CSQCD – VI, JINR – Dubna, 09/2017

From heavy-ion collisions to compact stars: EoS and relevance of the system size

Sylvain Mogliacci
University of Cape Town

Based on:

SM, WA Horowitz, I Kolbé / Preliminary / arXiv:17MM.XXXX

SM, JO Andersen, M Strickland, N Su, A Vuorinen / Published in JHEP / arXiv:1307.8098

Introduction

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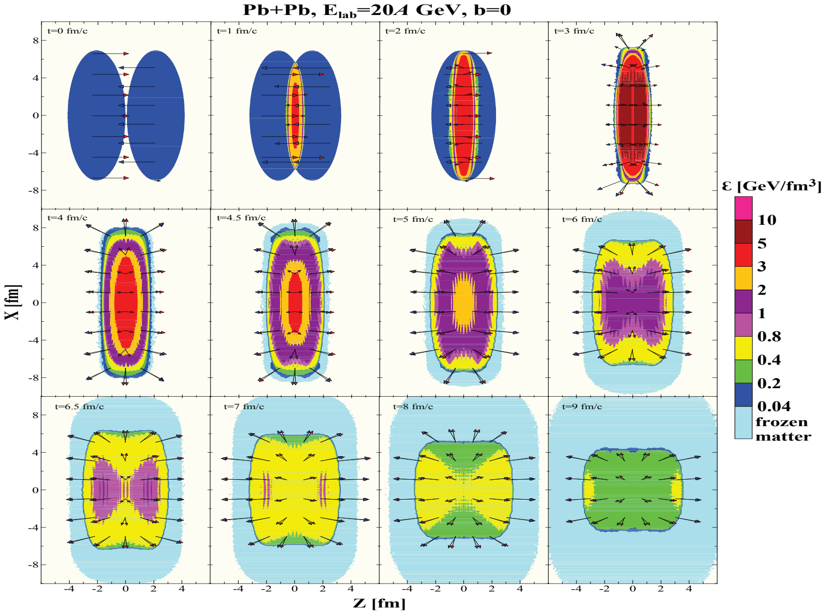
- On the deconfined side ($\neq$ system sizes $\sim$ possibly further $\neq$ EoS):
  Finite size correction for small systems $\rightleftharpoons$ Accounts better for DoF



From Yuri B. Ivanov's talk.

Phase Diagram... With a New Direction?

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HIC experiments and Proto-Neutron stars do not meet...
... if accounting for a size direction!

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But first, what about bare (not resummed) and conventional (infinite volume; no spatial compactification) perturbation theory...?

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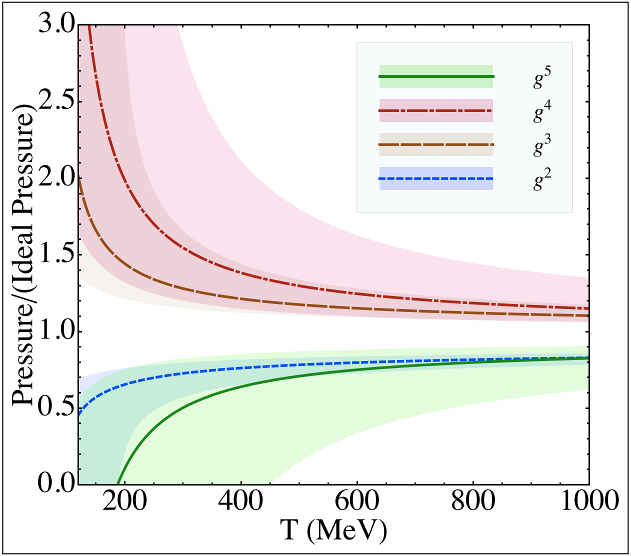

Finite density QCD Equation of State via resummed PT

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$\Rightarrow$ Gives the correct momentum scale contributions

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Note that $(\delta = 1) \Rightarrow (\mathcal{L}_{\rm HTLpt} = \mathcal{L}_{\rm QCD})$ hence adding $\mathcal{L}_{\rm HTL}$ shifts the ground state to an ideal gas of thermal (massive) quasiparticles and probes QCD!

