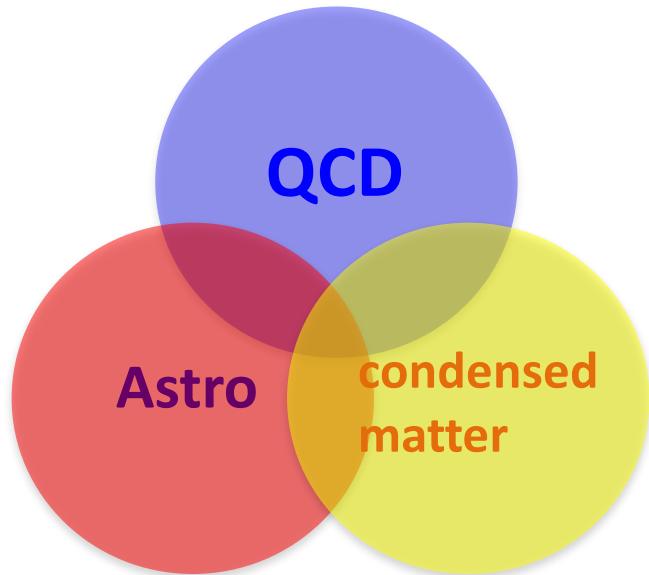


# ***QCD EoS in hadron-quark continuity***

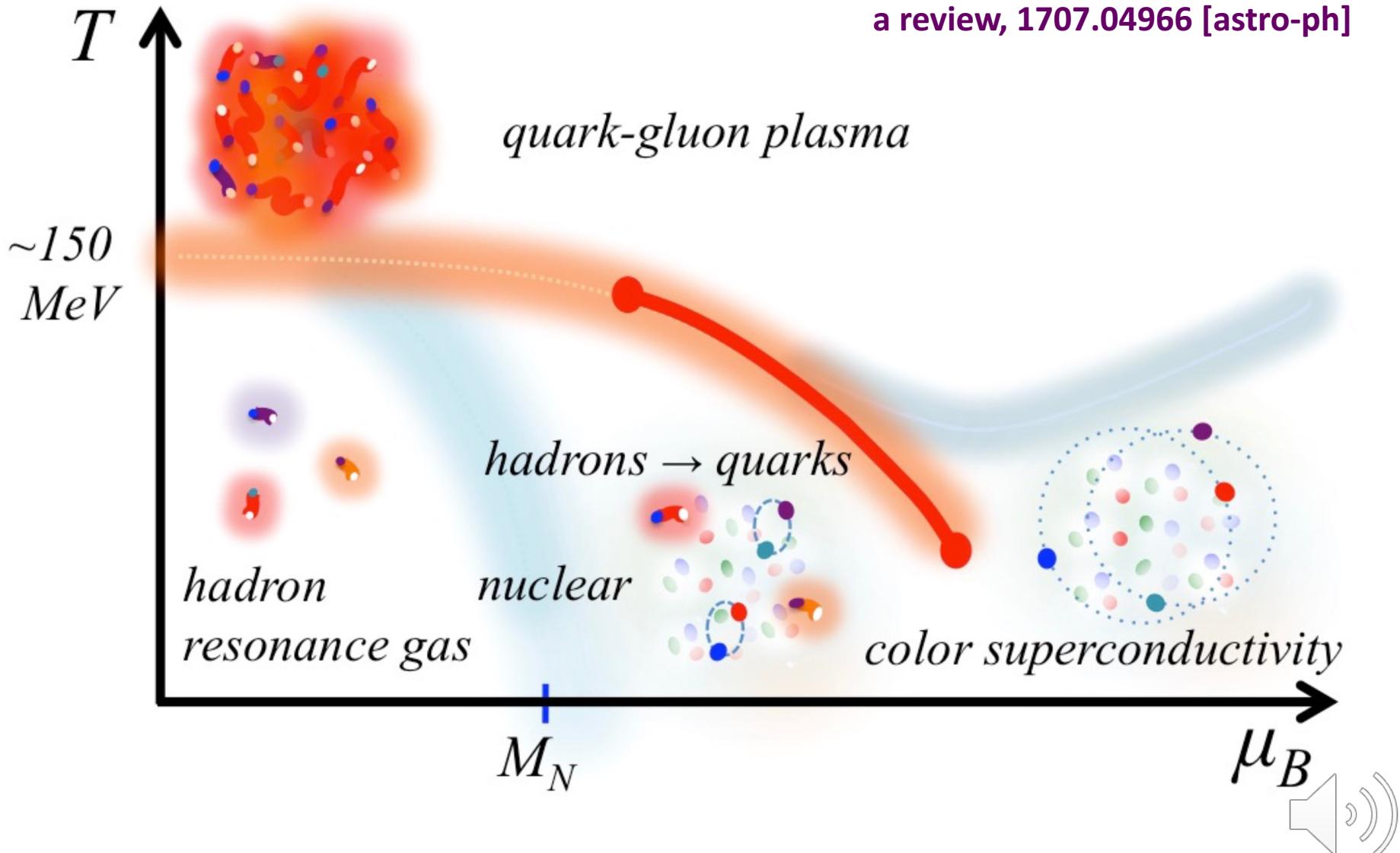
**Toru Kojo (CCNU)**

- TK, P.D. Powell, Y. Song, G. Baym  
[1412.1108 \[hep-ph\]](#) , PRD91, 045043 (2015)
- TK, [1508.1108 \[hep-ph\]](#), review in EPJA
- K. Fukushima & TK, [1509.1108](#), APJ817(2016)2
- TK, [1610.05486 \[hep-ph\]](#), PLB769 (2017) 14
- Baym-Hatsuda-TK-Powell-Song-Takatsuka,  
(review) [1707.04966 \[astro-ph\]](#)
- H.Zhang-D.Hou-TK-B.Qin, [1709.05654 \[hep-th\]](#)

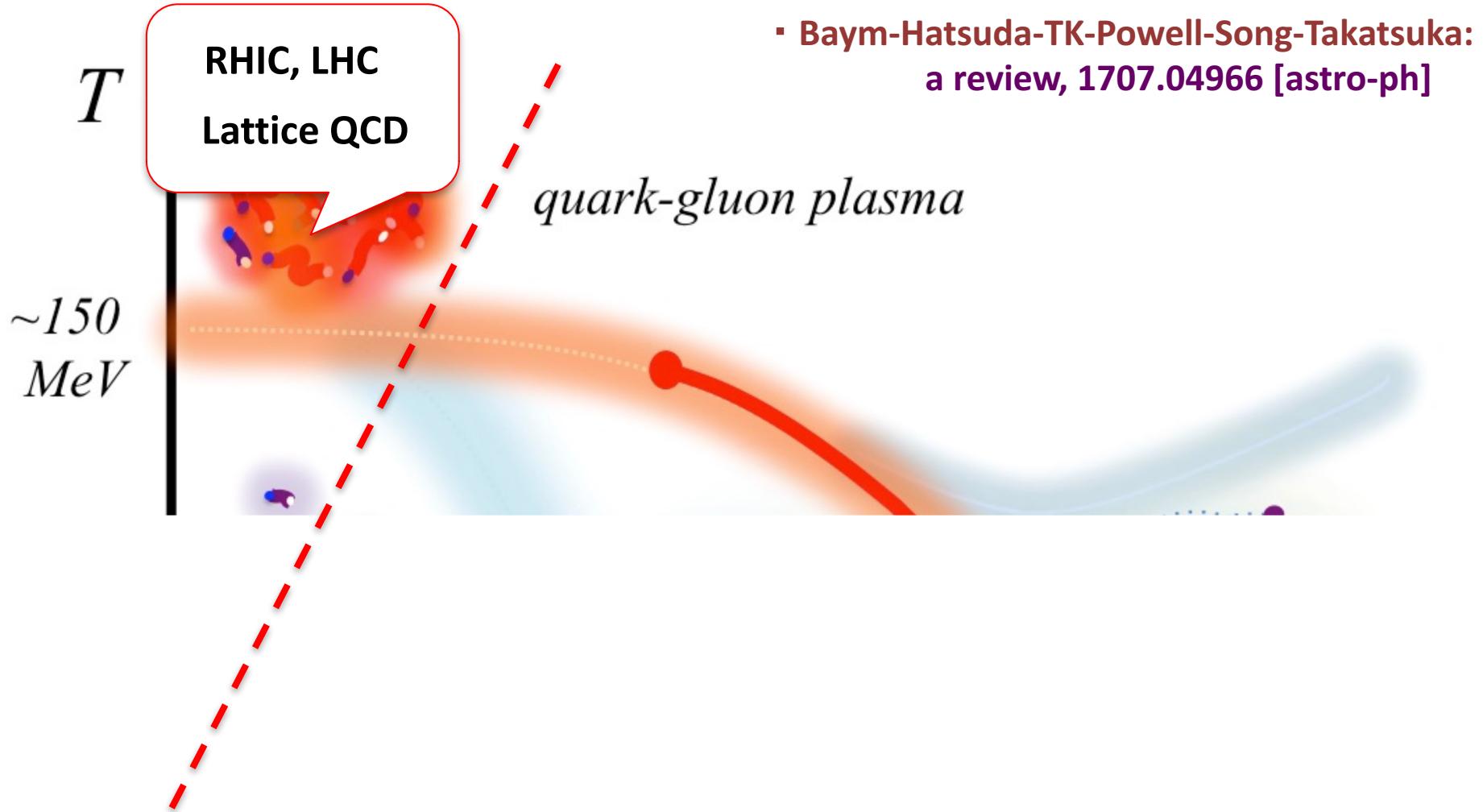


# Lattice + HIC ( $> 20 \text{ GeV}$ ) + HIC ( $< 20 \text{ GeV}$ ) + Astro

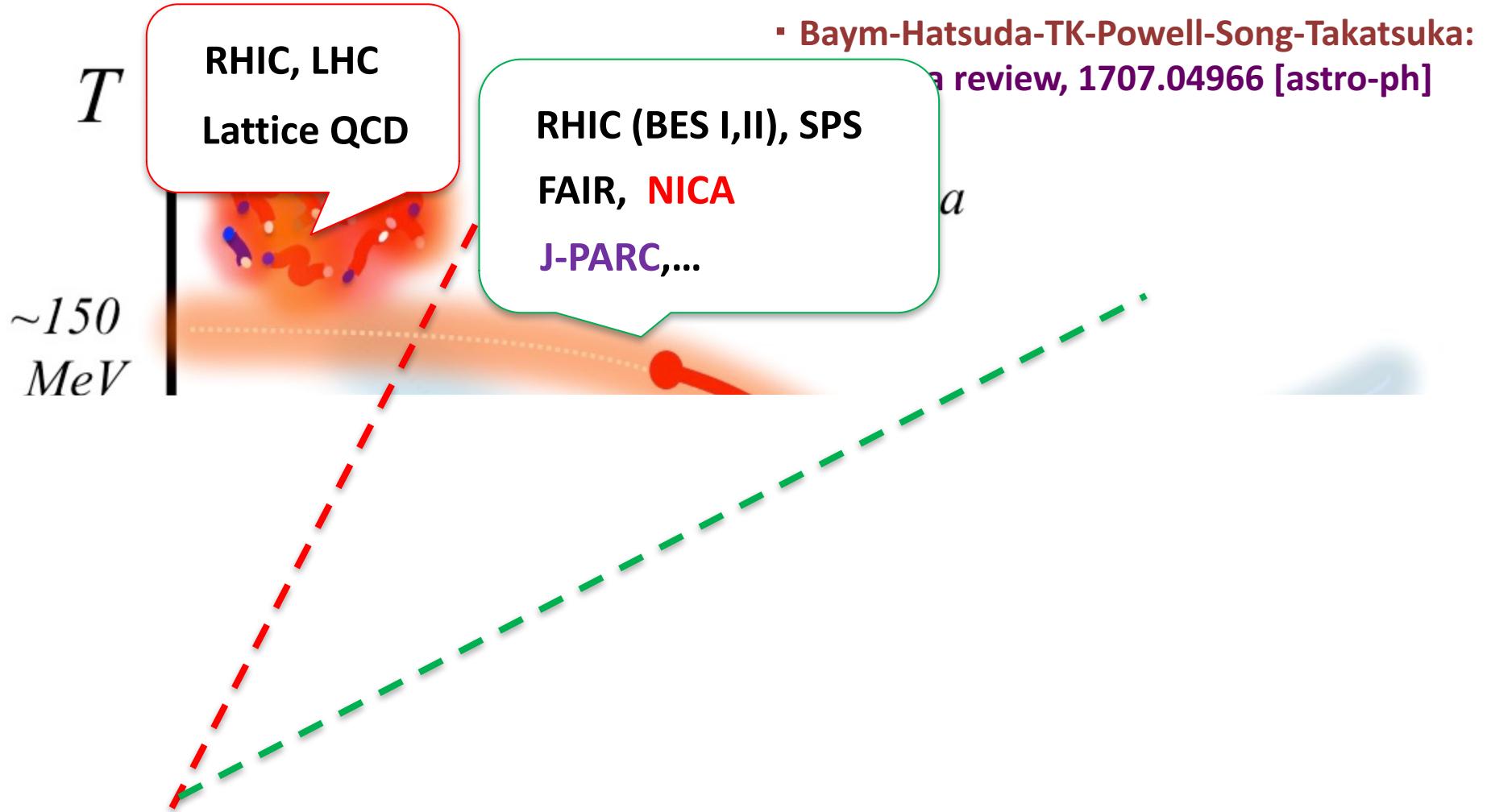
▪ Baym-Hatsuda-TK-Powell-Song-Takatsuka:  
a review, 1707.04966 [astro-ph]



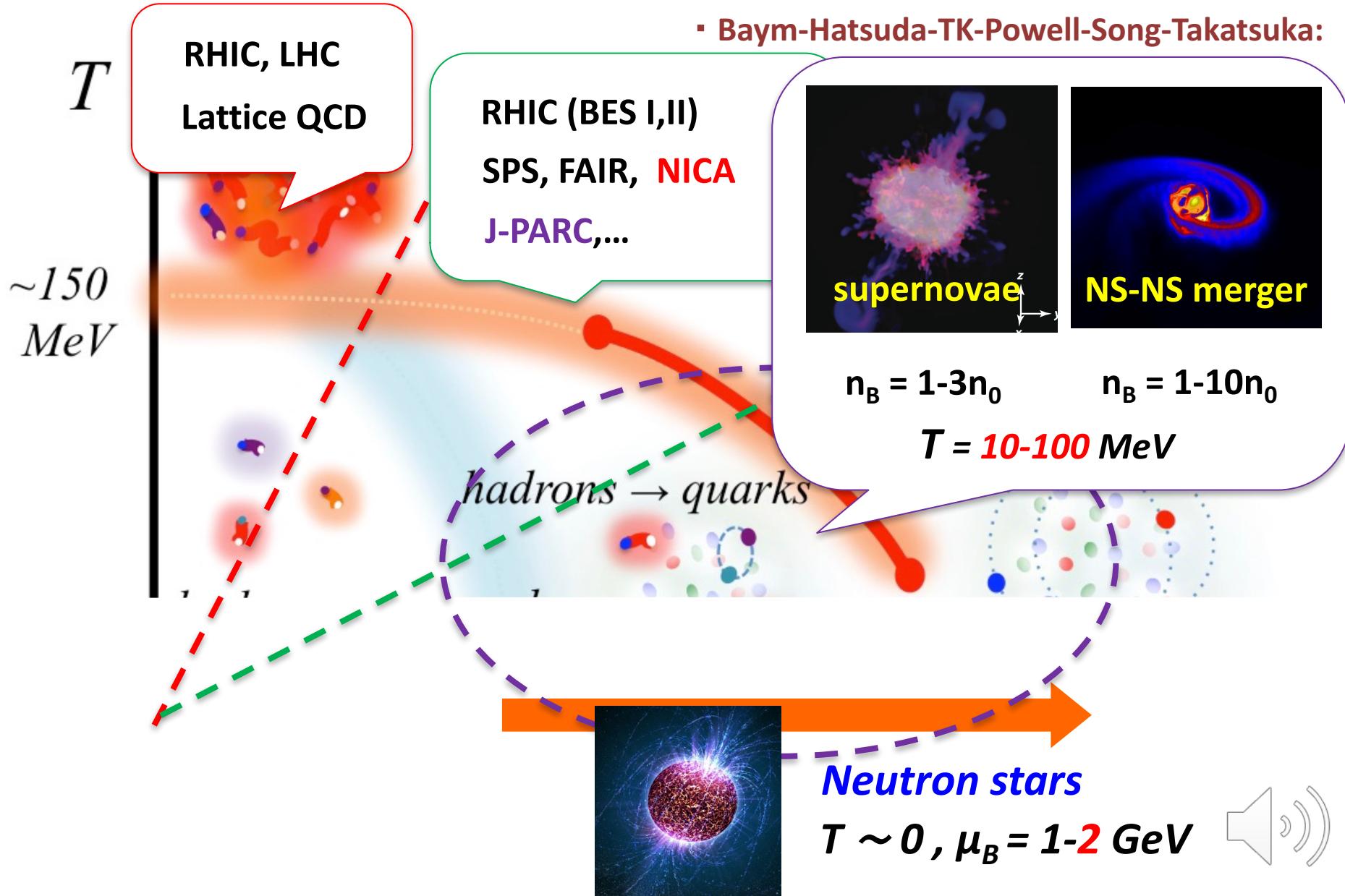
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# *Questions to be addressed*

- 1, Do we need 1<sup>st</sup> order P.T. to define quark matter?*
- 2, The nature of gluons in quark matter?*
- 3, The role of pairing effects on EoS?*



# *Outline*

*1, NS and nuclear constraints*

*2, 3-window modeling, quark EoS*

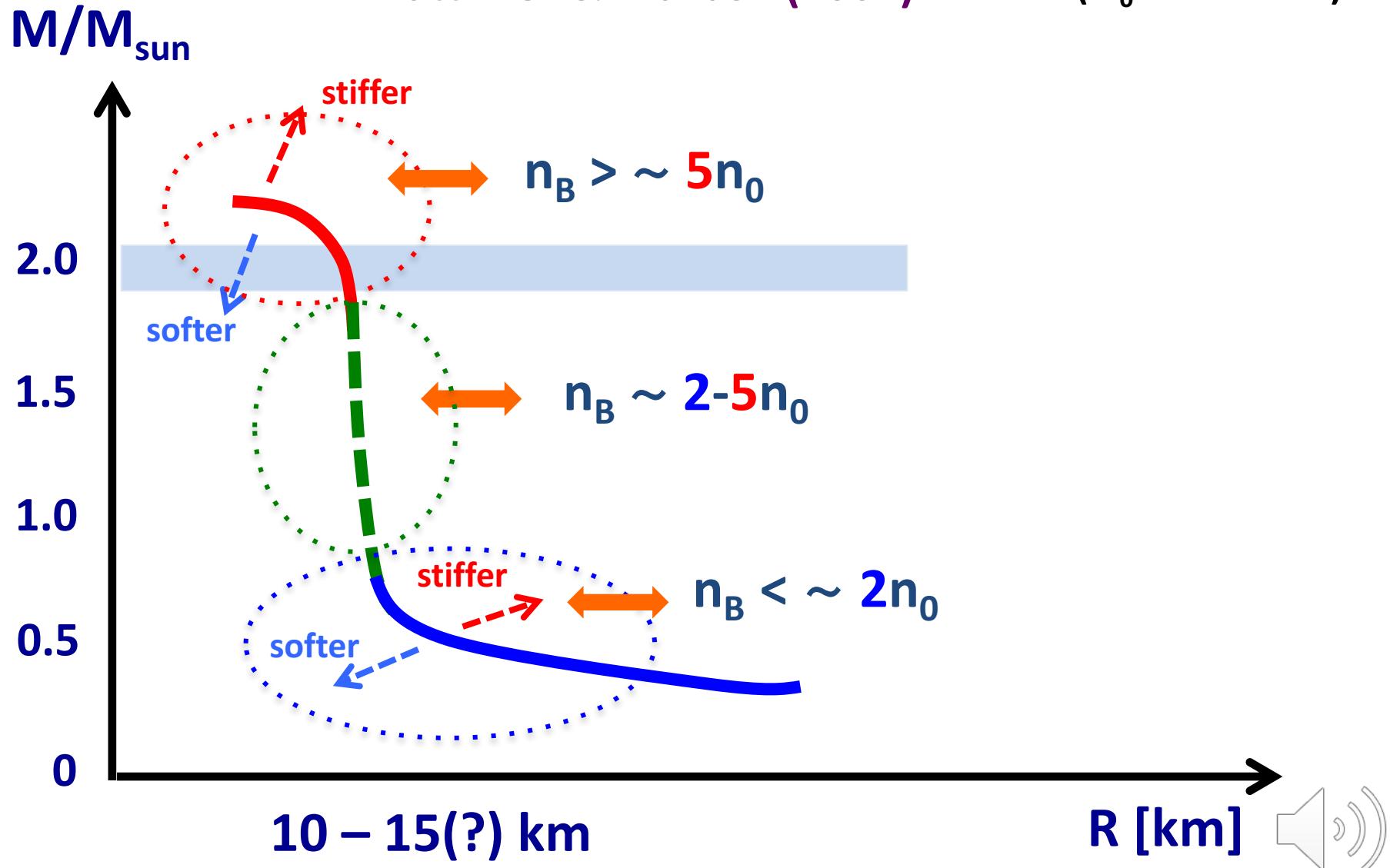
*3, Summary & outlook*



# *M-R relation & EoS*

Lattimer & Prakash (2001)

