gamma-1)}$$
$$P(\mu) = \kappa\left[(\mu - m_0)\frac{\Gamma-1}{\kappa\Gamma}\right]^{\Gamma/(\Gamma-1)}$$

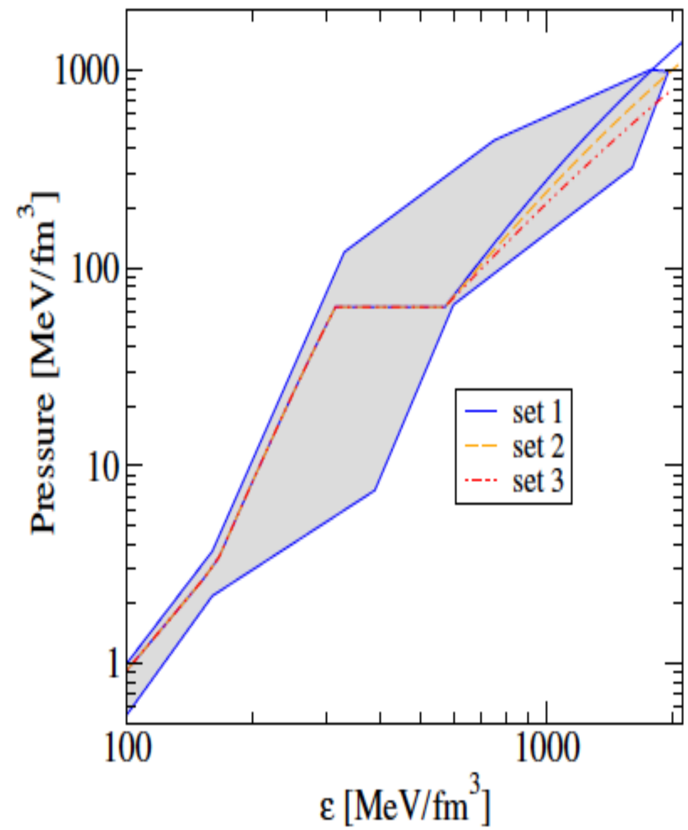
Maxwell construction:

$$P_1(\mu_{\rm crit}) = P_3(\mu_{\rm crit}) = P_{\rm crit}$$
$$\mu_{\rm crit} = \mu_1(n_{12}) = \mu_3(n_{23})$$

Seidov criterion for instability: $\frac{\Delta\varepsilon}{\varepsilon_{\rm crit}} \geq \frac{1}{2} + \frac{3}{3}\frac{P_{\rm crit}}{\varepsilon_{\rm crit}}$

Piecewise polytrope EoS – high mass twins?





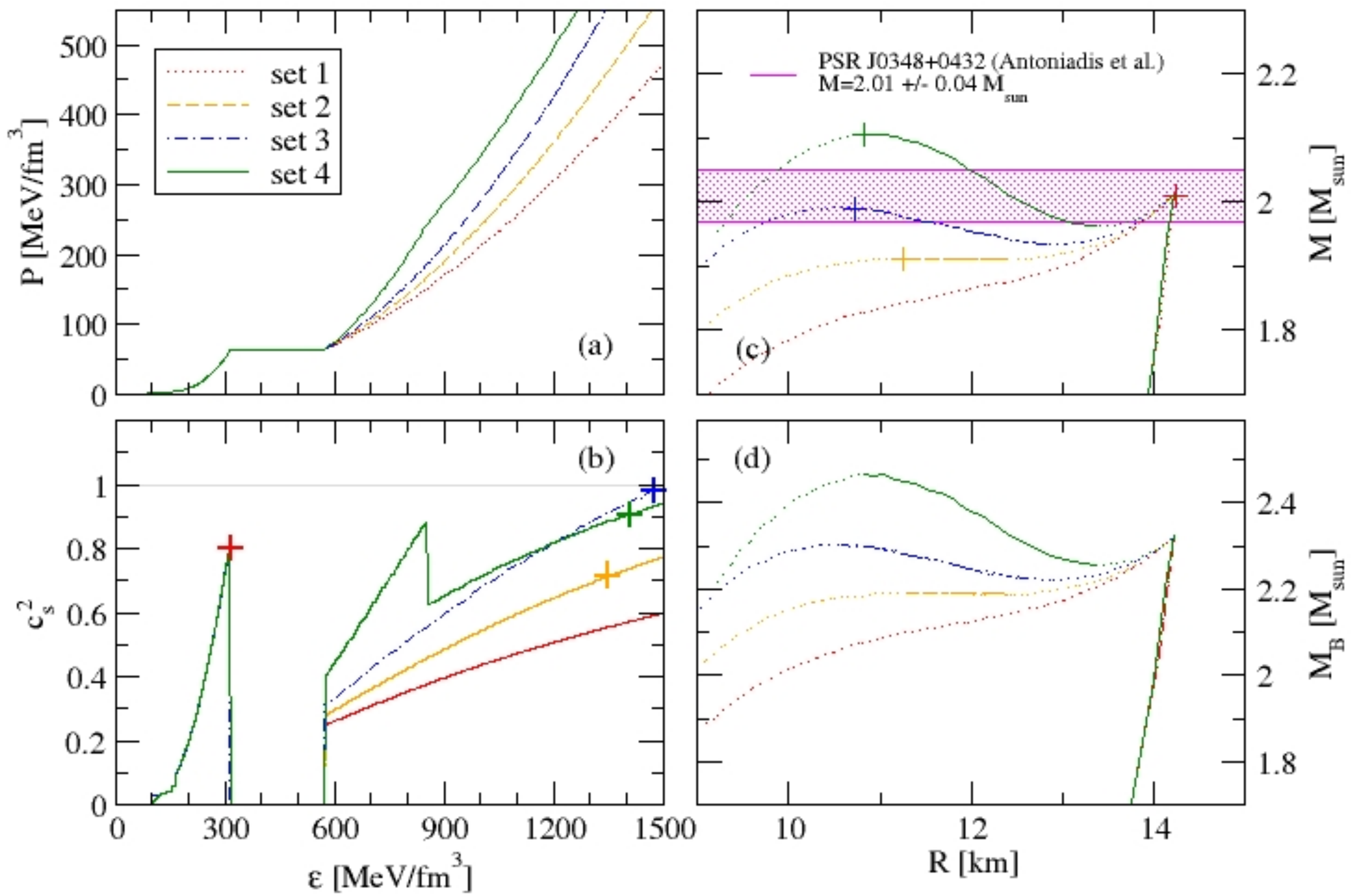
	Γ_3	κ_3 [MeV fm$^{3(\Gamma_3-1)}$]	$m_{0,3}$ [MeV]	M^{NS}_{max} [$M_\odot$]	M^{HS}_{max} [$M_\odot$]	M^{HS}_{min} [$M_\odot$]
set 1	3.12	447.16	1014.87	2.01	1.991	1.934
set 2	2.80	365.12	1004.88	2.01	1.910	1.909
set 3	2.50	302.56	991.75	2.01	-	-

Set with same onset of Phase transition:
P_crit = 68.18 MeV/fm^3
eps_crit = 318.26 MeV/fm^3
Δeps = 253.89 MeV/fm^3
n_12 = 0.32 fm^-3 ; n_23 = 0.53 fm^-3

Third family solutions in the 2M_sun mass range (HMT) exist !!

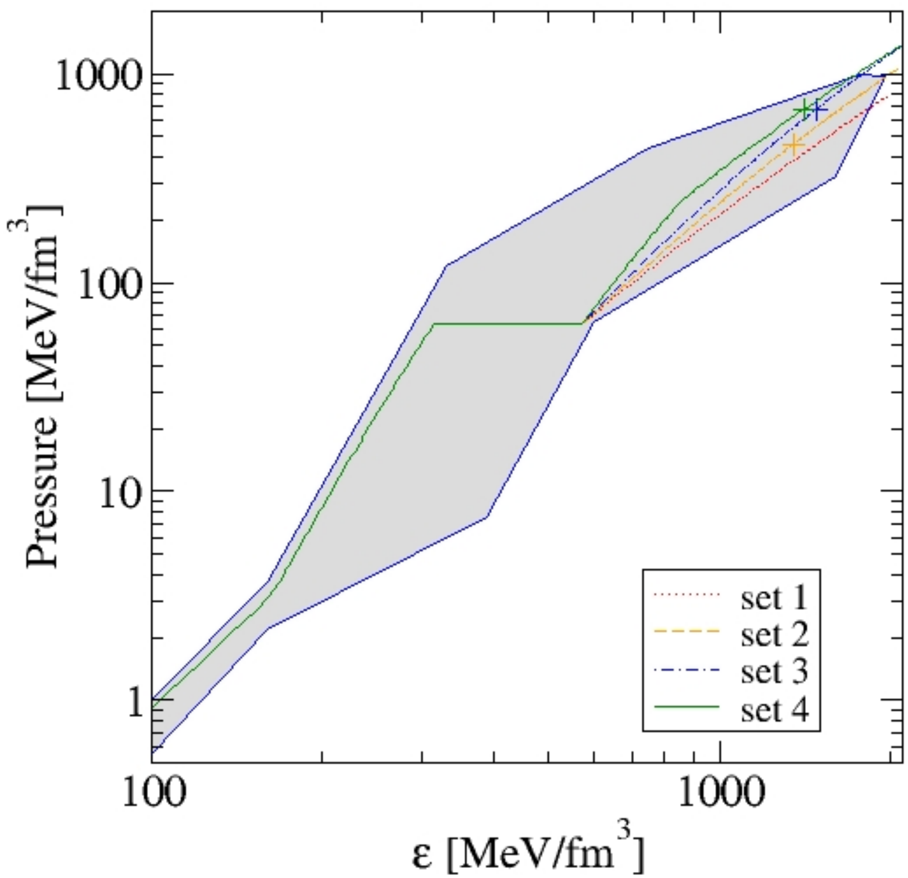
[D. Alvarez-Castillo & D.B. arxiv:1703.02681]

