PARTITION FUNCTIONS AND AUTOMORPHIC FORMS

Sunday, January 28, 2018 - Friday, February 2, 2018

BLTP JINR

Scientific Program

LECTURE COURSES

/about four one hour lectures each /

Hermann Nicolai (AEI, Golm, Germany) 2 lectures

ON K(E10) AND ITS FERMIONIC REPRESENTATIONS

In these lectures I will review basic properties of K(E10), the `maximal compact' subalgebra of the maximally extended hyperbolic Kac-Moody algebra E10, and its applications to the fermionic sector of maximal supergravity and M theory.

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John Duncan (Emory University, USA) 4 lectures

MOONSHINE IN NUMBER THEORY AND GEOMETRY

Moonshine arose in the 1970s as a collection of coincidences connecting modular functions to the monster simple group, which was newly discovered at that time. The effort to elucidate these connections led to new algebraic structures (e.g. vertex algebras and generalized Kac--Moody algebras) which have since found applications in representation theory, number theory, geometry and string theory. In this century the theory has been further enriched, with the discovery of connections between K3 surfaces and certain distinguished groups in the early part of this decade, and connections between sporadic simple groups and the arithmetic of modular abelian varieties in recent months. In these lectures we will review monstrous moonshine, explain the number theoretic foundations of umbral moonshine, and describe recent results which reveal a role for sporadic groups in arithmetic geometry. We will also review vertex algebra, and the known applications of that theory to these topics. There are many interesting open questions in this area.

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Seok Kim (CTP, Seoul National University, Korea) 4 lectures

SUPERCONFORMAL INDICES AND INSTANTON PARTITION FUNCTIONS

We discuss recent advances in the instanton partition functions of quantum field theories with 8 SUSY in spacetime dimensions d=4,5,6 (compactified to 4 dimensions for d=5,6). Apart from their original use (originally explored by Nekrasov) to describe the Seiberg-Witten theory, these partition functions played crucial roles recently to better understand aspects of strongly-coupled conformal field theories in d=4,5,6, often via various curved space partition functions (including "superconformal indices" or other partition functions). We shall also review some of these developments.

Slides: Lecture 1.pdf   Lecture 2.pdf Lecture 3.pdf Lecture 4.pdf

Yasuhiko Yamada (Kobe University, Japan) 4 lectures

THEORY AND APPLICATIONS OF THE ELLIPTIC PAINLEVE EQUATION

I will give an introduction on the theory of discrete Painleve equations focusing mainly on the elliptic difference case. The elliptic Painleve equation is the master case in the Sakai's scheme and has the largest affine Weyl group symmetry (E8). I will present the geometric picture behind the system, its Lax form and the tau-functions. The current status on some recent developments will also be reviewed.

Pierre Vanhove (Saclay, France)   4 lectures

FEYNMAN INTEGRALS AND MODULAR FORMS

Scattering amplitudes are central tools for analyzing fundamental interactions in particle physics. They are the bridge between the theoretical models and the experimental data. It is therefore of extreme theoretical and experimental importance to be able to evaluate scattering amplitudes for numerous physical processes. Despite being used for more than 60 years, the mathematical nature of scattering amplitudes is still poorly understood. Scattering amplitudes are expressed in terms of Feynman integrals. Feynman integrals have rich mathematical structures: algebraic geometry, motivic cohomology and modular forms. These mathematical structures provide powerful ways of evaluating analytically and numerically physical processes.

In this series of lecture we will present the recent progresses in the evaluation of Feynman integral. We will review the modern unitarity method and fundamental properties amplitudes in quantum field theory have to satisfy. We will then explain how Feynman integral can be understood as motivic periods. In the context of the special family of sunset integral, we will discuss the appearance of modular forms and some surprising connection with mirror symmetry.

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ADDITIONAL SURVEY LECTURES

Vyacheslav Nikulin (MIAN|University of Liverpool**) 1 lecture**

LORENTZIAN KAC-MOODY ALGEBRAS, I

Valery Gritsenko (University of Lille 1|IUF|NRU HSE) 1 lecture

LORENTZIAN KAC-MOODY ALGEBRAS, II

In our talks I, II we describe a new large class of Lorentzian Kac-Moody algebras. For all ranks, we classify 2-reflective hyperbolic lattices S with the group of 2-reflections of finite volume and with a lattice Weyl vector. They define the corresponding hyperbolic Kac-Moody algebras of restricted arithmetic type which are graded by S. For most of them we construct Lorentzian Kac-Moody algebras which give their automorphic corrections: they are graded by S, have the same simple real roots, but their denominator identities are given by automorphic forms with 2-reflective divisors. We give exact constructions of these automorphic forms as Borcherds products and, in some cases, as additive Jacobi liftings. See arXiv:1602.08359 (see Proceedings of the London Math. Society) for some details. The part I is based on the theory of hyperbolic lattices. The part II is based on the theory of automorphic forms and Borcherds products.

