Bayesian Analysis of hybrid EoS models with *M*-*R* data and phase transition construction mimicking mixed phase

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Outline

Outline

Bayesian analysis

What is Bayesian analysis?

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.
One of the most frequent case is analysis of probable values of model parameters.

Bayesian Analysis

Formulation of set of models (set of hypothesis):
π_i here $i = 0..N-1$

↓

Finding the *a priori* probabilities of the models:
$P(\pi_i) = 1/N \quad \text{for} \quad \forall i = 0..N-1$

↓

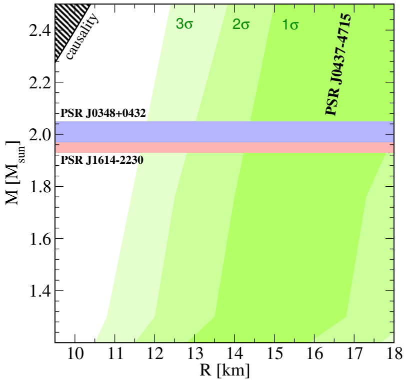
Calculating the coditional probabilities of the events:
$P(E\,|\vec{\pi}_i) = \prod_\alpha P(E_\alpha\,|\vec{\pi}_i)$,
where α is the index of the observational constraints.

↓

Calculating the *a posteriori* probabilities of the models:

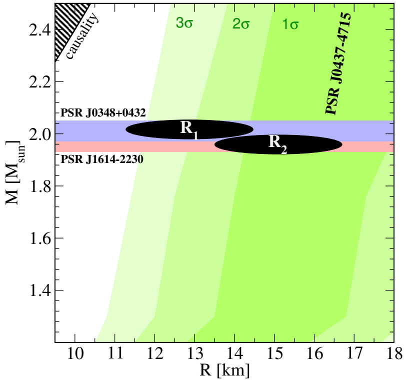
$$P(\vec{\pi}_i\,|E) = \frac{P(E|\vec{\pi}_i)P(\vec{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\vec{\pi}_j)P(\vec{\pi}_j)}$$

Mass and Radius Constraints



Radius and maximum mass constraints are given from PSR J0437-4715 (Bogdanov. Ast. J. **762**, 96) and PSR J0348+0432 (Antoniadis *et al.* Sci. **340**, 6131) correspondingly.

Fictitious Radius Constraints

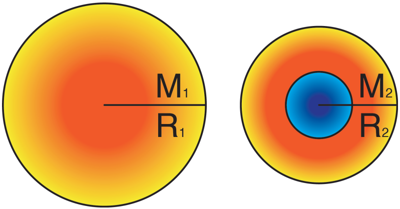


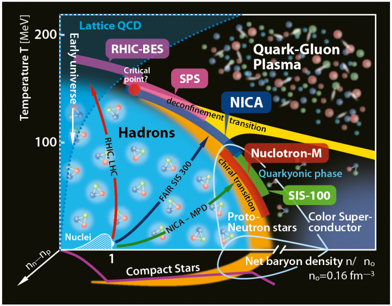
σ	R_1	R_2	R_1	R_2	R_1	R_2	R_1	R_2
1.5	11	13	11	15	13	15	15	11
1.0	11	13	11	15	13	15	15	11
0.5	11	13	11	15	13	15	15	11

We supposed that the inaccuracy of the fictitious measurements is normally distributed.

Fictitious Radius Constraints

What if we have twins





- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phase diagram?
  David Alvarez, *Supporting the existence of the QCD critical point by compact star observations*. 14:50 18 Sep 2017 (Monday), MPCS-2017, Yerevan, Armenia
- etc.

Hadronic matter EoS

Excluded-volume DD2

The excluded volume correction is applied at supersaturation densities and has the effect of stiffening the EoS. The available volume fraction Φ_N for the motion of nucleons at given density n:

$$\Phi_N = \begin{cases} 1 \, , & \text{if} \quad n \leq n_{\text{sat}} \\ \exp[-v|v|(n - n_{\text{sat}})^2/2] \, , & \text{if} \quad n > n_{\text{sat}} \, , \end{cases}$$

with $v = 16\pi r_N^3/3$ as the van-der-Waals excluded volume corresponding to a nucleon hard-core radius r_N, where $n_{\text{sat}} = 0.16 \ \mathrm{fm}^{-3}$ is the density in the interior of atomic nuclei. We introduce the dimensionless parameter $p = 10 \times v[\mathrm{fm}^3]$, taking values between $p = 0$ and $p = 80$ [S. Typel, EPJA 52 (3) (2016)].

