CHARACTERIZING HOT AND DENSE MATTER USING TSALLIS STATISTICS

TRAMBAK BHATTACHARYYA

UNIVERSITY OF CAPE TOWN

Boltzmann-Gibbs (BG) theory is not universal because it only applies to systems in states of thermodynamic equilibrium.

Many anomalous natural, and social systems exist for which *BG* statistical concepts appear to be inapplicable

Some of them can be handled using the techniques of Statistical Mechanics by introducing a more general entropy called the Tsallis (aka non extensive) entropy.

Non additive entropy: $S_q(\mathrm{A}+\mathrm{B}) = S_q(\mathrm{A})+S_q(\mathrm{B})+(1\text{-}q)\, S_q(\mathrm{A})S_q(\mathrm{B})$

- Non extensive when there is no/local correlation among the elements.

- Can be extensive when there exists (non-local) correlation for a special value of $q \neq 1$

And the $q \to 1$ (Boltzmann – Gibbs) Limit

Additive entropy: $S_{\mathrm{BG}}(\mathrm{A}+\mathrm{B}) = S_{BG}(\mathrm{A}) + S_{BG}(\mathrm{B})$.

- Can be extensive when there is no/local correlation among the elements.
- Can be non extensive when there exists (non-local) correlation

So, the more appropriate term would be ‘Tsallis non additive’ entropy in stead of ‘Tsallis non extensive entropy’.

Extremization of the Tsallis non additive entropy gives rise to the q-exponential probability distribution which is a generalization of the Boltzmann-Gibbs exponential distribution.

And we can derive the corresponding Tsallis q-thermodynamics using the following form of the Tsallis distribution function:

$$f = [1 + \beta(q-1)(E-\mu)]^{1/(q-1)}$$

C. Tsallis, J. Stat. Phys ***52****, 479(1988)*

For systems with spatio-temporal fluctuation in temperature:

$$q - 1 = \frac{\langle {\beta_{BG}}^2 \rangle - \langle \beta_{BG} \rangle^2}{\langle \beta_{BG} \rangle^2}; \qquad \beta = \langle \beta_{BG} \rangle$$

G. Wilk and Z. Włodarczyk, Phys. Rev. Lett. ***84****, 2770(2000)*

When temperature fluctuation dies down $f \rightarrow \mathrm{f}_{\mathrm{Boltzmann}}$

$$S = -gV \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left[f^q \ln_q f - f \right], \qquad T = \left. \frac{\partial \epsilon}{\partial s} \right|_n ,$$

$$N = gV \int \frac{\mathrm{d}^3p}{(2\pi)^3} f^q, \qquad \mu = \left. \frac{\partial \epsilon}{\partial n} \right|_s ,$$

$$\epsilon = g \int \frac{\mathrm{d}^3p}{(2\pi)^3} E\, f^q, \qquad n = \left. \frac{\partial P}{\partial \mu} \right|_T ,$$

$$P = g \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{p^2}{3E} f^q, \qquad s = \left. \frac{\partial P}{\partial T} \right|_\mu .$$

Thermodynamically consistent: J. Cleymans & D. Worku, J. Phys. G 39, 025006(2012)

Analytical Calculations of Tsallis Thermodynamic Variables:

Massless	Massive
	•Taylor's Series approximation •Mellin-Barnes

Analytical Calculations of Tsallis pressure in d space time dimension: m = 0

$$P_d = \int_0^\infty \frac{d^{d-1}p}{(2\pi)^d} \cdot \frac{p}{\left(1+\delta q\frac{p-\mu}{T}\right)^{\frac{1+\delta q}{\delta q}}} \qquad ;\ \delta q = q-1$$

$$P = \frac{gT^4}{6\pi^2}\frac{1}{(1-\delta q)(\frac{1}{2}-\delta q)(\frac{1}{3}-\delta q)},$$

Convergence of the integration for p→ ∞ **requires:**

$d < \frac{1+\delta q}{\delta q} \Longrightarrow \delta q < \frac{1}{d-1}$ In 4 dimension $\delta q < \frac{1}{3}$ or $q < \frac{4}{3}$

Analytical Calculations of Tsallis Thermodynamic Variables: m ≠ 0

Pressure: Taylor's series [up to O(q-1)]

$$P^B + (q-1)P^1,$$

$$P^B = \frac{g e^{\frac{\mu}{T}} T^4 a^2 K_2(a)}{2\pi^2},$$

$$P^1 = \frac{g e^{\frac{\mu}{T}} T^4}{4\pi^2}\Big[a^4 K_2(a) + 3a^3 K_3(a) - 2a^3 b K_3(a) + a^2 b^2 K_2(a) + 2a^2 b K_2(a)\Big]$$

$$a = \frac{m}{T};\ b = \frac{\mu}{T}$$

K: Modified Bessel Functions

T Bhattacharyya, J Cleymans, A Khuntia, P Pareek, R Sahoo EPJA 30, ***52*** *no. 2 (2016)*

Pressure: m$\neq$ 0, $\mu \neq 0$ using the Mellin-Barnes representation

$$P = g\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^2}{3E}\,f^q$$

Angular integral,
redefine p → k ≡ p/m

$$P = \frac{gm^4}{6\pi^2}\int_0^\infty \mathrm{d}k\left[\frac{k^4}{\sqrt{1+k^2}} \times \frac{1}{\left[\underbrace{\{1-\delta q\,\tfrac{\mu}{T}\}}_{Y} + \{\underbrace{\delta q\,\tfrac{m}{T}}_{X}\sqrt{1+k^2}\}\right]^{\overbrace{\tfrac{1+\delta q}{\delta q}}^{\lambda}}}\right]$$

$$\frac{1}{(X+Y)^{\lambda}} = \int_{\epsilon-i\infty}^{\epsilon+i\infty} dz/(2i\pi)\left[\frac{\Gamma(-z)\Gamma(\lambda+z)}{\Gamma(\lambda)}\frac{Y^{z}}{X^{\lambda+z}}\right]$$