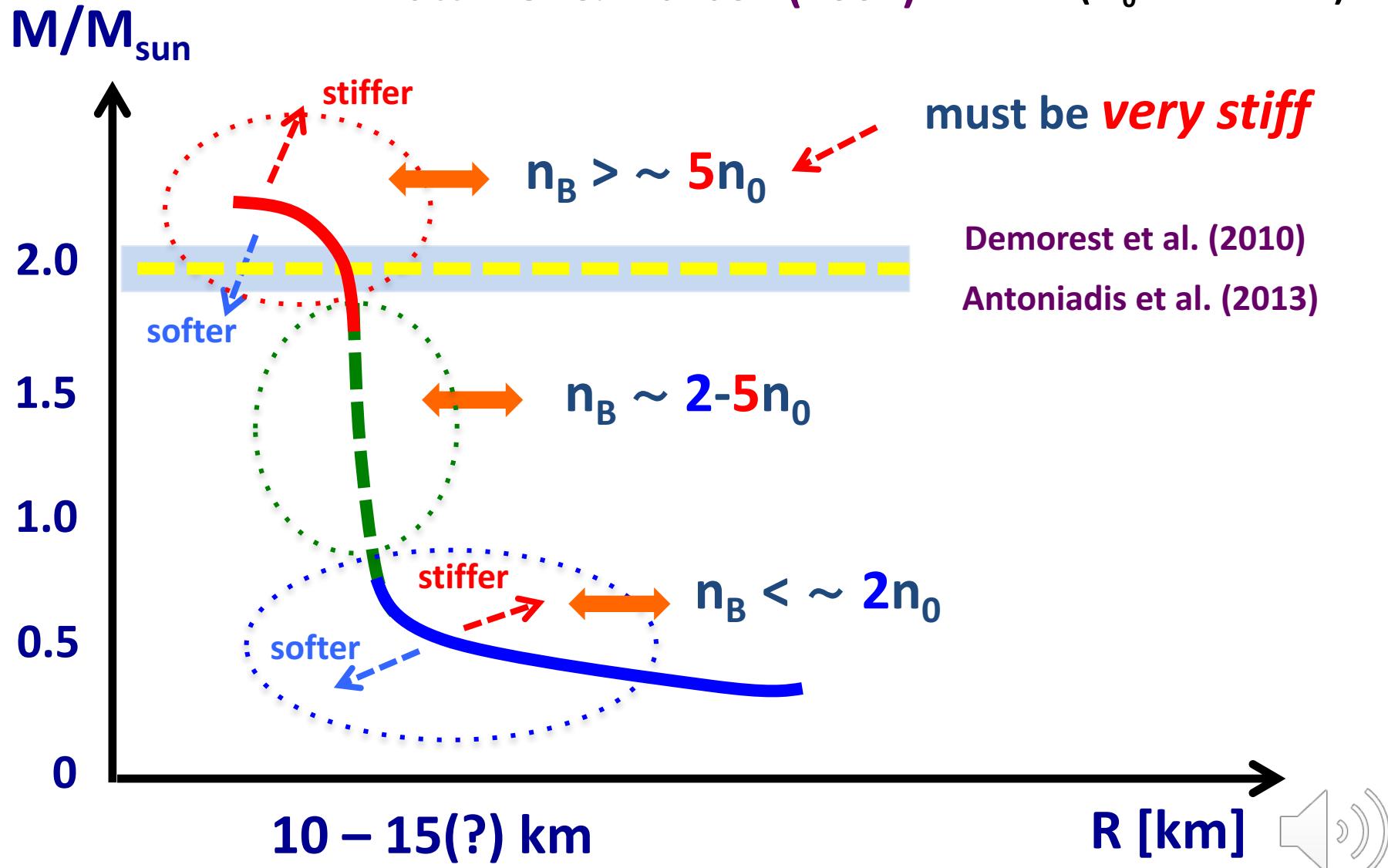
( $n_0 = 0.16 \text{ fm}^{-3}$ )



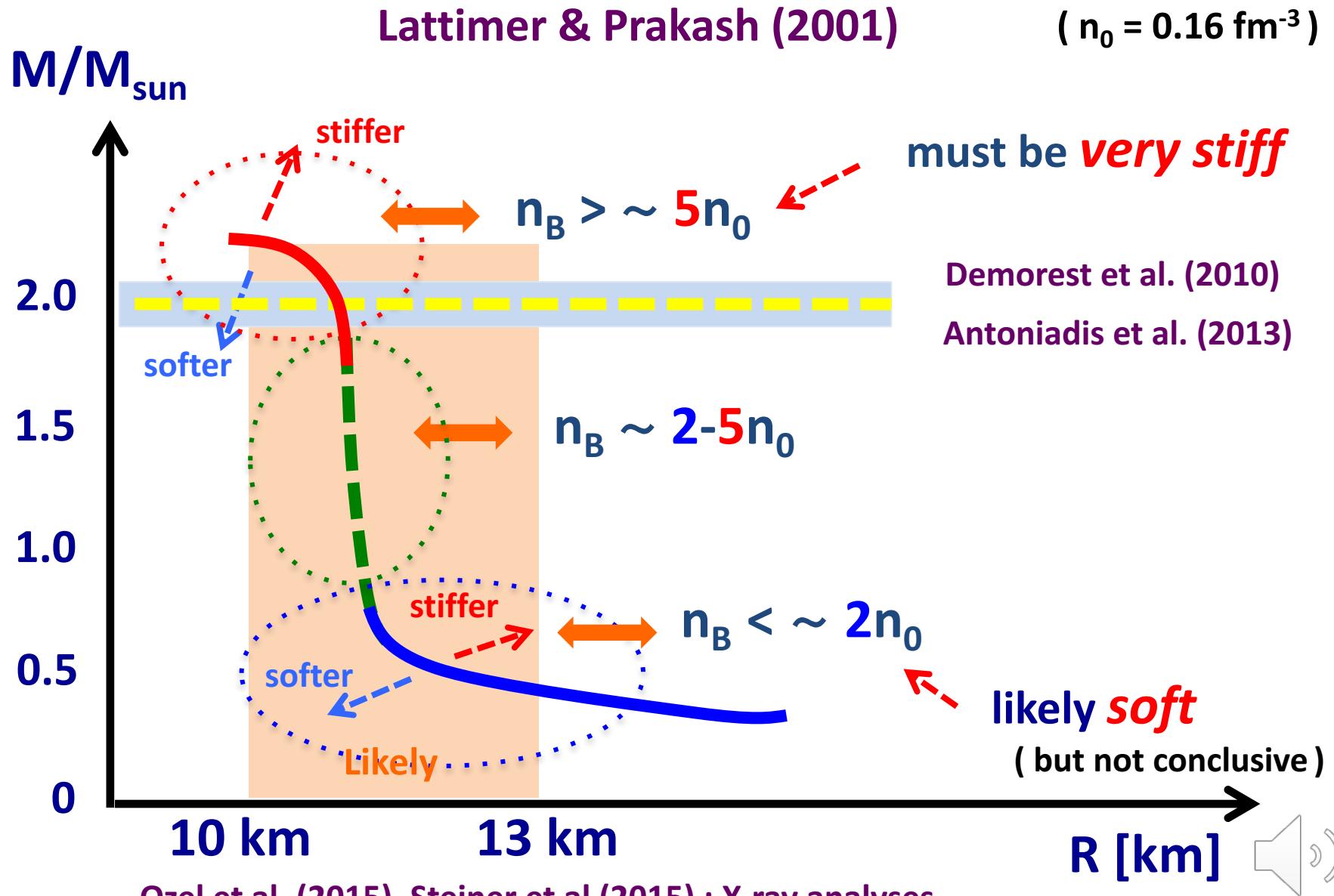
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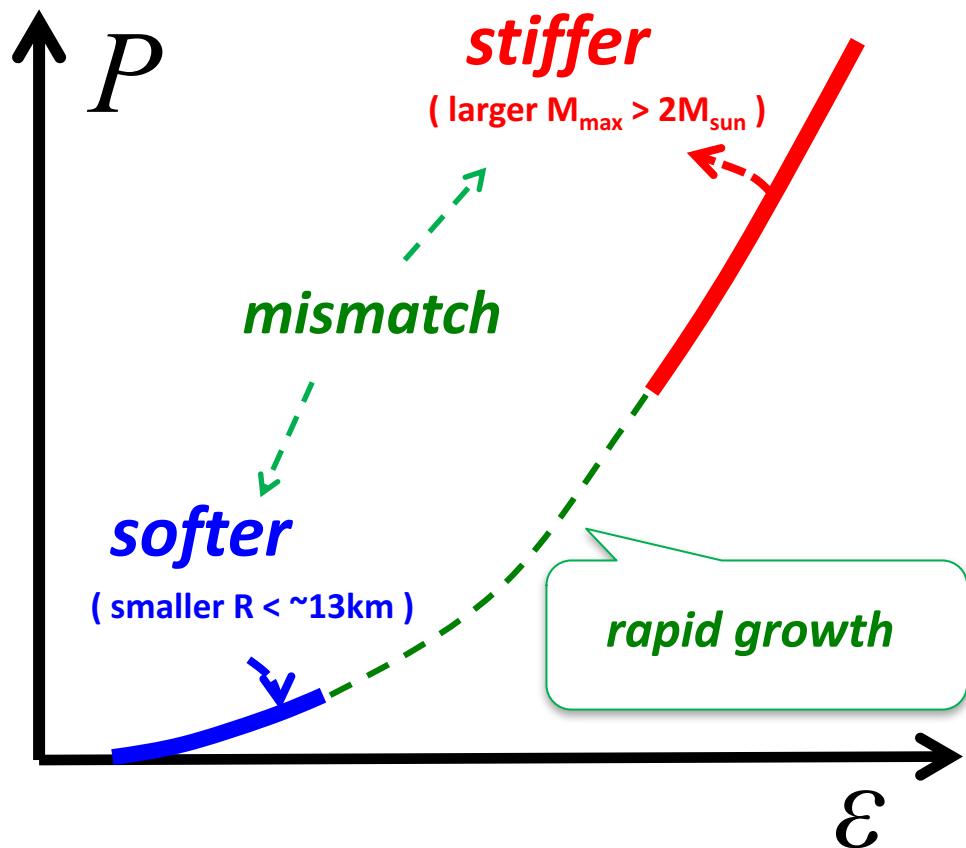
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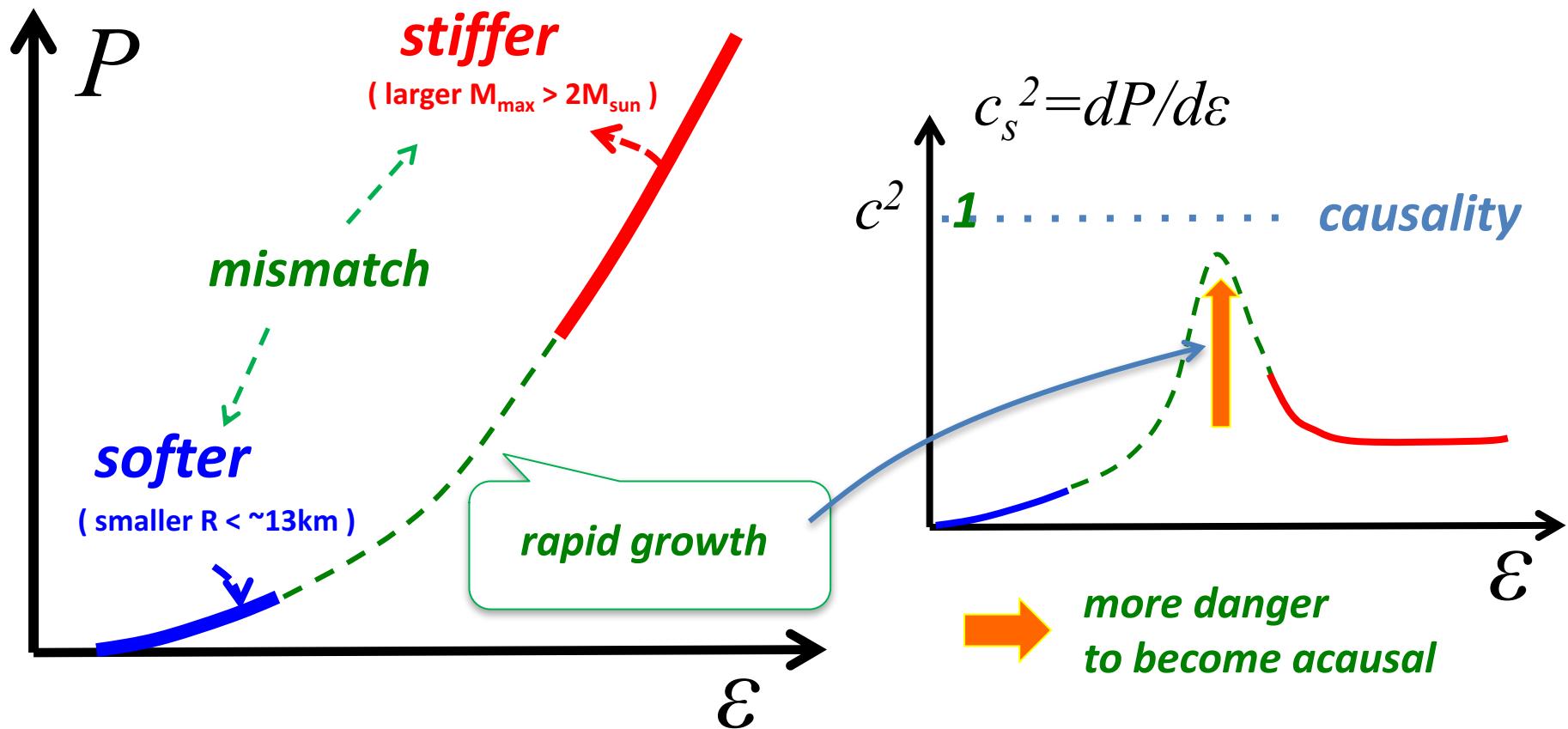
# M-R relation & EoS



# Causality constraint on $2n_0$ - $5n_0$ region



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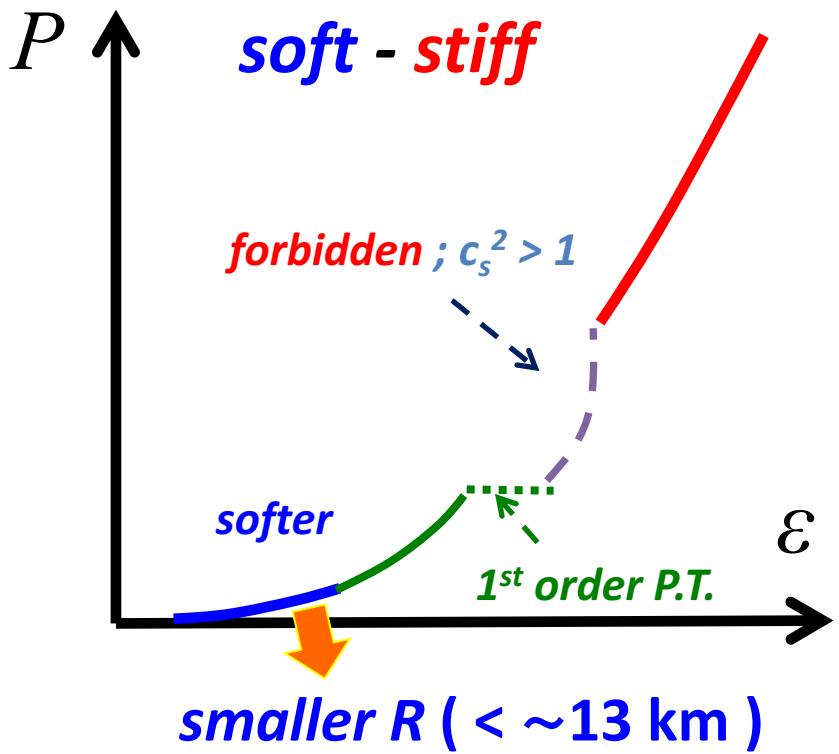


For **softer - stiffer EoS**  $\rightarrow$  **less freedom** for  $2n_0$ - $5n_0$  region



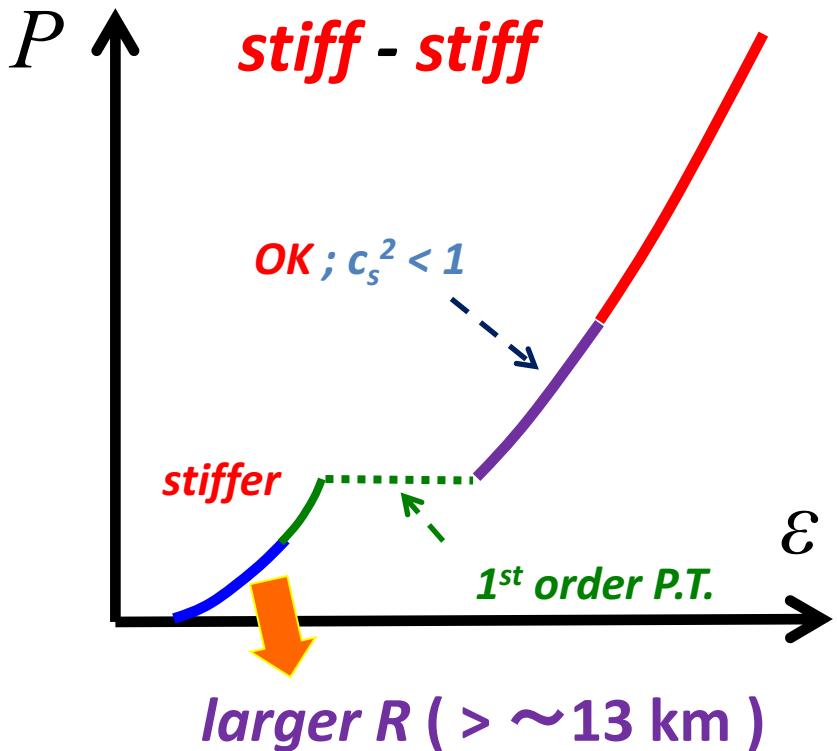
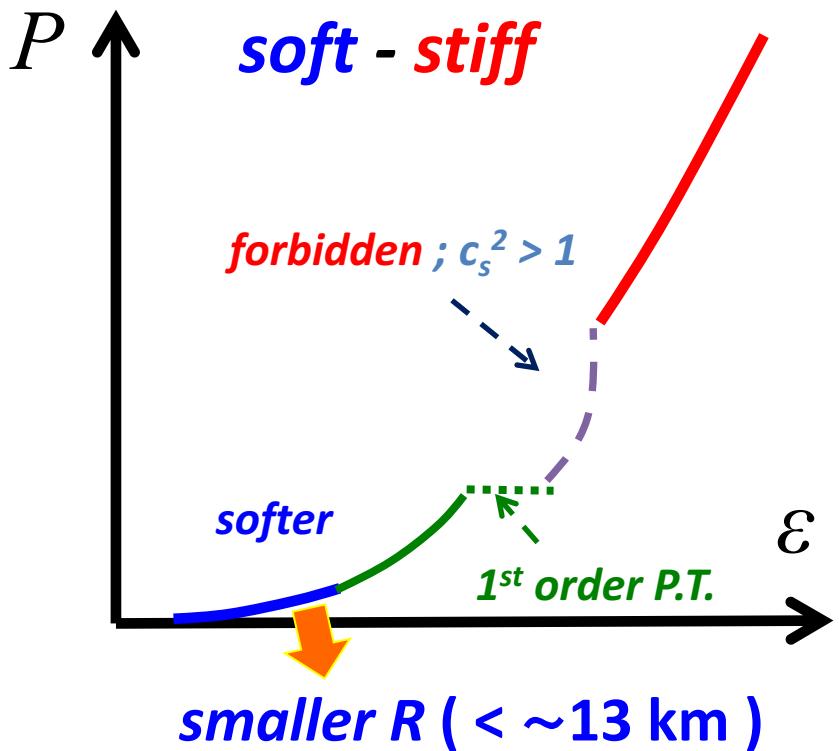
# Soft-Stiff v.s. Stiff-Stiff EoS

[more systematic analyses -> Han-Alford-Prakash 13]



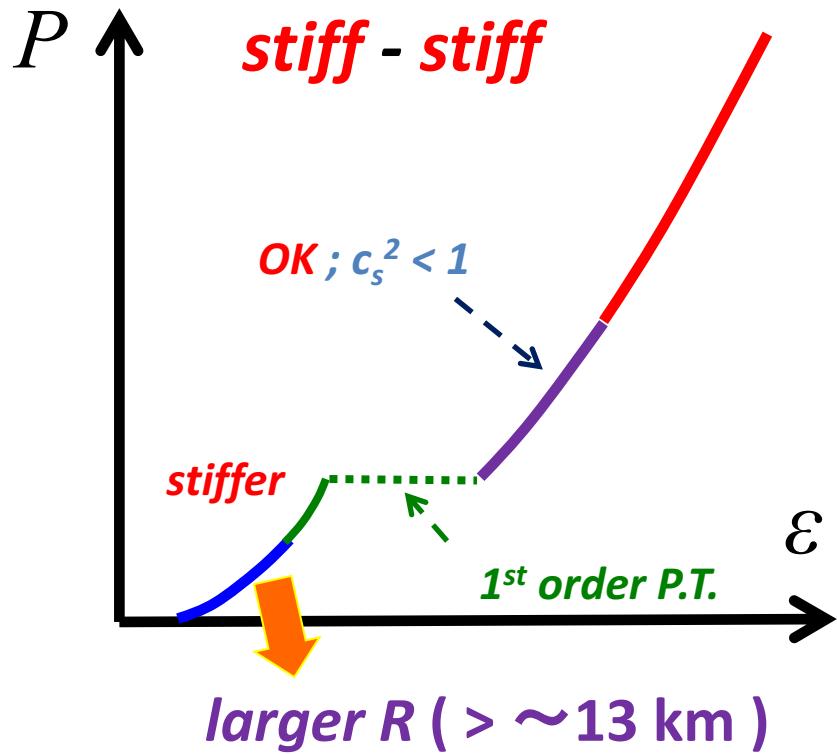
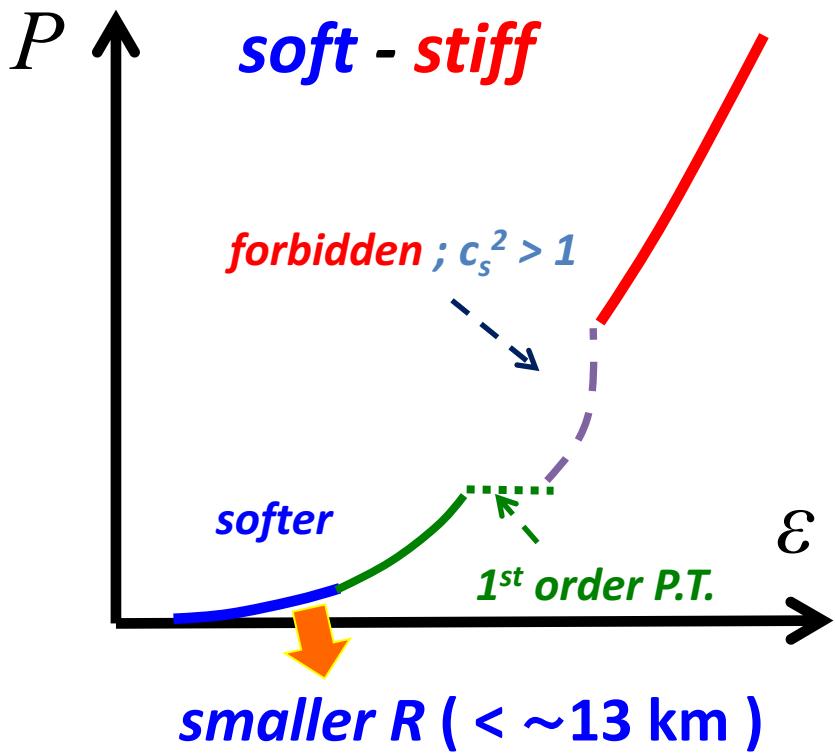
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[more systematic analyses -> Han-Alford-Prakash 13]



NS Radius

- nuclear EoS at  $(1.5\text{-}2) n_0$  (beyond ChEFT)
  - **strength of 1<sup>st</sup> P.T.**
- (weaker for smaller  $n_0$ )

# Small $R_{1.4}$ & soft EoS @ 1-2 $n_0$ ?

- *Thermal X-rays analyses for NS radii :*

- Suleimanov et al (2011) :  $> 13.9$  km
- Guillot et al. (2011) :  $9.1^{+1.3}_{-1.5}$  km
- Ozel & Freire (2015) :  $10.6 \pm 0.6$  km
- Steiner et al (2015) :  $12.0 \pm 1.0$  km

systematic uncertainties : distance to NS, atmosphere of NS, uniform T distributions,...



