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Confinement potential of the Cornell type from holographic approach to QCD

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Cornell potential

The detailed lattice simulations of the form of the heavy-quark potential yields

$$V(r) = -\frac{\kappa}{r} + \sigma r + \text{const}$$

G. S. Bali, *QCD forces and heavy quark bound states*, Phys. Rept. 343 (2001), 1-136, [hep-ph/0001312]

This result imposes a serious restriction on viable phenomenological approaches modeling the dynamics of non-perturbative strong interactions:
In the non-relativistic limit, they should be able to reproduce this potential

One of such promising approaches that passes the given test is the so-called **Soft-Wall (SW) holographic model**

Holographic approach to QCD
(= AdS/QCD approach)

The approach is motivated by the
AdS/CFT correspondence
in string theory

AdS/CFT correspondence (= gauge/gravity duality = holographic duality)

is a conjectured equivalence between a quantum gravity (in terms of string theory or M-theory) compactified on anti-de Sitter space (**AdS**) and a Conformal Field Theory (**CFT**) on AdS boundary

The most promoted example
(Maldacena, 1997 - *the most cited work in theoretical physics*!):

$$\boxed{\left\{\mathcal{N}=4\ SU(N_c)\ \text{SYM theory}\right\} = \left\{\text{IIB string theory in } AdS_5 \times S_5\right\}}$$

Or generally:

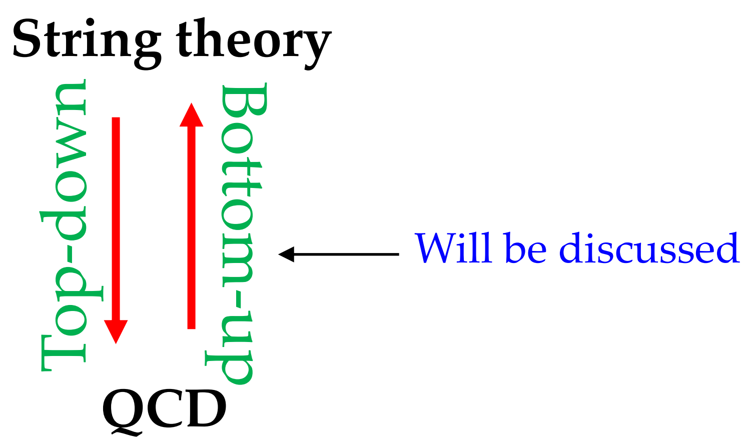
Strong coupling problems in non-abelian QFT	$\Longleftrightarrow$	Classical Problem in a curved higher-dimensional space
g^2, N_c		$g_s, \alpha' = l_s^2$ String coupling and string tension
$\lambda \equiv g^2 N$ T'Hooft coupling		

Source for major inspiration! (a great number of related models in the last 25 years)

<u>But still to be proven</u>…

AdS/QCD correspondence

AdS/QCD correspondence – a program for implementation of holographic duality for QCD following some recipies from the AdS/CFT correspondence



Phenomenological bottom-up AdS/QCD models

Typical ansatz: $$S = \int d^4x dz \sqrt{g}\, F(z) \mathcal{L} \qquad F(0) = 1$$

$$g = |\det g_{MN}|$$

AdS: $$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad z > 0$$

AdS/CFT: operators of 4D theory <-> fields in 5D theory

Vector mesons: $V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu q$ or $V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \vec{\tau} q$

$A_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \gamma_5 q$ or $A_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \gamma_5 \vec{\tau} q$

From the AdS/CFT recipes: $m_5^2 R^2 = (\Delta - J)(\Delta + J - 4) \qquad J = 0, 1$

Masses of 5D fields are related to the canonical dimensions of 4D operators!

In the given cases: $\Delta = 3,\ J = 1 \Rightarrow m_5^2 = 0$ gauge 5D theory!

Some applications

- ❑ Meson, baryon and glueball spectra
- ❑ Low-energy strong interactions (chiral dynamics)
- ❑ Hadronic formfactors
- ❑ Thermodynamic effects (QCD phase diagram)
- ❑ Description of quark-gluon plasma
- ❑ Condensed matter (high temperature superconductivity *etc.*)
- ❑ …

Deep relations with other approaches

- ➢ Light-front QCD
- ➢ QCD sum rules in the large-N_c limit
- ➢ Chiral perturbation theory supplemented by infinite number of vector mesons
- ➢ Renormalization group methods

Confinement $\Rightarrow$ linear Regge trajectories $\boxed{m_n^2 \sim n}$

Realization of linear Regge trajectories in the bottom-up holographic approach to QCD?