From a non-formal point of view, in these two talks we show that some hyperbolic Kac-Moody algebras have a so-called automorphic correction, i.e. one can construct a generalised hyperbolic Kac-Moody algebra with the same system of real simple roots but with an automorphic form as the Kac-Weyl-Borcherds denominator functions. This means that the new lorentzian algebra has a huge group of hidden symmetries. The class of such automorphic algebras is finite. We fully classified the class of all such algebras in all hyperbolic ranks with the maximal hyperbolic 2-Weyl groups and a Weyl vector with a negative square.

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Du Pei (Caltech) 1 lecture

BPS SPECTRA AND INVARIANTS FOR THREE- AND FOUR-MANIFOLDS

Much like harmonics of musical instruments, spectra of quantum systems contain wealth of useful information. In this lecture, I will discuss how to obtain new invariants of three- and four-manifolds from BPS spectra of quantum field theories. We will see, in concrete examples, how several well-known invariants of three- and four-manifolds can be realized as partition functions of supersymmetric gauge theories and computed using localization technique.

Vyacheslav Spiridonov (BLTP JINR and NRU HSE) 1 lecture

INTRODUCTION TO THE THEORY OF ELLIPTIC HYPERGEOMETRIC INTEGRALS

Superconformal indices of 4d supersymmetric field theories are described by elliptic hypergeometric integrals. I will describe the most important mathematical facts about this class of special functions and present some methods of proving identities for them supporting the Seiberg duality conjectures.

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TALKS BY PARTICIPANTS

D. Adler (NRU HSE)

THE RING OF WEAK JACOBI

FORMS FOR D_8 ROOT SYSTEM

In my talk I will consider weak Jacobi forms associated with lattices D_8. The main result states that the space of these forms is a free algebra with 9 generators over the ring of modular forms.

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P. Agarwal (Seul National University)

**N=1 LAGRANGIANS FOR GENERALIZED ARGYRES-DUGLAS THEORIES **

We find N=1 Lagrangian gauge theories that flow to generalized Argyres-Douglas theories with N=2 supersymmetry. We find that certain SU-quiver gauge theories flow to generalized Argyres-Douglas theories of type A_{k-1} A_{mk-1} and $(I_{m,km},S)$. We also find quiver gauge theories of SO/Sp gauge groups flowing to the (A_{2m-1},D_{2mk+1}), $(A_{2m},D_{2m(k-1)+k})$ and $D_{m(2k+2)}^{m(2k+2)}[m]$ theories.

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 L. Astrakhantsev (MIPT)

**MOVING MIRRORS IN 2 DIMENSIONAL QFT **

In my talk cases of ideal and nonideal moving mirrors in 2 dimensional Minkowski space-time will be considered. I will pay attention to Hamiltonian and total momentum of the system. Also I will consider corrections to the Keldysh propagator in case of nonideal mirror. In my talk I will consider weak Jacobi forms associated with lattices D_8 . The main result states that the space of these forms

is a free algebra with 9 generators over the ring of modular forms.

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M. Bershtein (Landau Inst.|Skoltech |NRU HSE)

**DIFFERENCE PAINLEVE EQUATIONS FRON 5d GAUGE THEORIES **

In the talk I review two relations between difference Painleve equations and supersymmetric 5d gauge theories. First, these equations can be constructed as deautonomization of cluster integrable systems.

Second, them can be solved using Nekrasov partition functions of the 5d gauge theories.

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A. Bolshov (MIPT)

GRASSMANIANS AND FORM FACTORS

The talk is about a representation of form factors in N=4 SYM as integrals over Grassmannian manifolds. A procedure for obtaining such a description will be described and illustrated by several examples.

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<font face="Calibri"><font face="Calibri">**L. Bork (ITEP, CFAP VNIIA)**

Integrands for form factors in N=4 SYM and ambitwistor strings.

We are going to discuss how to reconstruct loop integrands for form factors of different operator types in N=4 SYM from integrands of on-shell amplitudes via "gluing operation". This "gluing operation" was conjectured in context of construction of vertex operators for form factors in ambitwistor string models.

A. Chattopadhyaya (Indian Institute of Science)

Dyon degeneracies from Mathieu moonshine symmetry

We study Siegel modular forms associated with the theta lift of twisted elliptic genera of $K3$ orbifolded with g' corresponding to the conjugacy classes of the Mathieu group M_{24} . For this purpose we re-derive the explicit expressions for all the twisted elliptic genera for all the classes which belong to M_{23} □ M_{24}. We show that the Siegel modular forms satisfy the required properties for them to be generating functions of $1/4$ BPS dyons of type II string theories compactified on $K3\times T^2$ and

orbifolded by g' which acts as a $\mathbb{Z}_N$ automorphism on $K3$ together with a $1/N$ shift on a circle of T^2. In particular the inverse of these Siegel modular forms admit a Fourier expansion with integer coefficients together with the right sign as predicted from black hole physics. This observation is

in accordance with the conjecture by Sen and extends it to the partition function for dyons for all the 7 CHL compactifications. We construct Siegel modular forms corresponding to twisted elliptic genera whose twining character coincides with with the class $2B$ and $3B$ of M_{24} and show that they also satisfy similar properties. Apart from the orbifolds corresponding to the 7 CHL compactifications, the rest of the constructions are purely formal.