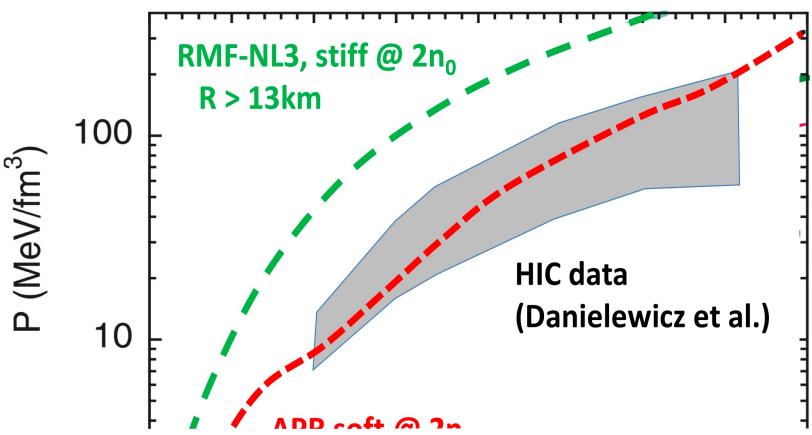
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- HIC* : (Danielewicz et al. 2002)



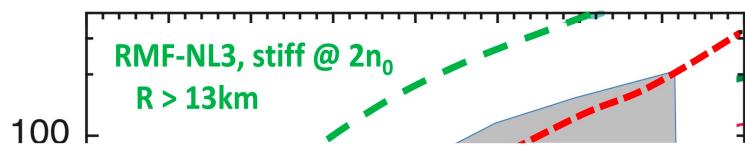
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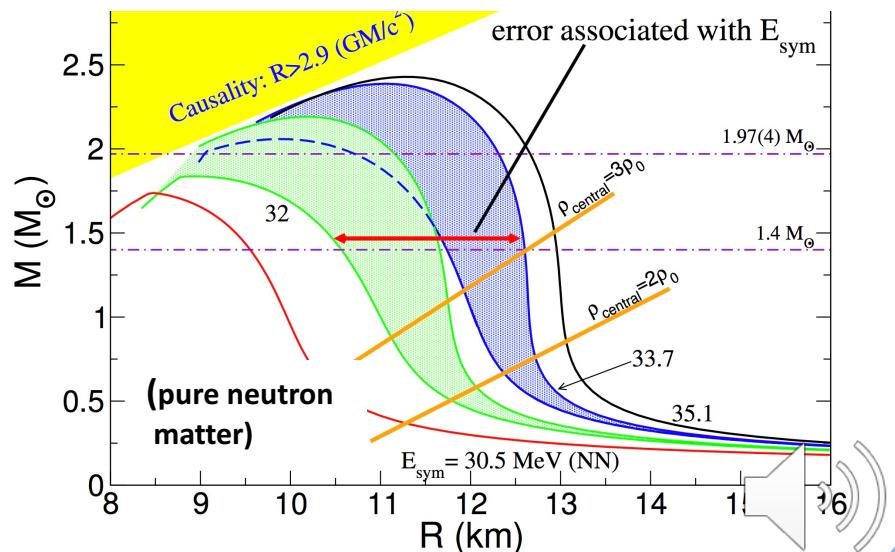
systematic uncertainties : distance to NS, atmosphere of NS, uniform T distributions,...

- HIC :** (Danielewicz et al. 2002)



- nuclear EoS extrapolation :** (Gandolfi et al. 2015)

(*sophisticated* potentials & Monte-Carlo)



# Small $R_{1.4}$ & soft EoS @ 1-2 $n_0$ ?

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systematic uncertainties + ... = NS radius + SNe radius + pulsars radius

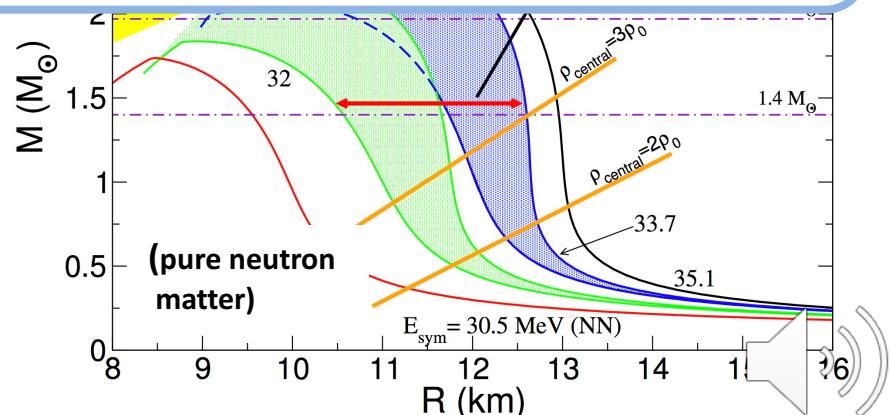
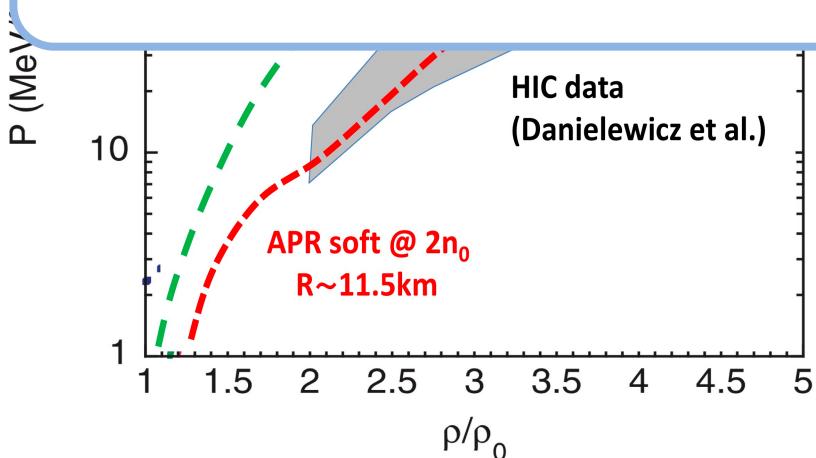


**Recent trends :**  $R < \sim 13$  km, soft EoS @ 1-2 $n_0$

(although each of them is not free from systematic uncertainties)

Based on this, below we try to construct **soft-stiff** EoS

[ No strong 1<sup>st</sup> order P.T; either **crossover** or weak 1<sup>st</sup> order ]



# *Outline*

*1, NS and nuclear constraints*

*2, 3-window modeling, quark EoS*

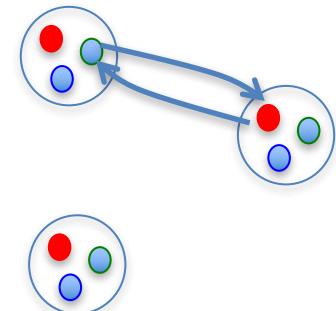
*3, Summary & outlook*



# 3-window modeling

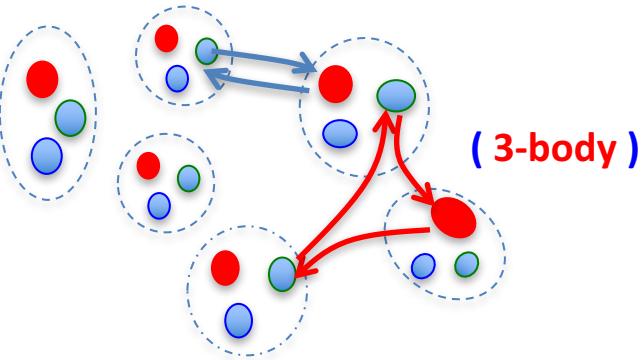
(Masuda-Hatsuda-Takatsuka 12)

- few meson exchange
- nucleons **only**



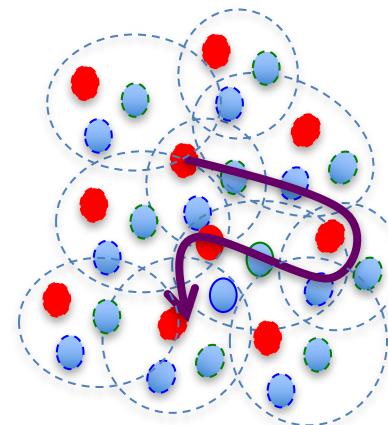
$\sim 2n_0$

- many-meson exchange  
( mobility --cf: Karsch-Satz '80 )
- structural change of hadrons



$\sim 5n_0$

- Baryons overlap
- Quark Fermi sea  
 $p_F \sim 400$  MeV



(pQCD)

$n_B$

$\sim 100n_0$



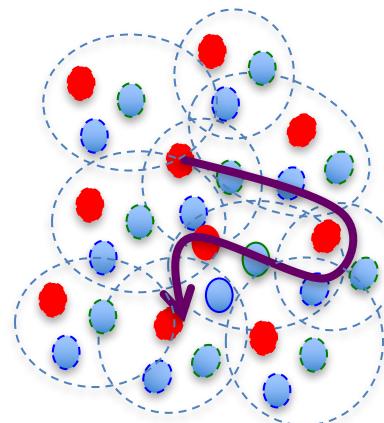
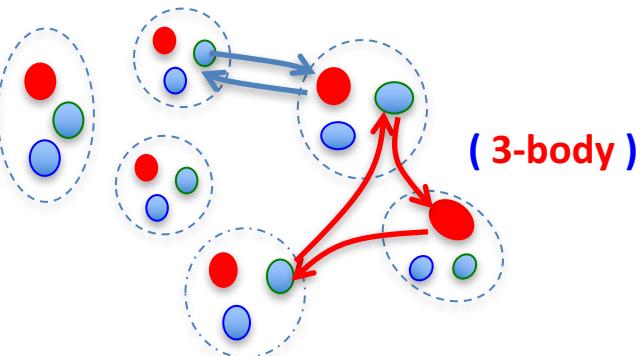
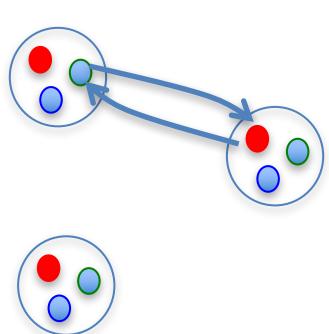
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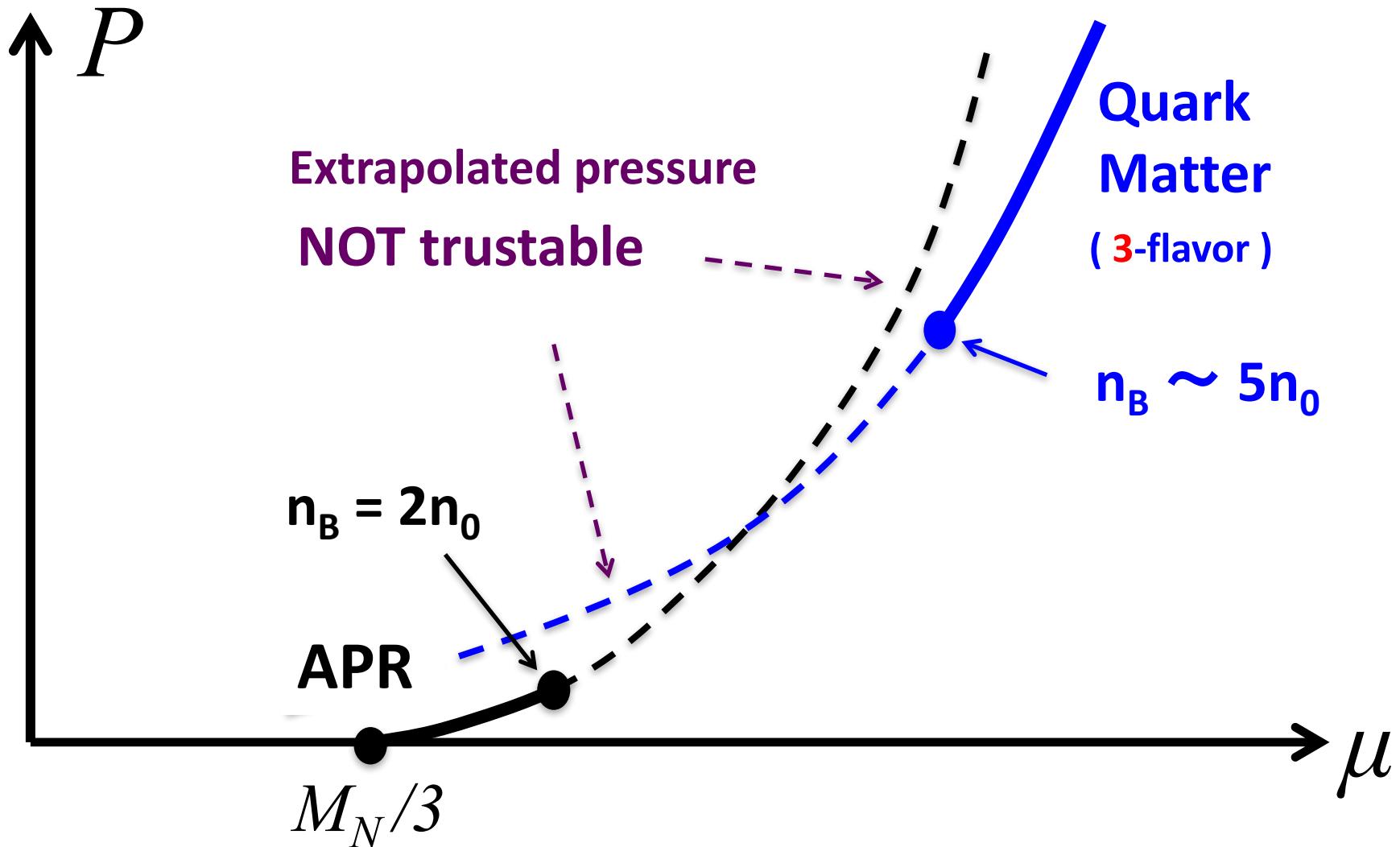
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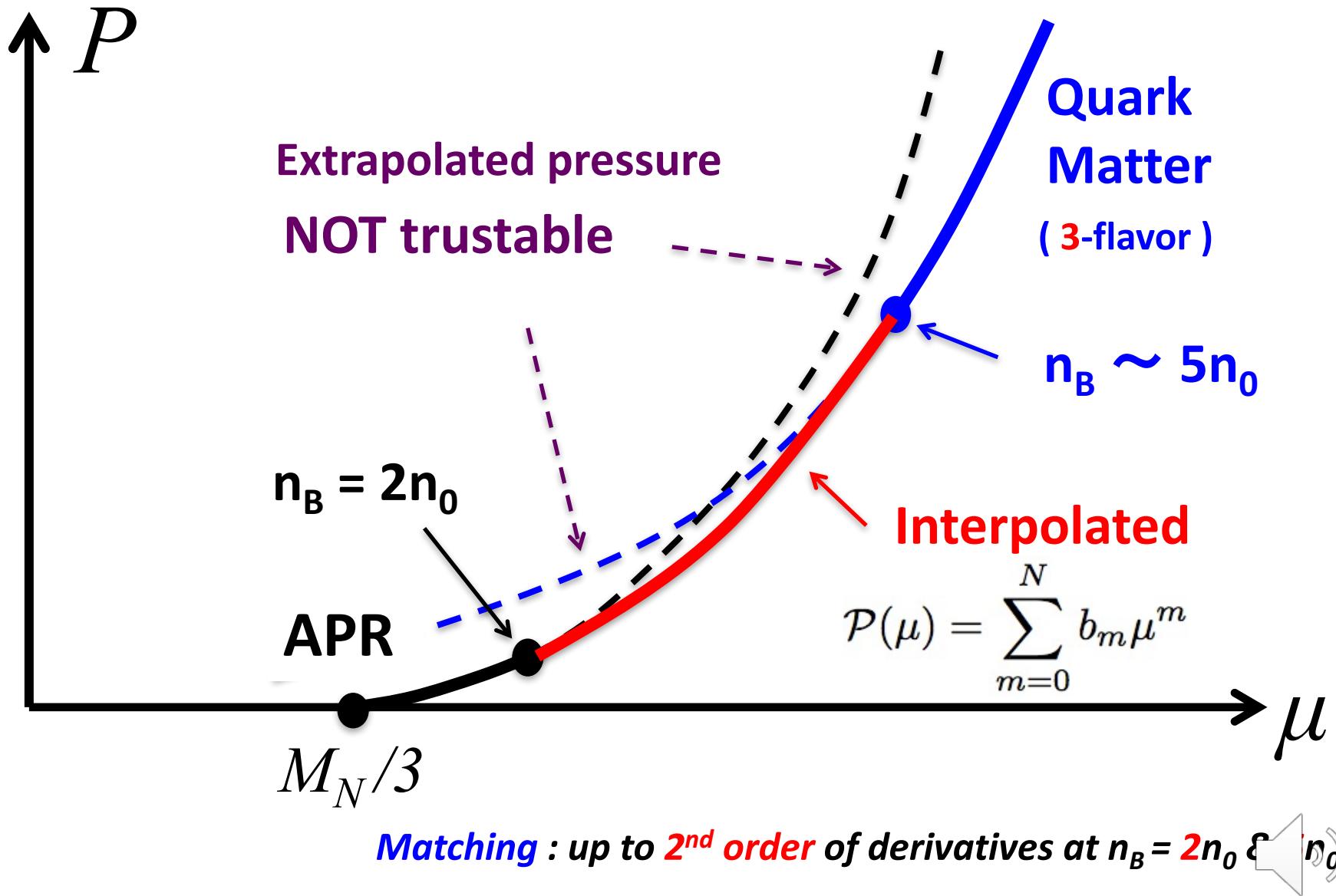
- Baryons overlap
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 $p_F \sim 400$  MeV

*APR**Interpolated EoS**Quark models* $\sim 2n_0$  $\sim 5n_0$  $\sim 100n_0$  $n_B$ 

# 3-window modeling : $P$ vs $\mu$



# 3-window modeling : $P$ vs $\mu$



# 3-flavor quark MF model : template

**Effective Hamiltonian** (inspired by hadron & nuclear physics):

$$\mathcal{H}_{\text{eff}} \sim \bar{\psi} \left[ -i\vec{\alpha} \cdot \vec{\partial} + m \right] \psi + \mathcal{H}_{\text{NJL}}^{\text{4Fermi+KMT}}$$

→ change in *Dirac sea*, beyond *no-sea approximation*

$$+ \mathcal{H}_{\text{conf}}^{3q \rightarrow B} \quad \xrightarrow{\hspace{1cm}} \quad \text{will be } \textcolor{red}{\text{ignored}} \text{ at } n_B > 5n_0$$

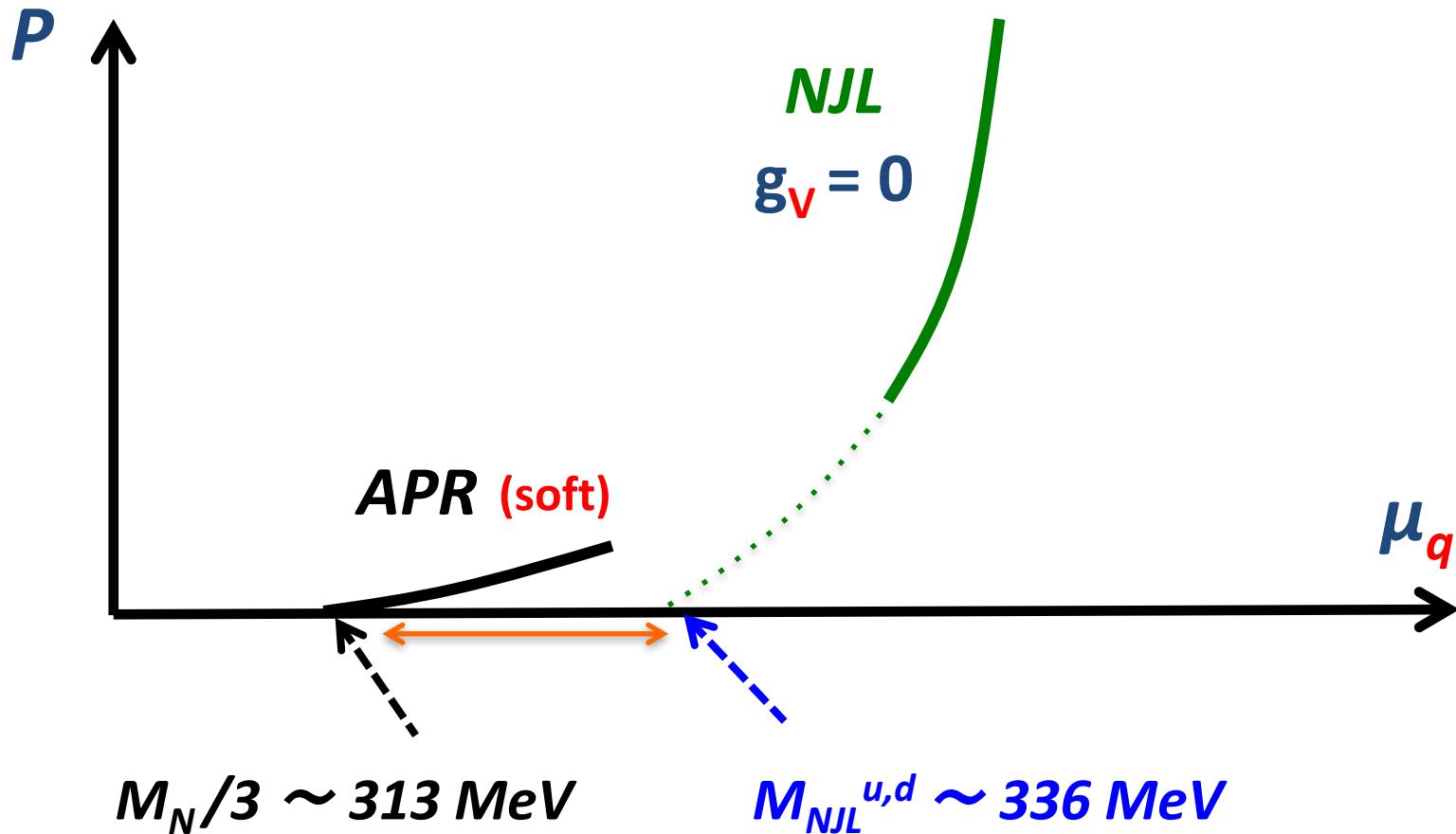
$$+ \mathcal{H}_{\text{OGE}} \quad \xrightarrow{\text{mag. part}} \quad - H \sum_{A,A'=2,5,7} \left( \bar{\psi} i \gamma_5 \lambda_A \tau_{A'} \psi_c \right)^2 \quad (\text{cf: } \textcolor{red}{N\text{-}\Delta \text{ splitting}})$$