Hadronic matter EoS

Symmetry energy for DD2

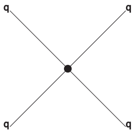
The symmetry energy $E_s(n)$ is defined as the difference in the energy per nucleon between pure neutron matter and symmetric matter in a uniform, infinite system. In RMF models the isovector ρ meson usually represents the only contribution to the isospin dependence of the interaction. Following [S. Typel, Phys. Rev. C **89**, 6, 064321 (2014)], we use here three parametrizations for the density-dependent ρ meson coupling ("soft", "medium" and "stiff").

$E_s(n)$	parametrization	
stiff	DD2+	DD2F+
medium	DD2	DD2F
soft	DD2−	DD2F−

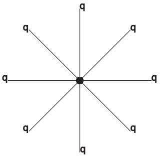
Quark matter EoS

Quark matter EoS with η_4 parameter

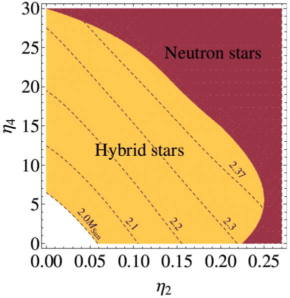
The quark matter was modelling by a two flavor Nambu-Jona- Lasinio (NJL) model with 8-quark interactions in the scalar and the vector channel [S. Benic, arxiv:1503.09145]. The η_2 was fixed [S. Benić, EPJA, 50:111 (2014)] and the η_4 – 8-quark vector NJL couplings parameter was variated from 0 to 30 with step 1 (so, $N_2 = 31$).

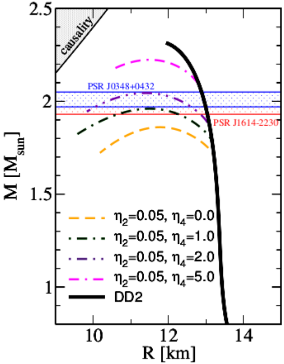


Dilute System



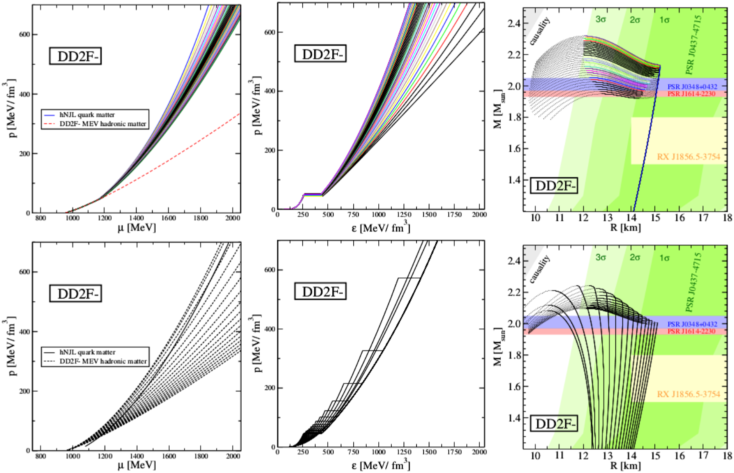
Dense System





Sanjin Benic. *EoS for dense matter with a QCD phase transition.*
15:00 26 Sep. 2017 (Tuesday), CSQCD-2017, Dubna, Russia

Hybrid matter EoS



Variations of the hybrid EoS for the DD2F$^-$ model. *Upper row.* The hadronic EoS is kept fixed while the quark EoS is allowed to vary for the parameters $\eta_4 = 0, 1, 2..30$.