Compact Star Twins



Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, to appear

Compact Star Twins

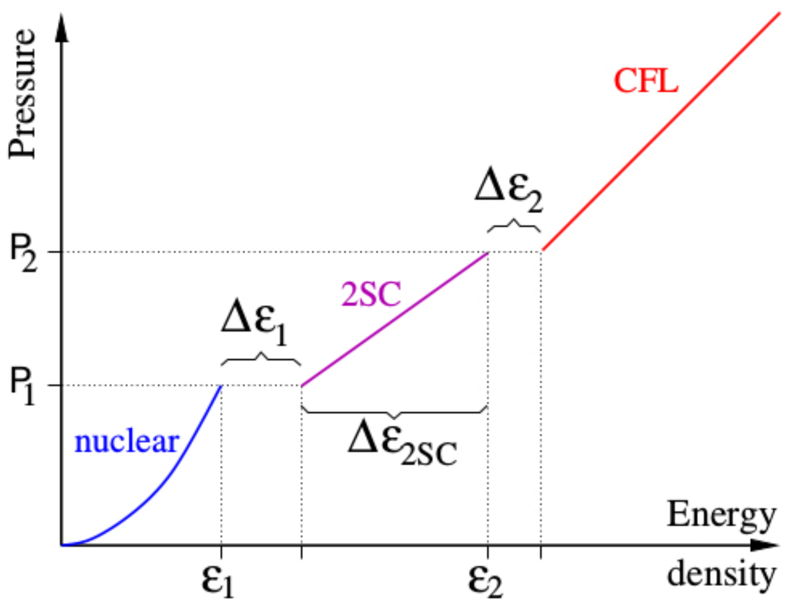


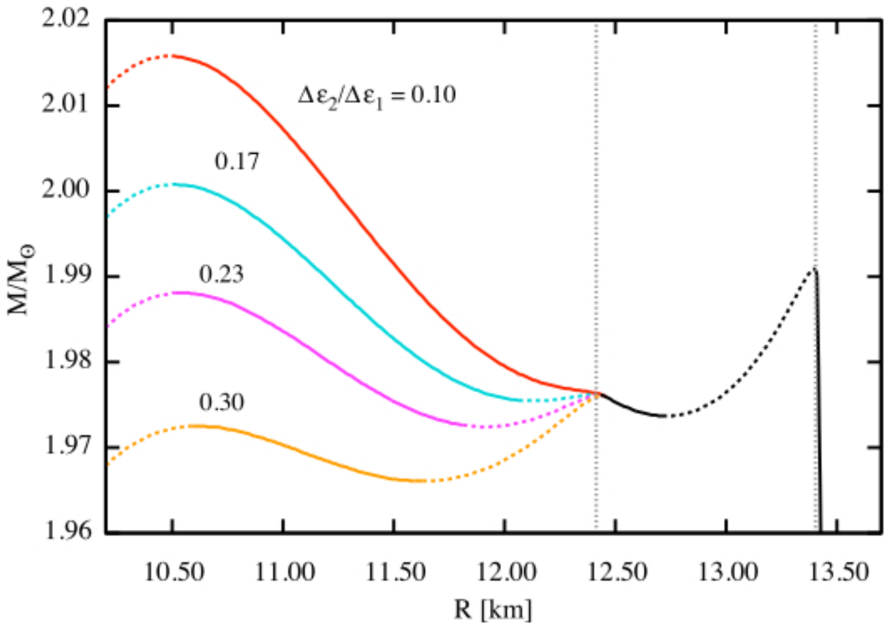
Gray region:
K. Hebeler, J. M. Lattimer, C. J. Pethick and A. Schwenk, Astrophys. J. **773**, 11 (2013).

Lines:
Four-polytrope model;
D.E. Alvarez-Castillo and D. Blaschke, High mass twins from multi-polytrope equations of state,
arXiv: 1703.02681v2, to appear

	Γ_3	κ_3 [MeV fm$^{3(\Gamma_3-1)}$]	$m_{0,3}$ [MeV]	M_{max}^{NS} [M$_\odot$]	M_{max}^{HS} [M$_\odot$]	M_{min}^{HS} [M$_\odot$]
set 1	2.50	302.56	991.75	2.01	-	-
set 2	2.80	365.12	1004.88	2.01	1.910	1.909
set 3	3.12	447.16	1014.87	2.01	1.991	1.934
set 4a	4.00	774.375	1031.815			
set 4b	2.80	548.309	958.553	2.01	2.106	1.961

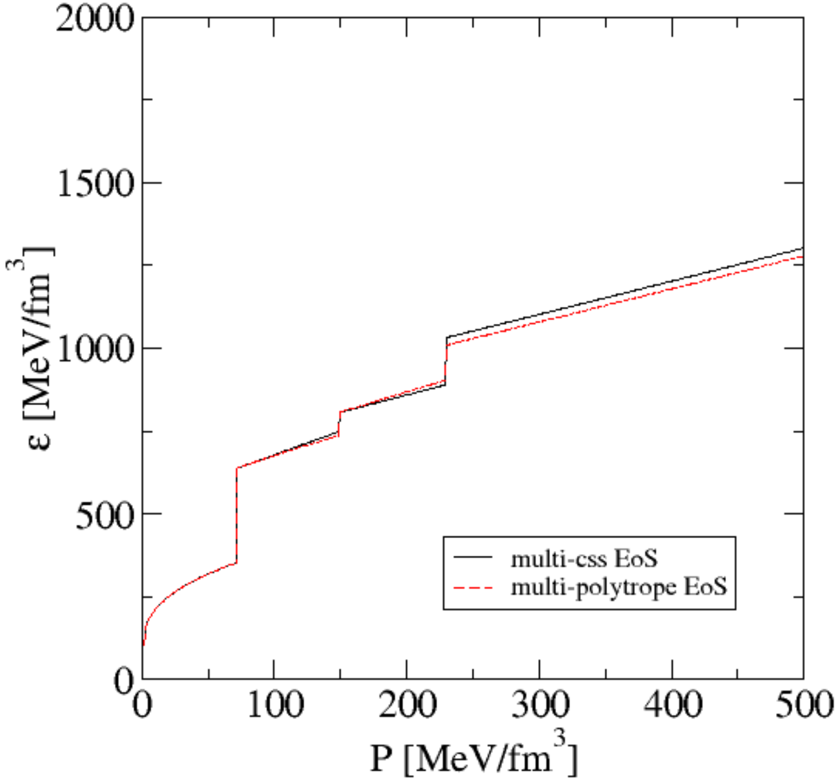
Compact Stars with Sequential QCD Phase Transitions

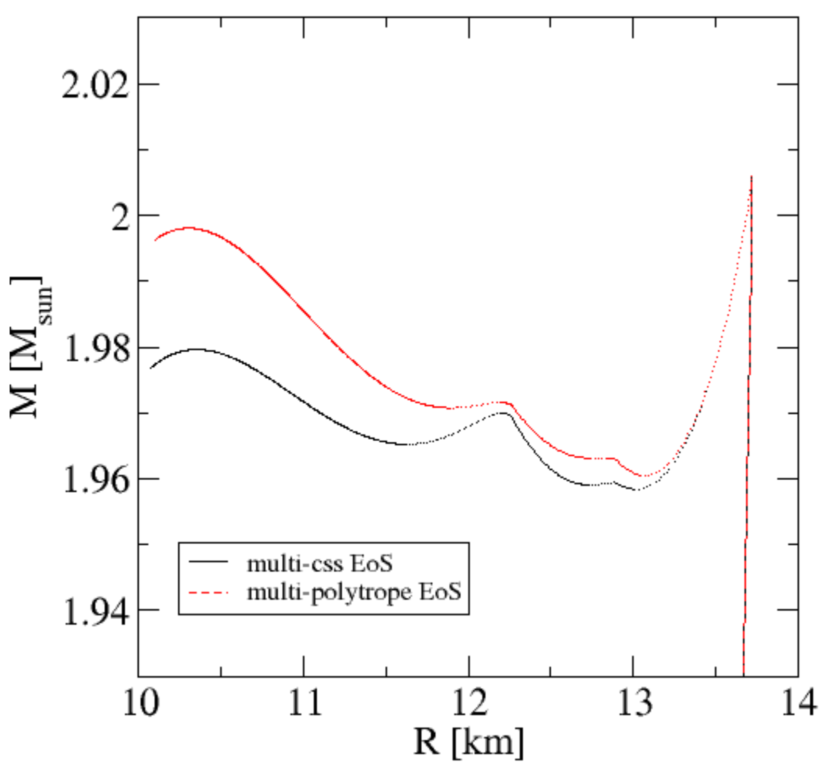




A. Sedrakian and M. Alford - 2017 - arXiv:1706.01592

A fifth family of compact stars at high mass





D. E. Alvarez-Castillo, D. B. Blaschke and H. Grigorian, i*n preparation*.

Quark substructure effects in baryonic matter

Excluded volume mechanism in the context of RMF models

Consider nucleons as hard spheres of volume $V_{\rm nuc}$, the available volume V_{av} for the motion of nucleons is only a fraction $\Phi = V_{av}/V$ of the total volume V of the system. We introduce

$$\Phi = 1 - v \sum_{i=n,p} n_i \ ,$$

with nucleon number densities n_i and volume parameter $v = \frac{1}{2}\frac{4\pi}{3}\left(2r_{\rm nuc}\right)^3 = 4V_{\rm nuc}$ and identical radii $r_{\rm nuc} = r_n = r_p$ of neutrons and protons. The total hadronic pressure and energy density are:

$$\begin{aligned} p_{\rm tot}(\mu_n, \mu_p) &= \frac{1}{\Phi} \sum_{i=n,p} p_i + p_{\rm mes} \ , \\ \varepsilon_{\rm tot}(\mu_n, \mu_p) &= -p_{\rm tot} + \sum_{i=n,p} \mu_i n_i \ , \end{aligned}$$

with contributions from nucleons and mesons depending on μ_n and μ_p. The nucleonic pressure

$$p_i = \frac{1}{4}\left(E_i n_i - m_i^* n_i^{(s)}\right) \ ,$$

contains the nucleon number densities, scalar densities and energies:

$$n_i = \frac{\Phi}{3\pi^3} k_i^3, \quad n_i^{(s)} = \frac{\Phi m_i^*}{2\pi^2}\left[E_i k_i - (m_i^*)^2 \ln \frac{k_i + E_i}{m_i^*}\right], \quad E_i = \sqrt{k_i^2 + (m_i^*)^2} = \mu_i - V_i - \frac{v}{\Phi} \sum_{j=p,n} p_j,$$

as well as Fermi momenta k_i and effective masses $m_i^* = m_i - S_i$. The vector V_i and scalar S_i potentials and the mesonic contribution $p_{\rm mes}$ to the total pressure have the usual form of RMF models with density-dependent couplings.

Benic, Blaschke, Alvarez-Castillo, Fischer, Typel, A&A 577, A40 (2015)

NJL model with multiquark interactions

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8\ ,\ \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2\ ,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\rm MF} = \bar{q}(i\partial\!\!\!/ - M)q + \tilde{\mu}_q\bar{q}\gamma^0 q - U\ ,$

$$M = m + 2\frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle + 4\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle\langle q^\dagger q\rangle^2\ ,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle - 4\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle\ ,$$

$$U = \frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle^2 + 3\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^4 - 3\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle^2 - \frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle^2 - 3\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^4\ .$$

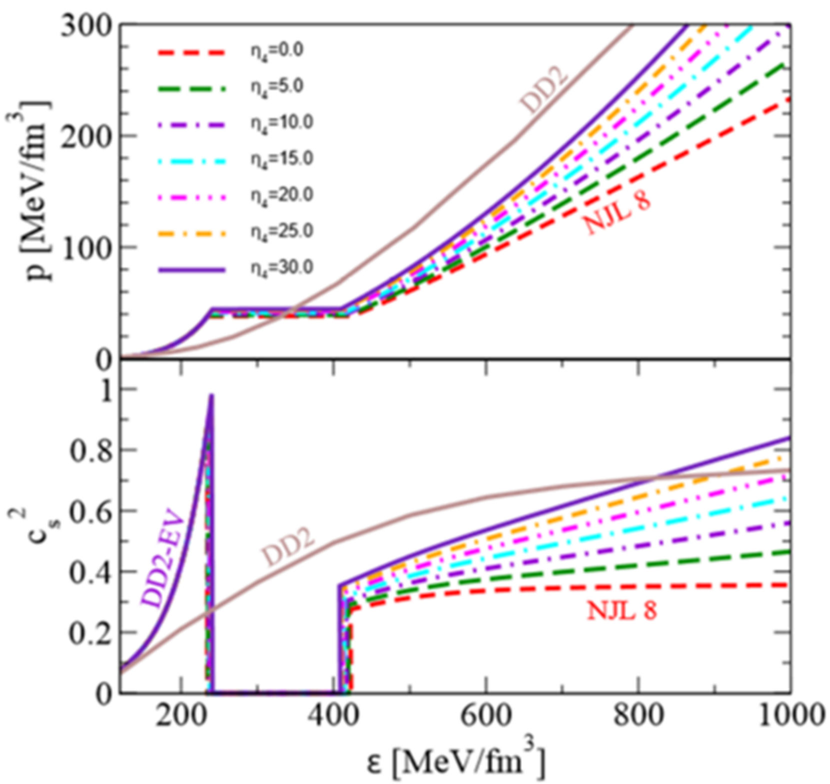
Thermodynamic Potential:

$$\Omega = U - 2N_f N_c \int \frac{d^3p}{(2\pi)^3}\Big\{E + T\log[1 + e^{-\beta(E-\tilde{\mu}_q)}] + T\log[1 + e^{-\beta(E+\tilde{\mu}_q)}]\Big\} + \Omega_0$$