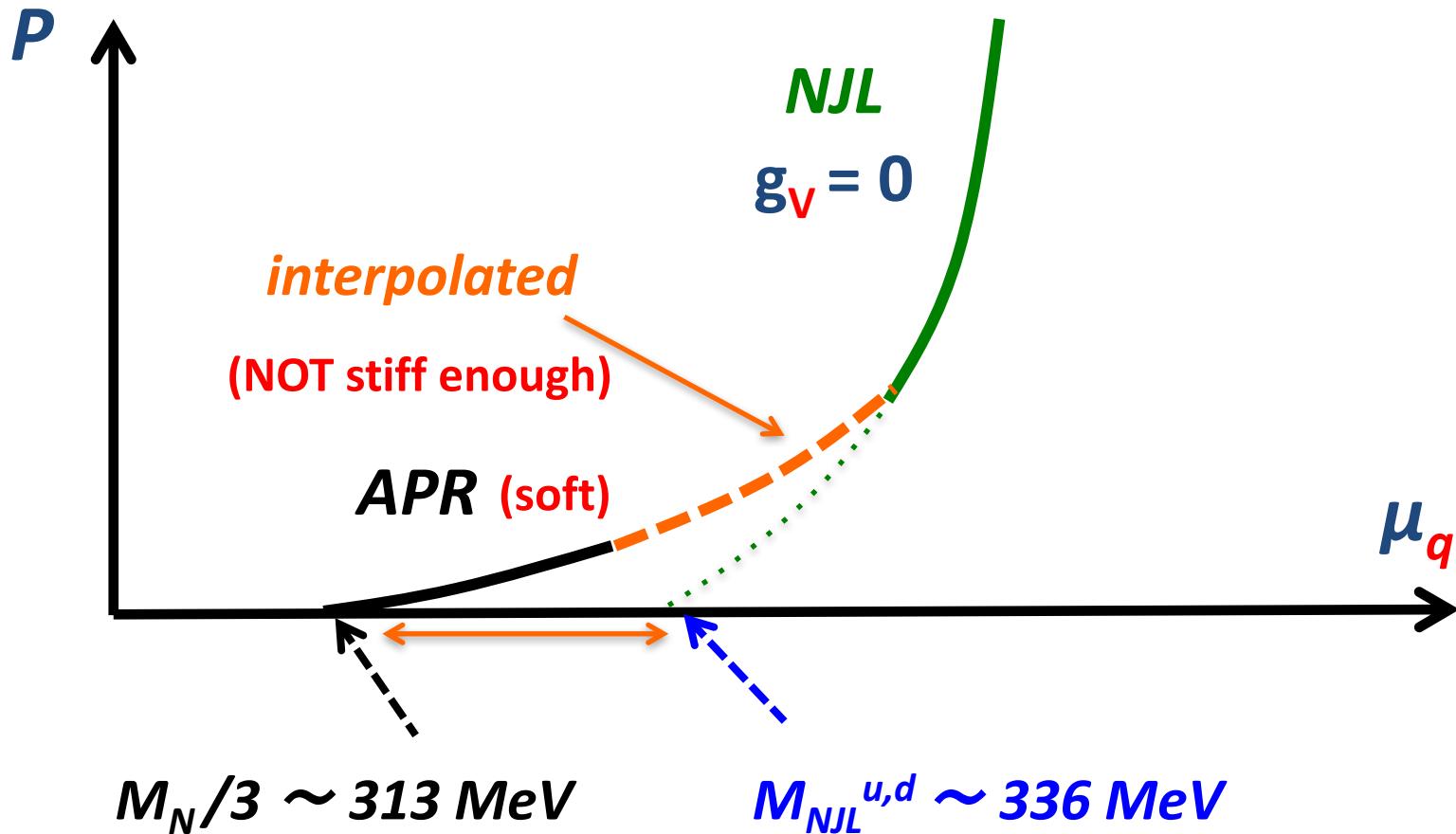
$$+ \mathcal{H}_{\text{nucl}} \quad \xrightarrow{\hspace{1cm}} \quad + g_V (\bar{\psi} \gamma_0 \psi)^2 \quad \sim \textcolor{blue}{\omega\text{-exchange}} \\ \textcolor{red}{(\text{repulsive})}$$

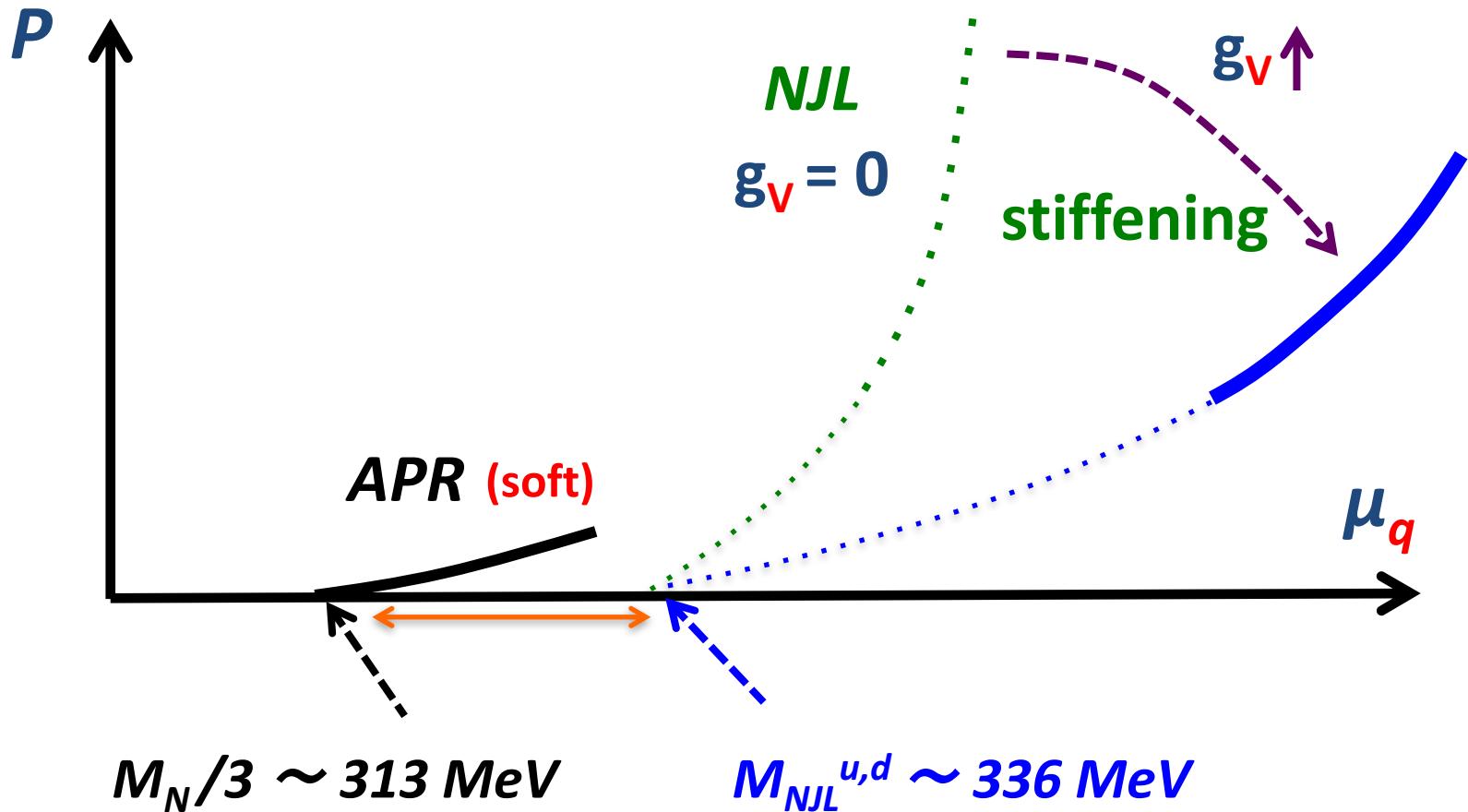
+ constraints (charge neutrality, β-equilibrium, color-neutrality)

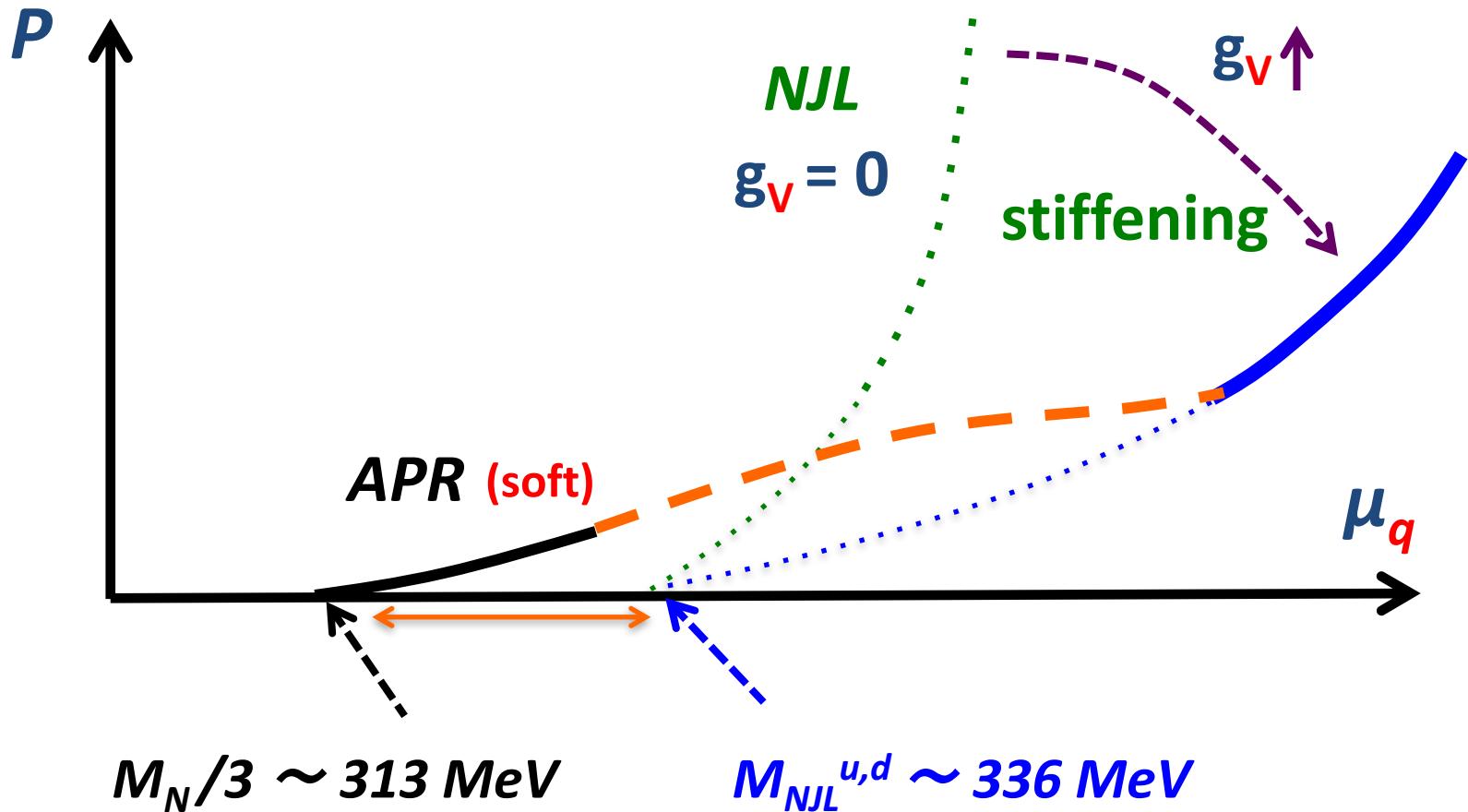
**Goal:** NS constraints →  $(G_s, H, g_V)_{@5-10n^c}$



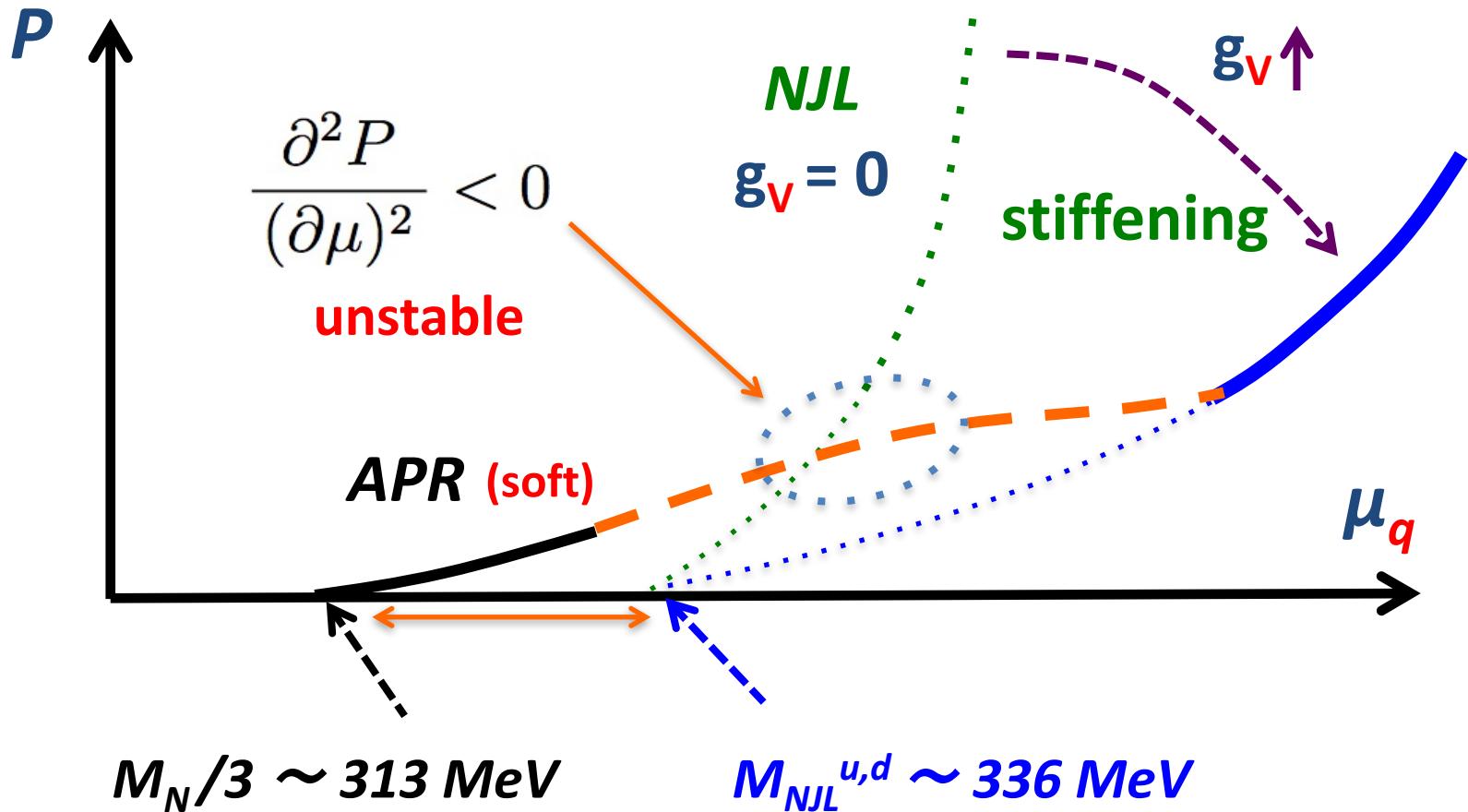
*minimal*

*minimal*

*minimal + vector int.*

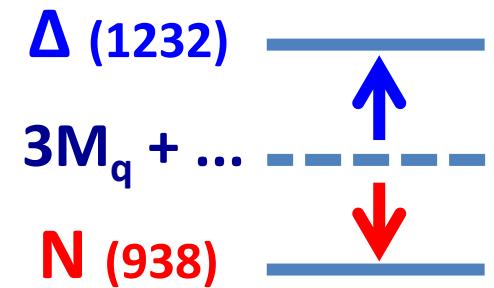
*minimal + vector int.*

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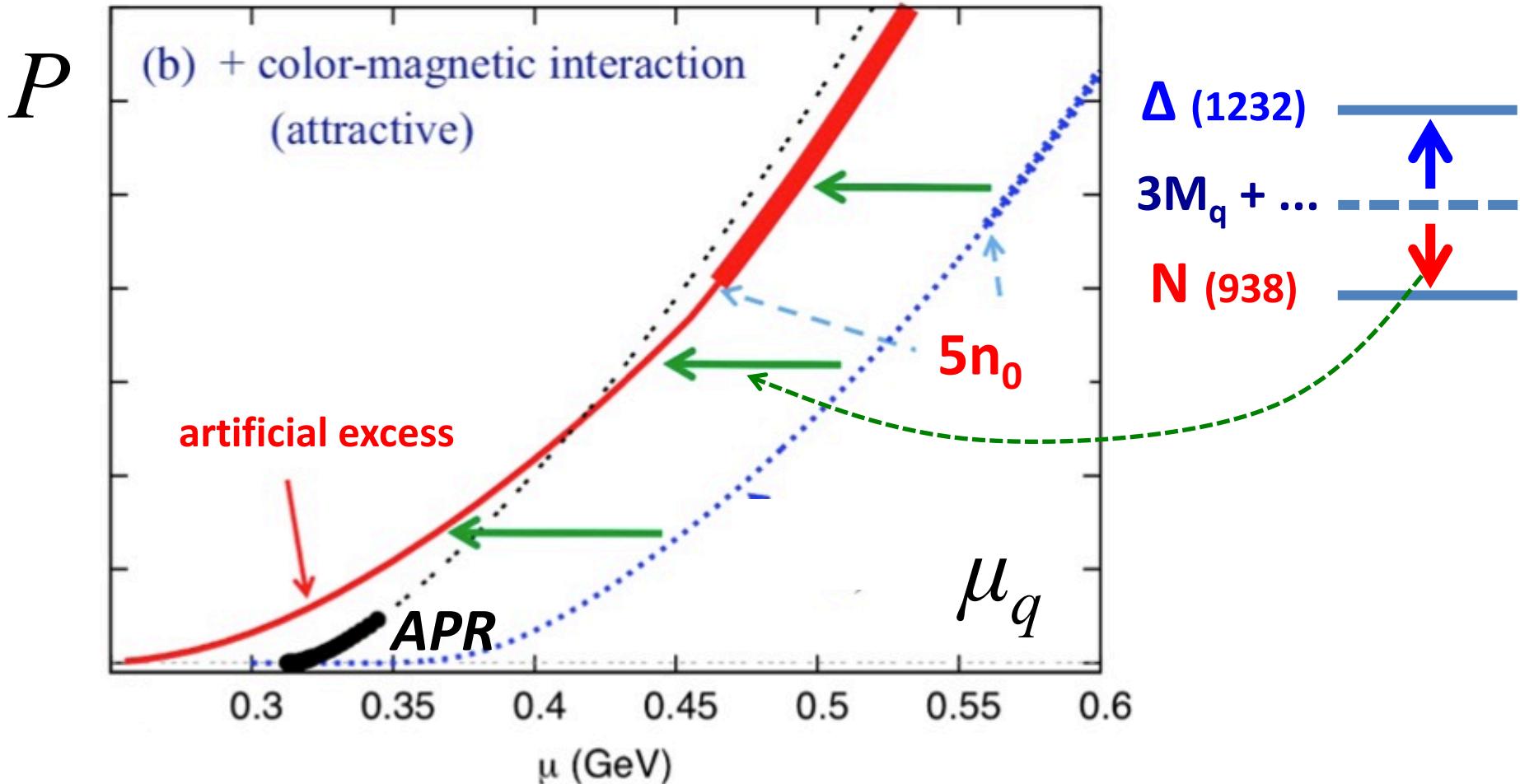
# + color magnetic interaction

(in *MF*, effects appear as diquark condensate)



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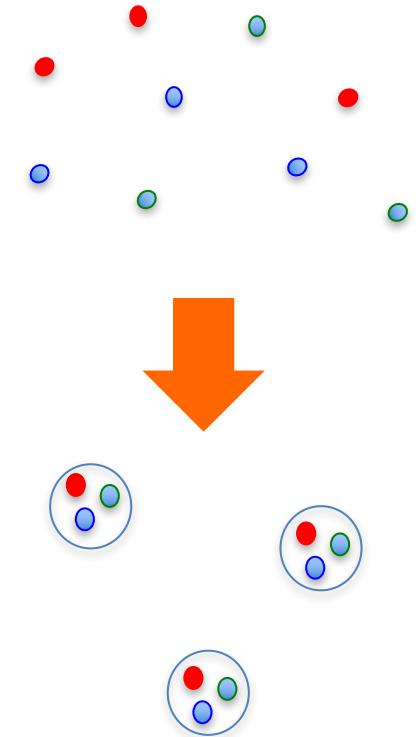


→ overall shift of  $P(\mu)$  toward lower  $\mu$



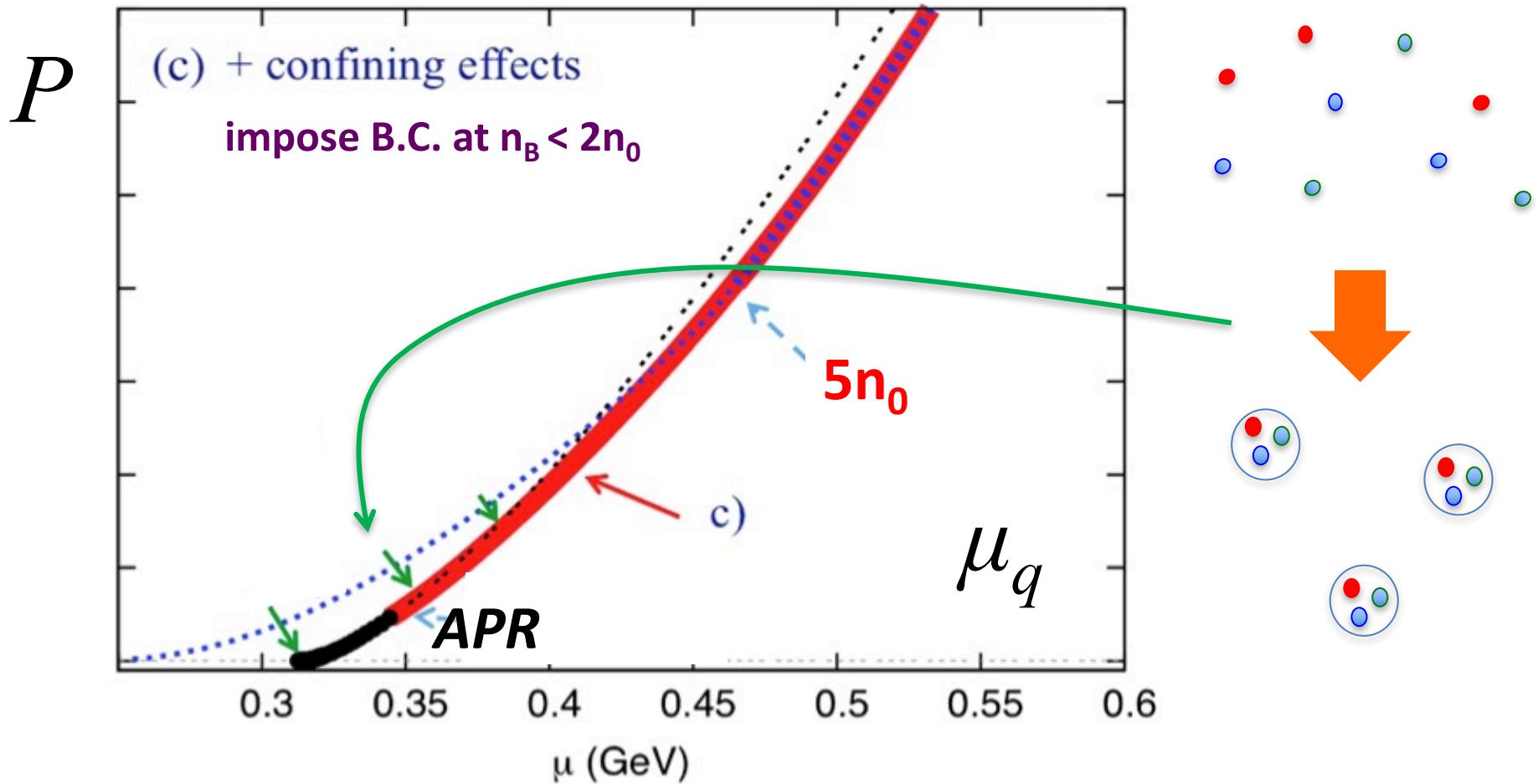
# + APR constraint at low density

( *mimic confining effects* )