$\mathrm{Re}(\lambda) > 0$ and $\mathrm{Re}(\epsilon) \in (-\mathrm{Re}(\lambda), 0)$

$$P_U = \frac{gm^4}{16\pi^{\frac{3}{2}}}\left(\frac{T}{\delta q m}\right)^{\frac{1+\delta q}{\delta q}}\left[\frac{\Gamma\left(\frac{1-3\delta q}{2\delta q}\right)}{\Gamma\left(\frac{1+2\delta q}{2\delta q}\right)} \times {}_2F_1\left(\frac{1+\delta q}{2\delta q}, \frac{1-3\delta q}{2\delta q}, \frac{1}{2}; \left(\frac{\delta q\mu - T}{\delta q m}\right)^2\right)\right.$$
$$\left. + 2\left(\frac{\delta q\mu - T}{\delta q m}\right) \times \frac{\Gamma\left(\frac{1-2\delta q}{2\delta q}\right)}{\Gamma\left(\frac{1+\delta q}{2\delta q}\right)} \times {}_2F_1\left(\frac{1+2\delta q}{2\delta q}, \frac{1-2\delta q}{2\delta q}, \frac{3}{2}; \left(\frac{\delta q\mu - T}{\delta q m}\right)^2\right)\right]$$

$$\delta q > \frac{T}{m+\mu} \qquad \text{for } m \geq \mu$$

T Bhattacharyya, J Cleymans, S Mogliacci Phys. Rev. D ***94,*** *094026 (2016)*

Energy density, pressure are part of the energy-momentum tensor of the medium.

evolution of the medium is dictated by the hydrodynamic equation which involves the energy momentum tensor.

This equation may involve transport coefficients like shear and bulk viscosities as the inputs.

Also, there is evolution of the heavy particles (e.g. charm/bottom quarks produced very early in high energy collisions)

Heavy particle transport inside the medium of (coloured) light particles: test particle problem in plasma physics.

$$HQ(\vec{p}, E_p) + LQ/g(\vec{q}, E_q) \rightarrow \quad HQ(\vec{p}', E_{p'}) + LQ/g(\vec{q}', E_{q'})$$

Heavy quarks are clean probes to study the coloured Medium because: they are produced very early, witnesses the whole evolution and is not a part of the medium being studied.

Evolution of the heavy quark distribution inside Quark Gluon Plasma which can be created in high energy /highly dense environment of hadrons is dictated by the Boltzmann Transport Equation (BTE) and the relative change in the distribution can be related to the experimental observables

Boltzmann Transport equation

$$p^{\mu}\partial_{\mu}f_a = C[f_a]$$

f_a is a power law distribution for incoming probe particle

$$\mathrm{C}[f_{test}] \coloneqq \int \text{dynamics} \bigotimes \text{phase space} \, [\coloneqq \cdots (f_a' \, f_b' - f_a f_b) \ldots]$$

$$f_a' f_b' - f_a f_b = e^{\{\ln(f_a')+\ln(f_b')\}} - e^{\{\ln(f_a)+\ln(f_b)\}}$$

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Relativistic nonextensive thermodynamics

A. Lavagno

Dipartimento di Fisica, Politecnico di Torino and INFN, Sezione di Torino, C.so Duca Degli Abruzzi 24, I-10129 Torino, Italy

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Abstract

Starting from the basic prescriptions of the Tsallis' nonextensive thermostatistics, i.e., generalized entropy and normalized q-expectation values, we study the relativistic nonextensive thermodynamics and derive a Boltzmann transport equation that implies the validity of the H-theorem where a local nonextensive four-entropy density is considered. Macroscopic thermodynamic functions and the equation of state for a perfect gas are derived at the equilibrium.

PACS: 05.20.Dd; 05.70.Ln; 05.90.+m; 25.75.-q

q Boltzmann Transport equation

$$p^{\mu}\partial_{\mu}\tilde{f} = C_q[f]$$

$$\tilde{f} = f^{q}(x,p) = [1+\beta(x)(q-1)(E-\mu)]^{q/(q-1)}$$

$$C_q[f] \coloneqq \int \text{dynamics} \bigotimes \text{phase space} \left[\coloneqq \cdots h_q(f_a' \, f_b') - h_q(f_a f_b) \ldots\right]$$

$$f_a' f_b' - f_a f_b \Rightarrow h_q(f_a' f_b') - h_q(f_a f_b) = e_q^{\{ln_q(f_a')+ln_q(f_b')\}} - e_q^{\{ln_q(f_a)+ln_q(f_b)\}}$$

	BTE	q-BTE
Evolving distribution	Evolution of power law test particle distribution	Evolution of Tsallis test particle distribution raised to the Tsallis parameter q
Collision term	Product of two single particle distribution functions $:= f_a\ f_b = e^{\{ln(f_a)+ln(f_b)\}}$	two particle distribution function which takes care of the correlation $:= h_q(f_a f_b) = e_q{}^{\{ln_q(f_a)+ln_q(f_b)\}}$

With the definitions of the q exponential

$$e_q(\mathrm{x}) = (1 - \delta q\ x)^{-\frac{1}{\delta q}}$$

and the q logarithm function

$$ln_q(\mathrm{x}) = \frac{1 - x^{-\delta q}}{\delta q}$$

We can show

$$h_q(f_a f_b) = \left\{1 - \delta q\left(\frac{1 - f_a^{-\delta q}}{\delta q} + \frac{1 - f_b^{-\delta q}}{\delta q}\right)\right\}^{-\frac{1}{\delta q}}$$

$$= f_a f_b + \delta q\ f_a f_b \log\left(f_a\right) \log\left(f_b\right) + O\left(\delta q^2\right)$$

Space averaging: BTE

We intend to average over the space dependence which comes through the probe particle distribution and will work with the momentum distribution only. We assume the medium to be homogeneous and the net external force on the system to be zero.