Vector of Parameters

Vector of Parameters

For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values $\overrightarrow{\pi} = \{p, \eta_4\}$:

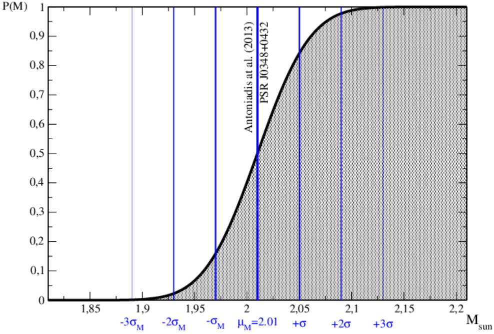
$$\overrightarrow{\pi}_i = \left\{p_{(k)}, \eta_{4(l)}\right\},$$

$i = 1 \dots N$ (here $N = N_1 \times N_2$), $i = N_2 \times k + l$ and $k = 0 \dots N_1 - 1$, $l = 0 \dots N_2 - 1$

Calculation of Probabilities

Conditional Probability of Mass Constraint for π_i

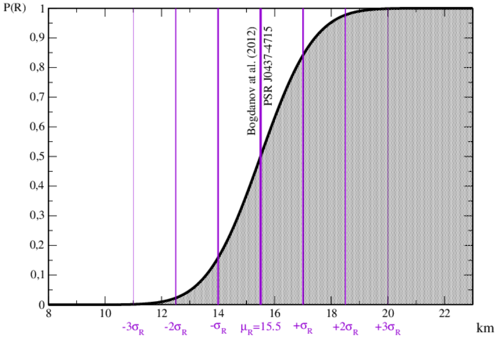
$P(E_A|\pi_i) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i.
$\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$ [Antoniadis et al. Science 340].



Calculation of Probabilities

Conditional Probability of Radius Constraint for π_i

$P\left(E_B \,|\pi_i\,\right) = \Phi(R_i, \mu_B, \sigma_B)$, here R_i is max radius given by π_i.
$\mu_B = 15.5 \text{ km}$ and $\sigma_B = 1.5 \text{ km}$ [Bogdanov/Hambaryan et al.].



Calculation of Probabilities

Probability of fictitious radius constraints

The probability to fulfil the constraint will correspond to the area where the configurations have values of radii predicted from the model when the masses are in the interval of the observational mass range $M_{G_j} \pm \Delta M_j$:

$$\begin{aligned} P\left(E_{F_\alpha} \,|\, \overrightarrow{\pi}_i\right) &= \Phi\left(R_\alpha(M_{G_\alpha} + \Delta M_\alpha; \overrightarrow{\pi}_i), \mu_{R_\alpha}, \sigma_{R_\alpha}\right) \\ &- \Phi\left(R_\alpha(M_{G_j} - \Delta M_\alpha; \overrightarrow{\pi}_i), \mu_{R_\alpha}, \sigma_{R_\alpha}\right) \end{aligned}$$

Because in some cases (mainly for the hybrid stars) the $R_\alpha(M_{G_\alpha}; \overrightarrow{\pi}_i)$ can be not uniquely defined functions, we have excluded possible overlaps of the boxes of the radius probability regions on the $M_{G_\alpha} \otimes R_\alpha$ plane to avoid the double counting.

Calculation of Probabilities

Probability of All Constraints for π_i

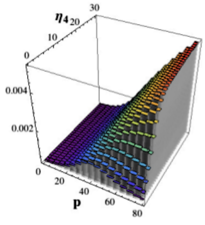
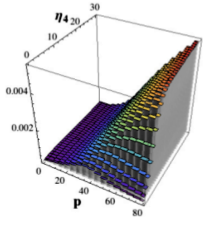
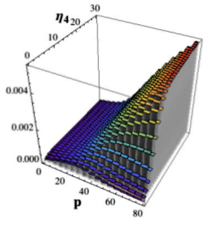
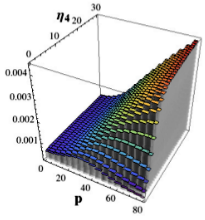
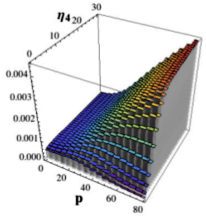
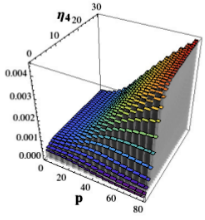
Taking to the account assumption that these measurements are independent on each other we can calculate complete conditional probability:

$$P\left(E \,|\, \overrightarrow{\pi}_i\right) = \prod_{\alpha} P\left(E_\alpha \,|\, \overrightarrow{\pi}_i\right).$$