Soft-wall holographic model

A. Karch, E. Katz, D. T. Son, M. A. Stephanov, PRD 74, 015005 (2006)

$$S = \int d^4x\, dz \sqrt{g}\, \underline{e^{-cz^2}}\, \mathcal{L}$$

"Dilaton" background

In a sense, the background in holographic action provides a phenomenological model for non-perturbative gluon vacuum in QCD

Alternative formulation of the SW holographic model:

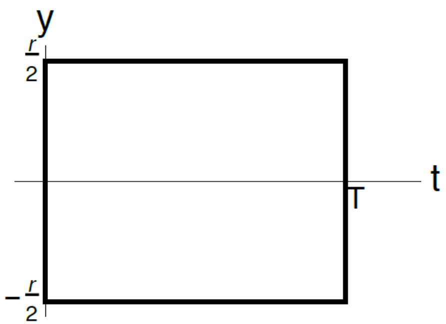
“Dilaton” background -> modified AdS metric (O. Andreev, PRD (2006))

$$g_{MN} = \mathrm{diag}\left\{\frac{R^2}{z^2}h, \ldots, \frac{R^2}{z^2}h\right\}, \qquad h = e^{-2cz^2}$$

This formulation is convenient to study the confinement properties. In particular, a Cornell like confinement potential for heavy quarks was derived (O. Andreev, V. Zakharov, PRD (2006))

But only in the case of background of the simplest vector SW model! **Generalizations?**

The main steps (J. Maldacena, PRL (1998)):
Consider the Wilson loop placed in the 4D boundary



$$T \to \infty \quad \Longrightarrow \quad \langle W(\mathcal{C})\rangle \sim e^{-TE(r)}$$

Alternatively $$\langle W(\mathcal{C})\rangle \sim e^{-S} \quad \Longrightarrow \quad \boxed{E = \frac{S}{T}}$$

area of a string world-sheet

The natural choice for the world-sheet area is the Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{\det g_{MN} \partial_\alpha X^M \partial_\beta X^N}$$

Choose $\xi_1 = t$ and $\xi_2 = y$

In the given model $$S = \frac{TR^2}{2\pi\alpha'} \int_{-r/2}^{r/2} dy \, \frac{h}{z^2} \sqrt{1 + z'^2}, \qquad z' = dz/dy$$

Omitting the details, the final result is

$$\boxed{r = 2\sqrt{\frac{\lambda}{c}} \int_0^1 dv \, \frac{h_0}{h} \frac{v^2}{\sqrt{1 - v^4 \frac{h_0^2}{h^2}}}} \qquad \boxed{E = \frac{R^2}{\pi\alpha'} \sqrt{\frac{c}{\lambda}} \int_0^1 \frac{dv}{v^2} \frac{h}{\sqrt{1 - v^4 \frac{h_0^2}{h^2}}}}$$

$$z_0 \equiv z\big|_{y=0}, \quad h_0 \equiv h\big|_{z=z_0}, \quad v \equiv \frac{z}{z_0}, \quad \lambda \equiv c z_0^2$$

Mass spectrum of vector SW model is $m_n^2 = 4|c|n, \qquad n = 1, 2, \ldots .$

Generalization to the arbitrary intercept, $m_n^2 = 4|c|(n + \underline{b})$

within this formulation, is achieved via (S.S. Afonin and T.D. Solomko, EPJC (2022))

$$h = e^{-2cz^2} \quad \rightarrow \quad h = e^{-2cz^2} U^4(b, 0, |cz^2|)$$

Tricomi function

The final result for this generalization is

$$r = 2\sqrt{\frac{\lambda}{c}} \int_0^1 dv \, \frac{U^4(b,0,\lambda)}{U^4(b,0,\lambda v^2)} \frac{v^2 e^{2\lambda(1-v^2)}}{\sqrt{1 - v^4 e^{4\lambda(1-v^2)} \frac{U^8(b,0,\lambda)}{U^8(b,0,\lambda v^2)}}},$$

$$E = \frac{R^2}{\pi\alpha'}\sqrt{\frac{c}{\lambda}} \left[\int_0^1 \frac{dv}{v^2} \left(\frac{e^{2\lambda v^2} U^4(b,0,\lambda v^2)}{\sqrt{1 - v^4 e^{4\lambda(1-v^2)} \frac{U^8(b,0,\lambda)}{U^8(b,0,\lambda v^2)}}} - D \right) - D \right]$$