For BTE, the averaging is trivial because the space dependent part (the incoming particle distribution) factorizes out

Space averaging: q BTE

For q BTE, space averaging is not so trivial ritual as the space dependent part, because of the correlation, is now intertwined with the homogeneous medium distribution

For details see: *T Bhattacharyya and J Cleymans arXiv: 1707.08425*

	BTE	q-BTE
Space averaging	Space dependent part factorizes out	No factorization because of correlation
Small momentum transfer limit	Expansion for small momentum transfer $\longrightarrow$ linear Fokker-Planck equation Inputs: $\longrightarrow$ linear Fokker-Planck drag/diffusion	Expansion for small momentum transfer $\longrightarrow$ non linear Fokker-Planck equation Inputs: $\longrightarrow$ non linear Fokker-Planck drag/diffusion

So, we take the small momentum transfer limit and compare with the form of the non linear Fokker Planck equation given below:

$$\frac{\partial f_p}{\partial t} = -\frac{\partial[A_{i,q} f_p]}{\partial p_i} + \frac{\partial}{\partial p_i}\frac{\partial[B_{ij,q} f_p^{1-\delta q}]}{\partial p_j}$$

*G Wolschin, Phys. Lett. B **569**, 67(2003), A. Lavagno Braz. Jour. of Phys.* ***35**, 516 (2005)*

Non linear Fokker Planck transport coefficients are given by:

$$A_{i,q} = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{q}'}{(2\pi)^3} \frac{d^3\mathbf{p}'}{(2\pi)^3} |\overline{M}|^2 (2\pi)^4 \delta^4(p + q - p^{'} - q^{'}) \;\; \mathcal{R}^1_{\mathbf{p},\mathbf{q}} \;\; (\mathbf{p} - \mathbf{p}')_i$$

$$B_{ij,q} = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{q}'}{(2\pi)^3} \frac{d^3\mathbf{p}'}{(2\pi)^3} |\overline{M}|^2 (2\pi)^4 \delta^4(p + q - p^{'} - q^{'}) \;\; \mathcal{R}^2_{\mathbf{p},\mathbf{q}} \;\; \frac{1}{2} (\mathbf{p}' - \mathbf{p})_i (\mathbf{p}' - \mathbf{p})_j$$

$\mathcal{R}_{p,q}^{1}$ and $\mathcal{R}_{p,q}^{2}$ are given by:

$$\mathcal{R}^1_{\mathbf{p},\mathbf{q}} = \left\{-\frac{2\pi r_0^3 g_{3,\mathbf{p},\mathbf{q}}^{-c+1}}{3a^3}\left[\{\psi^{(0)}(c-1)+\mathcal{G}_{\mathbf{p},\mathbf{q}}\}\{6\psi^{(0)}(c-1)\mathcal{G}_{\mathbf{p},\mathbf{q}}+\pi^2\}+2\mathcal{G}_{\mathbf{p},\mathbf{q}}\{3\psi^{(1)}(c-1)\mathcal{G}_{\mathbf{p},\mathbf{q}}+\mathcal{G}^2_{\mathbf{p},\mathbf{q}}\}+\Psi_{c-1}\right]\right.$$
$$\left.+\frac{8\pi g_{2,\mathbf{p}} r_0^3 g_{3,\mathbf{p},\mathbf{q}}^{-c}}{a^3}\frac{\Gamma(c)}{\Gamma(c-1)}\ {}_5F_4\left(1,1,1,1,c;2,2,2,2;-\frac{g_{2,\mathbf{p}}}{g_{3,\mathbf{p},\mathbf{q}}}\right)\right\} \quad (18)$$

where $\mathcal{G}_{\mathbf{p},\ \mathbf{q}} = -\log(g_{3,\mathbf{p},\ \mathbf{q}}) + \log(g_{2,\mathbf{p}}) + \gamma$ and

$$\Psi_{c-1} = \psi^{(0)}(c-1)^3 + 6\psi^{(0)}(c-1)\psi^{(1)}(c-1) + 2\psi^{(2)}(c-1) + 4\zeta(3) \quad (19)$$

$$g_{1,\mathbf{p}} = 1+\frac{E_{\mathbf{p}}\delta q}{T_p},\quad g_{2,\mathbf{p}} = \frac{\delta q}{T_p}\mathrm{Exp}(-a)E_{\mathbf{p}},\quad g_{3,\mathbf{p},\mathbf{q}} = 1+\frac{E_{\mathbf{p}}\delta q}{T_p}+\frac{E_{\mathbf{q}}\delta q}{T_q};$$

$\zeta(3) = 1.20205....$ is the zeta function, $\gamma = 0.57721...$ is the Euler-Mascheroni constant, $\psi^{(i)}s$ are the poly gamma functions and ${}_5F_4$ is the gauss hypergeometric function.

$$\mathcal{R}_{p,q}^{2} = f_p^{\delta q}\mathcal{R}_{p,q}^{1}$$

And arise due to the space averaging using the following temperature profile:

$$T(\mathbf{x};t) = \frac{T_p(t)}{\left[1 + \mathrm{Exp}\left\{a(t)\left(\frac{\sqrt{x^2+y^2+z^2}}{r_0(t)} - 1\right)\right\}\right]}$$

$T_p = 290$ MeV, $a = 5.99$ and $r_0 = 7.96$ fm

In the limit q→ 1, $\{\mathcal{R}^1_{p,q}, \mathcal{R}^2_{p,q}\} \rightarrow$ f(q) (medium particle distribution) and we get back the linear Fokker Planck transport coefficients