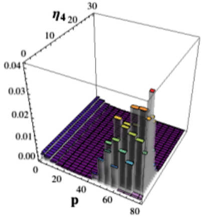
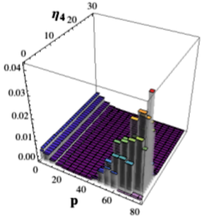
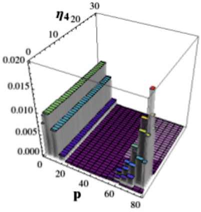
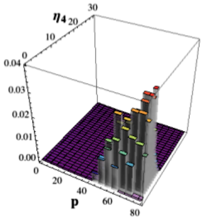
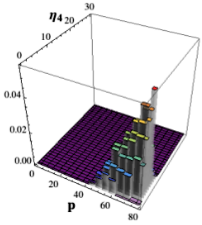
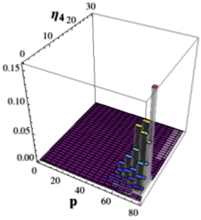
Calculation of *a posteriori* Probabilities of π_i

$$P\left(\overrightarrow{\pi}_i \,|\, E\right) = \frac{P\left(E \,|\, \overrightarrow{\pi}_i\right) P\left(\overrightarrow{\pi}_i\right)}{\sum_{j=0}^{N-1} P\left(E \,|\, \overrightarrow{\pi}_j\right) P\left(\overrightarrow{\pi}_j\right)}.$$

BA results with *M*-*R* data

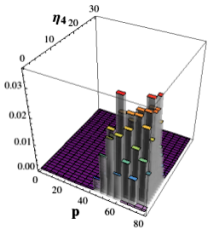
<table>
<tr><th>Symmetry energy →
EoS ↓</th><th>soft</th><th>medium</th><th>stiff</th></tr>
<tr><td>DD2F (semi-soft)</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>DD2 (stiff)</td><td>
</td><td>
</td><td>
</td></tr>
</table>

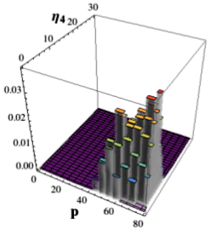
BA results with fictitious radius measuremets

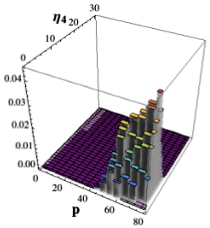
Radius error $\sigma \rightarrow$ R_A, R_D	1.5 km	1.0 km	0.5 km
11 km, 13 km			
11 km, 15 km			

BA results with fictitious measuremets

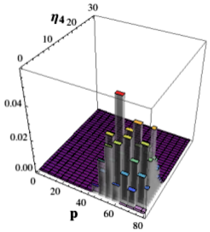
13 km, 15 km

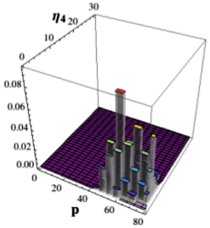


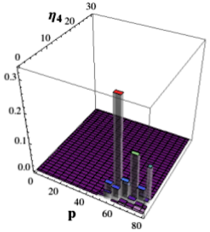




15 km, 11 km







Conclusions

- The region of the most probable models in the two-dimensional parameter space is sufficiently narrow, covering the ranges $40 < p < 80$ and $3 < \eta_4 < 7$.
- When fictitious radius measurements yield the smaller radius for the object with the slightly smaller mass, then the most probable value of the excluded volume parameter is lowered from $p = 80$ to the moderate $p \sim 50$, while the optimal stiffness of the quark matter remains unchanged, $\eta_4 \sim 5$.
- The developed BA tool has quote strong selective power even if the 5σ regions of the fictitious radius errors have deep overlapping.