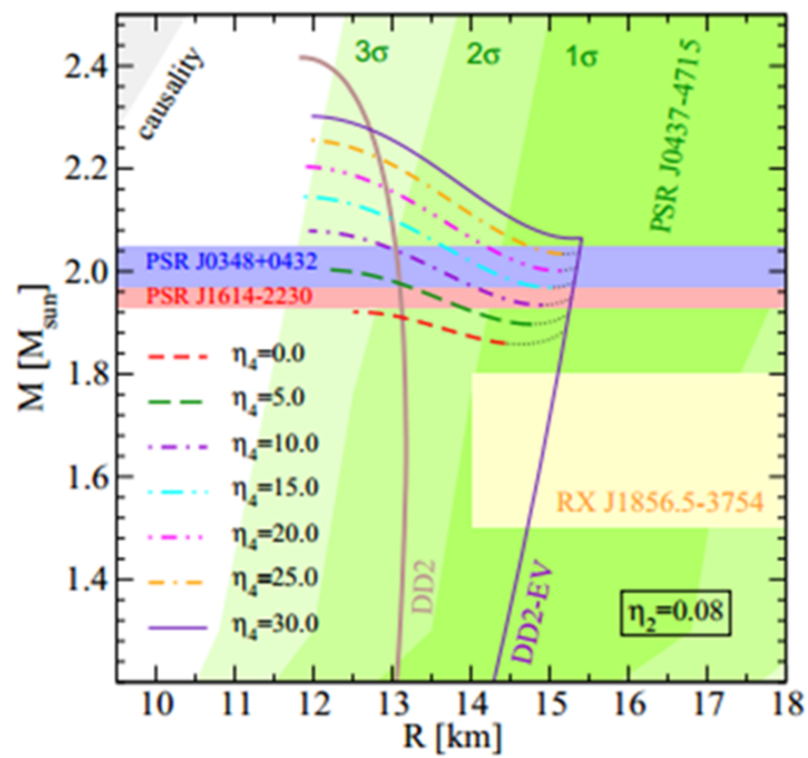
S. Benic - Eur.Phys.J. A50 (2014) 111, arXiv:1401.5380

Neutron Star Twins

Equation of State



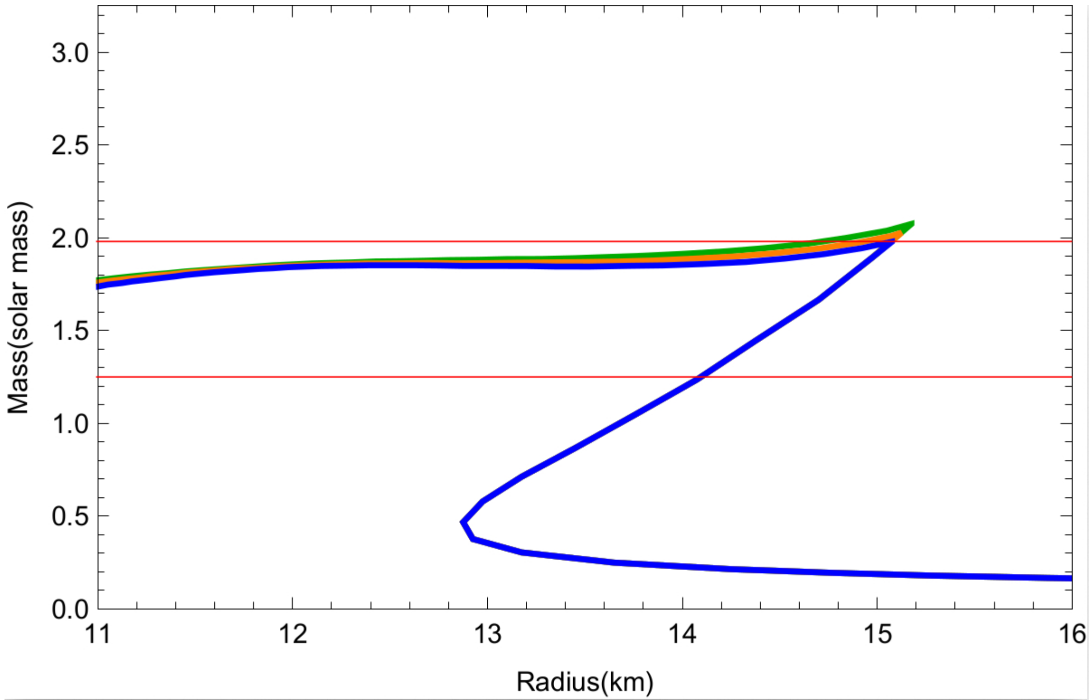
Mass-Radius Relation



Benic, Blaschke, Alvarez-Castillo, Fischer, Typel:
A&A 577, A40 (2015) - arXiv:1411.2856 (2014)

Neutron Star Twins

Nonlocal NJL models



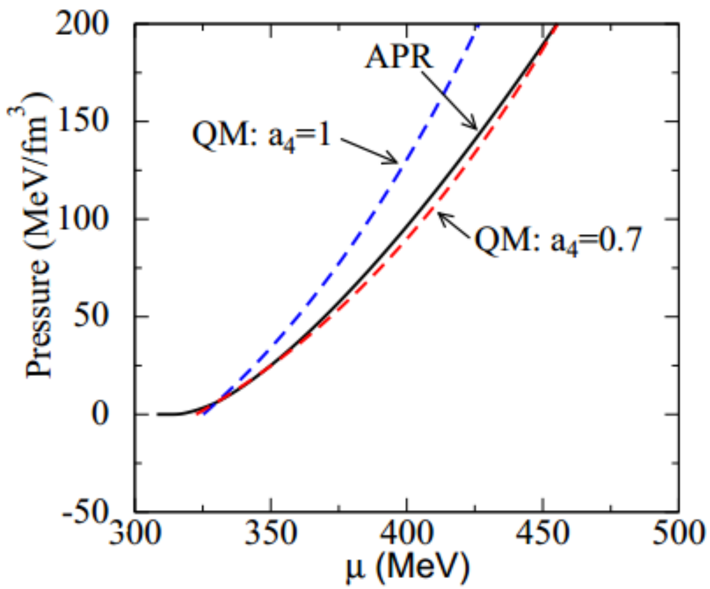
$$S_E = \int d^4x \, \left\{ \bar{\psi}(x) \left(-i\partial\!\!\!/ + m_c \right) \psi(x) - \frac{G_S}{2} j_S^f(x) j_S^f(x) - \frac{H}{2} \left[j_D^a(x) \right]^\dagger j_D^a(x) \right\}$$

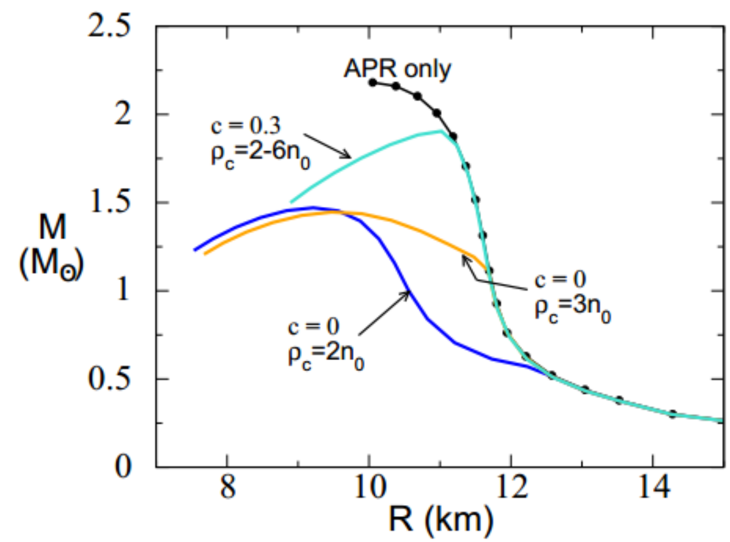
$$j_S^f(x) = \int d^4z \; g(z) \; \bar{\psi}(x + \frac{z}{2}) \; \Gamma_f \, \psi(x - \frac{z}{2}) \, , \;\; j_D^a(x) = \int d^4z \; g(z) \; \bar{\psi}_C(x + \frac{z}{2}) \; i\gamma_5 \tau_2 \lambda_a \; \psi(x - \frac{z}{2})$$

Grunfeld, Alvarez-Castillo, Blaschke, Pagura

Work in Progress

Avoiding Masquerades





$$\Omega_{\rm QM} = -\frac{3}{4\pi^2}\, a_4\, \mu^4 + \frac{3}{4\pi^2} a_2\, \mu^2 + B_{\rm eff}$$