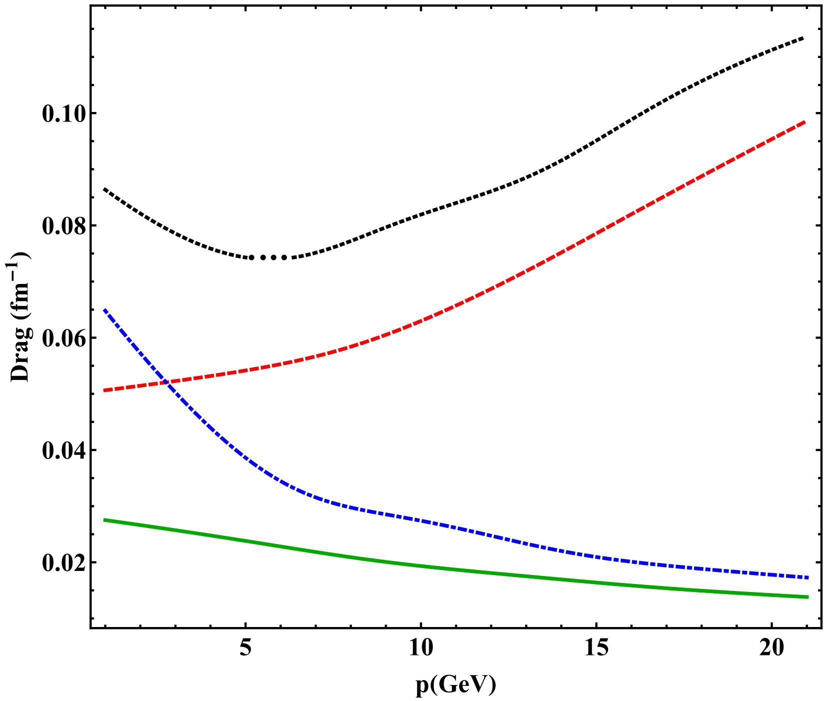

FIG. 3. Variation of the extensive and non extensive drag coefficients with momentum of the incoming heavy quark. The dotted (black) line represents the non-extensive drag for the charm quark and the dashed (red) line represents that for the bottom quark. The dot-dashed (blue) line and the solid (green) lines are the extensive drag coefficients for the charm and the bottom quark respectively.

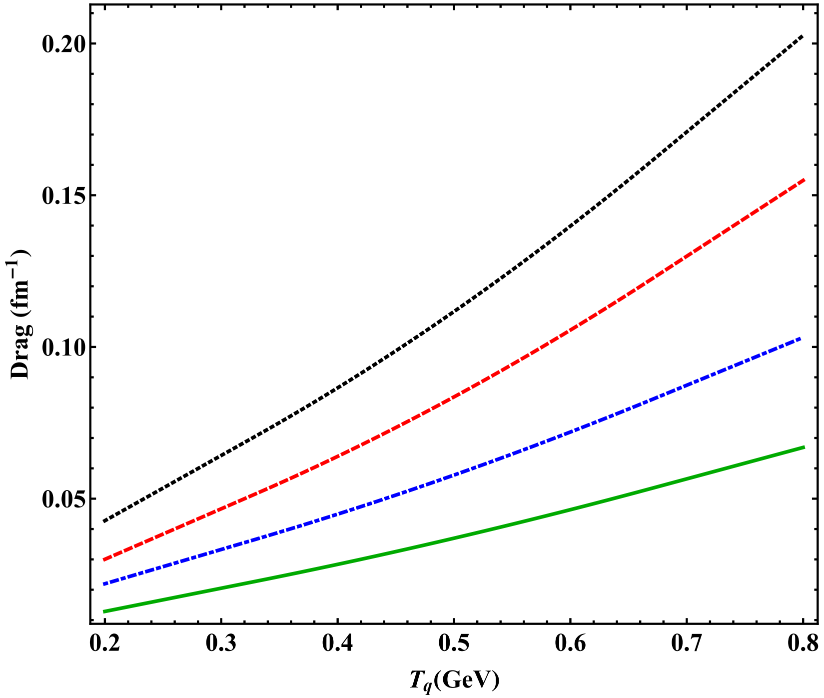

FIG. 4. Variation of the extensive and non extensive drag coefficients with temperature of the medium. The dotted (black) line represents the non-extensive drag for the charm quark and the dashed (red) line represents that for the bottom quark. The dot-dashed (blue) line and the solid (green) lines are the extensive drag coefficients for the charm and the bottom quark respectively.

T Bhattacharyya and Jean Cleymans arXiv: 1707.08425

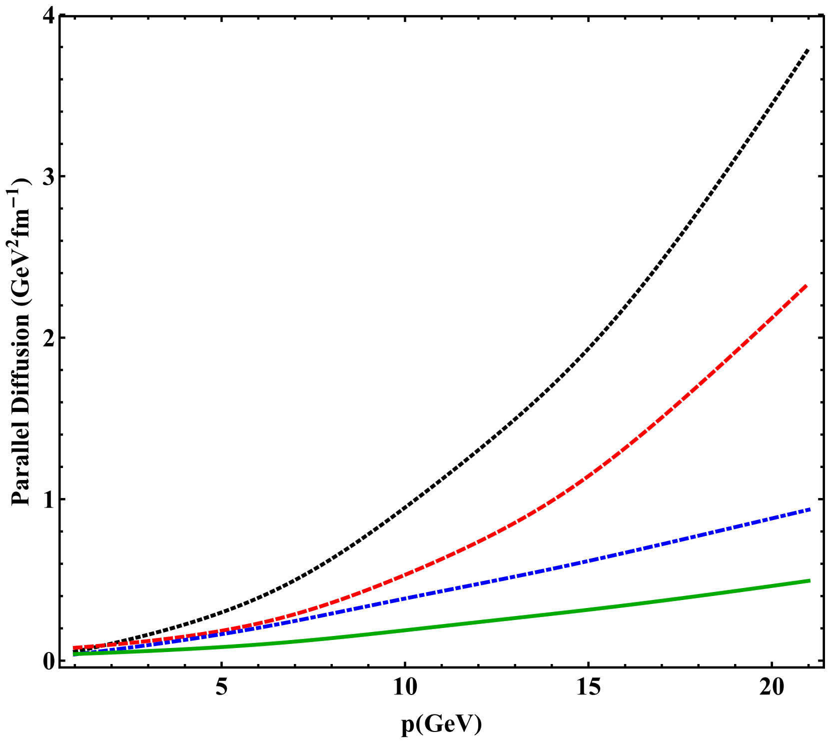

FIG. 5. Variation of the extensive and non extensive parallel diffusion coefficient with momentum of the incoming heavy quark. The dotted (black) line represents the non-extensive for the charm quark and the dashed (red) line represents that for the bottom quark. The dot-dashed (blue) line and the solid (green) lines are the extensive drag coefficients for the charm and the bottom quark respectively.