# + APR constraint at low density

( *mimic confining effects* )

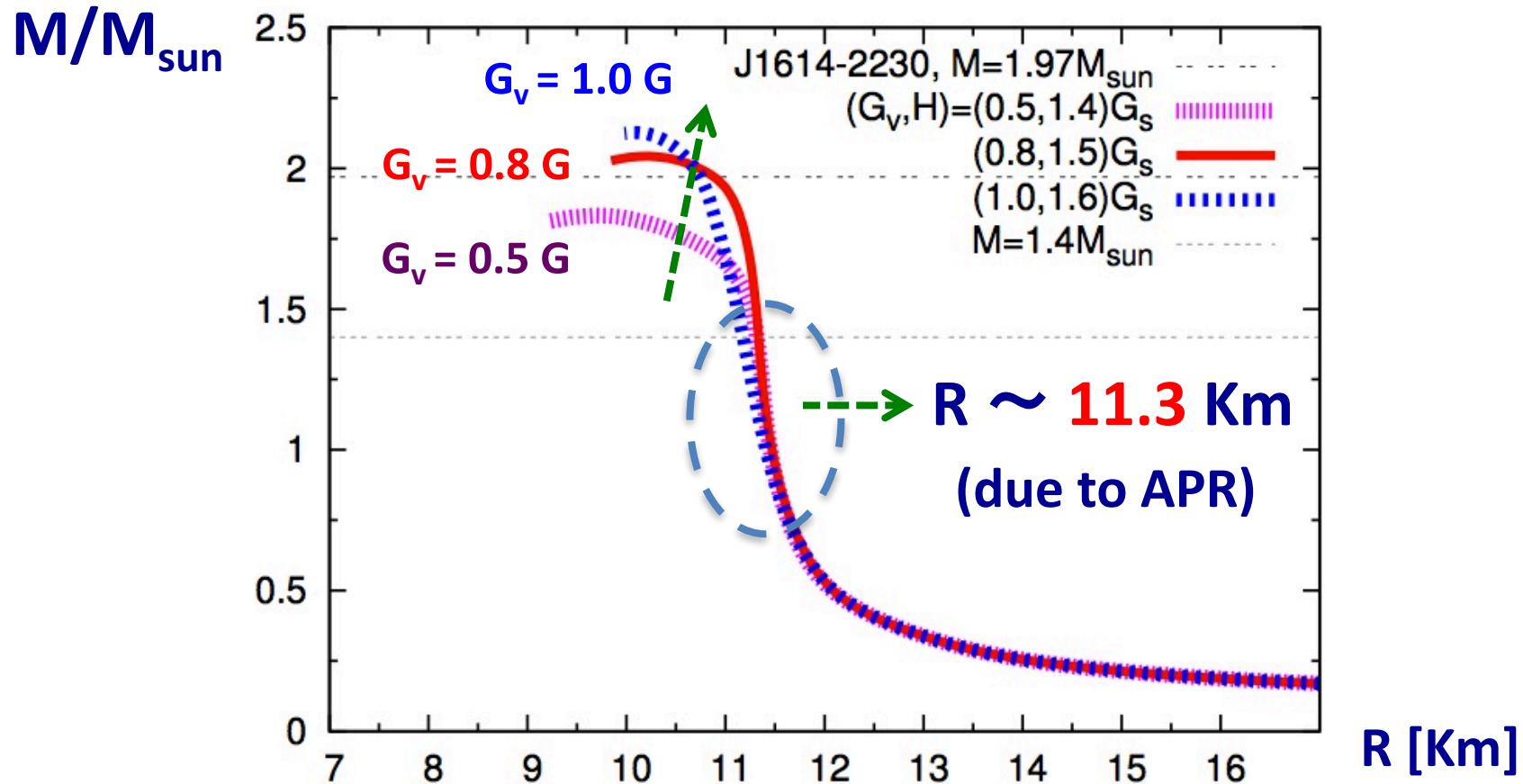


→ discard *artificial* excess of  $P$  at  $n_B < \sim 5n_0$

( *like Polyakov loop effects in hot QCD* )



# M-R curves



we need :

$$G_s \sim G_v \sim H @ n_B = 5-10 n_0 \rightarrow O(G_s^{\text{vac}})$$



# Gluons behind models

- **Statement :**

To  $n_B \sim 5 - 10 n_0$ , gluons should be  
as **non-perturbative** as in the vacuum

*If NOT*

- NJL parameters (at  $n_B \sim 5n_0$ ) :  $(G_s, G_\nu, H) \ll G_s^{\text{vac}}$
- The (gluonic) bag constant of  $O(\Lambda_{\text{QCD}}^4)$  must be included
- **Too much** strange quarks;  $m_s \sim 100 \text{ MeV}$

→ Troubles with the  **$2M_{\text{sun}}$**  constraint



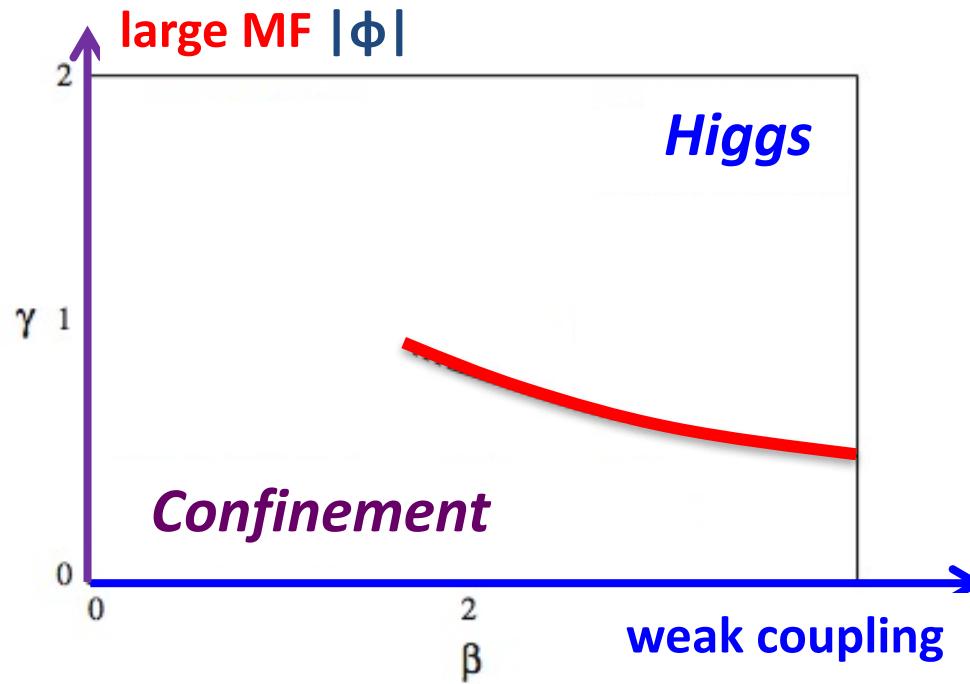
# Crossover : gauge dynamics

e.g.) Higgs model ***with const. amplitude***

[ Fradkin-Shenkar 79 ]

$$S = \beta \sum_{\text{plaq}} \frac{1}{2} \text{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x + \hat{\mu})]$$

amp. of Higgs  $|\phi|$       phase of Higgs



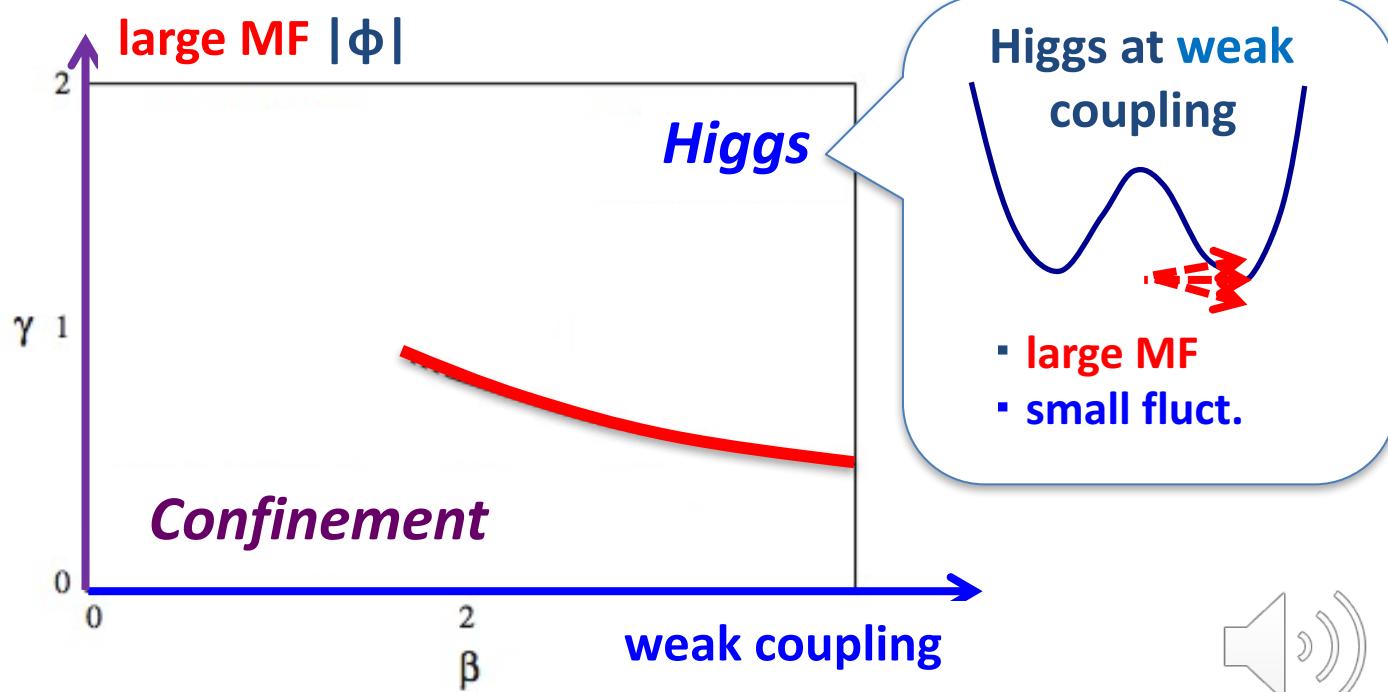
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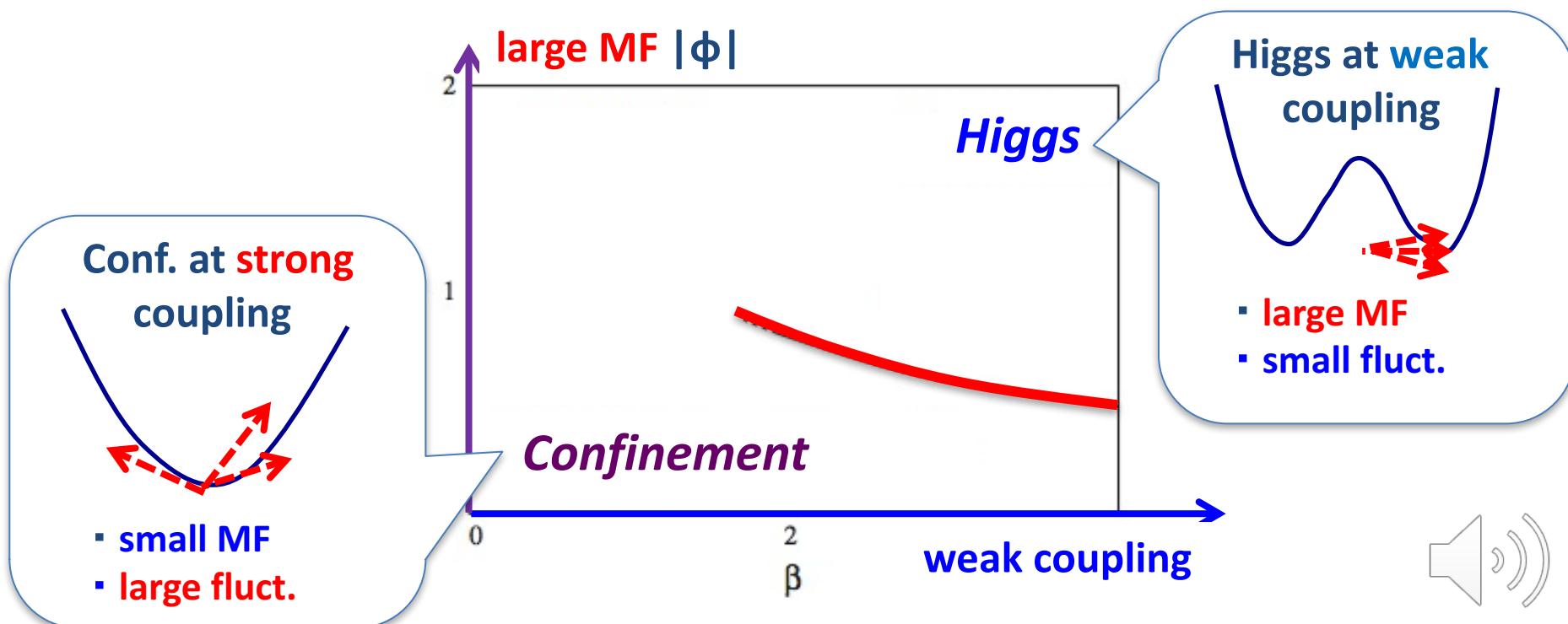
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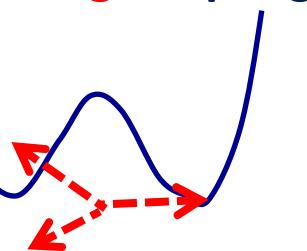
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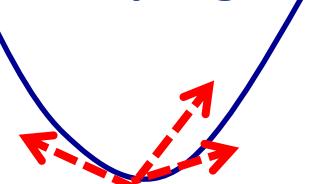
# Crossover : gauge dynamics

Conf.-Higgs  
at **strong coupling**



- large MF
- large fluct.

Conf. at **strong coupling**



- small MF
- large fluct.

**Model with const. amplitude**

amp. of Higgs  $|\phi|$

$$\frac{1}{2}$$

$$\text{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x + \hat{\mu})]$$

[ Fradkin-Shenkar 79 ]

phase of Higgs

large MF  $|\phi|$

Higgs

Higgs at weak coupling

- large MF
- small fluct.

Confinement

0

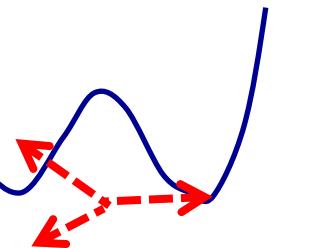
$\beta$

weak coupling



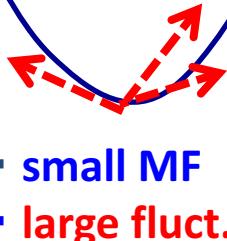
# Crossover : *aauae dvnamics*

Conf.-Higgs  
at **strong coupling**



- large MF
- large fluct.

Conf. at **strong coupling**



- small MF
- large fluct.

Model with

$$\frac{1}{2} \text{Tr}[UU]$$

Conjecture:

To complete the hadron-quark continuity picture **to the level of gauge dynamics**, the strong coupling is necessary to bypass conf-Higgs transition at weak coupling.

large MF |d

Higgs

Higgs at weak coupling

- large MF
- small fluct.

Confinement

weak coupling

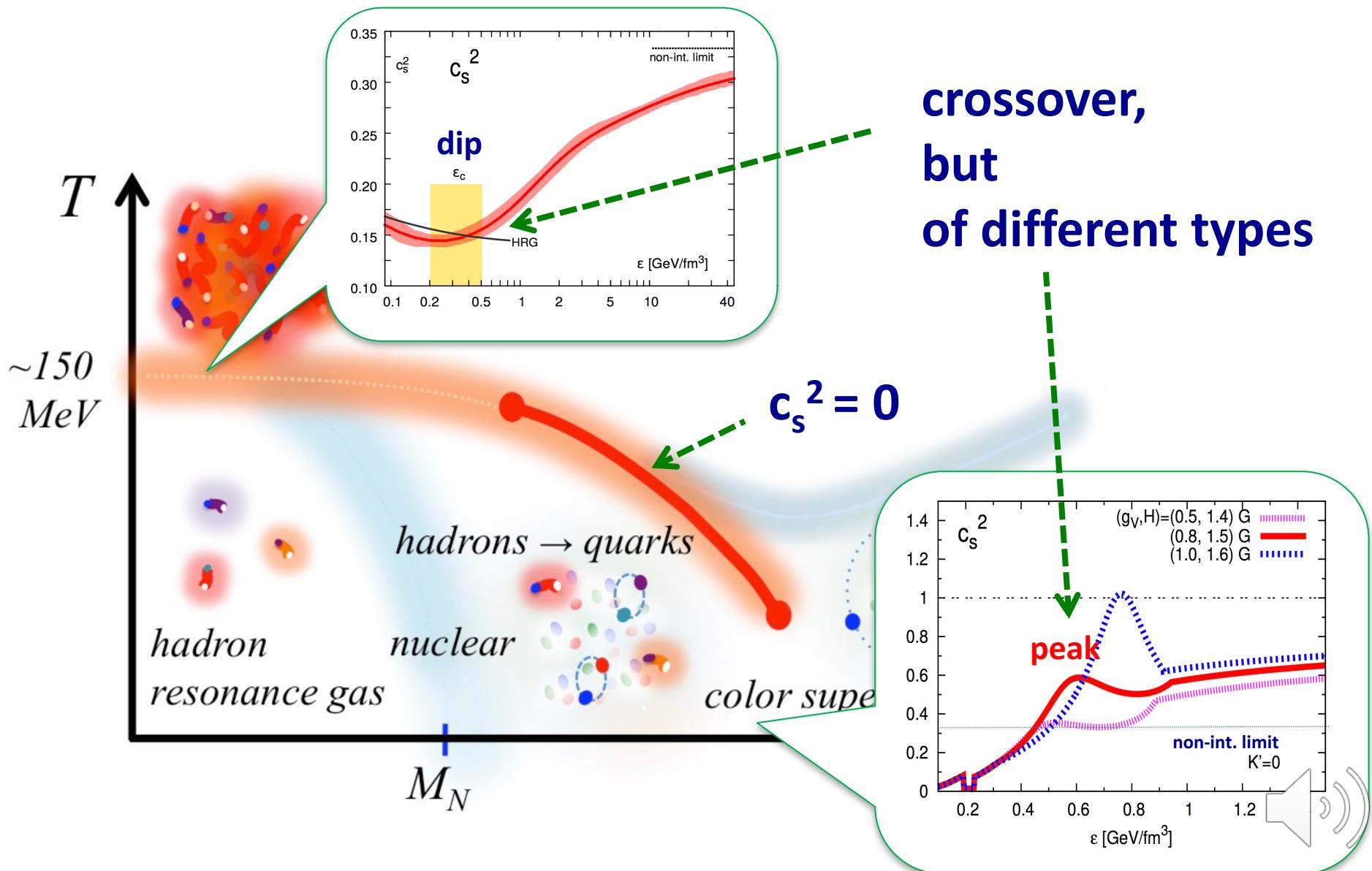
0

2

$\beta$



# Speed of sound



# Summary

- Small NS radii &  $M_{\max} \sim 2M_{\text{sun}}$  & causality  
→ **crossover** or weak 1<sup>st</sup> order P.T. for hadron-quark transition
- Systematic uncertainties in radii estimates, but in near-future NICER and aLIGO will estimate R to **5-10%** accuracy.
- Quark models **inside** hadrons may be extrapolated to **5-10n<sub>0</sub>**.  
intermediate-short range correlations (chiral & color-magnetic int.)
- Crossover in **hot** QCD and **dense** QCD are likely different:
  - hot QCD: smooth, but **rapid change** from HRG to QGP picture
  - dense QCD: smooth, strongly int. **hadrons** ~ strongly int. **quarks**  
“quark-hadron duality” or “quarkyonic” (quark matter with non-pert. gluons)