$$a_4 \equiv 1 - c\ ,$$

Alford et al. - Astrophys.J.629:969-978, 2005 - arXiv:nucl-th/0411016

Avoiding reconfinement

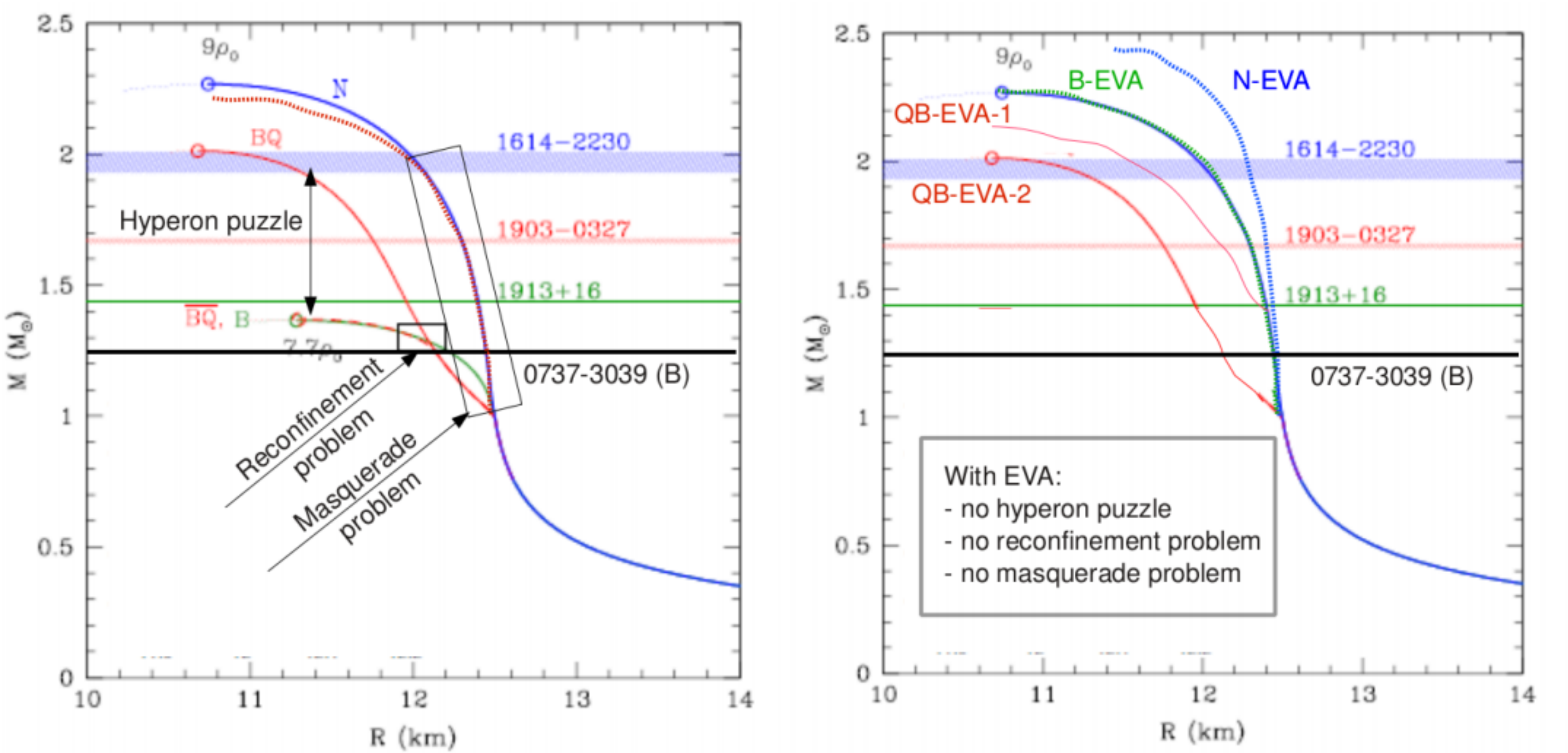
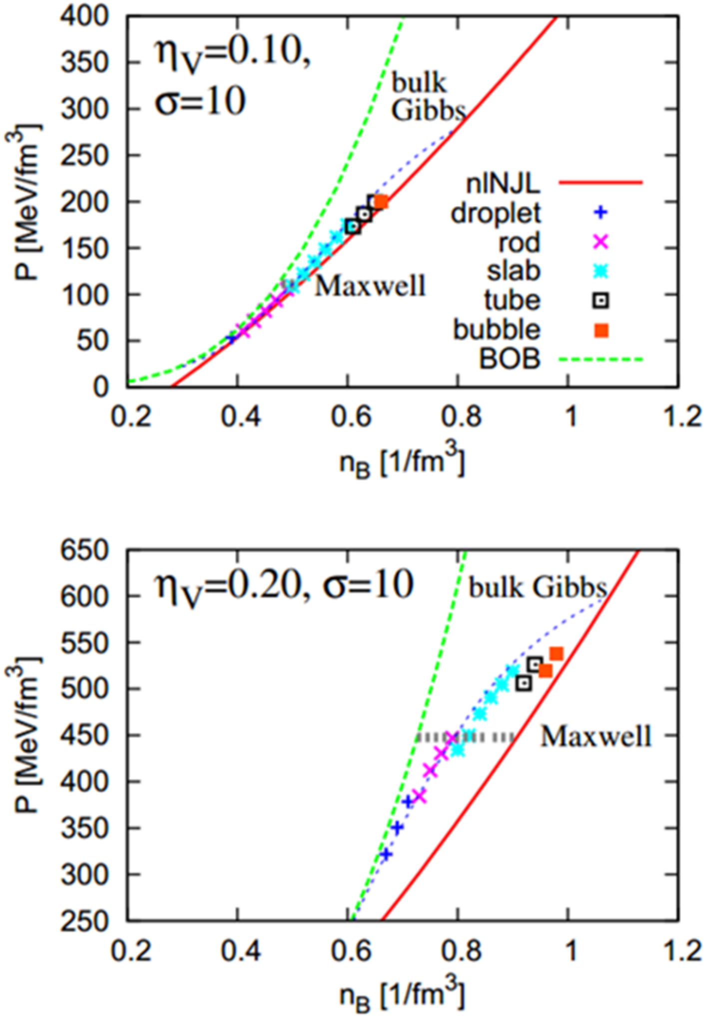


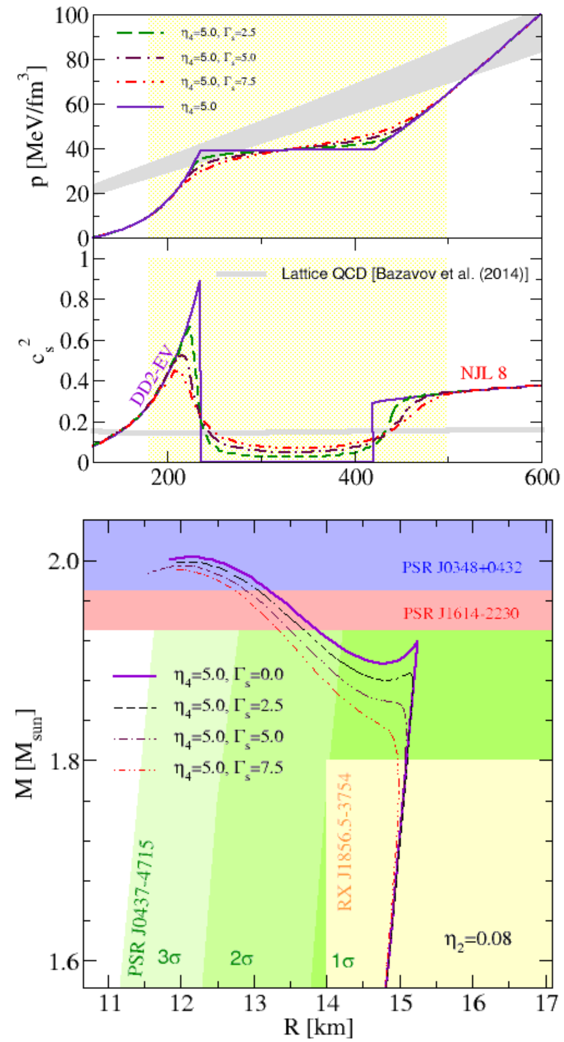
FIGURE 1. Mass-radius sequences for different model equations of state (EoS) illustrate how the three major problems in the theory of exotic matter in compact stars (left panel) can be solved (right panel) by taking into account the baryon size effect within a excluded volume approximation (EVA). Due to the EVA both, the nucleonic (N-EVA) and hyperonic (B-EVA) EoS get sufficiently stiffened to describe high-mass pulsars so that the hyperon puzzle gets solved which implies a removal of the reconfinement problem. Since the EVA does not apply to the quark matter EoS it shall be always sufficiently different from the hadronic one so that the masquerade problem is solved.

e, Alvarez-Castillo - AIP Conf. Proc. 1701, 020013 (2015) - arXiv:1503.03834

Pasta phases in hybrid stars

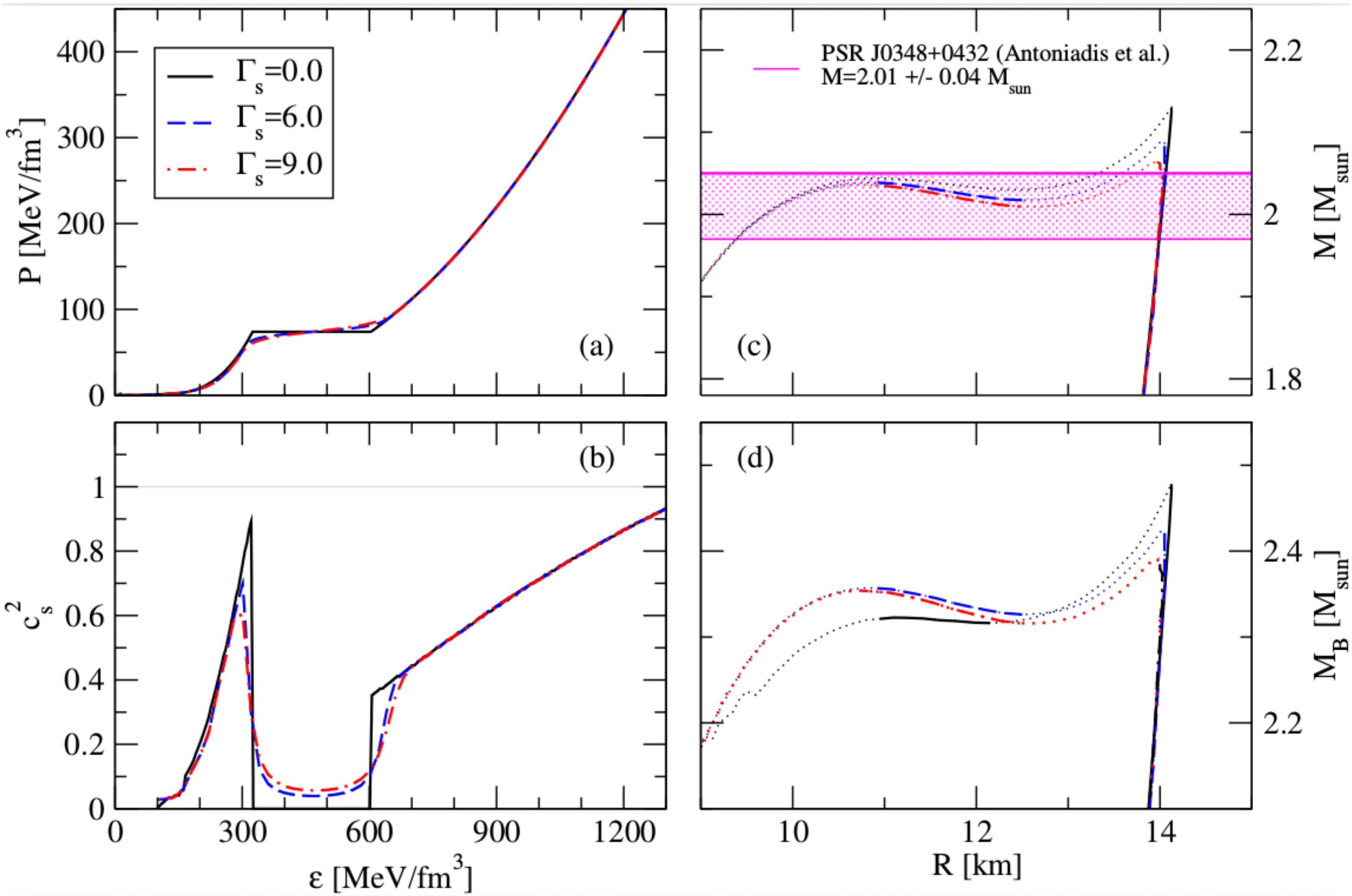


Yasutake et al., Phys. Rev. C 89, 065803 (2014)
arXiv:1403.7492



Alvarez Castillo, Blaschke, Phys. Part. Nucl. 46 (2015)

Pasta phases in hybrid stars



D. Alvarez Castillo, D. Blaschke, S. Typel
Proceedings of STARS2017, arXiv: 1709.08857

Astrophysical Applications

What can we learn from the inspiral II

- Waveforms incl. finite-size effects are described by tidal deformability (how a star reacts on an external tidal field)
- Offer possibility to constrain EoS because tidal deformability depends on EoS

$$\Lambda \equiv \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5$$

- Corresponding to ~10 % error in radius R for nearby events (<100Mpc) (e.g. Read et al. 2013)
- Note: faithful templates to be constructed

R/M compactness (EoS dependent)

k_2 tidal love number (EoS dependent)

Computing the love number/tidal deformability

Extension of a standard TOV solver (i.e. numerically an integration of coupled ODEs):

Ansatz for the metric including a l=2 perturbation

$$
\begin{aligned}
ds^2 \;=\; & -e^{2\Phi(r)}\left[1+H(r)Y_{20}(\theta,\varphi)\right]dt^2 \\
& +e^{2\Lambda(r)}\left[1-H(r)Y_{20}(\theta,\varphi)\right]dr^2 \\
& +r^2\left[1-K(r)Y_{20}(\theta,\varphi)\right]\left(d\theta^2+\sin^2\theta d\varphi^2\right)
\end{aligned}
$$