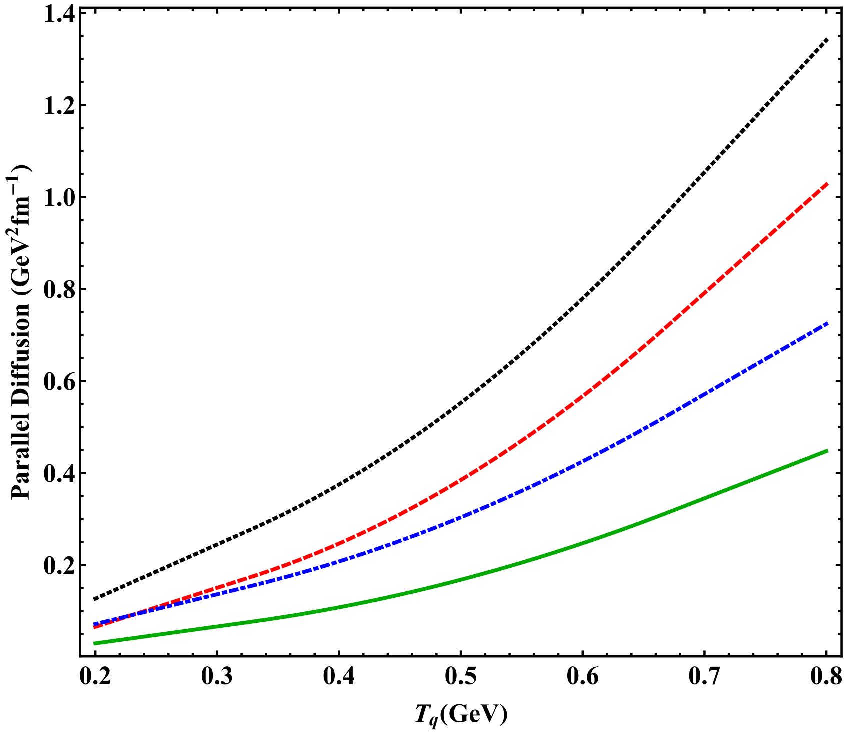

FIG. 6. Variation of the extensive and non extensive parallel diffusion coefficients with temperature of the medium. The dotted (black) line represents the non-extensive drag for the charm quark and the dashed (red) line represents that for the bottom quark. The dot-dashed (blue) line and the solid (green) lines are the extensive drag coefficients for the charm and the bottom quark respectively.

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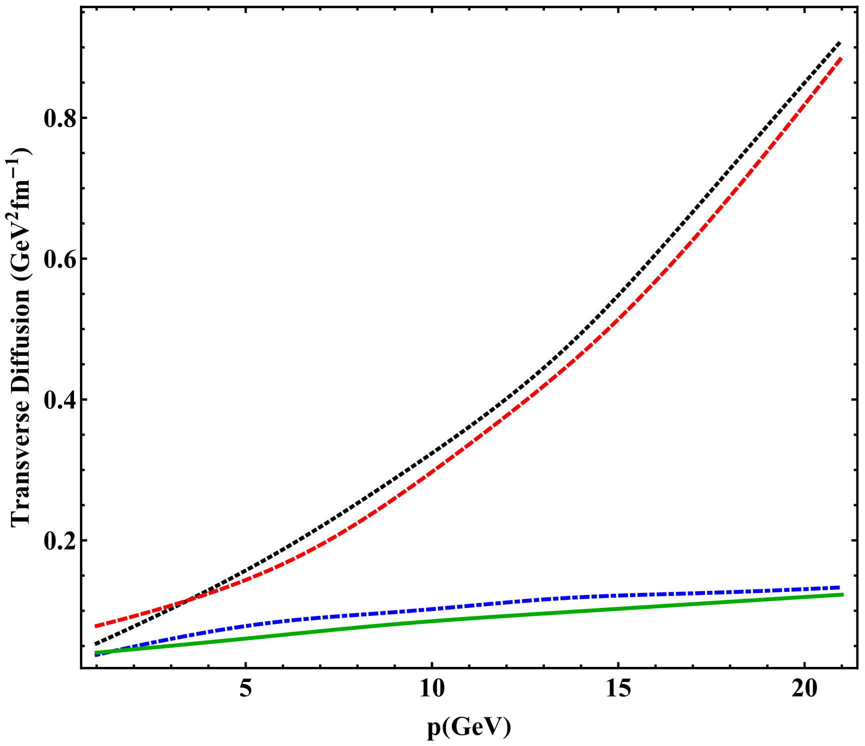

FIG. 7. Variation of the extensive and non extensive transverse diffusion coefficient with the momentum of the incoming heavy quark. The dotted (black) line represents the non-extensive transverse diffusion for the charm quark and the dashed (red) line represents that for the bottom quark. The dot-dashed (blue) line and the solid (green) lines are the extensive transverse diffusion coefficients for the charm and the bottom quark respectively.

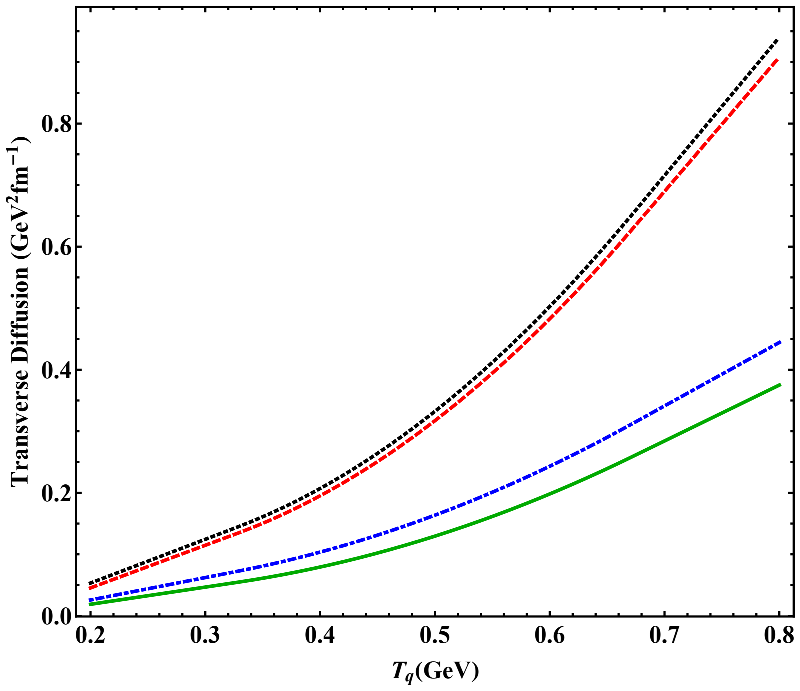

FIG. 8. Variation of the extensive and non extensive transverse diffusion coefficients with the temperature of the medium. The dotted (black) line represents the non-extensive transverse diffusion for the charm quark and the dashed (red) line represents that for the bottom quark. The dot-dashed (blue) line and the solid (green) lines are the extensive transverse diffusion coefficients for the charm and the bottom quark respectively.