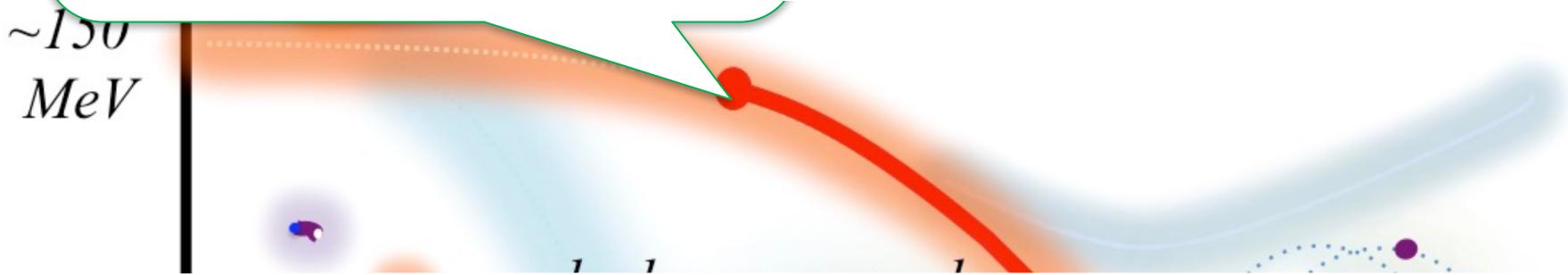


*Back up*

# Questions relevant for continuity picture

**RHIC, NICA, FAIR, J-PARC, ...**

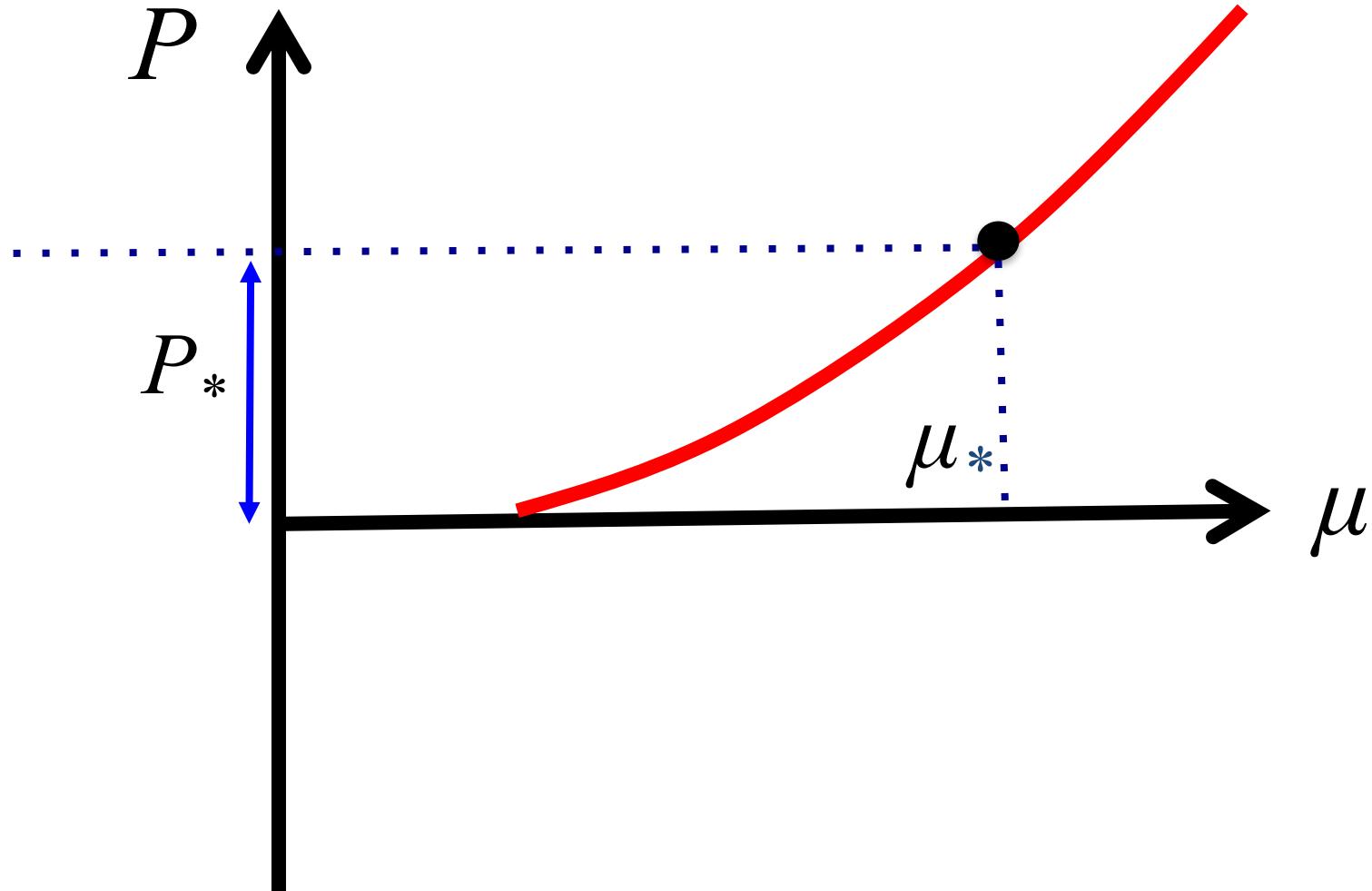
- CEP exists or not?
- If exists, how strong is the 1<sup>st</sup> order phase transition?  
(How much chiral cond. after P.T.?)



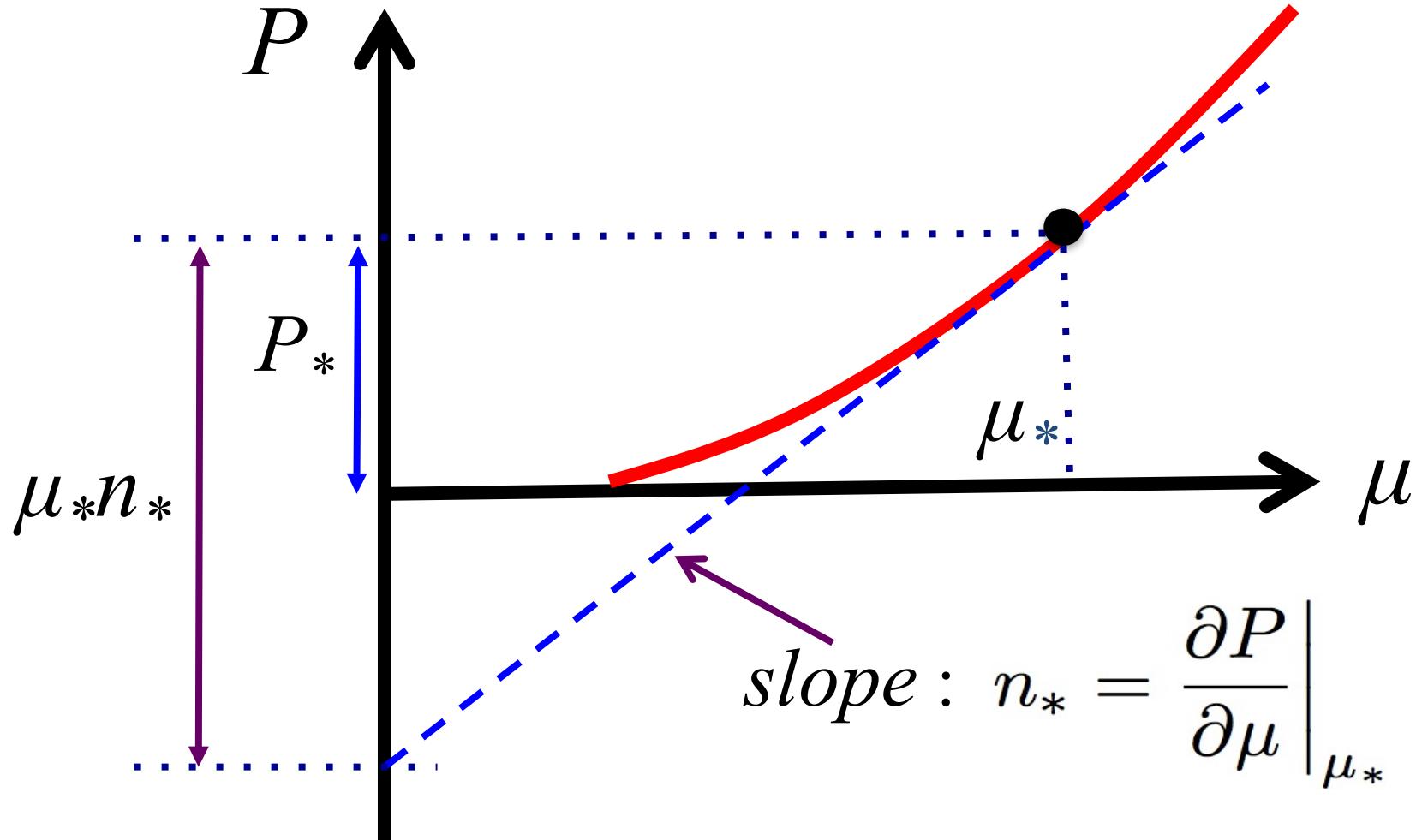
**NICER, aLIGO, VIRGO,...**

- How large is NS radius?

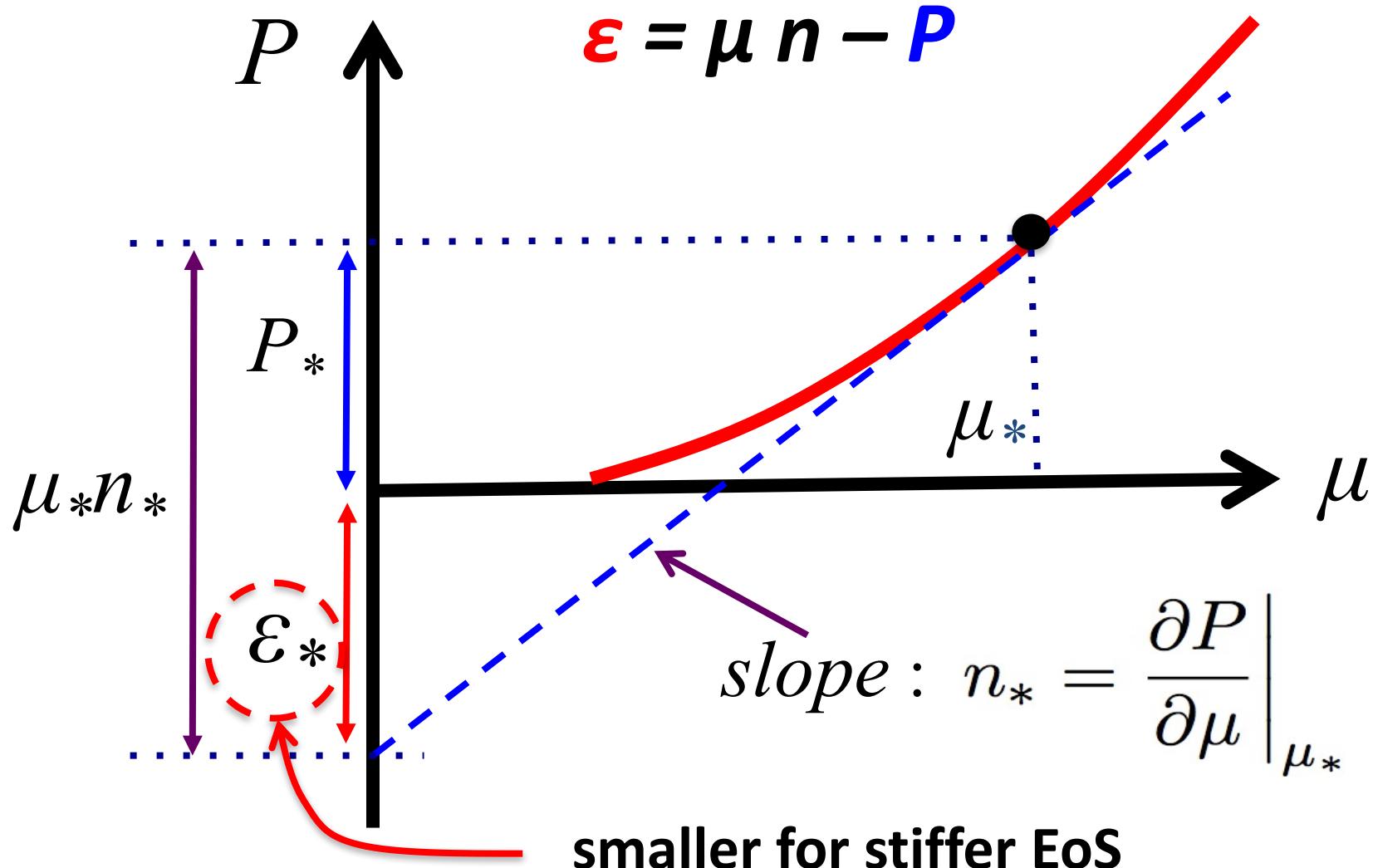
# How stiff EoS looks like in $P(\mu)$ curves



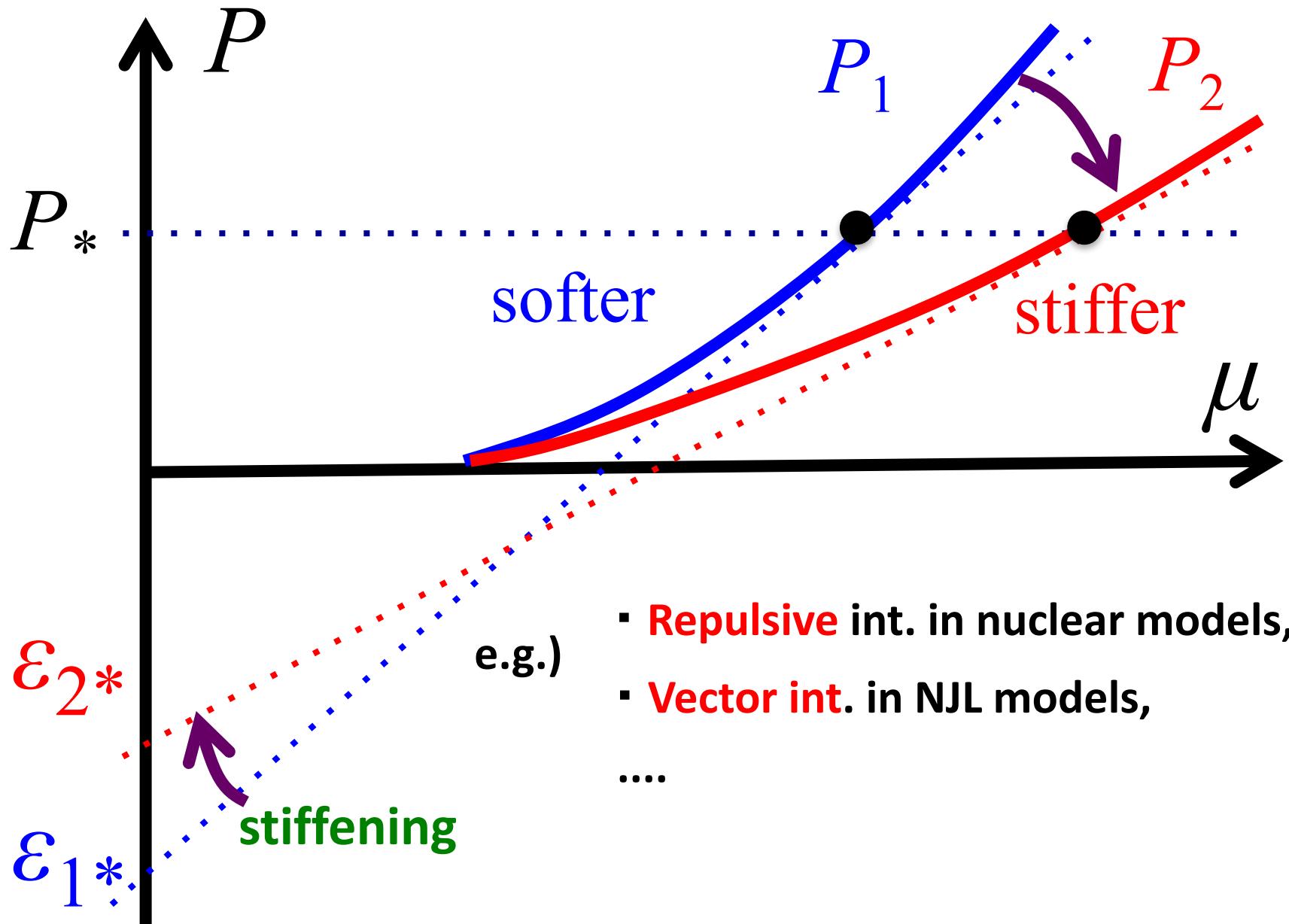
# How stiff EoS looks like in $P(\mu)$ curves



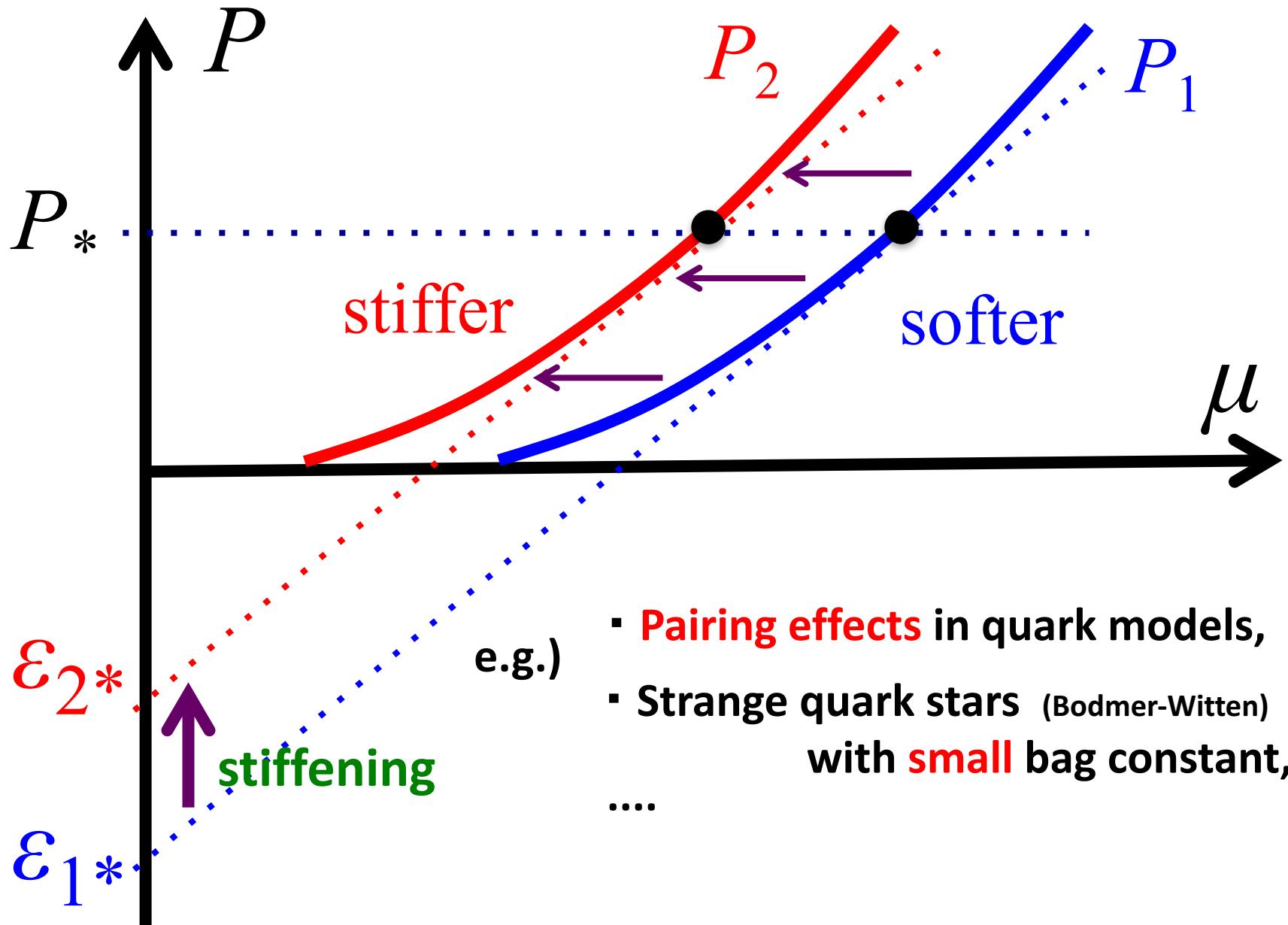
# How stiff EoS looks like in $P(\mu)$ curves



# Example of stiffening 1

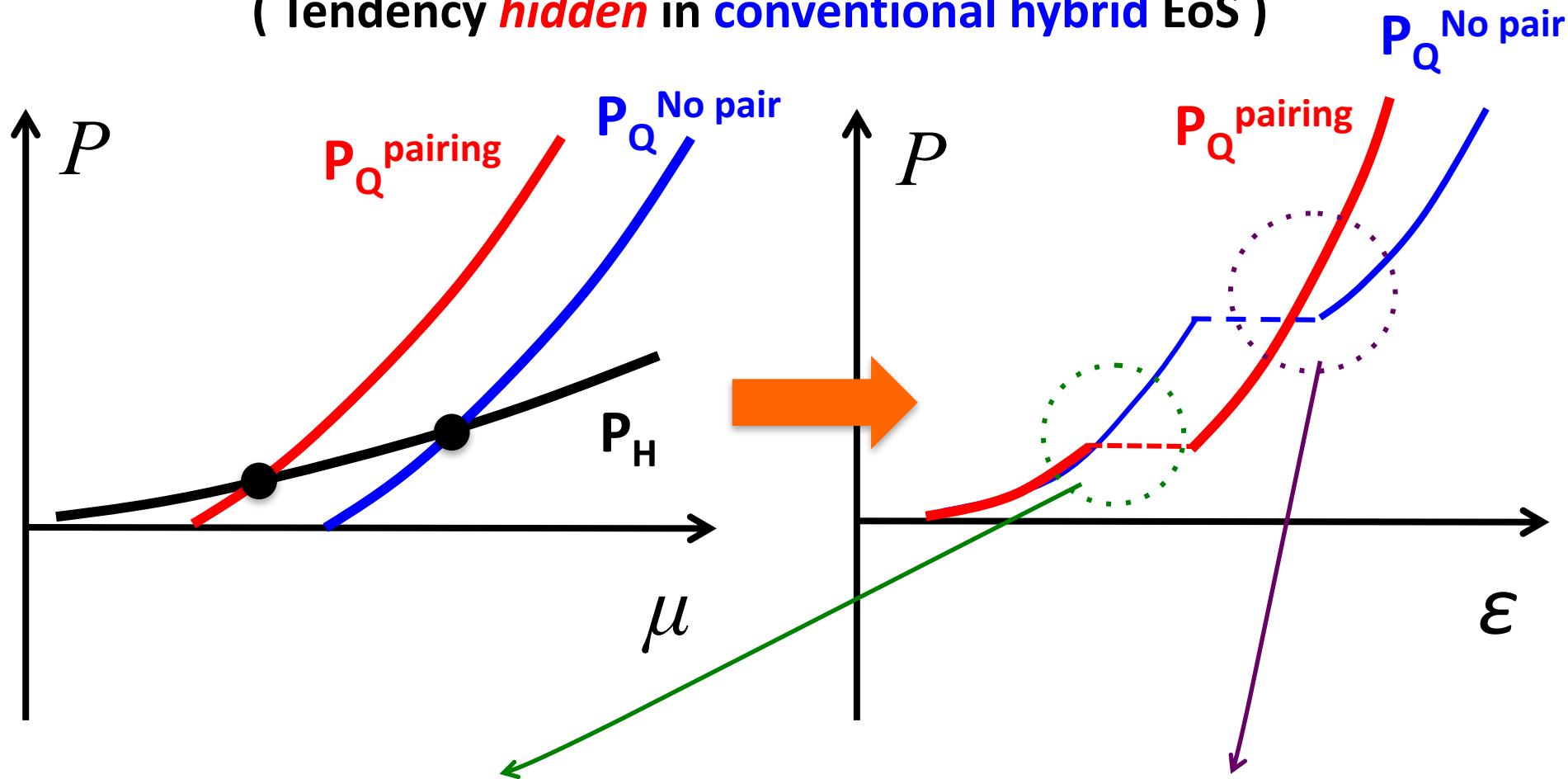


## Example of stiffening 2

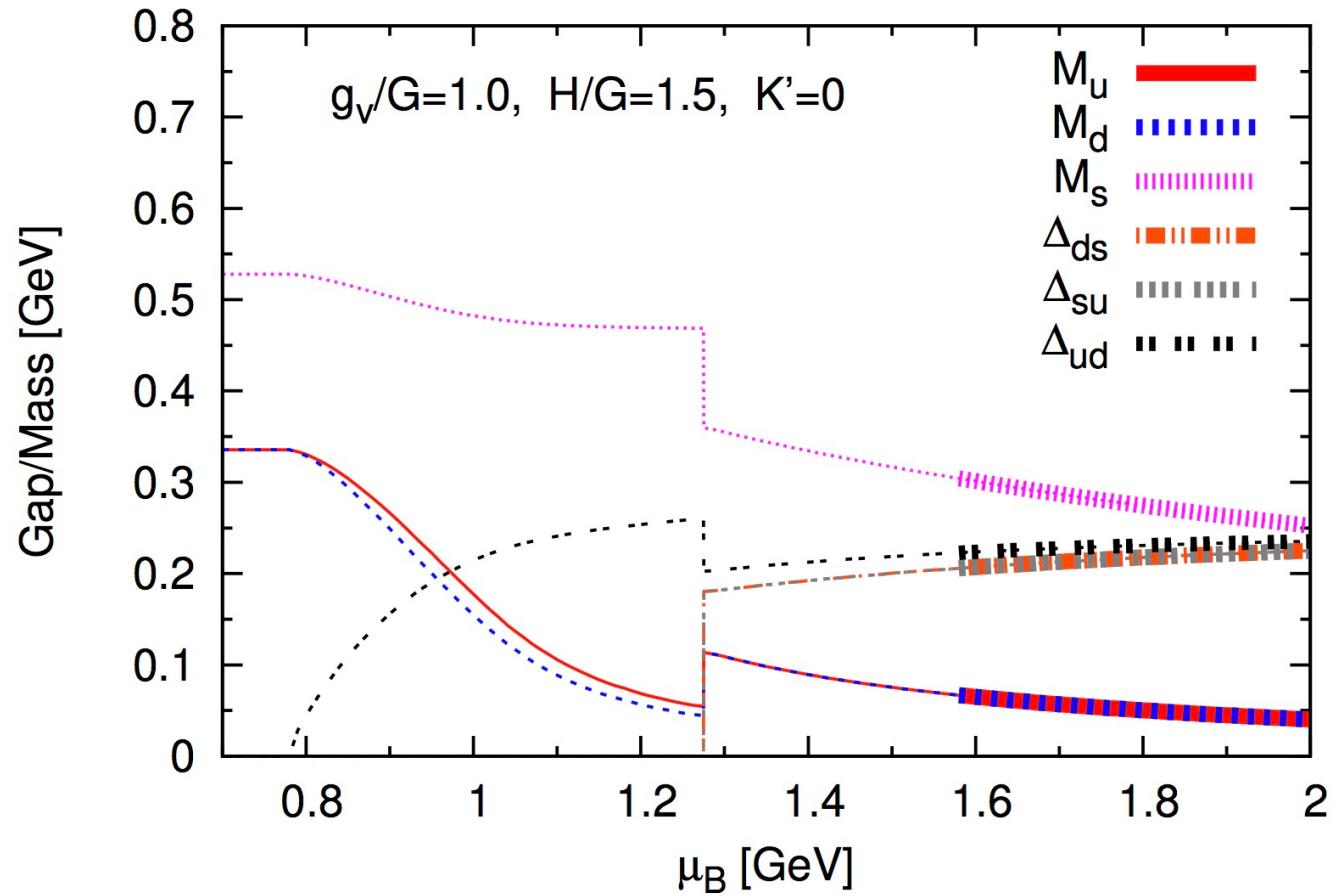


# *“Pairing” can stiffen EoS*

( Tendency *hidden* in conventional hybrid EoS )