LOW-ORDER CUMULANTS

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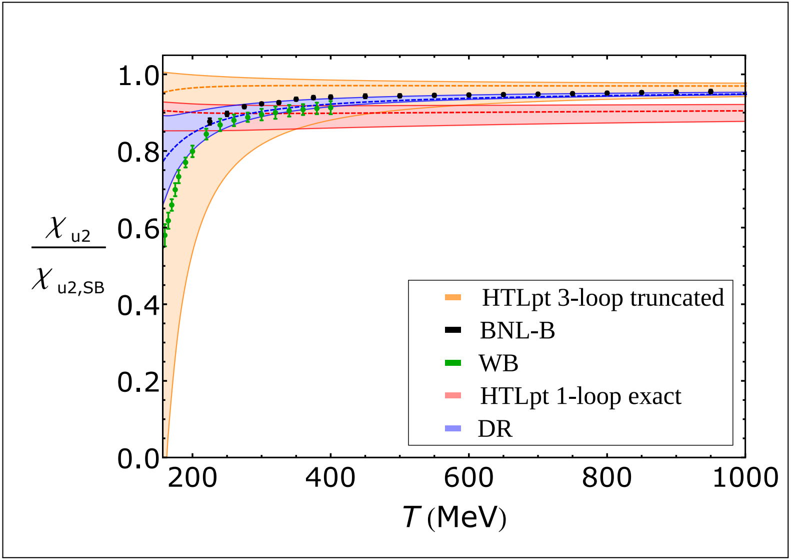

$$\chi_{B4} = \Big(\chi_{u4} + \chi_{d4} + \chi_{s4} + 4\,\chi_{u3\,d} + 4\,\chi_{u3\,s} + 4\,\chi_{d3\,u} + 4\,\chi_{d3\,s} + 4\,\chi_{s3\,u} + 4\,\chi_{s3\,d} + 6\,\chi_{u2\,d2} + 6\,\chi_{d2\,s2} + 6\,\chi_{u2\,s2} + 12\,\chi_{u2\,ds} + 12\,\chi_{d2\,us} + 12\,\chi_{s2\,ud}\Big)/81$$

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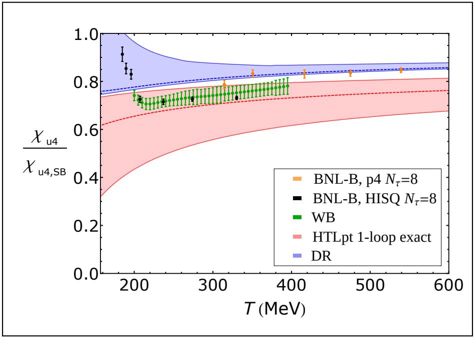

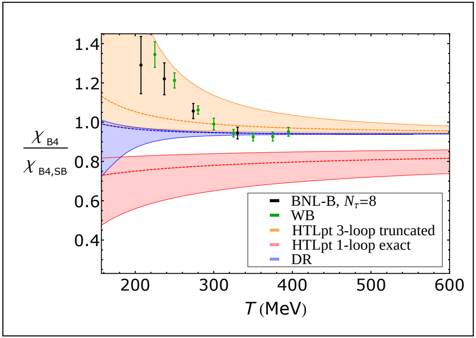

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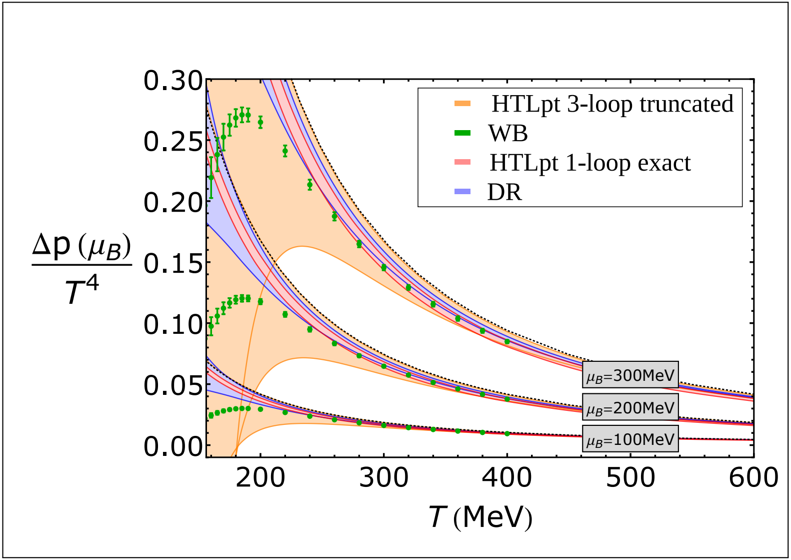
Which is nothing but a Taylor series containing all order cumulants:

$$\begin{aligned}\Delta p(T) &= \sum_{i,j,k,\ldots=1}^{\infty} \left. \frac{\partial^{i+j+k+\cdots} p(T, \{\mu_u, \mu_d, \mu_s, \ldots\})}{\partial \mu_u^i \partial \mu_d^j \partial \mu_s^k \ldots} \right|_{\{\mu_f\}=0} \times \frac{\mu_u^i \mu_d^j \mu_s^k \ldots}{i!\; j!\; k! \ldots} \\ &= \sum_{i,j,k,\ldots=1}^{\infty} \chi_{u_i\, d_j\, s_k\, \ldots} \times \frac{\mu_u^i \mu_d^j \mu_s^k \ldots}{i!\; j!\; k! \ldots}\end{aligned}$$

QCD Pressure at Finite μ_B



Finite size correction for QGP deconfined systems

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- Spatial compactification(s): Boundary ensuring a geometric confinement

$\Longrightarrow$ DoF exist only within the QGP region!

Now, finally, the preliminary results!

The life in between infinite parallel planes distant from L

S-B Corrections: (4-loop) Interaction Vs. Finite Size

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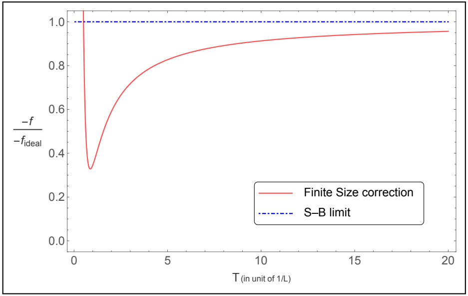

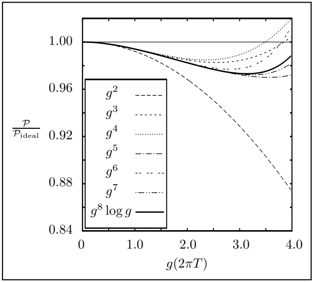

From Jens O. Andersen et al, JHEP 0908 (2009) 066.

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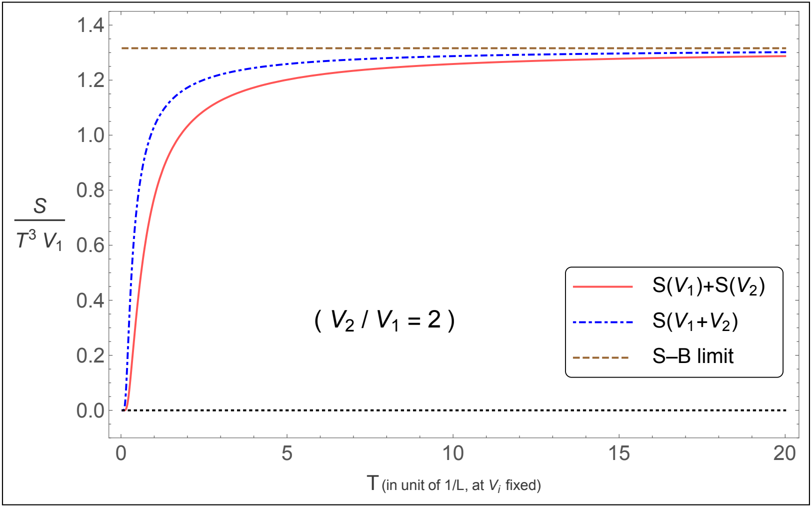

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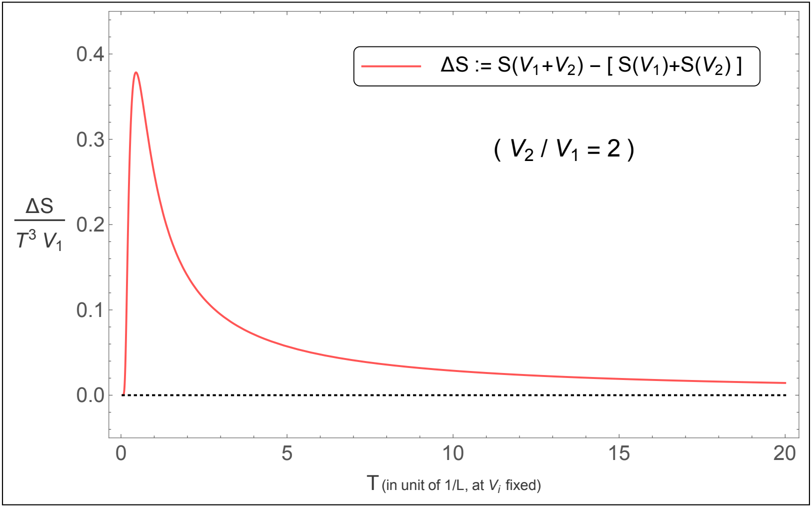

