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Inflation in the scalar-tensor theory of gravity with a non-minimal kinetic coupling

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Scalar-tensor theory of gravity with a non-minimal kinetic coupling

- Action:

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi}-\frac{1}{2}[g^{\mu\nu}-\kappa G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}-V(\phi)\right)$$

$$V=V_0\phi^\alpha$$

- Field equations:

$$3H^2=4\pi\dot{\phi}^2(1+9\kappa H^2)+8\pi V_0\phi^\alpha$$

$$2\dot{H}+3H^2=-4\pi\dot{\phi}^2(1-\kappa(2\dot{H}+3H^2+4H\ddot{\phi}\dot{\phi}^{-1}))+8\pi V_0\phi^\alpha$$

$$(\ddot{\phi}+3H\dot{\phi})+3\kappa(H^2\ddot{\phi}+2H\dot{H}\dot{\phi}+3H^3\dot{\phi})=-V_0\alpha\phi^{\alpha-1}$$

S. V. Sushkov, Phys. Rev. D85, 123520 (2012).
C. Germani and A. Kehagias, Phys.Rev.Lett. 105:011302 (2010)

Slow-roll conditions

- Slow-roll parameters:

$$\epsilon_0 = -\frac{\dot{H}}{H^2} \qquad k_0 = 12\pi\kappa\dot{\phi}^2$$

$$\epsilon_1 = \frac{\dot{\epsilon}_0}{H\epsilon_0} \qquad k_1 = \frac{\dot{k}_0}{H k_0}$$

- Field equations under the slow-roll conditions :

$$\dot{\phi} = -\frac{\alpha V_0 \phi^{\alpha-1}}{2\sqrt{6\pi V_0}\phi^{\alpha/2}(1+8\pi\kappa V_0\phi^\alpha)}$$

$$\dot{H} = -\frac{\alpha^2 V_0 \phi^{\alpha-2}}{6(8\pi\kappa V_0\phi^\alpha + 1)}$$

$$\frac{3H^2}{8\pi} = V_0\phi^\alpha$$

Granda, L.N., Jimenez, D.F., Eur. Phys. J. C 79, 772 (2019).

Slow-roll conditions

- Condition of end of inflation:

$$\frac{\alpha^2}{16\pi\phi_E^2(1+8\pi\kappa V_0\phi_E^\alpha)} = 1$$

- Number of e-folds:

$$N = \int_{\phi_E}^{\phi_I} \frac{H}{\dot{\phi}} d\phi = -\frac{4\pi}{\alpha}(\phi_E^2 - \phi_I^2) - \frac{64\pi^2\kappa V_0}{\alpha(\alpha+2)}(\phi_E^{\alpha+2} - \phi_I^{\alpha+2})$$

Parameters of primodal spectrum

- The tensor/scalar ratio, spectral index, the power spectra for the scalar perturbations:

$$r = \frac{P_T(k)}{P_\xi(k)} = 16\epsilon_0 = \frac{\alpha^2}{\pi\phi_I^2(1 + 8\pi\kappa V_0\phi_I^\alpha)}$$