Following Hinderer et al. 2010

Integrate standard TOV system:

$$
\begin{aligned}
e^{2\Lambda} &= \left(1-\frac{2m_r}{r}\right)^{-1}, \\
\frac{d\Phi}{dr} &= -\frac{1}{\epsilon+p}\frac{dp}{dr}, \\
\frac{dp}{dr} &= -(\epsilon+p)\frac{m_r+4\pi r^3 p}{r(r-2m_r)}, \\
\frac{dm_r}{dr} &= 4\pi r^2\epsilon.
\end{aligned}
$$

EoS to be provided ε(p)

And additional eqs. for perturbations:

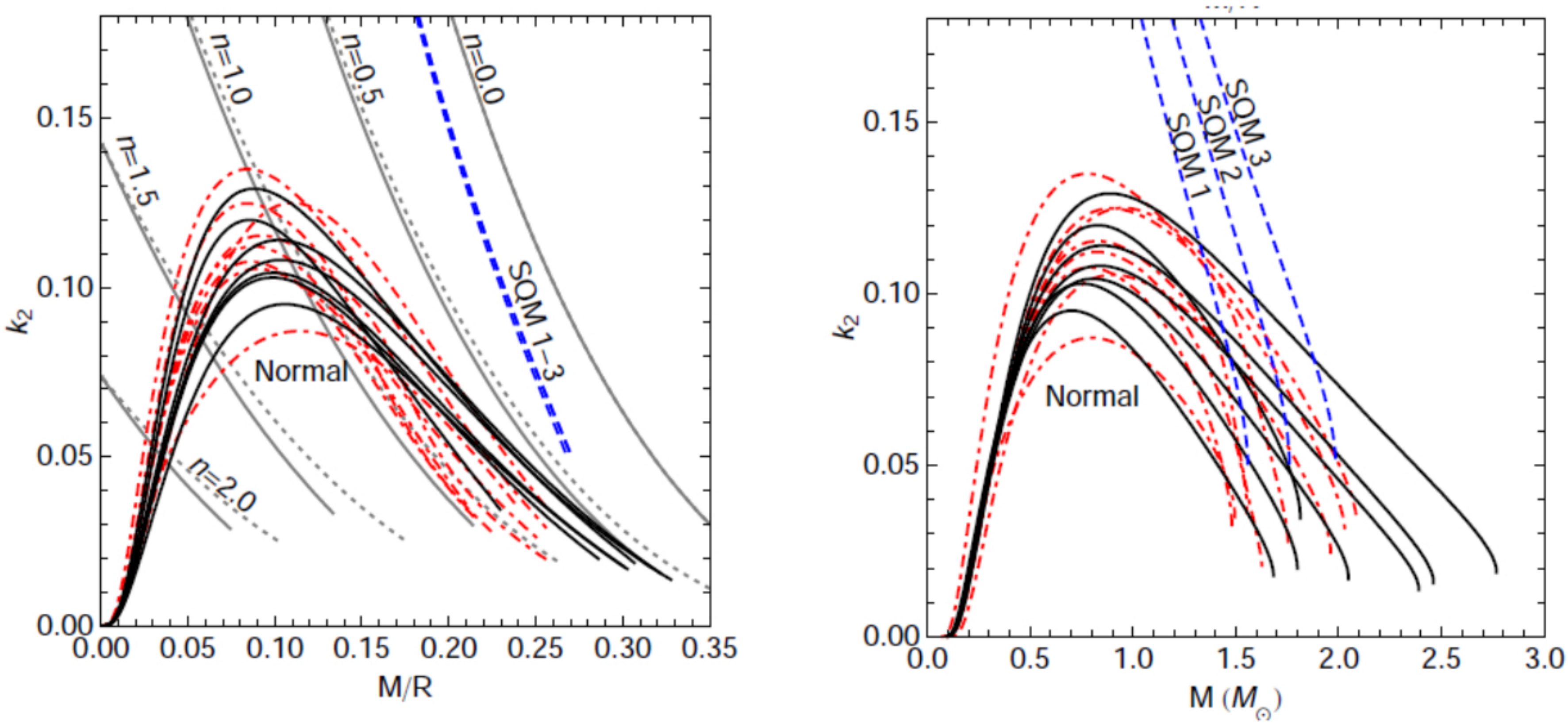
$$
\frac{dH}{dr} = \beta \qquad (11)
$$

$$
\begin{aligned}
\frac{d\beta}{dr} = & \; 2\left(1-2\frac{m_r}{r}\right)^{-1}H\left\{-2\pi\left[5\epsilon+9p+f(\epsilon+p)\right]\right. \\
& \left. +\frac{3}{r^2}+2\left(1-2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2}+4\pi rp\right)^2\right\} \\
& +\frac{2\beta}{r}\left(1-2\frac{m_r}{r}\right)^{-1}\left\{-1+\frac{m_r}{r}+2\pi r^2(\epsilon-p)\right\}.
\end{aligned}
$$

(K(r) given by H(r))

Note: Although multidimensional problem – computation in 1D since absorbed in Y20

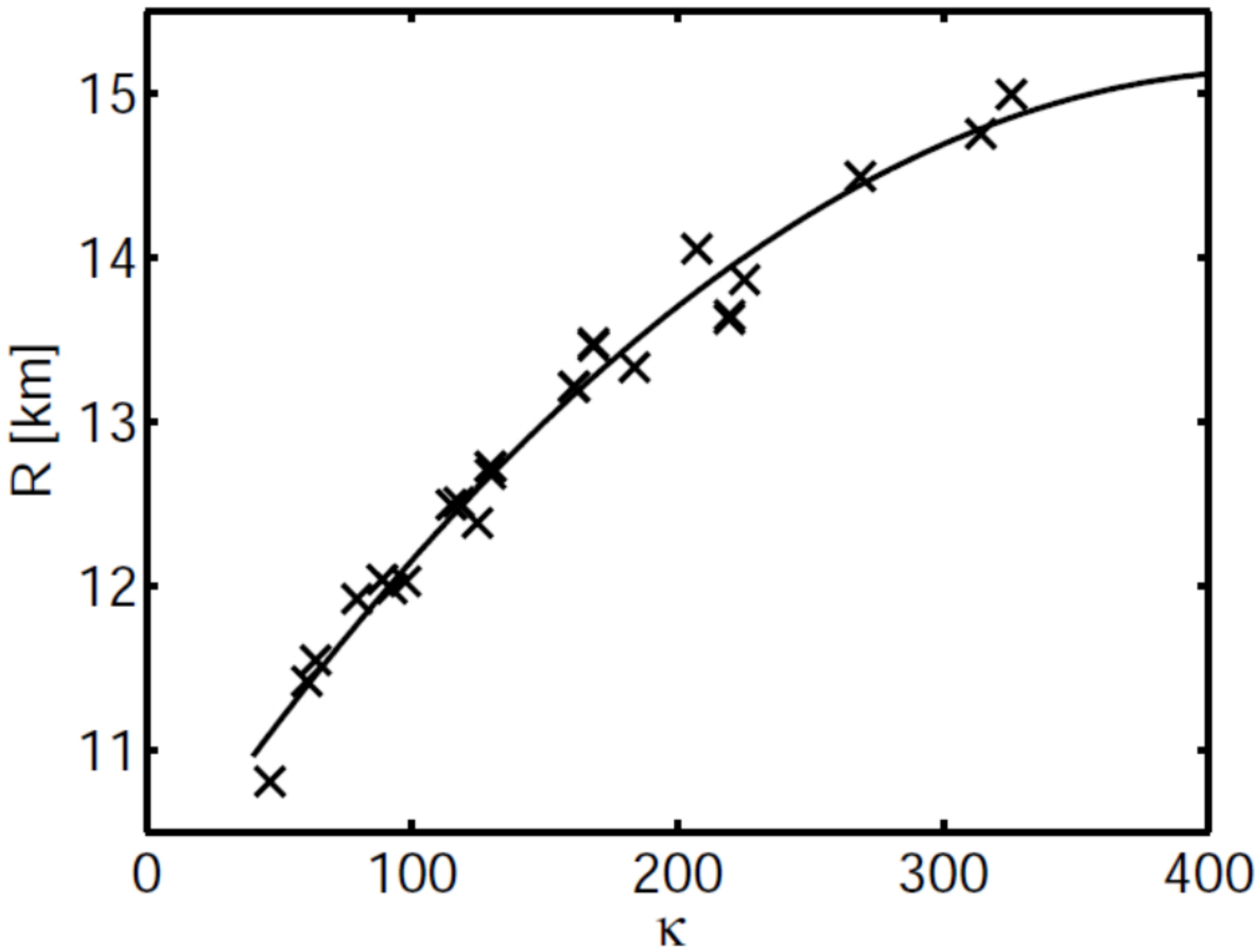
Love number



For fixed compactness k_2 depends on EoS => tidal deformability is not a unique function of compactness for different EoSs