Isochoric Speed of Sound Vs. Temperature

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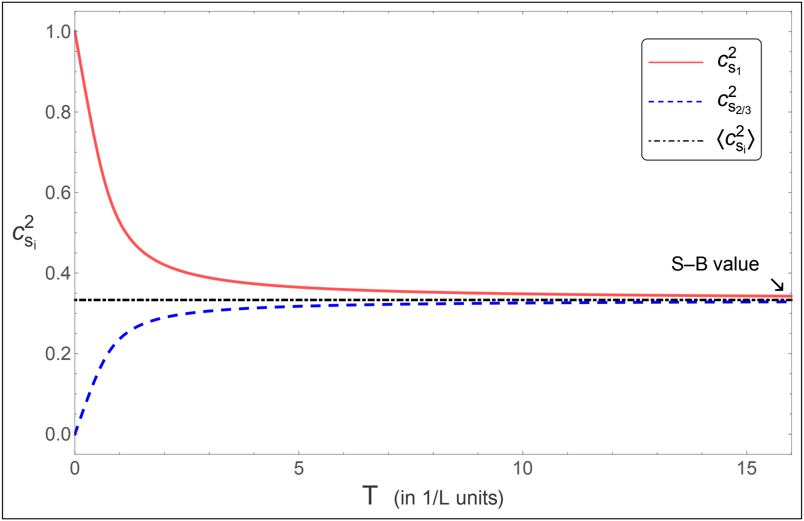

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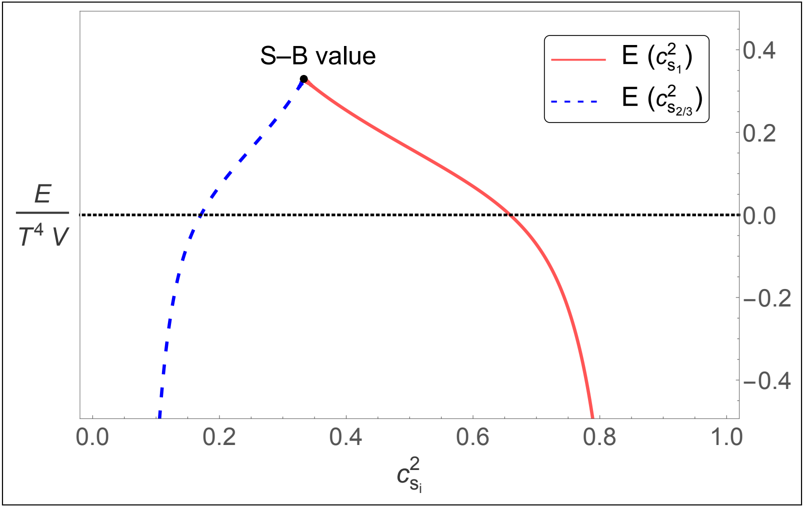

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THANKS A LOT FOR YOUR ATTENTION!

Backup slides

Backup: Some Notation

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At one-loop, contributions coming from, e.g., the quarks read:

$$p_{q_f}(T,\mu) = 2 \sum\!\!\!\!\!\!\int_{\{K\}} \log\left[A_S^2(i\widetilde{\omega}_n+\mu_f,k) - A_0^2(i\widetilde{\omega}_n+\mu_f,k)\right]$$

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With A_S and A_0:

$$A_0(i\widetilde{\omega}_n+\mu_f,k) \equiv i\widetilde{\omega}_n+\mu_f - \frac{m_{q_f}^2}{i\widetilde{\omega}_n+\mu_f}\,\widetilde{\mathcal{T}}_K(i\widetilde{\omega}_n+\mu_f,k)$$

$$A_S(i\widetilde{\omega}_n+\mu_f,k) \equiv k + \frac{m_{q_f}^2}{k}\left[1-\widetilde{\mathcal{T}}_K(i\widetilde{\omega}_n+\mu_f,k)\right]$$