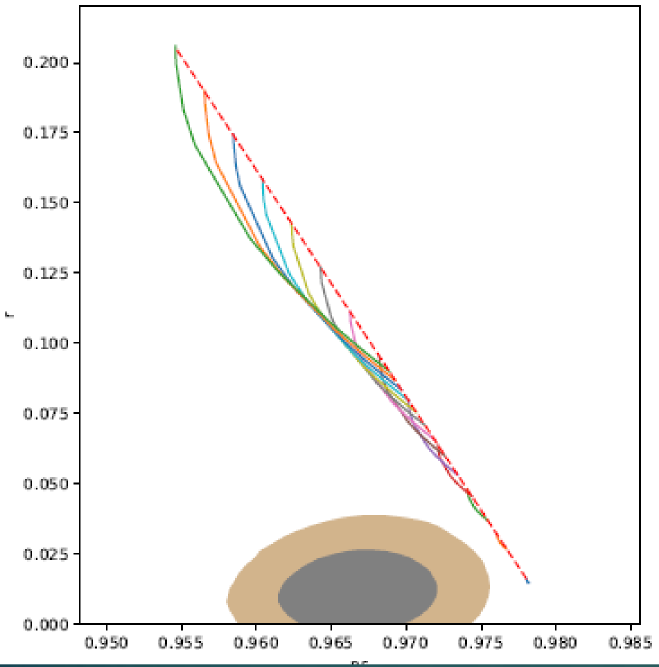
$$n_s - 1 = \frac{d\ln P(\xi)}{d\ln k} = -2\epsilon_0 - \epsilon_1 = -\frac{\alpha^2}{8\pi\phi_I^2(1 + 8\pi\kappa V_0\phi_I^\alpha)} - \frac{\alpha(1 + 8\pi\kappa V_0\phi_I^\alpha + 4\pi\alpha\kappa V_0\phi^\alpha)}{4\pi\phi_I^2(1 + 8\pi\kappa V_0\phi_I^\alpha)^2}$$

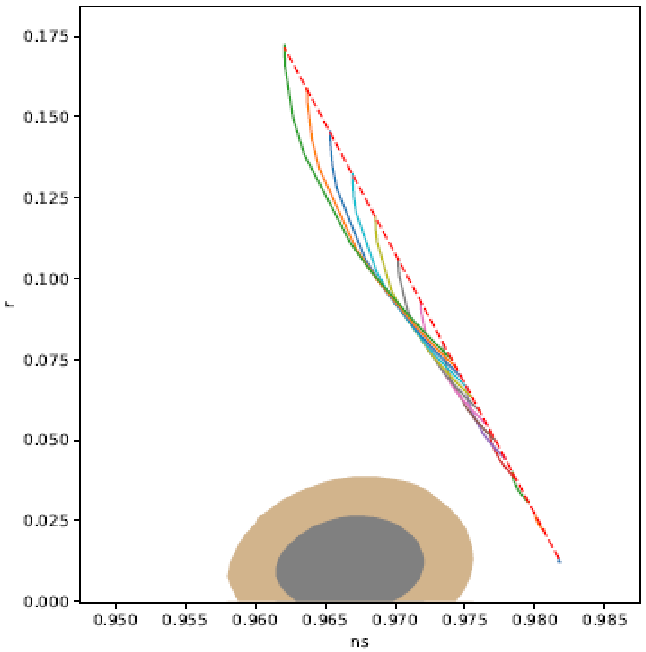
$$P_\xi \approx \frac{H^2}{8\pi^2}\frac{1}{\epsilon_0} = \frac{16V_0\phi_I^{\alpha+2}(1 + 8\pi\kappa V_0\phi_I^\alpha)}{3\alpha^2}$$

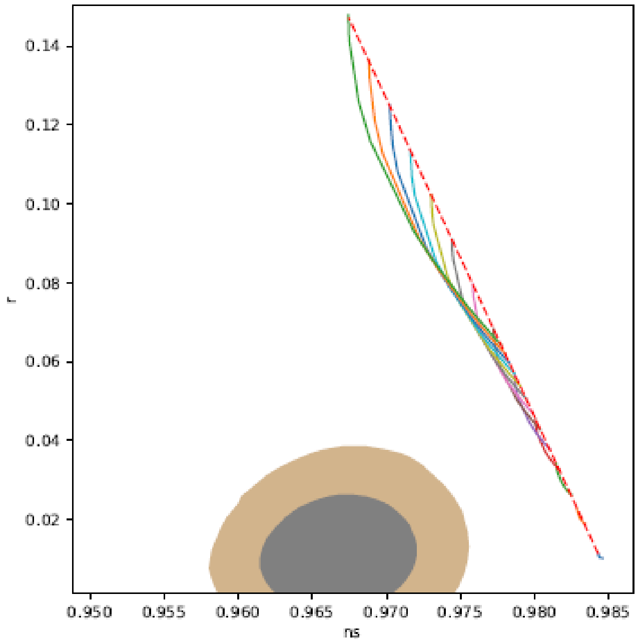
Shinji Tsujikawa, Phys. Rev. D 85, 083518
Granda, L.N., Jimenez, D.F., Eur. Phys. J. C 79, 772 (2019).