Ayriyan, Alvarez, Benic, Blaschke, Grigorian, Typel. Acta Phys. Pol. B **10**(3) (2017) 799-804

Alvarez, Ayriyan, Benic, Blaschke, Grigorian, Typel. Eur. Phys. J. A (2016) **52**:69

Ayriyan, Alvarez, Blaschke, Grigorian. J. Phys. CS **668** (2016) 012038

Ayriyan, Alvarez, Blaschke, Grigorian, Sokolowski. Phys. Part. Nucl. **46**(5), 2015, 854-857

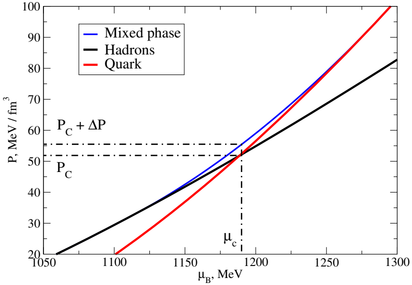
Blaschke, Grigorian, Alvarez, Ayriyan. J. Phys. CS **496** (2014), 012002

Outline

Motivation

Usually modeling of possible hadron-quark phase transition are made with use of the so-called Maxwell construction, where the two phases are assumed to be separated. However due to surface tension effects [D. Voskresensky et al. Nuc. Phys. A **723** (2003)] in the mixed phase with structure (pasta) could be thermodynamically preferred [T. Tatsumi et al. arXiv:1107.0804]. Thus a simple model of such a mixed phase equation of state parametrized by impact of structures in mixed phase of the pressure ΔP will be very useful.

Idea and Formalism



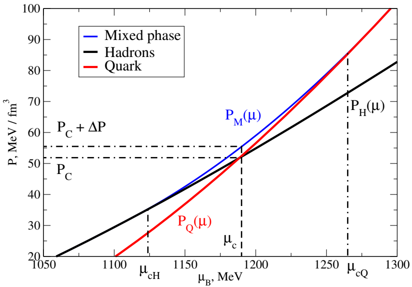
$P_M(\mu_c) = P_c + \Delta P$, where $P_c = P_H(\mu_c) = P_Q(\mu_c)$.

Using this ansatz one can assume that the pressure of mixed phase $P_M(\mu)$ could be expressed as the following function of the chemical potential

$$P_M(\mu) = \alpha(\mu - \mu_c)^p + \beta(\mu - \mu_c)^q + (1 + \Delta_p)P_c,$$

here p and q for instance are non-negative numbers, currently $\{p = 2, q = 1\}$.

Indea and Formalism



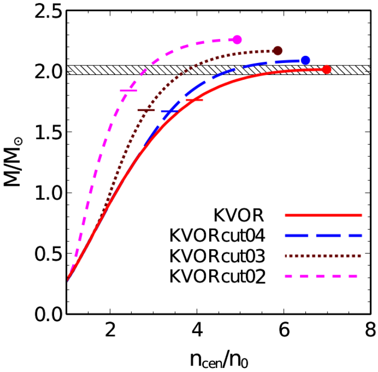
$$P_M(\mu_{cH}) = P_H(\mu_{cH}), \qquad P_M(\mu_{cQ}) = P_Q(\mu_{cQ}),$$
$$n_M(\mu_{cH}) = n_H(\mu_{cH}), \qquad n_M(\mu_{cQ}) = n_Q(\mu_{cQ}).$$

$$\alpha = \frac{1}{\mu_c - \mu_{cH}} \left(\frac{P_Q(\mu_{cQ}) - P_H(\mu_{cH})}{\mu_{cQ} - \mu_{cH}} + \frac{\Delta P - P_Q(\mu_{cQ})}{\mu_c - \mu_{cQ}} \right)$$
$$\beta = \frac{\Delta P - P_Q(\mu_{cH})}{\mu_c - \mu_{cH}} + \frac{\Delta P - P_H(\mu_{cQ})}{\mu_c - \mu_{cH}} + \frac{P_Q(\mu_{cQ}) - P_L(\mu_{cH})}{\mu_{cQ} - \mu_{cH}}$$

Hadronic matter

For the hadronic pase the well known KVOR equation of state [Kolomeitsev & Voskresensky, Nuc. Phys. A 759 (2005)] with modification of stiffness is taken.