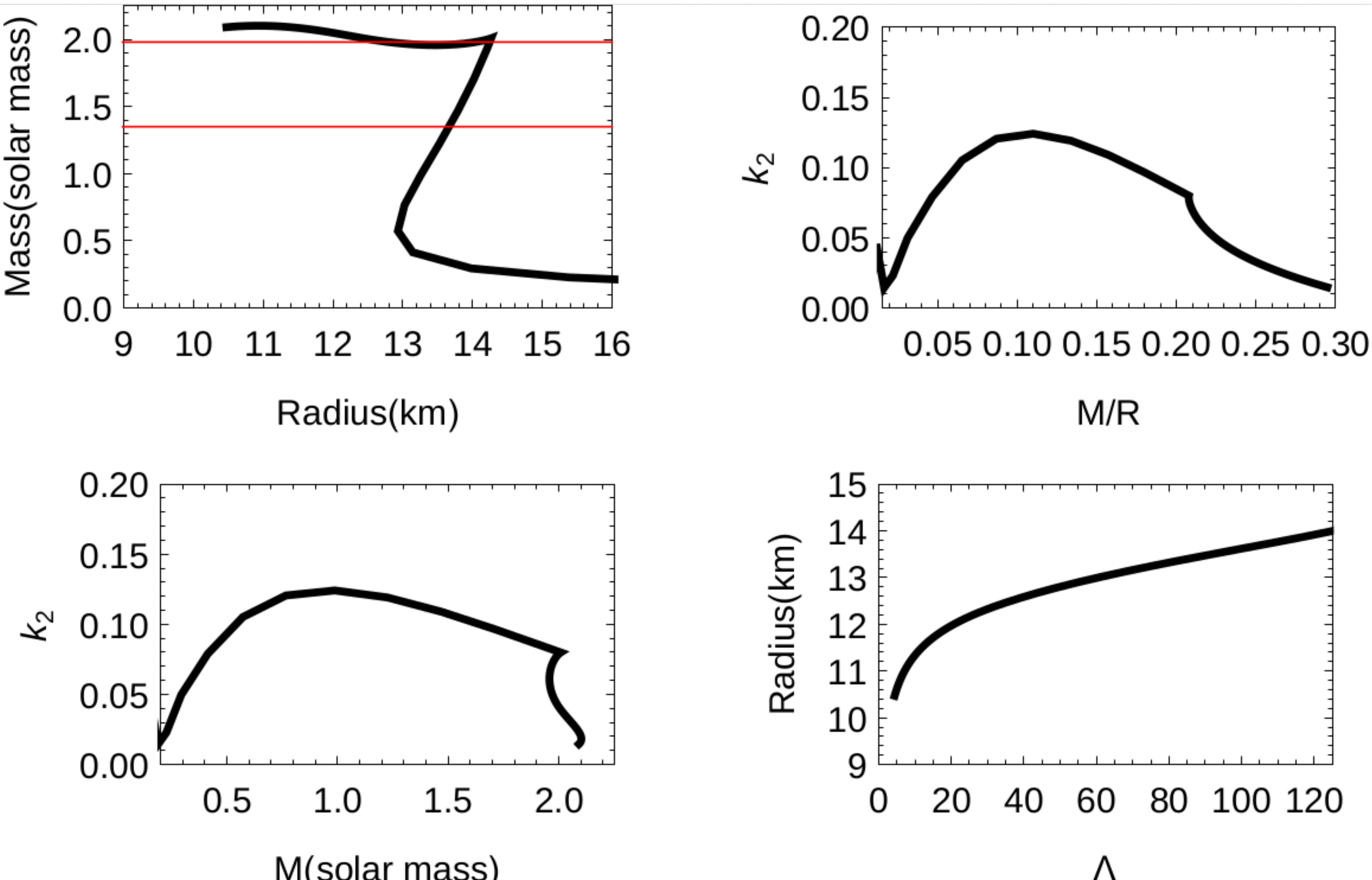
Hinderer et al. 2010

Tidal deformability vs radius

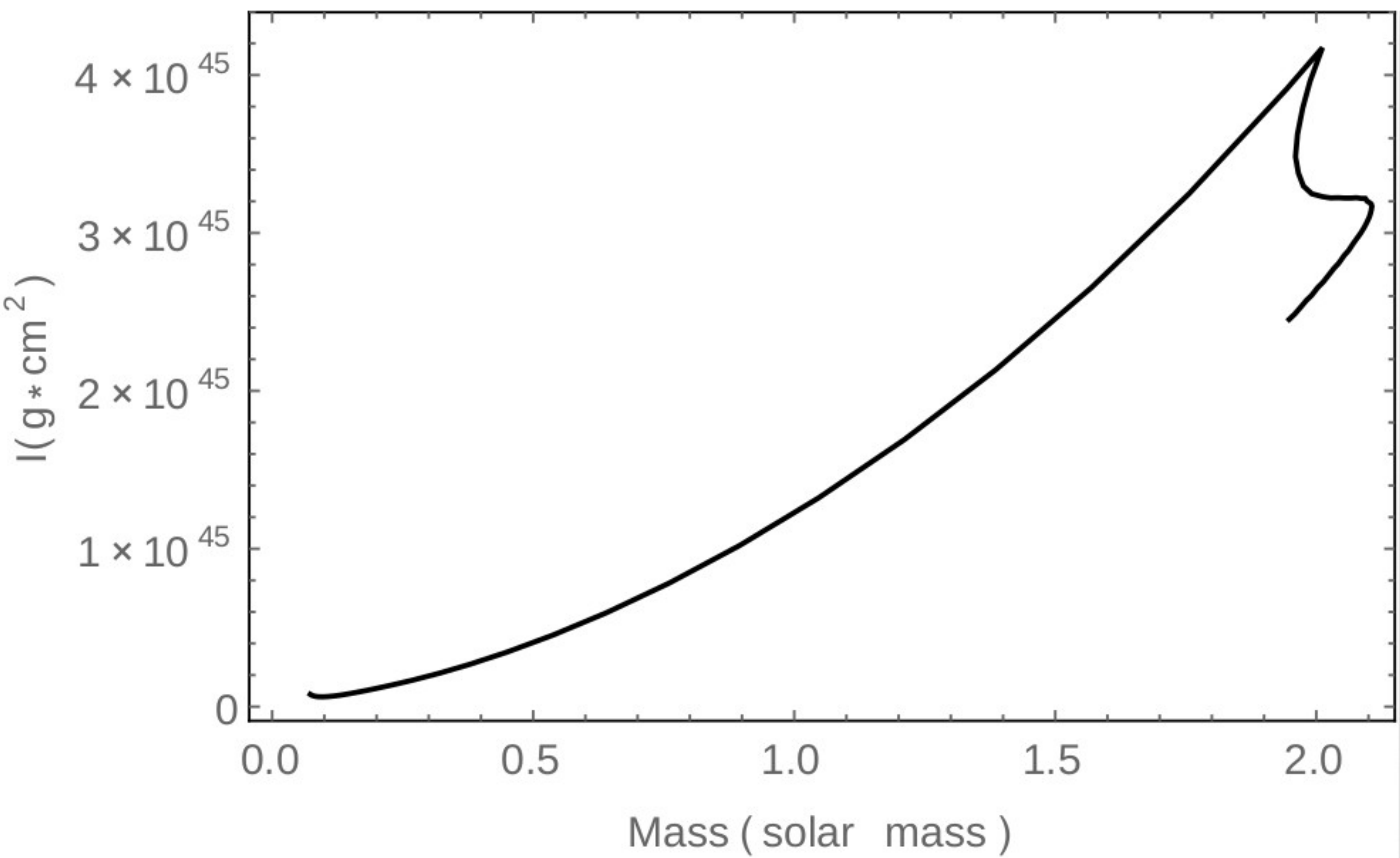


1.35 Msun stars with many different EoS, Bauswein 2015 unpublished, max dev. 314 meters

Neutron Star Twins

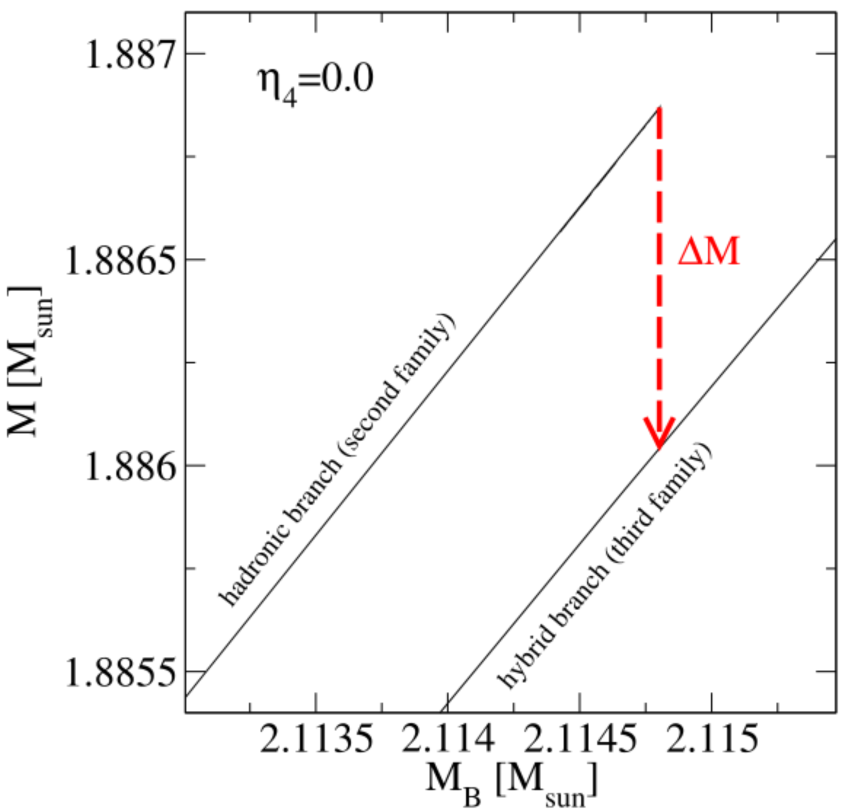


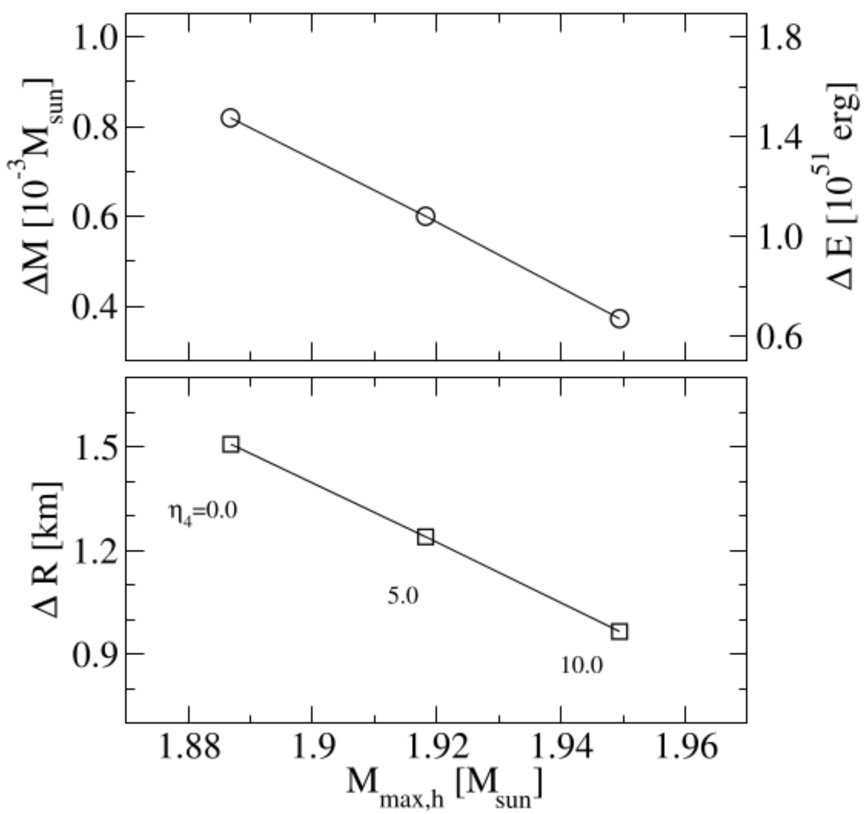
Neutron Star Twins



Energy bursts from deconfinement

(Problem: no neutrino trapping yet)





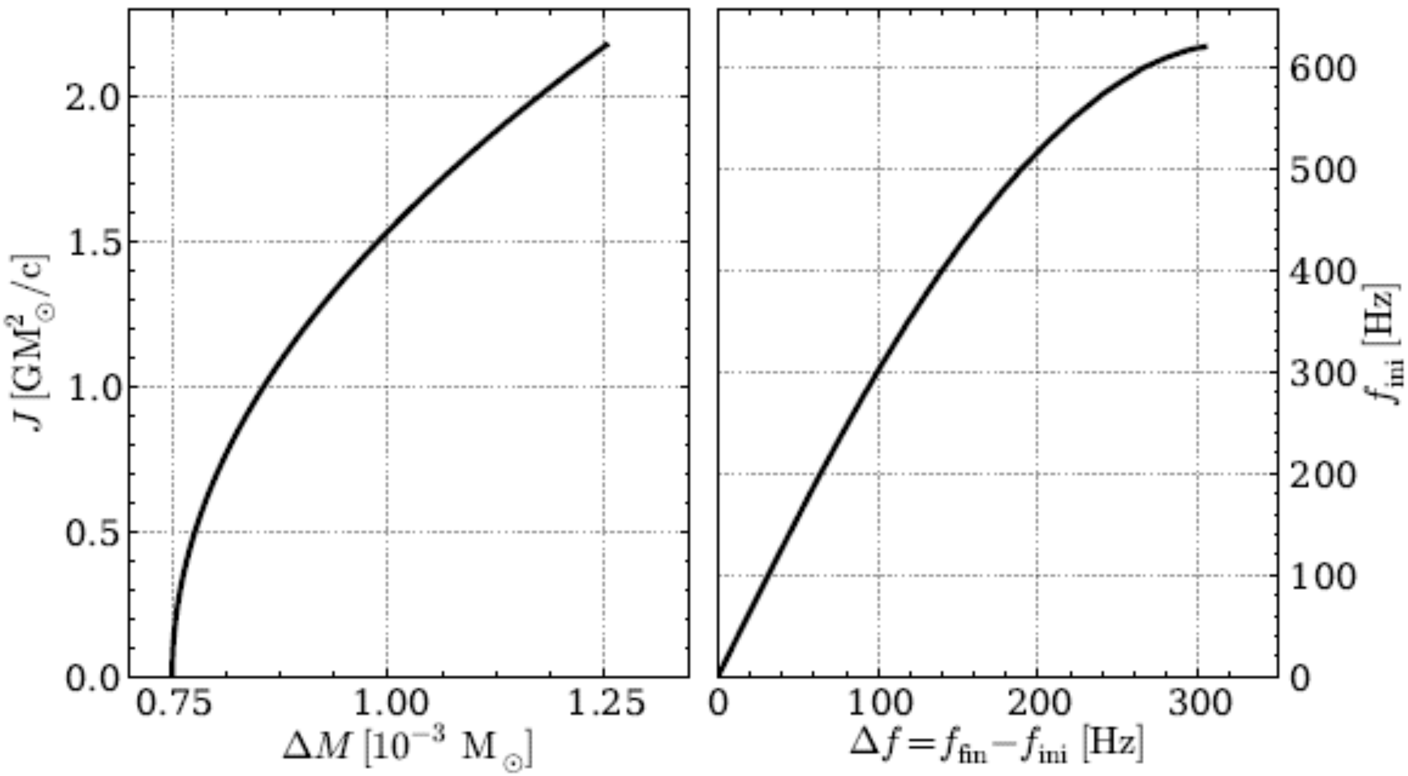
Alvarez-Castillo, Bejger, Blaschke, Haensel, Zdunik (2015), arXiv:1401.5380