T Bhattacharyya and Jean Cleymans arXiv: 1707.08425

Possible extension to dense matter

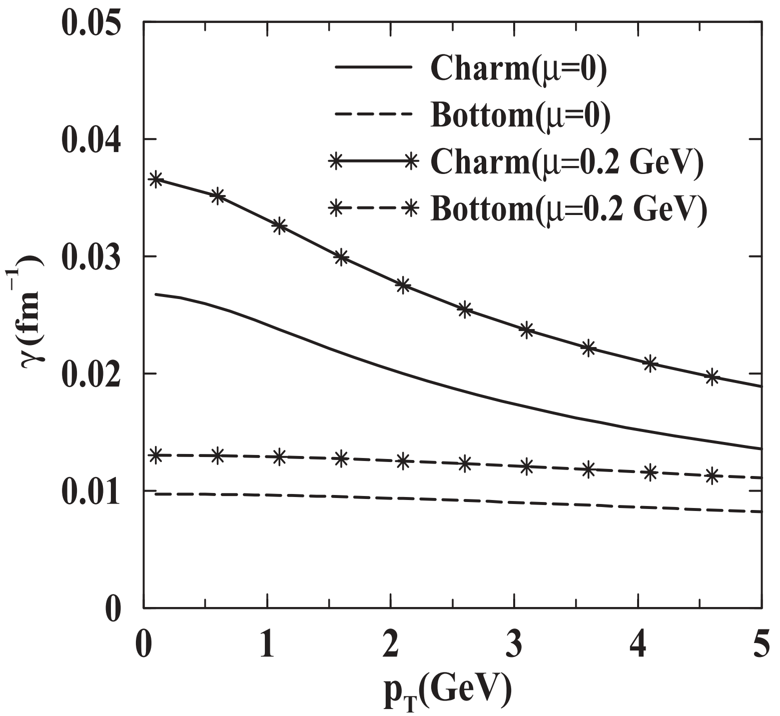

FIG. 5. Variation of drag coefficient with p_T for $T = 200\,\text{MeV}$.

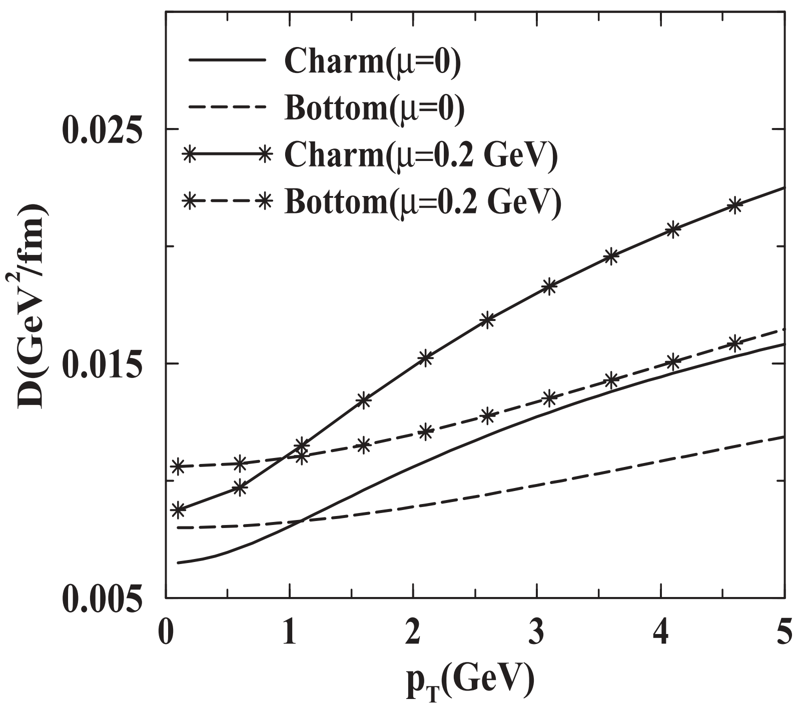

FIG. 7. Variation of diffusion coefficient with p_T for $T = 200\,\text{MeV}$.

PHYSICAL REVIEW C **80**, 054916 (2009)

Probing quark gluon plasma properties by heavy flavors

Santosh K. Das, Jan-e Alam,* and Payal Mohanty
Variable Energy Cyclotron Centre, 1/AF, Bidhan Nagar, Kolkata-700064, India
(Received 3 September 2009; published 30 November 2009)

The Fokker-Planck (FP) equation has been solved to study the interaction of nonequilibrated heavy quarks with the quark gluon plasma expected to be formed in heavy ion collisions at Relativistic Heavy Ion Collider energies. Solutions of the FP equation have been convoluted with the relevant fragmentation functions to obtain the D and B meson spectra. Results are compared with experimental data measured by the STAR Collaboration. It is found that the present experimental data cannot distinguish p_T spectra obtained from the equilibrium versus the nonequilibrium charm distributions. Data at lower p_T may play a crucial role in making the distinction between the two. The nuclear suppression factor R_{AA} for nonphotonic single-electron spectra resulting from semileptonic decays of hadrons containing heavy flavors has been evaluated using the present formalism. It is observed that the experimental data on the nuclear suppression factor of nonphotonic electrons can be reproduced within this formalism by enhancing the perturbative QCD cross sections by a factor of 2, provided that the expansion of the bulk matter is governed by the velocity of sound $c_s \sim 1/\sqrt{4}$. The ideal-gas equation of state fails to reproduce the data even with enhancement of the perturbative QCD cross sections by a factor of 2.