[K.A. Maslov, E.E. Kolomeitsev, D.N. Voskresensky, Nucl.Phys. A950 (2016)]

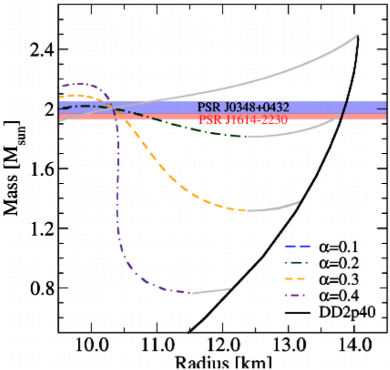


Konstantin Maslov. *Charged ρ-meson condensate in neutron stars within RMF models*. 14:20 18 Sep 2017 (Monday), MPCS-2017, Yerevan, Armenia

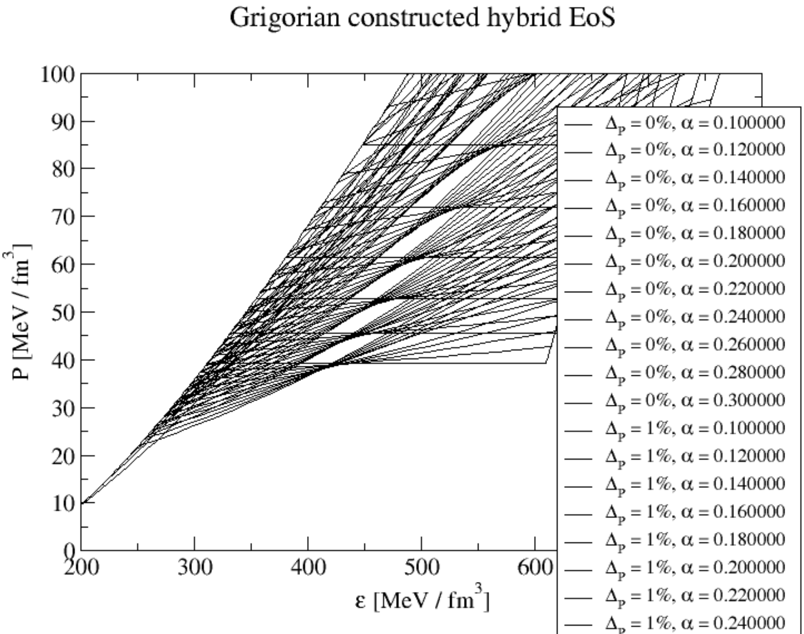
Quark matter

For the quark phase the density functional for quark matter model is used with available volume fraction parameter α (varied in the range [0.1..0.3]).

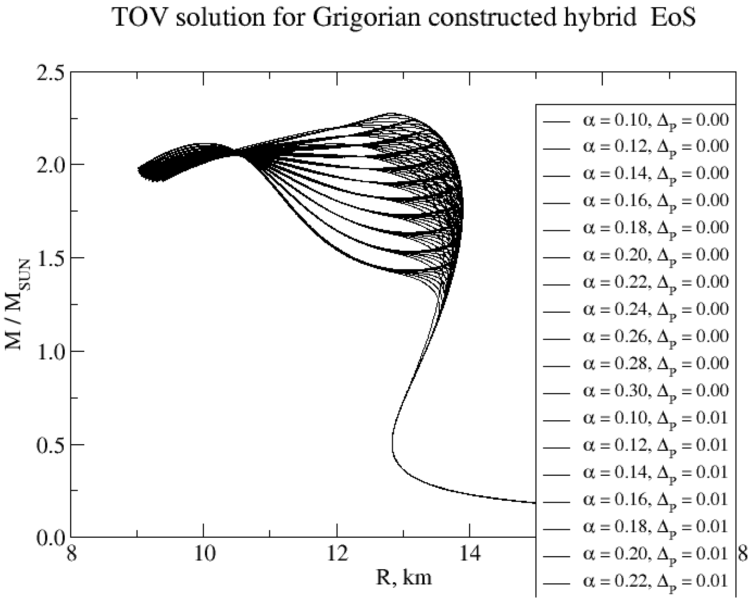
[M. Kaltenborn, N.-U. Bastian, D. Blaschke, Phys. Rev. D (accepted), arXiv:1701.04400]