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Where the HTL function $\widetilde{\mathcal{T}}_K$ can be represented as:

$$\widetilde{\mathcal{T}}_K(i\widetilde{\omega}_n+\mu_f,k) = {}_2F_1\left(\frac{1}{2},1;\frac{3}{2}-\epsilon;\frac{k^2}{(i\widetilde{\omega}_n+\mu_f)^2}\right)$$

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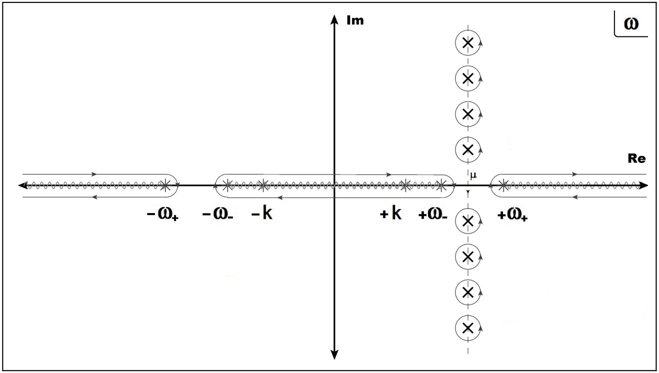

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- Running of the coupling: HTLpt/DR $\rightarrow$ 1/2-loop perturbative running

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- Running of the coupling: HTLpt/DR $\rightarrow$ 1/2-loop perturbative running
- m_{D}, m_{q_f} mass parameters: Mainly their weak coupling values at 1/2-loop
- QCD scale: Matching the running to lattice value at a reference scale

  $\Rightarrow$ Gives $\Lambda_{\overline{\mathrm{MS}}}^{\mathrm{HTLpt/DR}} = 176/283 \pm 30$ MeV to be "conservative"

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- Relevant to nowadays experiments at RHIC [Tannenbaum, arXiv:1201.5900], LHC [Müller, ARNPS **62** (2012)], FAIR [Heuser, NPA **904-905** (2013)] and NICA [Kekelidze et al., NPA **904-905** (2013)]:

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- Lattice data from:

  BNL-B [Bazavov et al., PRD **88** (2013) and PRL **111** (2013); Schmidt, JPCS **432** (2013) and NPA **904-905** (2013)]

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- Truncated 3-loop HTLpt results from:

  [Haque et al., PRD **89** (2014)]

Backup: HTLpt/DR Higher Order Cumulant

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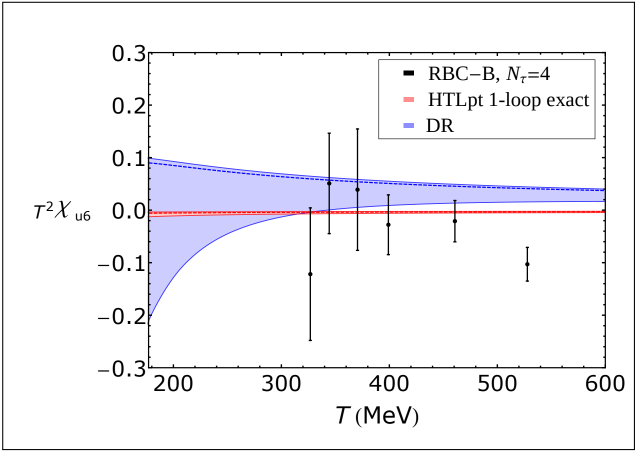

Backup: HTLpt/DR Ratios of Cumulants

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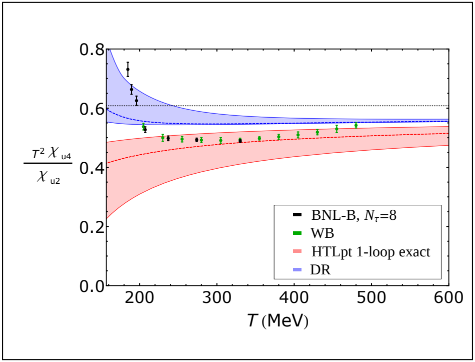
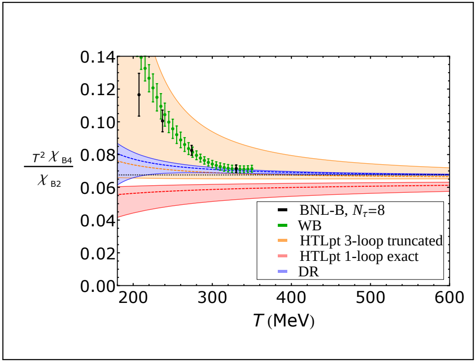
Recall that:

$$\chi_{B4} = \Big(\chi_{u4} + \chi_{d4} + \chi_{s4} + 4\,\chi_{u3\,d} + 4\,\chi_{u3\,s} + 4\,\chi_{d3\,u} + 4\,\chi_{d3\,s} + 4\,\chi_{s3\,u} + 4\,\chi_{s3\,d} + 6\,\chi_{u2\,d2} + 6\,\chi_{d2\,s2} + 6\,\chi_{u2\,s2} + 12\,\chi_{u2\,ds} + 12\,\chi_{d2\,us} + 12\,\chi_{s2\,ud}\Big)/81$$

$$\chi_{B2} = \Big(\chi_{u2} + \chi_{d2} + \chi_{s2} + 2\,\chi_{ud} + 2\,\chi_{ds} + 2\,\chi_{us}\Big)/9$$





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$$-\frac{T^{1+2\alpha}}{2\prod_{i=1}^{c}\left(L_i\right)}\times\left(\frac{\bar{\Lambda}^2 e^{\gamma_E}}{4\pi}\right)^{2-\frac{D}{2}}\times$$

$$\times\sum_{n\in\mathbb{Z}^1}\ \sum_{\boldsymbol{k}\in\mathbb{N}^c}\int\frac{\mathrm{d}^{D-1-c}\boldsymbol{p}}{(2\pi)^{D-1-c}}\left[\frac{1}{\left(\omega_n^2+\sum_{i=1}^{c}\omega_{k_i}^2+\boldsymbol{p}^2+m^2\right)^{\alpha}}\right]$$

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- Analytically continuing the above, say for $c=3$ and $m\neq0$, gives such a (out of many different possible) representation(s) for the free-energy:

$$\bar{f}^{(3)}_{\mathrm{R}}(T,L_1,L_2,L_3;m_{\mathrm{R}}) = -\frac{T}{8L_1L_2L_3}\times\log\left(1-e^{-\frac{m_{\mathrm{R}}}{T}}\right) - \frac{m_{\mathrm{R}}T}{8\pi L_1L_2}\times\sum'_{(s,s_1)\in\mathbb{Z}^2\backslash\{0\}}\left[\frac{K_1\left(\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_3)^2s_1^2}\right)}{\sqrt{s^2+(2TL_3)^2s_1^2}}\right]$$
$$-\frac{m_{\mathrm{R}}T}{8\pi L_3}\times\sum'_{(s,s_1)\in\mathbb{Z}^2\backslash\{0\}}\left[\frac{K_1\left(\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_2)^2s_1^2}\right)}{L_1\sqrt{s^2+(2TL_2)^2s_1^2}}+\frac{K_1\left(\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_1)^2s_1^2}\right)}{L_2\sqrt{s^2+(2TL_1)^2s_1^2}}\right]$$
$$+\frac{T^3}{8\pi L_1}\times\sum'_{(s,s_1,s_2)\in\mathbb{Z}^3\backslash\{0\}}\left[\frac{e^{-\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_2)^2s_1^2+(2TL_3)^2s_2^2}}\left(1+\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_2)^2s_1^2+(2TL_3)^2s_2^2}\right)}{\left(s^2+(2TL_2)^2s_1^2+(2TL_3)^2s_2^2\right)^{3/2}}\right]$$
$$+\frac{T^3}{8\pi L_2}\times\sum'_{(s,s_1,s_2)\in\mathbb{Z}^3\backslash\{0\}}\left[\frac{e^{-\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_3)^2s_2^2}}\left(1+\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_3)^2s_2^2}\right)}{\left(s^2+(2TL_1)^2s_1^2+(2TL_3)^2s_2^2\right)^{3/2}}\right]$$
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$$-\frac{m_{\mathrm{R}}^2T^2}{4\pi^2}\times\sum'_{(s,s_1,s_2,s_3)\in\mathbb{Z}^4\backslash\{0\}}\left[\frac{K_2\left(\frac{m_{\mathrm{R}}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2+(2TL_3)^2s_3^2}\right)}{\left(s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2+(2TL_3)^2s_3^2\right)}\right], \tag{66}$$