→ *Softening* at *low  $n_B$*  & *stiffening* at *high  $n_B$*



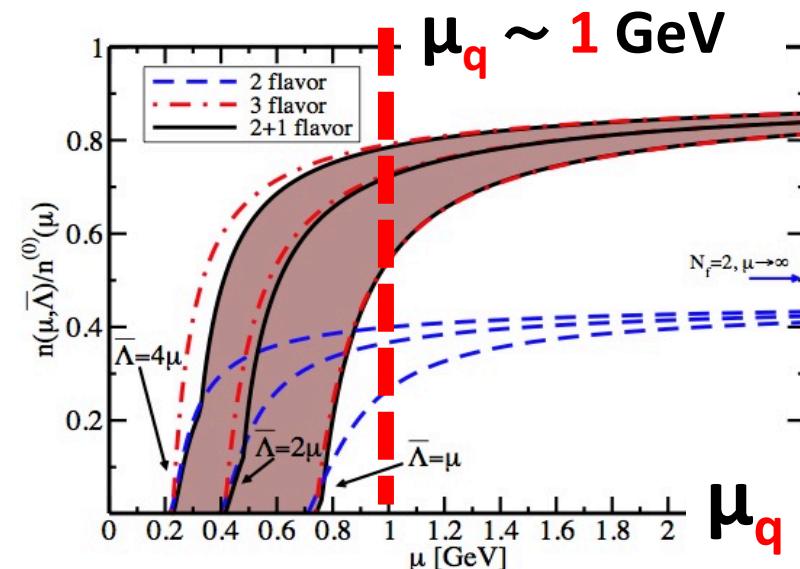
# Theoretical guides at $N_c=3$

- 3-loop *pQCD at large  $\mu_q$*

[ Freedman-McLerran 78; Baluni 78  
Kurkela-Romatschke-Vuorinen 09, ... ]

- large  $\alpha_s$  corrections at  $\mu_q < 1 \text{ GeV}$

→ soft gluons important at  $n_B < 100 n_0$



- Nuclear calculations ( ChEFT+many-body ) at small  $\mu_q$*

- reliable at  $n_B \sim n_0$

[ Akmal et al. (APR) 98; Gandolfi et al. 12, ... ]

- At  $n_B > 2n_0$ 
  - *convergence* problems :  $\langle V_{\text{2-body}} \rangle \sim \langle V_{\text{3-body}} \rangle \sim \dots$
  - *hyperon softening*, unless introducing ad hoc repulsion
  - *changes in hadron w.f. & Dirac sea negligible?*

# Nuclear EoS : convergence ?

Many-body interaction (APR-A18+UIX case)

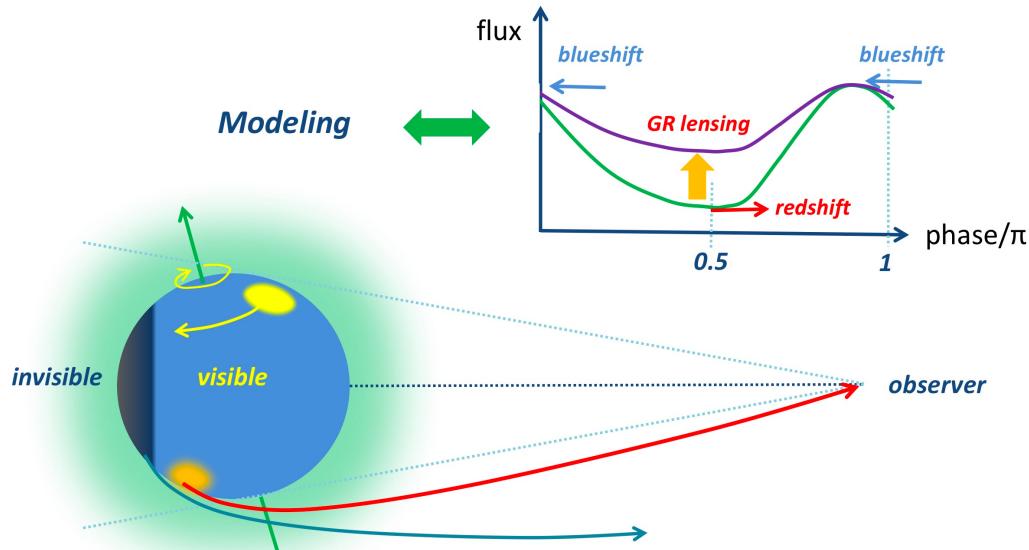
$n_B$	2 –body int.		3 –body int.		4 –body int. (our guess)
$n_0$	$\langle v_{ij}^\pi \rangle$	$\langle v_{ij}^R \rangle$	$\langle V_{ijk}^{2\pi} \rangle$	$\langle V_{ijk}^R \rangle$	
$n_0$	-4.1	-29.9	1.2	4.5	small
$2 n_0$	-25.1	-36.4	-17.4	30.6	marginal
$3 n_0$	-35.7	-44.7	-34.1	78.0	
$4 n_0$	-52.2	-41.1	-76.9	160.3	large

*grow rapidly !!*

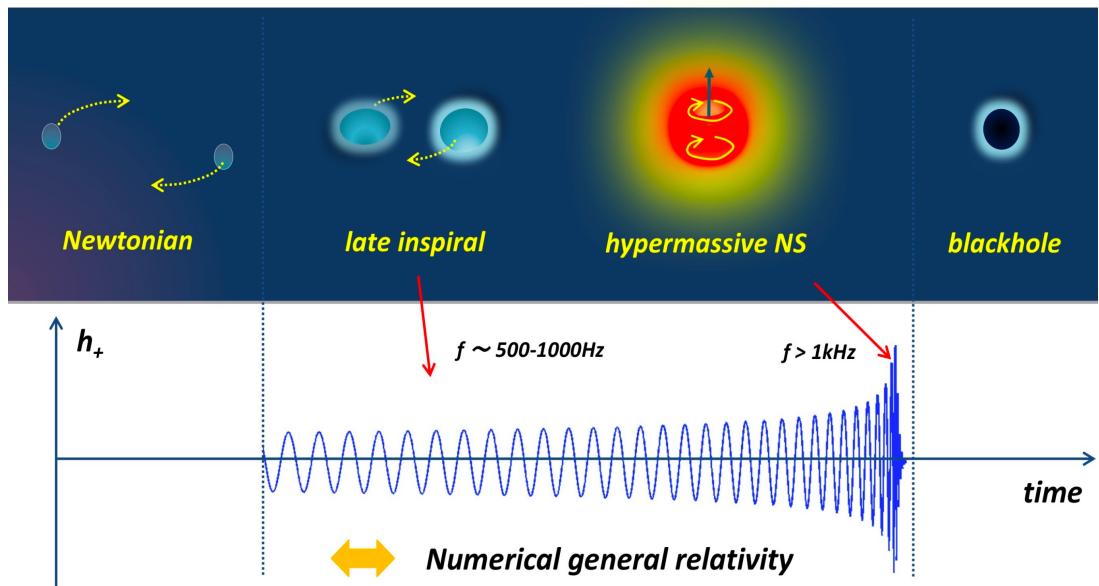
$$\langle V_{N\text{-body}} \rangle \sim c_N (n_B / n_0)^N$$

# Near future NS radius measurements

- NICER (2017~) :
  - timing analyses of hot spots*
  - $R \& M/R \rightarrow 5\text{-}10\% \text{ accuracy}$



- aLIGO (2015~) : GWs from NS-NS mergers

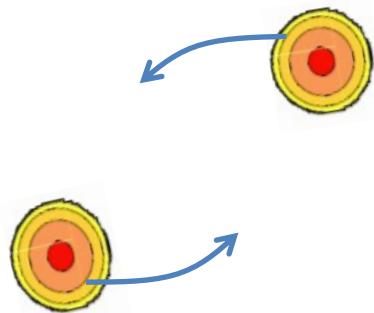


*tidal deformation of stars*

$R \& M/R \rightarrow 5\text{-}10\% \text{ accuracy}$

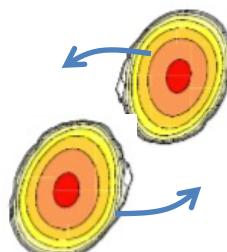
# *GW from NS-NS mergers* ( 0.1-10 (?) events / year)

*Early inspiral*



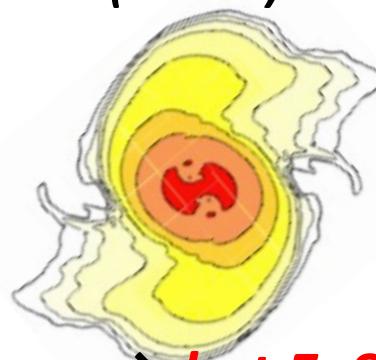
$\rightarrow M_1$  &  $M_2$   
spins

*Tidally deformed  
phase*



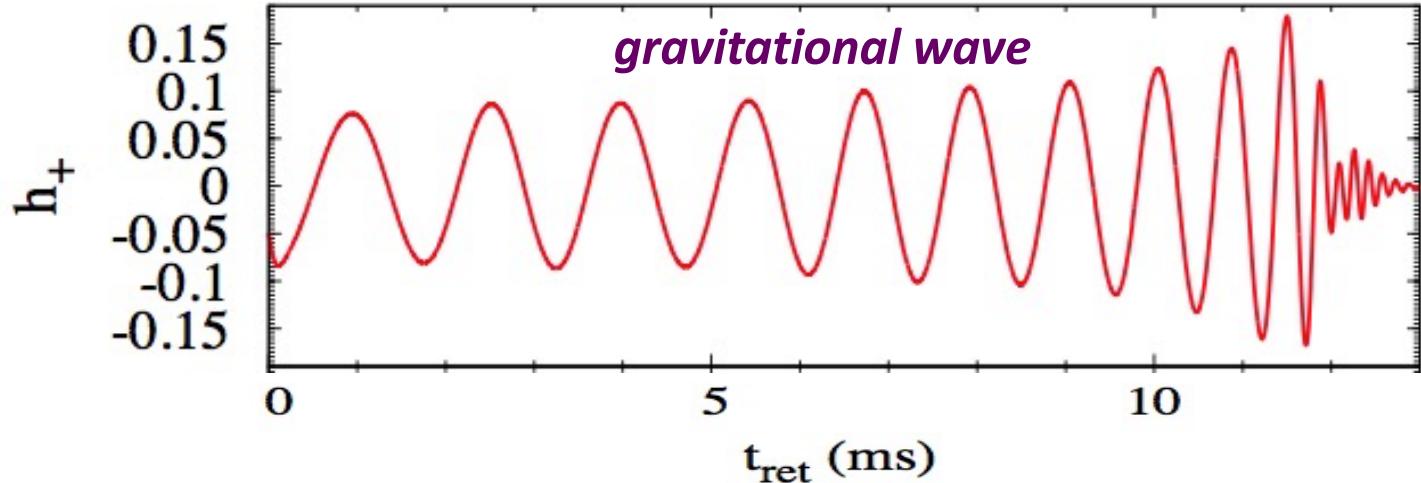
$\rightarrow R$  (deformability  
& compactness)

*Hyper Massive NS  
(HMNS)*

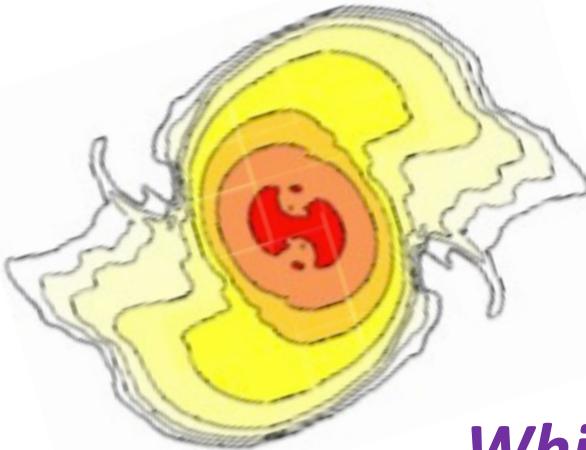


$BH$

( aLIGO,  
VIRGO, ...)



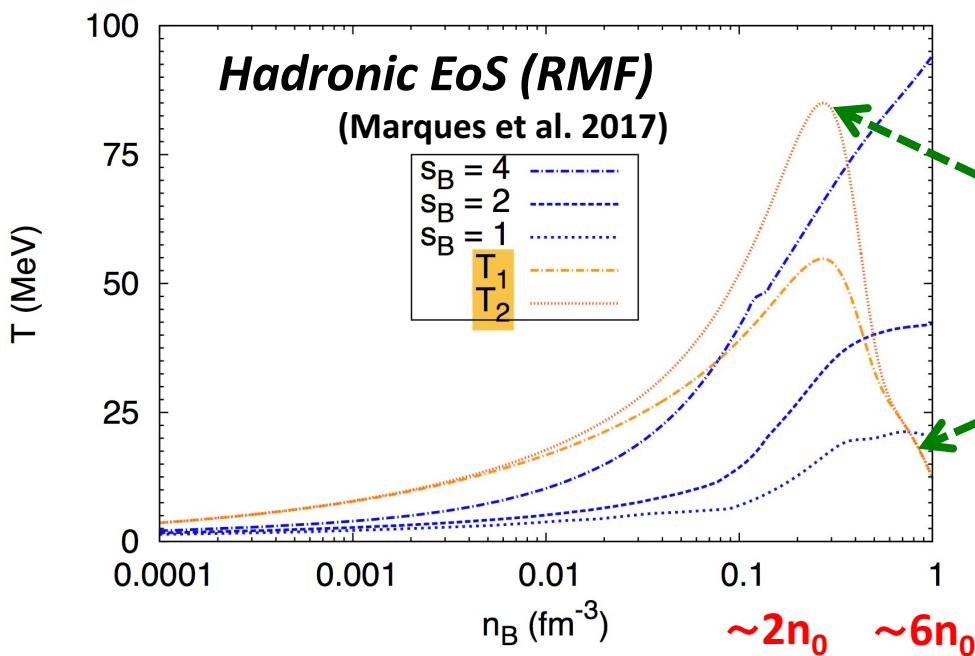
# Hyper massive NS (HMNS)



*Differential rotation & thermal pressure*

→ stars of  $2-3M_{\text{sun}}$  can survive for  $\sim 10\text{ms}$

*Which density region is hot?*



For typical *hadronic EoS*  
coupled to *GR simulations* :

$T = 30-100 \text{ MeV at } n_B \sim 2n_0$

$T = 10-20 \text{ MeV at } n_B \sim 5n_0$

NOTE: profiles depends on *EoS*

# *Hot EoS for post mergers*

- Almost all GR simulations use hot *nuclear* EoS  
 [Shen-EoS (Shen et al.), SLy EoS (Lattimer-Swetsky), ...]

- *Hot quark matter EoS (for  $n_B > 5n_0$ )*

## *Normal* quark matter

- *pQCD EoS* (gapless quarks & gapped gluons) [Kurkela-Vuorinen '16]
- *3-window EoS* (gapless quarks) [Masuda-Hatsuda-Takatsuka '15]
- ...

$$\text{gapless quarks} \rightarrow \Delta P(T) \sim p_F^2 T^2 \quad (>> T^4)$$

*This work* → *Gapped* quark matter, *Color-Flavor-Locked (CFL)*

$$\text{For } T < \Delta ; \quad \Delta P(T) \sim T^4 + \dots$$

neutrinos, photons, NG modes

# NG mode contributions (CFL color-super phase)

[Son-Stephanov 2000, Bedaque-Schafer 2002, ...]

- setup consistent with T=0 NS descriptions

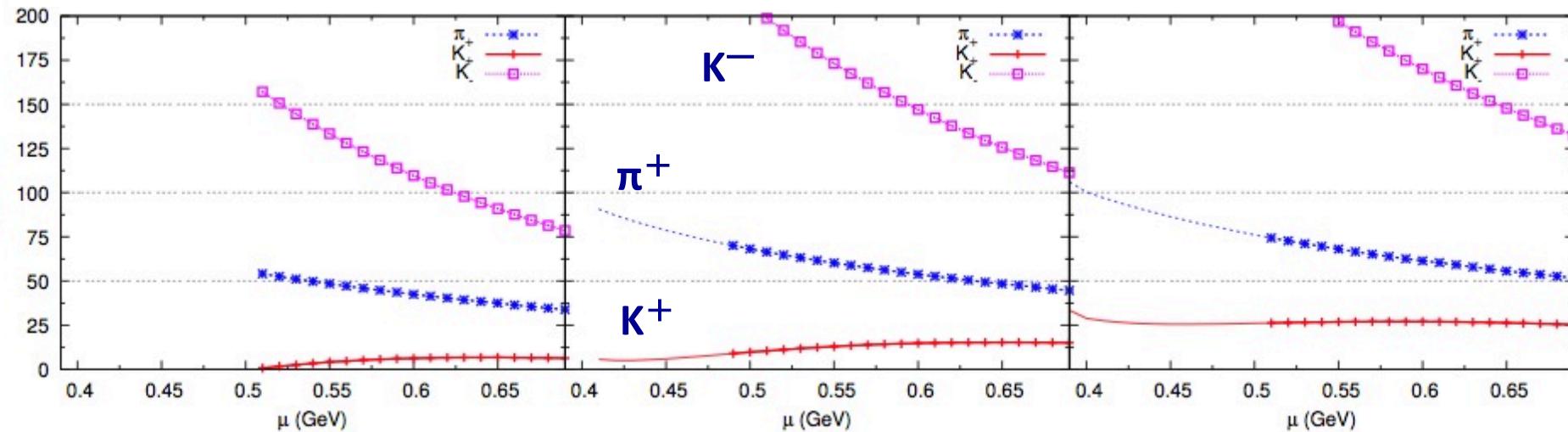
- explicit sym. breaking, mass &  $U_A(1)$
- neutrality conditions
- coexistence of chiral and diquark condensates
- keep “pa”, “pp”, “aa” contributions to be consistent with gap eq.