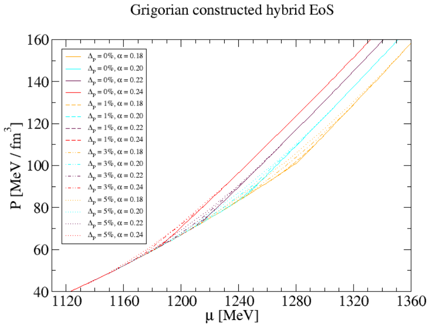
Results

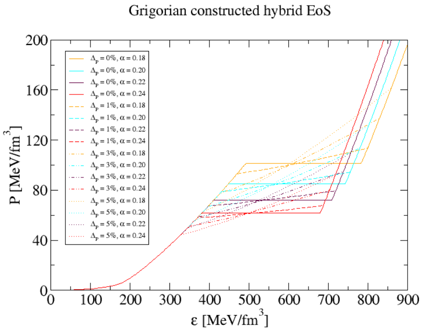

Grigorian constructed hybrid EoS
P [MeV / fm^3]
ε [MeV / fm^3]
100
90
80
70
60
50
40
30
20
10
0
200
300
400
500
600
Δ_p = 0%, α = 0.100000
Δ_p = 0%, α = 0.120000
Δ_p = 0%, α = 0.140000
Δ_p = 0%, α = 0.160000
Δ_p = 0%, α = 0.180000
Δ_p = 0%, α = 0.200000
Δ_p = 0%, α = 0.220000
Δ_p = 0%, α = 0.240000
Δ_p = 0%, α = 0.260000
Δ_p = 0%, α = 0.280000
Δ_p = 0%, α = 0.300000
Δ_p = 1%, α = 0.100000
Δ_p = 1%, α = 0.120000
Δ_p = 1%, α = 0.140000
Δ_p = 1%, α = 0.160000
Δ_p = 1%, α = 0.180000
Δ_p = 1%, α = 0.200000
Δ_p = 1%, α = 0.220000
Δ_p = 1%, α = 0.240000

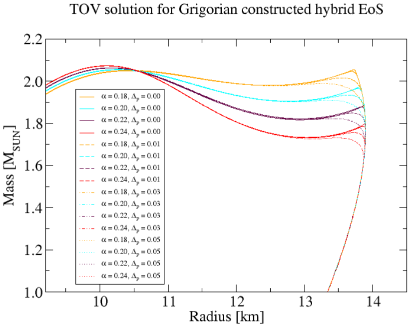
Results

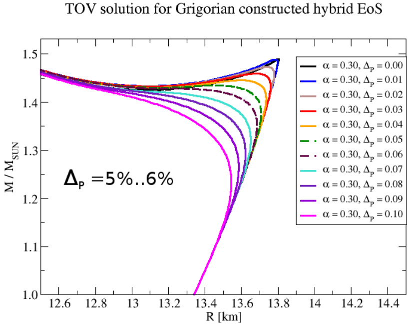

TOV solution for Grigorian constructed hybrid EoS
2.5
2.0
1.5
1.0
0.5
0.0
M / M_SUN
8
10
12
14
8
R, km
α = 0.10, Δp = 0.00
α = 0.12, Δp = 0.00
α = 0.14, Δp = 0.00
α = 0.16, Δp = 0.00
α = 0.18, Δp = 0.00
α = 0.20, Δp = 0.00
α = 0.22, Δp = 0.00
α = 0.24, Δp = 0.00
α = 0.26, Δp = 0.00
α = 0.28, Δp = 0.00
α = 0.30, Δp = 0.00
α = 0.10, Δp = 0.01
α = 0.12, Δp = 0.01
α = 0.14, Δp = 0.01
α = 0.16, Δp = 0.01
α = 0.18, Δp = 0.01
α = 0.20, Δp = 0.01
α = 0.22, Δp = 0.01