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Summary and conclusion

Broadly, with the help of Tsallis Statistics we discussed about the inputs to study the evolution of a medium and that of a probe particle passing through that medium when correlation is present.

Analytical calculation of the thermodynamic variables of hot and dense 'ideal' massive and massless Tsallis gas

Computing drag/diffusion coefficients of heavy quarks correlated with the medium particles

Drag and diffusion are substantially modified in presence of correlation

In dense system, we expect increase in the transport coefficients

BACKUP SLIDES

Systems (temp T_1) connected to a finite heat bath (temp T, entropy S, heat capacity C):

$$q - 1 = \frac{1}{C};\ T_1 = Te^{-\frac{S}{C}}$$

T. S. Biró, G. G. Barnaföldi, and P. Ván Eur. Phys. J. A ***49****, 110 (2013)*

Theoretical and Mathematical Physics, **174**(3): 386–405 (2013)

STATISTICAL FIELD THEORY OF A NONADDITIVE SYSTEM

© **A. I. Olemskoi,* O. V. Yushchenko,† and A. Yu. Badalyan†**

Following are Tsallis thermodynamic variables for m = 0, μ ≠ 0

$$P = \frac{gT^4}{6\pi^2}\frac{\left(1-\delta q\,\frac{\mu}{T}\right)^{\frac{3\delta q-1}{\delta q}}}{(1-\delta q)(\frac{1}{2}-\delta q)(\frac{1}{3}-\delta q)},$$

$$\epsilon = \frac{gT^4}{2\pi^2}\frac{\left(1-\delta q\,\frac{\mu}{T}\right)^{\frac{3\delta q-1}{\delta q}}}{(1-\delta q)(\frac{1}{2}-\delta q)(\frac{1}{3}-\delta q)},$$

$$s = \frac{gT^3}{6\pi^2}\frac{\left(4-\frac{\mu}{T}-\delta q\,\frac{\mu}{T}\right)\left(1-\delta q\,\frac{\mu}{T}\right)^{\frac{2\delta q-1}{\delta q}}}{(1-\delta q)(\frac{1}{2}-\delta q)(\frac{1}{3}-\delta q)},$$

$$n = \frac{gT^3}{2\pi^2}\frac{\left(1-\delta q\,\frac{\mu}{T}\right)^{\frac{2\delta q-1}{\delta q}}}{(1-\delta q)(\frac{1}{2}-\delta q)}.$$

*T Bhattacharyya, J Cleymans, S Mogliacci Phys. Rev. D **94,** 094026 (2016)*

Kondo cloud of single heavy quark in cold and dense matter

Shigehiro Yasui[1,*]

[1] *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

The Kondo effect is a universal phenomena observed in a variety of fermion systems containing a heavy impurity particle whose interaction is governed by the non-Abelian interaction. At extremely high density, I study the Kondo effect by color exchange in quark matter containing a single heavy (charm or bottom) quark as an impurity particle. To obtain the ground state with the Kondo effect, I introduce the condensate mixing the light quark and the heavy quark (Kondo cloud) in the mean-field approximation. I estimate the energy gain by formation of the Kondo cloud, and present that the Kondo cloud exhibits the resonant structure. I also evaluate the scattering cross section for the light quark and the heavy quark, and discuss its effect to the finite size quark matter.

PACS numbers: 12.39.Hg,21.65.Qr,12.38.Mh,72.15.Qm

Introduction.— Heavy (charm or bottom) quarks play the important role for studying the properties of nuclear matter and quark matter, whose dynamics is governed by Quantum Chromodynamics (QCD). They are good probes, because (i) due to their heavy masses [1], the heavy quark dynamics would be less affected by light quarks, and (ii) the heavy-quark symmetry for heavy flavor and spin makes the dynamics simple and provides a systematic understanding of spectroscopy and reaction [2, 3]. However, the impurity particle injected as a "probe" sometimes happens to cause a drastic changes of the medium properties. The Kondo effect is one of the well-known examples, as a few number of impurity particles can change the thermodynamic and transportation properties of the medium [4–6]. In this article, I discuss the Kondo effect in quark matter at low temperature and high density, called the QCD Kondo effect [7–10]. The QCD Kondo effect will be useful to study of high density matter, because heavy quarks can be produced by initial gluon dynamics in relativistic heavy ion collisions in accelerator facilities such as GSI-FAIR [11] and J-PARC and/or by high energy neutrino reactions in interior cores of neutron stars [10].

Historically, the Kondo effect was discovered to explain the logarithmic enhancement of electric resistance of metals containing a few magnetic (spin) impurities at low temperature [12]. Kondo found that the spin-exchange (non-Abelian) interaction between the conducting electrons and the spin impurities strongly enhances the ef-

10] as heavy impurity particles, although their energy scales are different from the electron systems in laboratory [22]. In the strong interaction, in fact, several types of non-Abelian interaction are present: the spin exchange with SU(2) symmetry (e.g. for $\bar{D}_s$, $\bar{D}_s^*$ mesons) and the isospin exchange with SU(2) symmetry (e.g. for a $\bar{D}$ meson) in nuclear matter, and the color exchange with SU(3) symmetry (e.g. for a c quark) in quark matter. The Fermi surface and the loop effect exists naturally in nuclear/quark matter at low temperature. Hence, the condition for realizing the Kondo effect is satisfied.

The Kondo effect in nuclear/quark matter provides us with basic knowledge for heavy hadron/quark dynamics: (i) heavy-hadron–nucleon (heavy-quark–light-quark) interaction, (ii) modification of impurity properties by medium and (iii) change of nuclear/quark matter by impurity effect [23].

In the present study, I discuss the QCD Kondo effect in quark matter. So far, there have been perturbative analyses [7–9], and recently a non-perturbative study has been performed by the mean-field approximation for the ground state [10] [24]. The mean-field approach is simple, but it provides us with understanding the essential properties of the Kondo effect [25–30]. In this study, I consider the situation that there is a single heavy quark in quark matter, and apply the mean-field approach to study the ground state.