- spectra in RPA (results consistent with EFT) [ Basler-Buballa '10, TK16 ]

weak coupling ←

*setup for NS constraints*

→ strong coupling

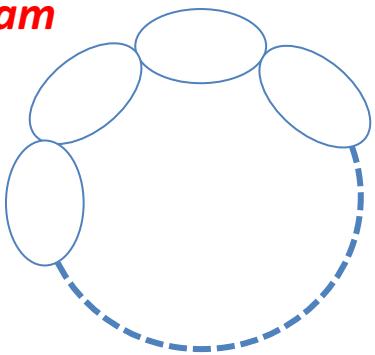


most NG modes > 50 MeV; light K; more massive at stronger coupling

# Thermodynamics (beyond low T regime)

*Ring diagram*

*connected  
Green's  
function*



*NG bosons (bound states)*

*pre-formed pairs ( $p-a$ ,  $p-p$ ,  $a-a$  pairs)*

*decaying pairs (continuum)*

*very important to keep (see below)*

*The phase shift rep. of thermodynamic-potential :*

[Beth-Uhlenbeck 1939, Dashen-Ma-Bernstein 1969]

$$\Omega_X(T, \mu) = \int \frac{d\vec{q}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[ \underline{\omega} + \overline{T \ln \left( 1 - e^{-\frac{\omega-\mu_X}{T}} \right) + T \ln \left( 1 - e^{-\frac{\omega+\mu_X}{T}} \right)} \right] \frac{\partial \delta_X(\omega, \vec{q})}{\partial \omega}$$

$$\mathcal{G}/\mathcal{G}_0 = |\mathcal{G}/\mathcal{G}_0| e^{i\underline{\delta(\omega, \vec{q})}}$$

*full/free  
Green's function*

*phase shift*

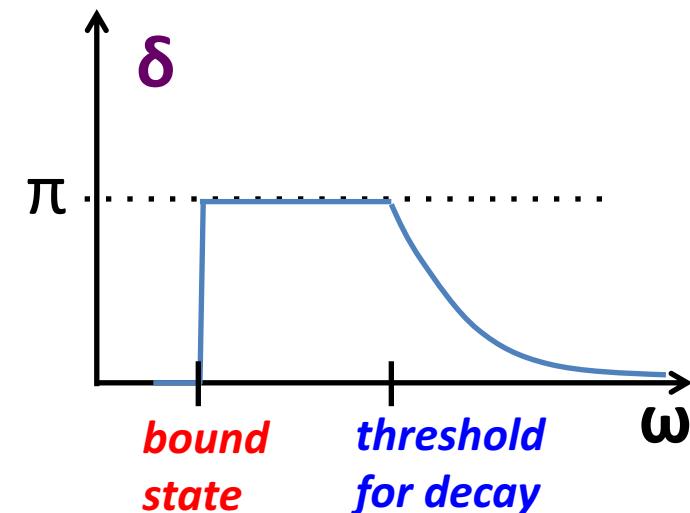
## Constraint: Levinson's theorem

$$\mathcal{G}/\mathcal{G}_0 = |\mathcal{G}/\mathcal{G}_0| e^{i\underline{\delta(\omega, \vec{q})}}$$

*Meaning: Total num. of states does not change by interactions*

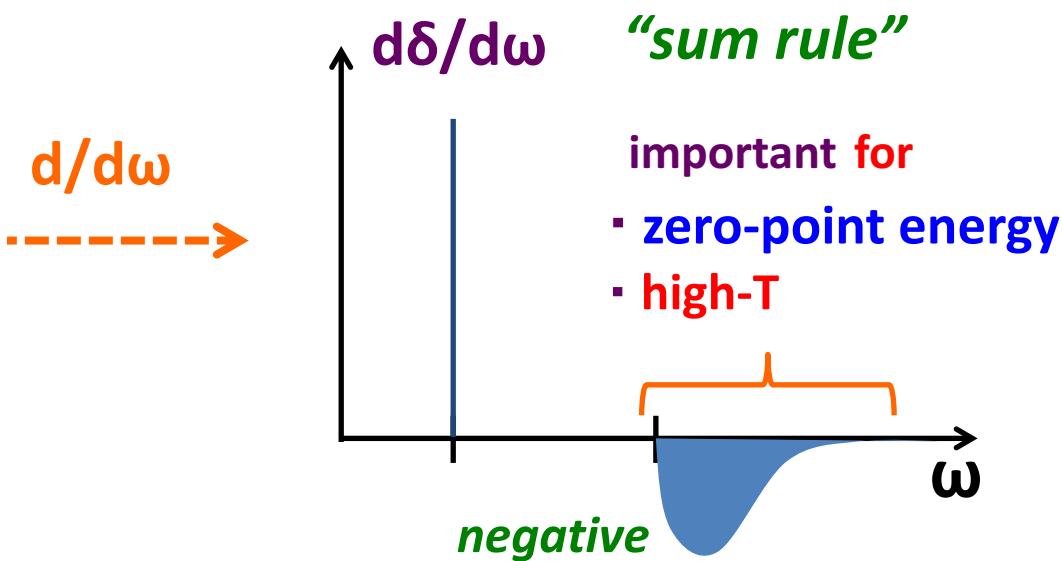
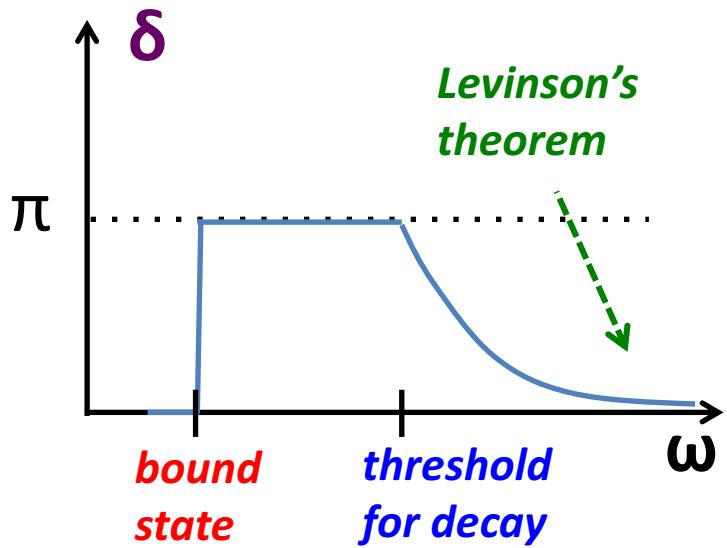
$$\begin{aligned} 0 &= \int_0^\infty dE \operatorname{Tr} [\operatorname{Im} \mathcal{G} - \operatorname{Im} \mathcal{G}_0] \\ &= \int_0^\infty dE \partial_E \operatorname{Tr} [\operatorname{Im} \ln \mathcal{G}^{-1} / \mathcal{G}_0^{-1}] \\ &= -\operatorname{Tr} [\delta(\infty) - \delta(0)] \end{aligned}$$

*invariant*



# Phase shifts & Levinson's theorem

$$\Omega_X(T, \mu) = \int \frac{d\vec{q}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[ \underline{\omega + T \ln \left( 1 - e^{-\frac{\omega - \mu_X}{T}} \right)} + T \ln \left( 1 - e^{-\frac{\omega + \mu_X}{T}} \right) \right] \frac{\partial \delta_X(\omega, \vec{q})}{\partial \omega}$$

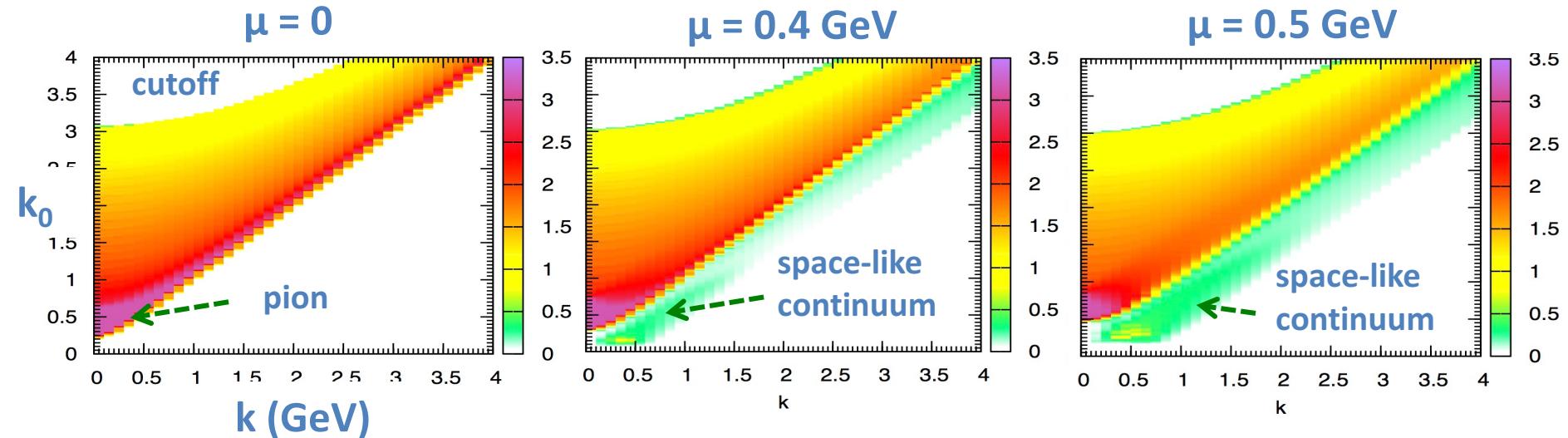


origin:  $\mathcal{G}^{\text{conn.}} = \underline{\mathcal{G}^{\text{full}} - \mathcal{G}_0}$

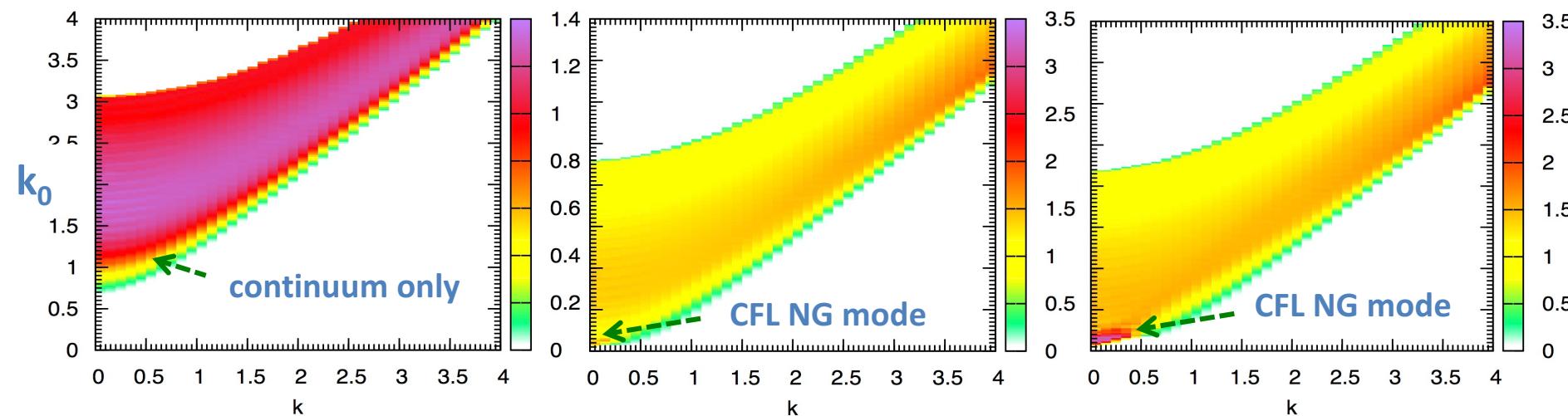
Pressure from low  $E$  and high  $E$  cancel one another;  
taming a meson (diquark) gas at high  $T$

# Phase shift $\delta(k_0, k)$ : e.g. $\pi$ -channel

particle-hole, particle-antiparticle

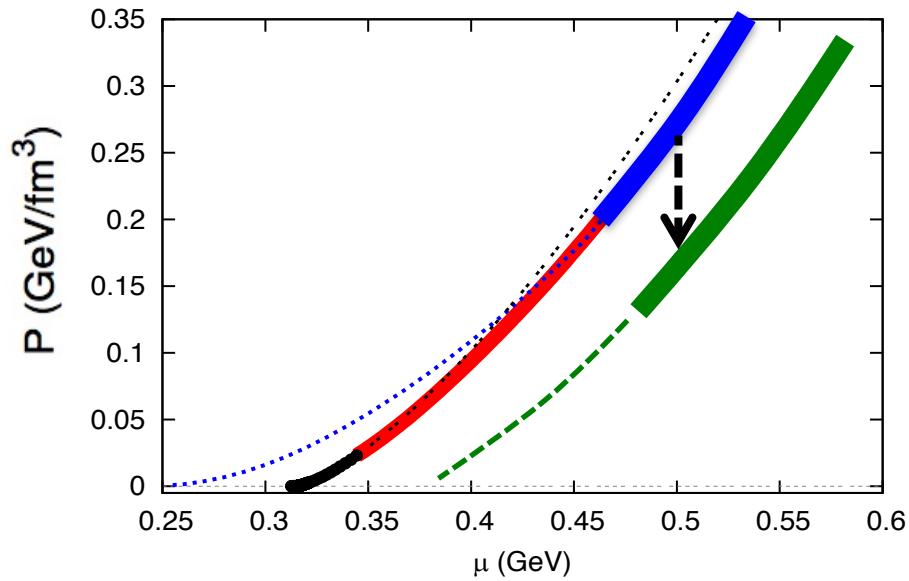


particle-particle, hole-hole



# *Discussion : Bag constant ?*

$P_{NJL}$  @ 5  $n_0 \rightarrow$  only 200 - 400 MeV fm $^{-3}$



If  $B_g \sim \Lambda_{QCD}^4$  appears @ 5 $n_0$   
EoS  $\rightarrow$  impossible to pass  
any constraints

Together with  $G_V \sim H \sim G_s^{vac}$ , we claim :

Gluons **should remain non-perturbative** to  $n_B \sim 5\text{-}10 n_0$

# *Discussion : Bag constant ?*

Def:  $\mathcal{B} \equiv \epsilon_{pert}^{vac} - \epsilon_{full}^{vac} \sim \Lambda_{\text{QCD}}^4 > 0$

- Energy **gain** by **non-pert. effects** ;  
e.g.) ChSB in Dirac sea, gluon condensation, ...
- 

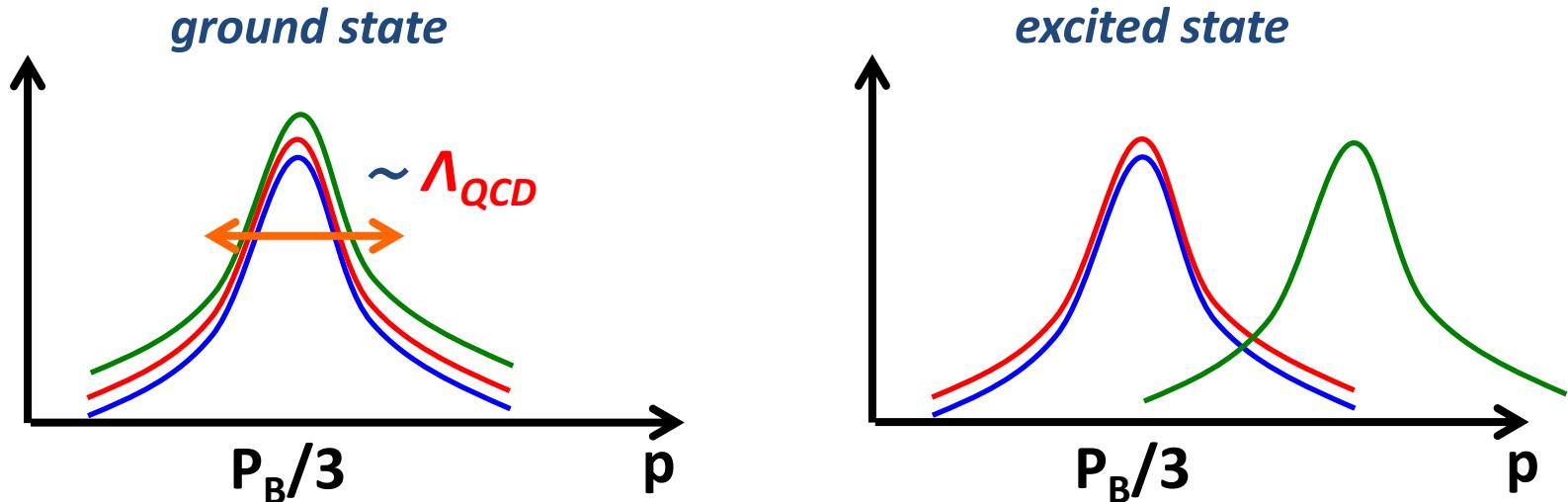
*If  $\mu$  is large enough :* ( softening )

- Loss of **non-pert. effects**  $\rightarrow$  
$$\begin{cases} \epsilon_{\text{matter}} \rightarrow \epsilon_{\text{matter}} + \mathcal{B} \\ P_{\text{matter}} \rightarrow P_{\text{matter}} - \mathcal{B} \end{cases}$$
- NJL takes into account the **vac. contributions only partially** ;  
it **misses** contributions from **gluonic** one,  $B_g$

# Discussion 3: Hyperon problems ?

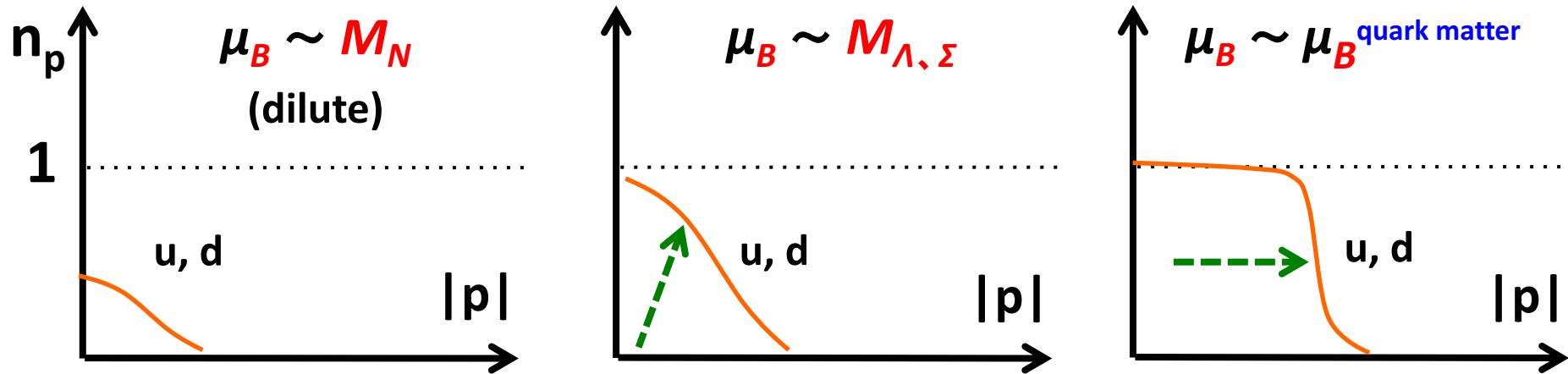
**How did we avoid hyperon softening ?**

- $\mu_B^{th}$  for strangeness :  $\left\{ \begin{array}{ll} \mu_B \sim 3M_s \sim 1.5 \text{ GeV} & \text{(quark picture)} \\ \mu_B \sim \mu_\Lambda, \mu_\Sigma \sim 1.1-1.2 \text{ GeV} & \text{(hadron picture)} \\ (\text{uds, uus, ...}) & \end{array} \right.$
- **A quark w.f. for a baryon** (e.g. Isgur-Kahl)



# Discussion 3: Hyperon problems ?

- Quark descriptions of hadronic matter :



How to put hyperons ??

- $M_{\Lambda, \Sigma}$  at *low P* is *rejected* by quark Pauli blocking on (u,d)
- $M_{\Lambda, \Sigma}$  at *high P* avoid the blocking, but *is energetic*

[ Note: this argument becomes *more powerful* at higher  $n_B$  ]

# *Several branches*

- Confined, *but chiral symmetric* matter ( many papers ... )
    - have been *challenged* by many model calculations [ Glozman et al. 2007, .... ]

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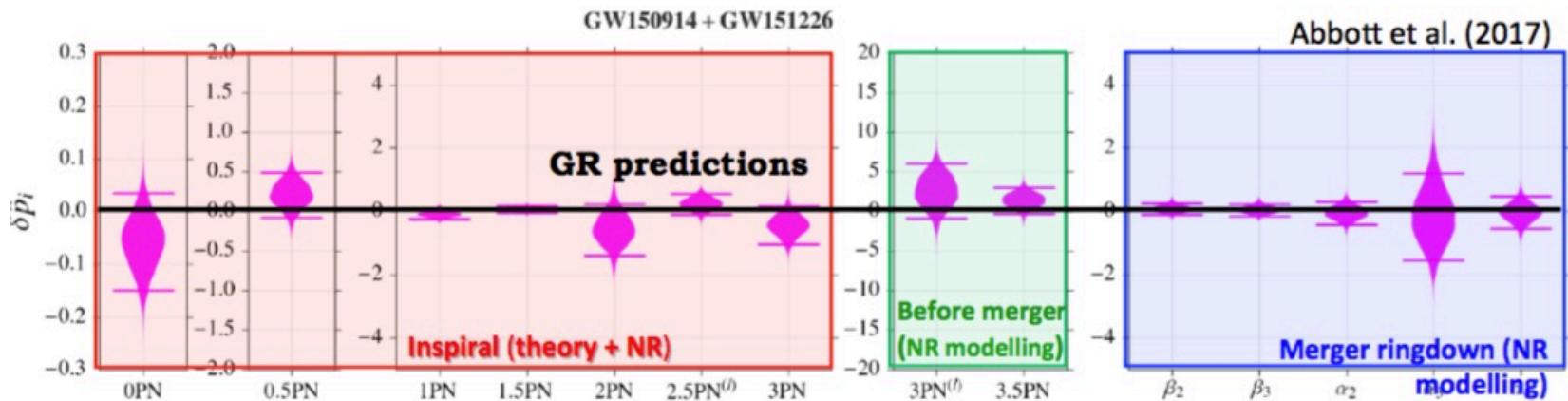
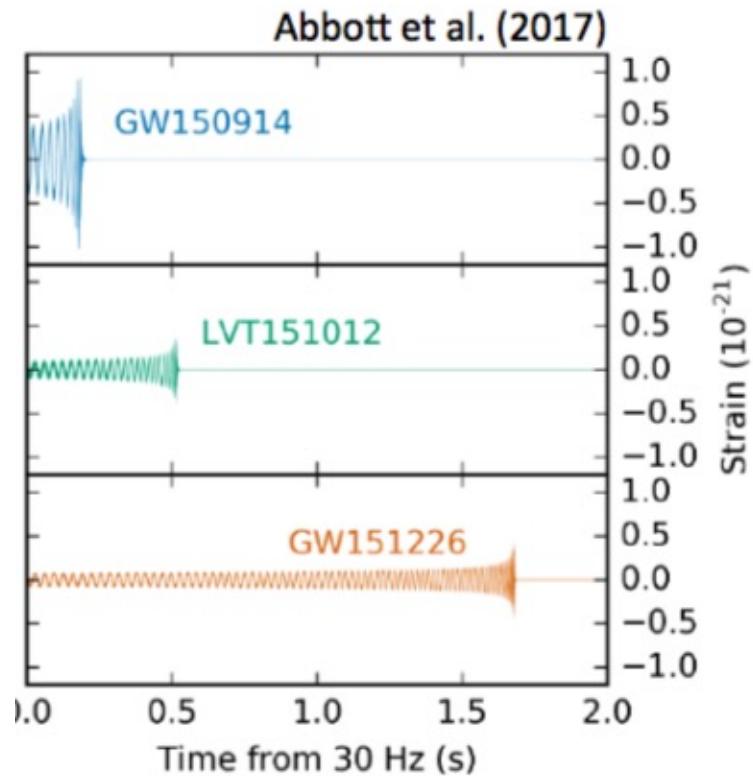
  - Confined, *inhomogeneous* chiral SSB ( still ongoing ... )
    - Skyrme crystals, ...
    - Chiral density wave (1-D periodic structure) [ Carignano-Nickel-Bubbala ]
    - Quarkyonic Chiral Spirals
    - Interweaving Chiral Spirals
- [ TK-Hidaka-Fukushima  
-McLerran-Pisarski-Tsvelik 09-11 ]
- 
- Reinterpretation of *Hadron-Quark Continuity*
    - Original proposal : Schafer-Wilczek
    - CSC in quarkyonic matter & NS context [ Fukushima-TK '15 ]

# *GW150914 : the discovery of GWs*

**BH-BH mergers**

→ larger amplitudes

**The wave patterns:  
consistent with  
the general relativity  
in strong field regimes**



# Frequency spectrum

*GR simulations*, Hotokezaka et al. 2016