Results


Grigorian constructed hybrid EoS
P [MeV / fm^3]
μ [MeV]
$\Delta_p = 0\%, \alpha = 0.18$
$\Delta_p = 0\%, \alpha = 0.20$
$\Delta_p = 0\%, \alpha = 0.22$
$\Delta_p = 0\%, \alpha = 0.24$
$\Delta_p = 1\%, \alpha = 0.18$
$\Delta_p = 1\%, \alpha = 0.20$
$\Delta_p = 1\%, \alpha = 0.22$
$\Delta_p = 1\%, \alpha = 0.24$
$\Delta_p = 3\%, \alpha = 0.18$
$\Delta_p = 3\%, \alpha = 0.20$
$\Delta_p = 3\%, \alpha = 0.22$
$\Delta_p = 3\%, \alpha = 0.24$
$\Delta_p = 5\%, \alpha = 0.18$
$\Delta_p = 5\%, \alpha = 0.20$
$\Delta_p = 5\%, \alpha = 0.22$
$\Delta_p = 5\%, \alpha = 0.24$


Grigorian constructed hybrid EoS
P [MeV/fm^3]
ε [MeV/fm^3]
$\Delta_p = 0\%, \alpha = 0.18$
$\Delta_p = 0\%, \alpha = 0.20$
$\Delta_p = 0\%, \alpha = 0.22$
$\Delta_p = 0\%, \alpha = 0.24$
$\Delta_p = 1\%, \alpha = 0.18$
$\Delta_p = 1\%, \alpha = 0.20$
$\Delta_p = 1\%, \alpha = 0.22$
$\Delta_p = 1\%, \alpha = 0.24$
$\Delta_p = 3\%, \alpha = 0.18$
$\Delta_p = 3\%, \alpha = 0.20$
$\Delta_p = 3\%, \alpha = 0.22$
$\Delta_p = 3\%, \alpha = 0.24$
$\Delta_p = 5\%, \alpha = 0.18$
$\Delta_p = 5\%, \alpha = 0.20$
$\Delta_p = 5\%, \alpha = 0.22$
$\Delta_p = 5\%, \alpha = 0.24$


TOV solution for Grigorian constructed hybrid EoS
Mass [M_{SUN}]
Radius [km]
$\alpha = 0.18, \Delta_p = 0.00$
$\alpha = 0.20, \Delta_p = 0.00$
$\alpha = 0.22, \Delta_p = 0.00$
$\alpha = 0.24, \Delta_p = 0.00$
$\alpha = 0.18, \Delta_p = 0.01$
$\alpha = 0.20, \Delta_p = 0.01$
$\alpha = 0.22, \Delta_p = 0.01$
$\alpha = 0.24, \Delta_p = 0.01$
$\alpha = 0.18, \Delta_p = 0.03$
$\alpha = 0.20, \Delta_p = 0.03$
$\alpha = 0.22, \Delta_p = 0.03$
$\alpha = 0.24, \Delta_p = 0.03$
$\alpha = 0.18, \Delta_p = 0.05$
$\alpha = 0.20, \Delta_p = 0.05$
$\alpha = 0.22, \Delta_p = 0.05$
$\alpha = 0.24, \Delta_p = 0.05$


TOV solution for Grigorian constructed hybrid EoS
M / M_{SUN}
R [km]
Δ_p =5%..6%
$\alpha = 0.30, \Delta_p = 0.00$
$\alpha = 0.30, \Delta_p = 0.01$
$\alpha = 0.30, \Delta_p = 0.02$
$\alpha = 0.30, \Delta_p = 0.03$
$\alpha = 0.30, \Delta_p = 0.04$
$\alpha = 0.30, \Delta_p = 0.05$
$\alpha = 0.30, \Delta_p = 0.06$
$\alpha = 0.30, \Delta_p = 0.07$
$\alpha = 0.30, \Delta_p = 0.08$
$\alpha = 0.30, \Delta_p = 0.09$
$\alpha = 0.30, \Delta_p = 0.10$

Summary and conclusion

- A simple one parameter phase transition construction mimicking a structured mixed phase is presented.
- Such phase transition allows third family.
- Limit of additional pressure at critical point allowing twins is found to be around $\Delta_p = 6\%$.
- This construction could be easy generalized for the case of finite temperatures.

Thanks for your attention!