Energy bursts from deconfinement

(case with rotation)

$$\Delta E^{\rm rot} = E^{\rm rot}_{\rm fin} - E^{\rm rot}_{\rm ini} = \frac{1}{2} J \left(\Omega_{\rm fin} - \Omega_{\rm ini}\right)$$

For J=2 GM_sun/c frequency changes by 240 Hz and Delta E^rot ~ 2x10^52 erg ...



Bejger, Blaschke, Haensel, Zdunik, Fortin, A&A 600 (2017) A39, arXiv:1608.07049

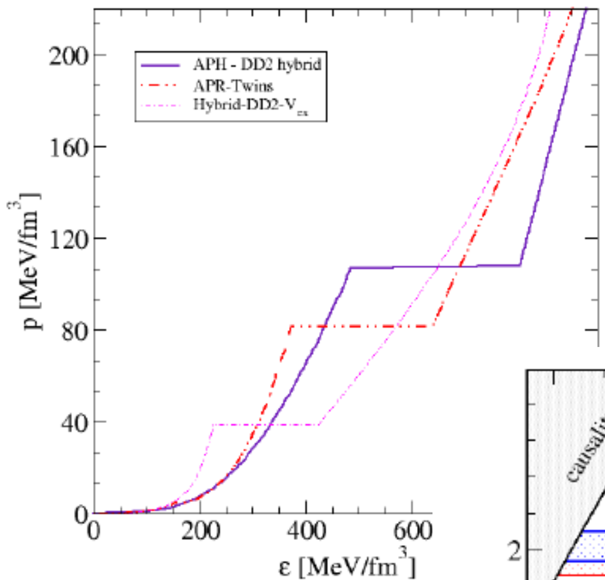
NICA White Paper – selected topics ...

Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

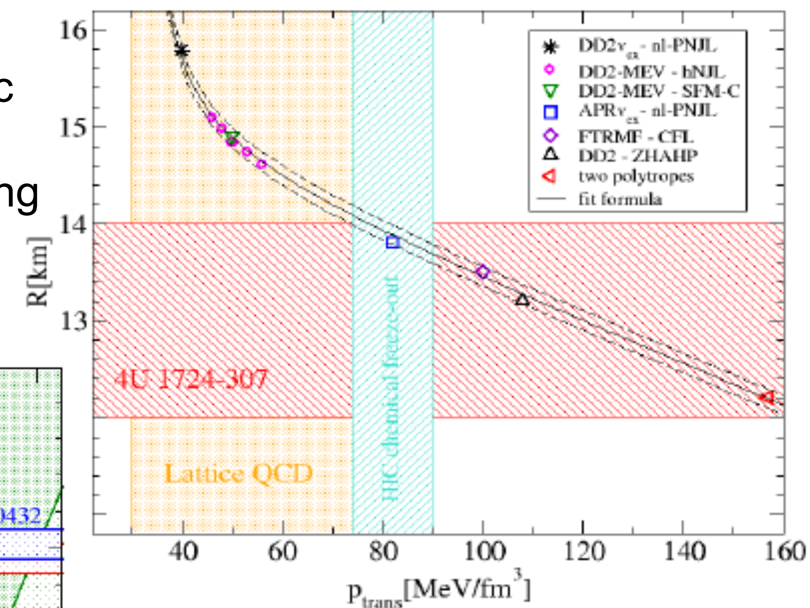
Neutron star mass limit at $2M_{\odot}$ supports the existence of a CEP

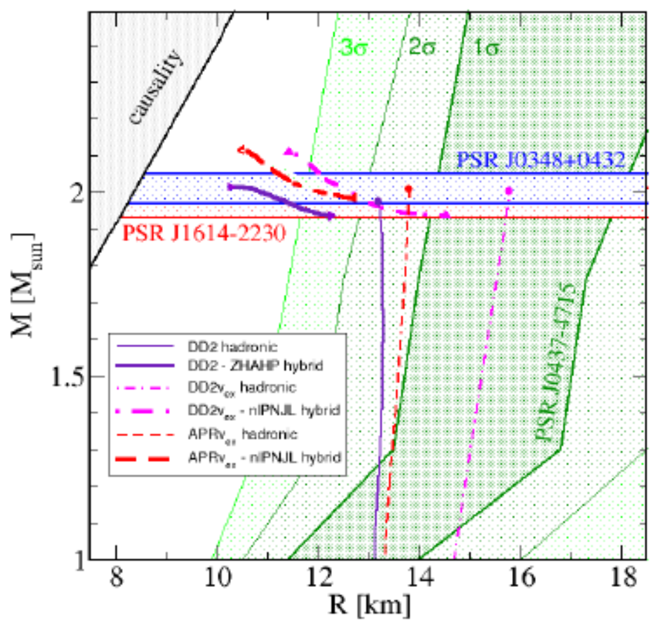
Eur. Phys. J. A 52, no. 8, 232 (2016)

D. Alvarez-Castillo[1,a], S. Benic[2,b], D. Blaschke[1,3,4], Sophia Han[5,6], and S. Typel[7]



Endpoint of hadronic Neutron star config. At 2Msun, then strong Phase transition





Strong phase transition

High-mass twin stars

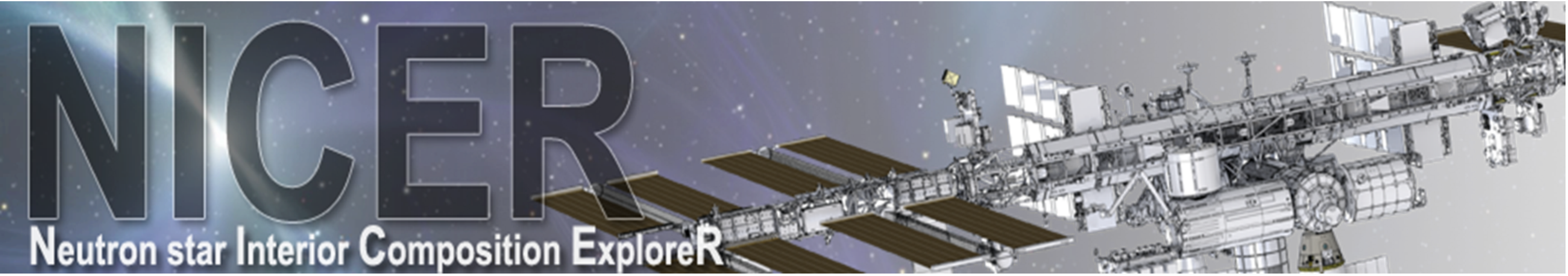
Universal transition pressure ?

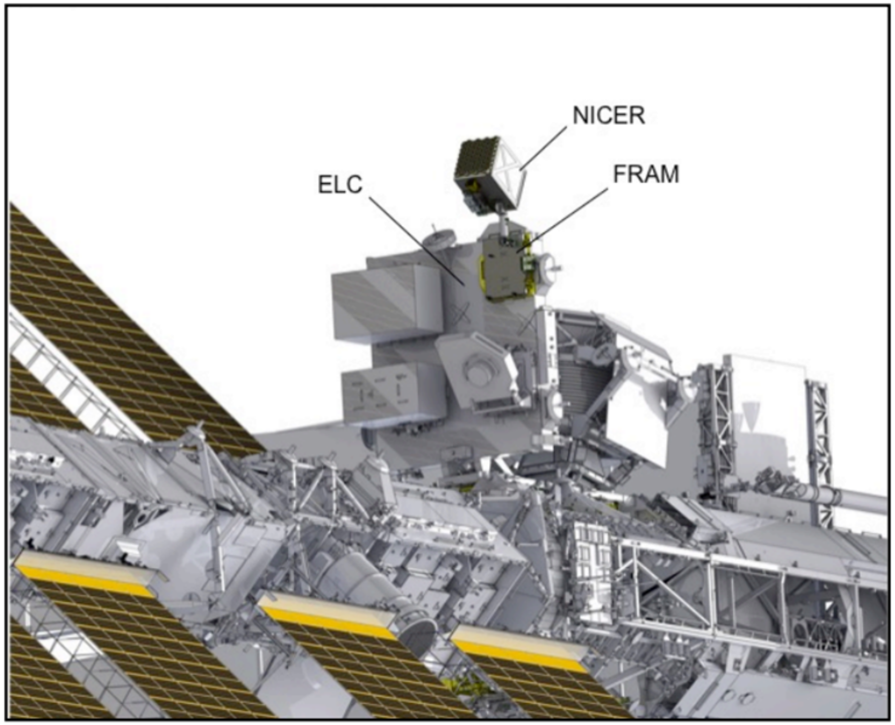
Petran & Rafelski, PRC 88, 021901

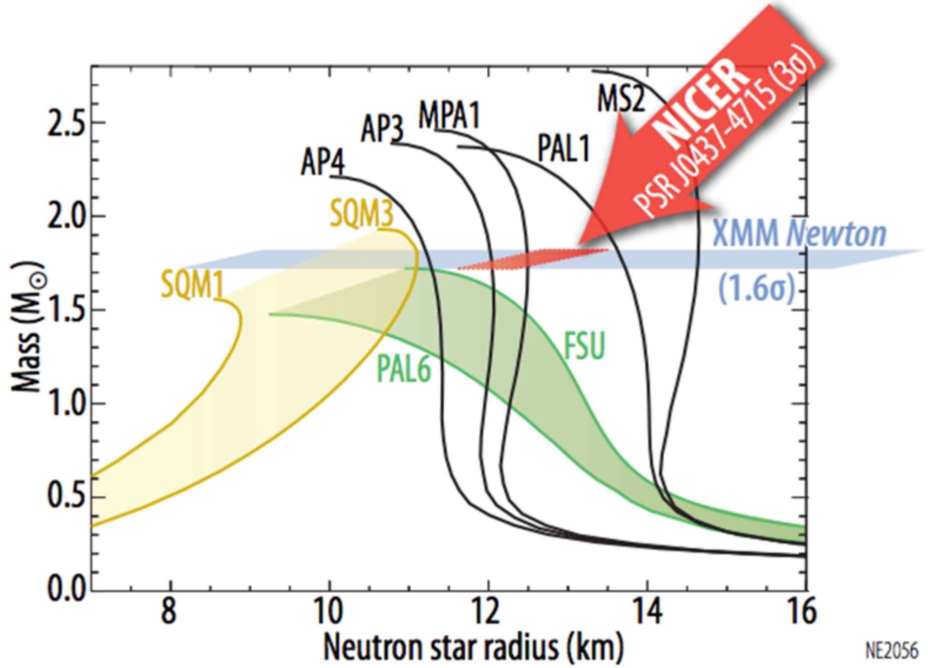
P_{trans} = 82 +/- 8 MeV/fm^3

Perspectives and future work

- Extension of the EoS to finite T: applications to supernovae and heavy ion collisions
- Gravitational wave signal estimation
- Moments of inertia: I love Q relations
- Radio emission description and dynamical collapse for the twins
- Pasta phases inclusion into the Bayesian Analysis for detection assessment