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A. Chekmenev (LPI)

Conformal Lagrangians from the (formal) near boundary analysis of AdS gauge fields

A simple generating procedure allowing to obtain Lagrangians of conformal massless and massive fields of mixed-symmetry type in explicit form is presented. The approach originates from the analysis of equations on leading boundary value of AdS field in the case of odd-dimensional AdS space.

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A. Chowdhury (ITP, TU Wien)

Calabi-Yau manifolds and sporadic groups

A few years ago a connection between the elliptic genus of the $K3$ manifold and the largest Mathieu group M_{24} was proposed. We study the elliptic genera for Calabi-Yau manifolds of larger dimensions and discuss potential connections between the expansion coefficients of these elliptic genera and sporadic groups. While the Calabi-Yau 3-fold case is rather uninteresting, the elliptic genera of certain Calabi-Yau d-folds for $d>3$ have expansions that could potentially arise from underlying sporadic symmetry groups. We explore such potential connections by calculating twined elliptic genera for a large number of Calabi-Yau 5-folds that are hypersurfaces in weighted projected spaces,

for a toroidal orbifold and two Gepner models. [arXiv:1711.09698]

Slides.pdf

I. Gahramanov

Supersymmetric indices, elliptic hypergeometric functions and integrability

In this talk, we will review connection of integrable statistical models to supersymmetric indices and elliptic hypergeometric functions. The supersymmetric index is one of the efficient tools in the study of supersymmetric gauge theories providing the most rigorous mathematical check of supersymmetric dualities. The supersymmetric index of a four-dimensional supersymmetric gauge theory is expressed in terms of elliptic hypergeometric integrals. Such integrals are the new class of special functions and are of interest both in mathematics and in physics. Another intriguing physical interpretation of these integral identities stems from the role they play as the Yang-Baxter equation for certain integrable two-dimensional statistical models. As a result, one can construct a correspondence between supersymmetric gauge theories and integrable models such that the two-dimensional spin lattice is the quiver diagram, the partition function of the lattice model is the supersymmetric index and the Yang-Baxter equation expresses the index identity for dual pairs.

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A. Lobashev (MSU)

Kontsevich integral in topological models of gravity

We consider the procedure of Kontsevich integral calculation in topological Poincare gauge gravitational theory. The Lagrangian of this theory is the sum of two parts - one is& the Chern-Simons term for local Lorentz connection and other is the Chern-Simons term for tetrads field. The Kontsevich integral is not defined for non-compact gauge groups. We consider the local morphism between Poincare gauge gravitational theory and gauge gravitational theory of SO(4) gauge gravitational theory and reduce the result of calculation by means Wigner-Ihnonue algorithm. We discuss the application of obtained results to construction of knot invariants and some connections with theory of theta functions.

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R. Matias

Replication and sporadic groups

Abstract: In this talk we explore the Hecke Algebra of the group $\Gamma_0(2)+$ in order to define a new form of replication which turns out to respect the power map structure of 2.B, where B is the Baby Monster Sporadic Group, in the same way that usual replication respects the power map structure in the Monster Group.

K. Sun (SISSA, Trieste)

Blowup equations for refined topological strings

Gottsche-Nakajima-Yoshioka K-theoretic blowup equations characterize the Nekrasov partition function of five dimensional N=1 supersymmetric gauge theories compactified on a circle, which via geometric engineering correspond to the refined topological string theory on *SU(N)* geometries. In this paper, we study the K-theoretic blowup equations for general local Calabi-Yau threefolds. We find that both vanishing and unity blowup equations exist for the partition function of refined topological string,

and the crucial ingredients are the r fields introduced in our previous paper. These blowup equations are in fact the functional equations for the partition function and each of them results in infinite identities among the refined free energies. Evidences show that they can be used to determine the full refined BPS invariants of local Calabi-Yau threefolds. This serves an independent and sometimes more powerful way to compute the partition function other than the refined topological vertex in the A-model and the refined holomorphic anomaly equations in the B-model. We study the modular properties of the blowup equations and provide a procedure to determine all the vanishing and unity r fields from the polynomial part of refined topological string at large radius point. We also find that

certain form of blowup equations exist at generic loci of the moduli space. This talk is based on arXiv:1711.09884

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A. Zadora (MSU)

Dilaton black holes, integrability and coupling constant quantization

It is known that dilatonic dyonic black holes support coupling constant quantization in extreme limit. We show that this quantization rule may be generalized to a wider class of black holes and to p-branes. First terms of the inifinite tower of quantization sequence correspond to integrable Toda-like systems with algebras A_2 (sl(3,R)), B_2 (so(5)) and exceptional algebra G_2.

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