Here $D = U^4(b, 0, 0)$

The same calculation can be made for the scalar SW model, where

$$h = e^{2cz^2/3} U^{4/3}(b, -1, cz^2)$$

$$r = 2\sqrt{\frac{\lambda}{c}} \int_0^1 dv \, \frac{U^{4/3}(b, -1, \lambda)}{U^{4/3}(b, -1, \lambda v^2)} \frac{v^2 e^{2\lambda(1-v^2)/3}}{\sqrt{1 - v^4 e^{4\lambda(1-v^2)/3} \frac{U^{8/3}(b,-1,\lambda)}{U^{8/3}(b,-1,\lambda v^2)}}},$$

$$E = \frac{R^2}{\pi\alpha'}\sqrt{\frac{c}{\lambda}} \left[\int_0^1 \frac{dv}{v^2} \left(\frac{e^{2\lambda v^2/3} U^{4/3}(b, -1, \lambda v^2)}{\sqrt{1 - v^4 e^{4\lambda(1-v^2)/3} \frac{U^{8/3}(b,-1,\lambda)}{U^{8/3}(b,-1,\lambda v^2)}}} - D \right) - D \right]$$

Here $D \equiv U^{4/3}(b, -1, 0)$

Comparison with phenomenology

$$V(r) = -\frac{\kappa}{r} + \sigma r + C$$

Typical phenomenological values of the potential parameters:

- $C \approx -0.3\,\mathrm{GeV}$
- For charmonium: $\kappa \approx 0.25$, $\sigma \approx 0.21\,\mathrm{GeV}^2$
- For charmonium and bottomonium (works better at small distances): $\kappa \approx 0.51$, $\sigma \approx 0.18\,\mathrm{GeV}^2$ ← **standard value of (420 MeV)^2**

Comparison with the lattice results in SU(3) gauge theory, where $E(0.5\,\mathrm{fm}) = 0$; quenched: $\sigma = 0.18\,\mathrm{GeV}^2$, $\kappa = 0.295$; un-quenched: $\kappa = 0.36$.

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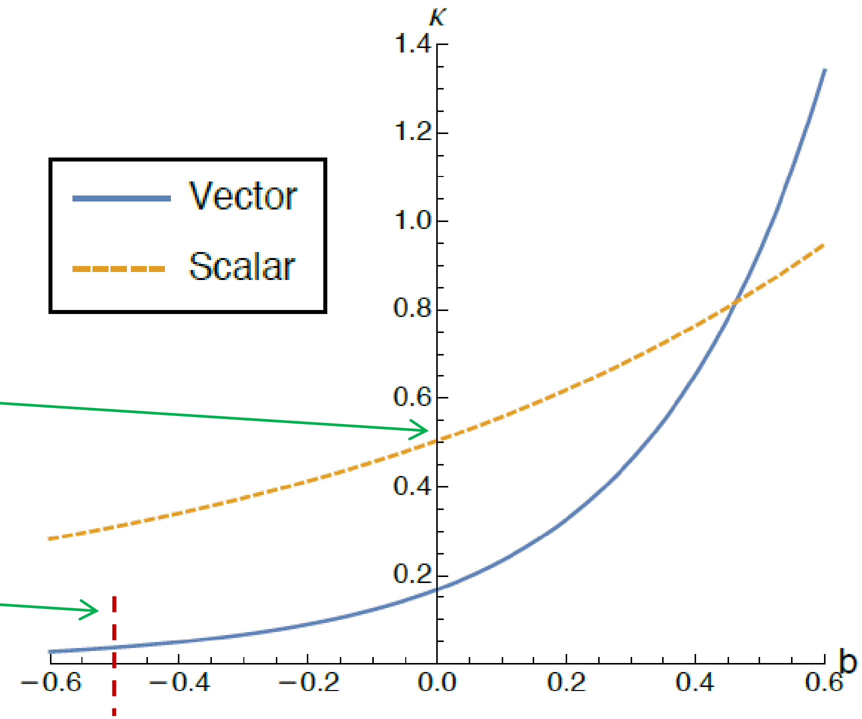
For fixed $\sigma_\infty = 0.18\,\mathrm{GeV}^2$

$\kappa \approx 0.51$

phenomenology



The meaning of ***b = 0*** in the scalar case?