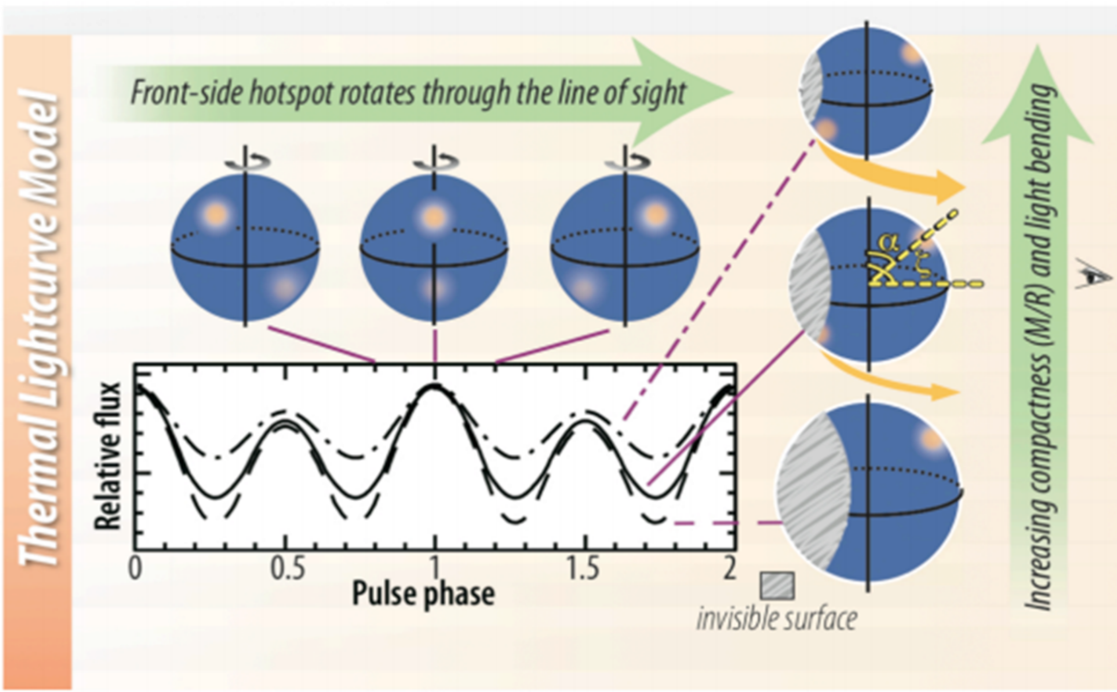


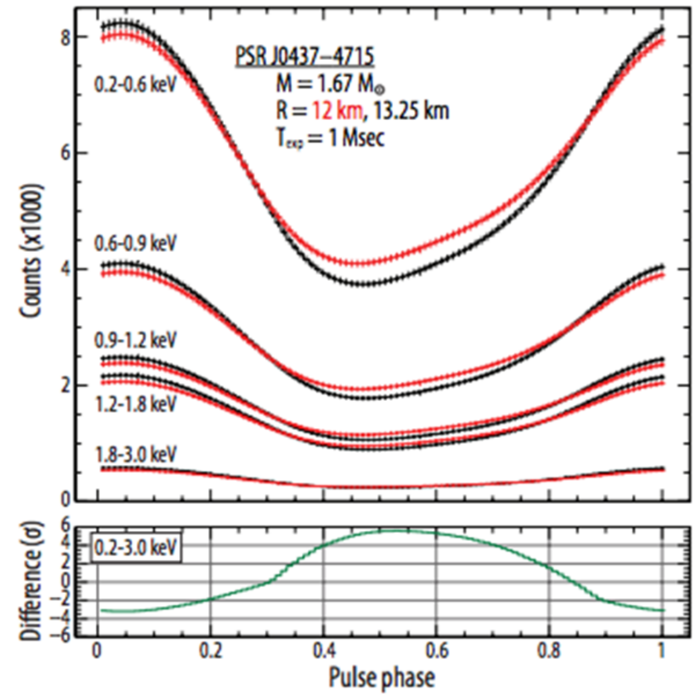


NICER 2017

Gendreau, K. C., Arzoumanian, Z., & Okajima, T. 2012, Proc. SPIE, 8443, 844313


NICER
Neutron star Interior Composition ExploreR


Thermal Lightcurve Model
Front-side hotspot rotates through the line of sight
Increasing compactness (M/R) and light bending
Relative flux
Pulse phase
0
0.5
1
1.5
2
α
ζ
invisible surface

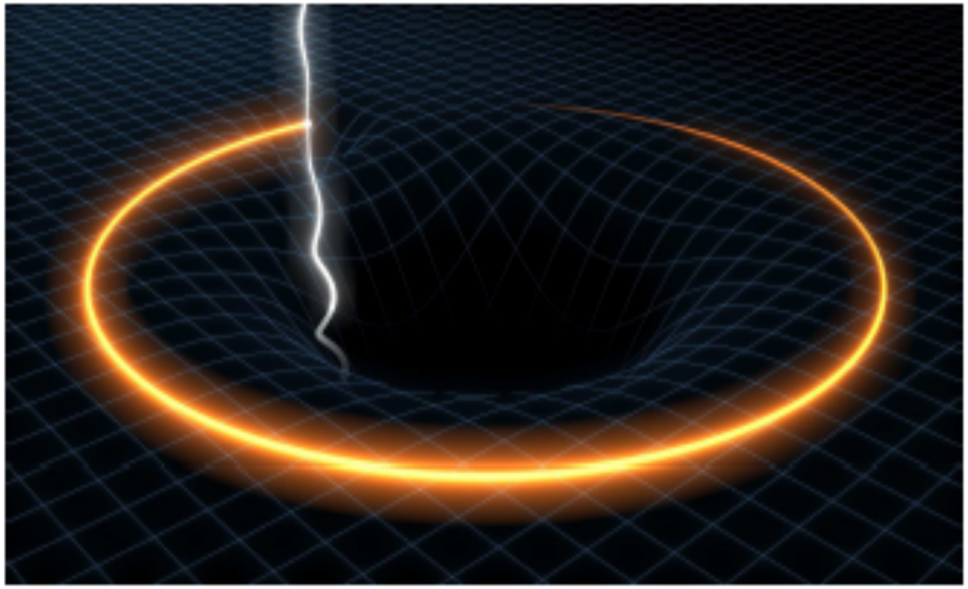

PSR J0437−4715
M = 1.67 M⊙
R = 12 km, 13.25 km
T_exp = 1 Msec
0.2-0.6 keV
0.6-0.9 keV
0.9-1.2 keV
1.2-1.8 keV
1.8-3.0 keV
Counts (x1000)
0.2-3.0 keV
Difference (σ)
Pulse phase

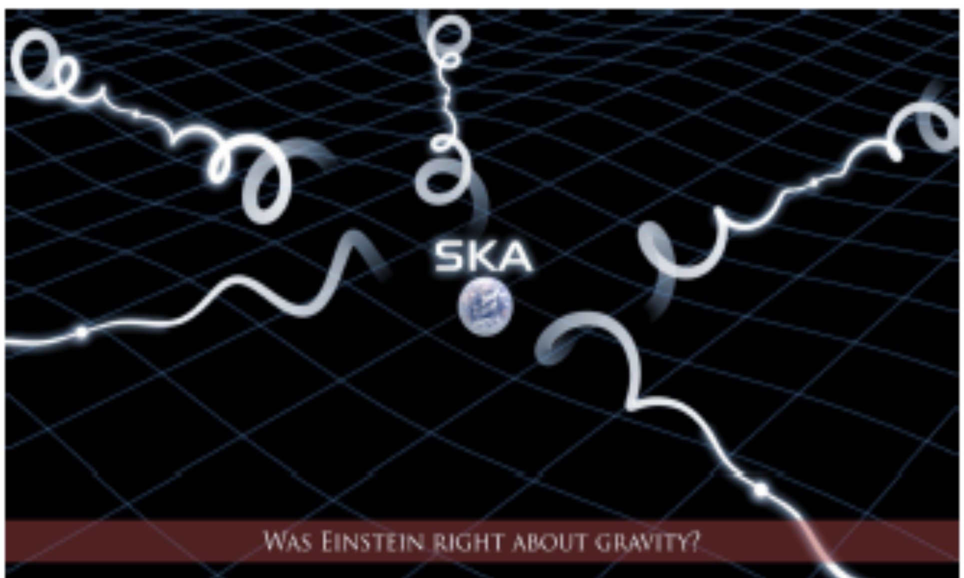
Hot Spots

Perspectives for new Instruments?

The Future: SKA - Square Kilometer Array

The Future: SKA - Square Kilometer Array





SKA Facts:

- The dishes of the SKA will produce 10 times the global internet traffic
- The data collected by the SKA in a single day would take nearly two million years to playback on an ipod
- The SKA will be so sensitive that it will be able to detect an airport radar on a planet 50 light years away

Discovery Potential:

- Find a Pulsar - Black Hole Binary
- Constrain Einstein Gravity
- Gravitational waves

Conclusions

- Three of the fundamental puzzles of compact star structure, the hyperon puzzle, the masquerade problem and the reconfinement problem may likely be all solved by accounting for the compositeness of baryons (by excluded volume and/or quark Pauli blocking) on the hadronic side and by introducing stiffening effects on the quark matter side of the EoS.

- Given the knowledge from lattice QCD that at zero baryon density the QCD phase transition proceeds as a crossover, twins would then support the existence of a CEP in the QCD phase diagram.

Conclusions

- Excluded volume effects (quark Pauli blocking) stiffen high-density nuclear matter and trigger an early deconfinement transition, thus play an important role for the M-R relations and cooling properties of compact stars.
-
- High mass neutron star twins robust against the appearance of pasta phases in the quark-hadron interface.
-
- Energy bursts via deconfinement feasible for the twins.
-
- Possible universal phase transition pressure.

Gracias

Announcement of the 11th BONN workshop on

– Formation and Evolution of Neutron stars –

- December 11+12, 2017
- MPIfR/AIfA University of Bonn, Auf dem Hügel 69–71, 53121 Bonn
- **Programme**:
  Start: 11:00 Monday, Dec. 11, 2017
  End: 16:00 Tuesday, Dec. 12, 2017

  Dinner Monday night is optional (at your own expense – @19:00 at a local restaurant)

- The topic for this 11th meeting is: ***Neutron Stars in Future Research***
  There will be a total of six sessions over two days:
  – Highlights and General News on Neutron Stars (New Research Results)
  – Accreting Neutron Stars
  – Millisecond Pulsars and their Applications I.
  – Millisecond Pulsars and their Applications II.
  – Supernovae, Young Neutron Stars and the Equation-of-state
  – Neutron Stars as Gravitational Wave Sources