In the mean-field approach, I define the Kondo cloud by the condensate with light quark ψ and the heavy quark

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Diffusion of charmed quarks in the quark-gluon plasma

Benjamin Svetitsky*

Department of Nuclear Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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We calculate the classical drag and diffusion coefficients for a charmed quark propagating in the quark-gluon plasma. Both coefficients turn out rather large, so that (1) a charmed quark created when the plasma is hot will be stopped before propagating 1 fm and (2) subsequent diffusion will be fast. The first effect should serve to *increase* the yield of J/ψ mesons in relativistic heavy-ion col-

$$R(\mathbf{p},t) \equiv \left[\frac{\partial f}{\partial t}\right]_{\text{collisions}}$$

$$= \int d^3k[w(\mathbf{p}+\mathbf{k},\mathbf{k})f(\mathbf{p}+\mathbf{k}) - w(\mathbf{p},\mathbf{k})f(\mathbf{p})] \ .$$

$$w(\mathbf{p}+\mathbf{k},\mathbf{k})f(\mathbf{p}+\mathbf{k}) \approx w(\mathbf{p},\mathbf{k})f(\mathbf{p}) + \mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}}(wf)$$

$$+ \tfrac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(wf) \ ,$$

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Derivation of nonlinear Fokker-Planck equations by means of approximations to the master equation

Evaldo M. F. Curado[1,*] and Fernando D. Nobre[1,2,†]
[1]*Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro—RJ, Brazil*
[2]*Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, Campus Universitário, Caixa Postal 1641, 59072-970 Natal, Rio Grande do Norte, Brazil*
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Nonlinear Fokker-Planck equations (FPEs) are derived as approximations to the master equation, in cases of transitions among both discrete and continuous sets of states. The nonlinear effects, introduced through the transition probabilities, are argued to be relevant for many real phenomena within the class of anomalous-diffusion problems. The nonlinear FPEs obtained appear to be more general than some previously proposed (on a purely phenomenological basis) ones. In spite of this, the same kind of solution applies, i.e., it is shown that the time-dependent Tsallis's probability distribution is a solution of both equations, obtained either from discrete or continuous sets of states, and that the corresponding stationary solution is, in the infinite-time limit, a stable solution.

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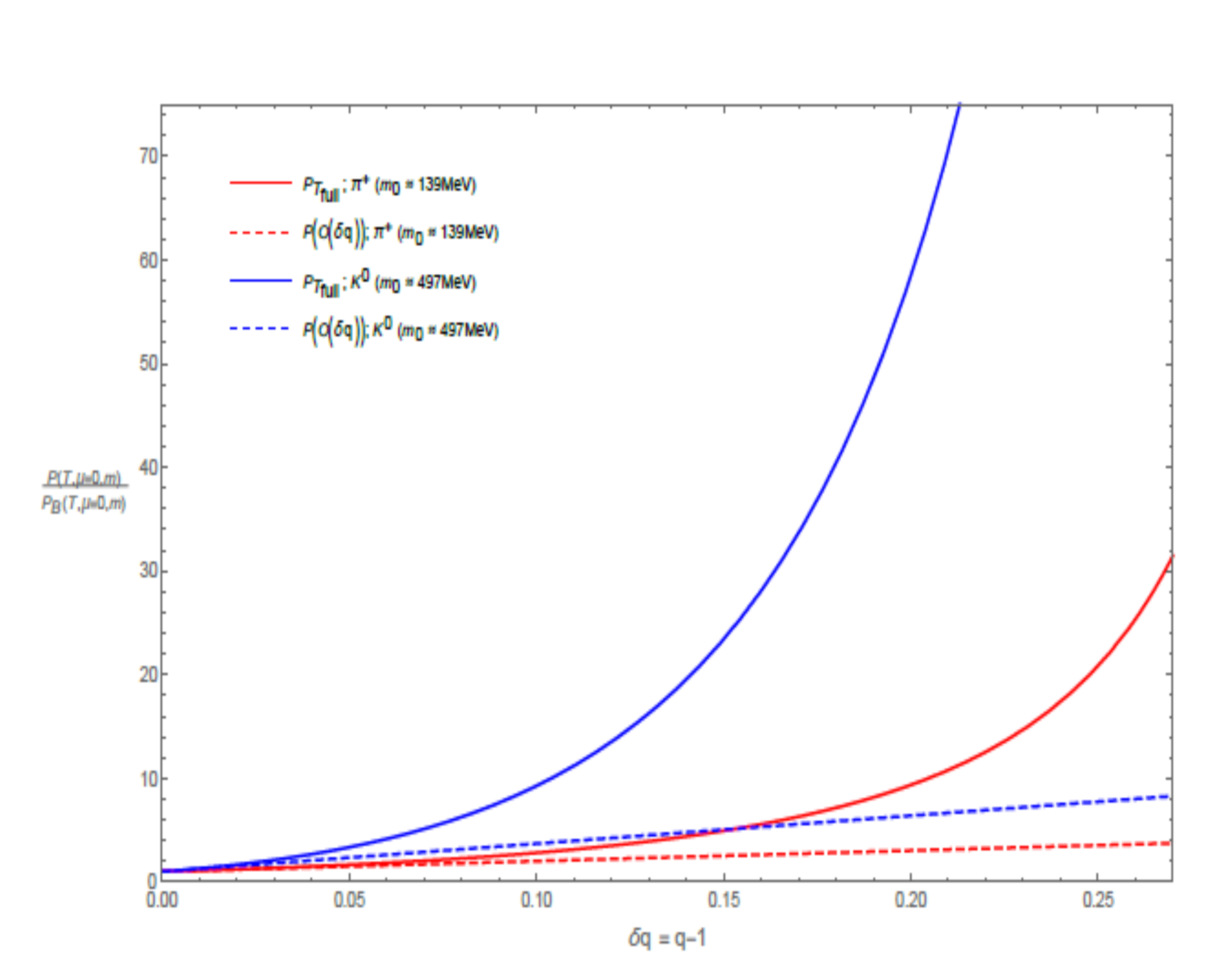

S Mogliacci Poster presentation (SAIP 2016)