The scalar SW spectrum: $m_n^2 = 4c(n + \Delta/2 + b), \qquad n = 0, 1, 2, \ldots$

Interpolating operator for scalar glueball: $\boxed{\beta G_{\mu\nu}^2} \implies \boxed{\Delta = 4}$

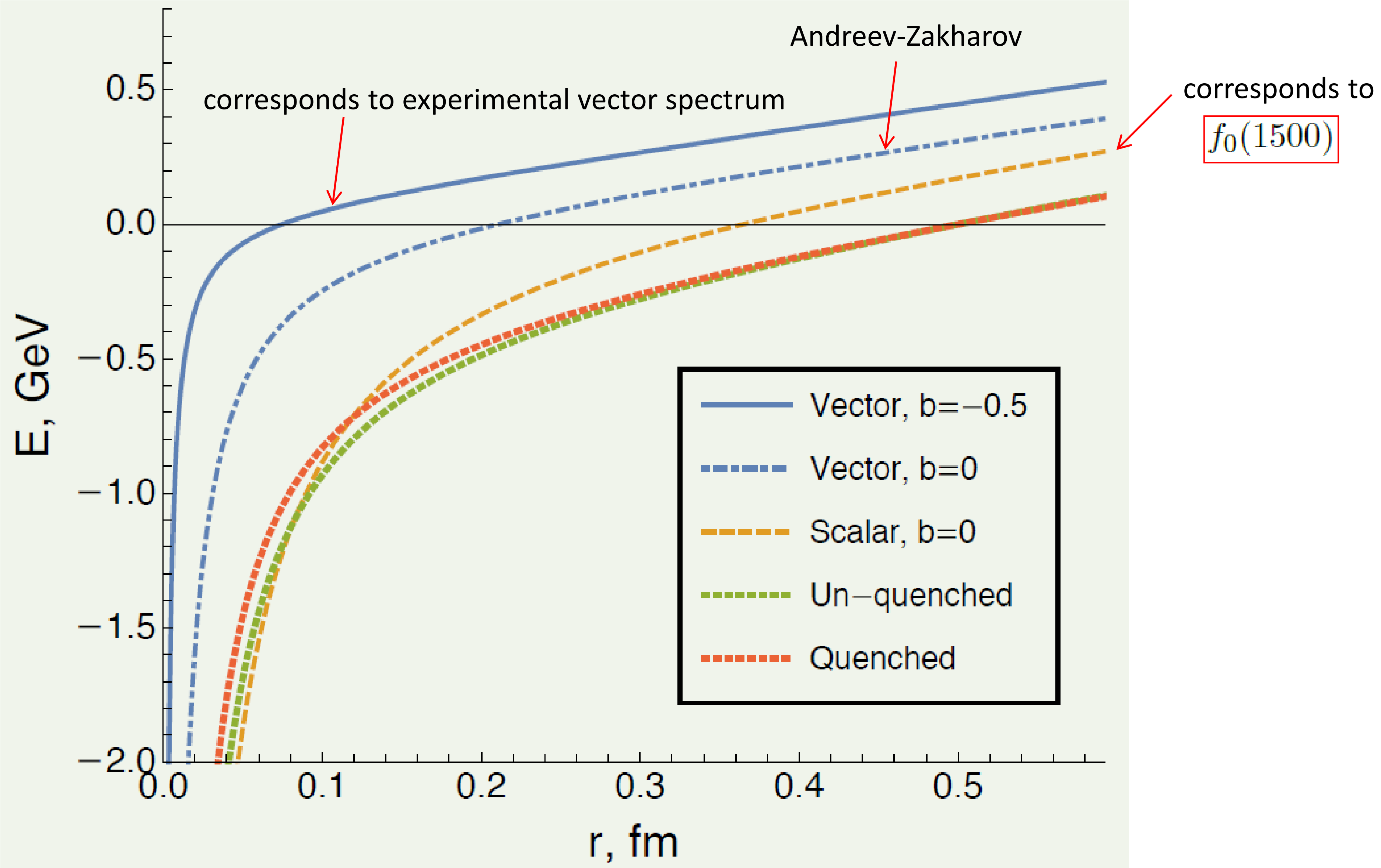
The SW spectrum for vector mesons: $m_n^2 = 4|c|(n + b), \qquad n = 1, 2, \ldots$

where the phenomenology gives $b \approx -0.5$

$\implies$ Prediction for the first scalar glueball: $\boxed{m_s \approx 2m_\rho}$

A natural candidate is the scalar meson $\boxed{f_0(1500)}$

The final plots



The background of scalar SW model gives a good *quantitative* description, while the vector one reproduces only a *qualitative* behavior!

Conclusions

- Within the framework of Soft Wall holographic model, the Cornell potential is derived as a function of intercept of linear Regge spectrum for the vector and scalar "dilaton" backgrounds

- The scalar background leads to a *quantitative* consistency with phenomenology and lattice simulations, the agreement in the vector case is *qualitative* only

- By-product: The overall consistency of our holographic description of confinement potential seems to confirm the glueball nature of the scalar meson $f_0(1500)$

- The obtained results provide a new model demonstration for the dominance of the vacuum scalar sector in confinement physics of strong interactions