Predictions of theory and observational limits

- N = 50
- N = 60
- N = 70







P. A. R. Ade et al. [BICEP and Keck], Phys. Rev. Lett. 127 (2021) no.15, 151301

Asimptotics.

- Asimptotics

$$\lim_{\kappa V_0\to\infty} r = \frac{16\alpha}{2N(\alpha+2)+\alpha}$$
$$\lim_{\kappa V_0\to\infty} n_s = 1-\frac{4(\alpha+1)}{2N(\alpha+2)+\alpha}$$

$$\Rightarrow \quad n_s - 1 = -\frac{(2N-1)r+16}{16N}$$

- Case with $\kappa V_0 = 0$

$$r = \frac{4\alpha}{\frac{\alpha}{4}+N}$$
$$n_s = 1-\frac{\alpha+2}{2N+\frac{\alpha}{2}}$$

$$\Rightarrow \quad n_s - 1 = -\frac{(2N-1)r+16}{16N}$$

Case with $\kappa < 0$. Field equations.

- Let's consider case with $V(\phi) = 0$, field equations:

$$3H^2 = 4\pi\dot{\phi}^2(1 + 9\kappa H^2)$$

$$2\dot{H} + 3H^2 = -4\pi\dot{\phi}^2(1 - \kappa(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}))$$

$$(\ddot{\phi} + 3H\dot{\phi}) + 3\kappa(H^2\ddot{\phi} + 2H\dot{H}\dot{\phi} + 3H^3\dot{\phi}) = 0$$

Case with $\kappa < 0$. Number of e-folds.

- Solution of field equations:

$$\sqrt{-\kappa}\ln\left(\left(\frac{1+\sqrt{-3\kappa}H}{1-\sqrt{-3\kappa}H}\right)^{\frac{1}{\sqrt{3}}}\left(\frac{1-3\sqrt{-\kappa}H}{1+3\sqrt{-\kappa}H}\right)^{\frac{1}{2}}\right)+\frac{1}{3H}=t+const$$

- Number of e-folds:

$$N=\int_{t_i}^{t_e}\frac{1+9\kappa H^2-54\kappa^2H^4}{-3H(1+9\kappa H^2)(1+3\kappa H^2)}dH=$$

$$ln(H(t_e)^{-\frac{1}{3}}(1+9\kappa H(t_e)^2)^{\frac{1}{6}}(1+3\kappa H(t_e)^2)^{-\frac{1}{3}})-$$
$$ln(H(t_i)^{-\frac{1}{3}}(1+9\kappa H(t_i)^2)^{\frac{1}{6}}(1+3\kappa H(t_i)^2)^{-\frac{1}{3}})$$

Case with $\kappa < 0$. Condition of end of inflation

- Condition of end of inflation:

$$\epsilon_0 = 1$$

$$\frac{3(1+9\kappa H(t_e)^2)(1+3\kappa H(t_e)^2)}{1+9\kappa H(t_e)^2-54\kappa^2 H(t_e)^4} = 1,$$

$$\kappa H(t_e)^2 = -\frac{27-\sqrt{513}}{54} \approx -0.081.$$

Case with $\kappa < 0$. Initial conditions.

- Condition for obtaining inflation in more than *N* e-folds :

$$\ln(H(t_i)^{-\frac{1}{3}}(1+9\kappa H(t_i)^2)^{\frac{1}{6}}(1+3\kappa H(t_i)^2)^{-\frac{1}{3}}) < q - N$$

where

$$q = \ln(H(t_e)^{-\frac{1}{3}}(1+9\kappa H(t_e)^2)^{\frac{1}{6}}(1+3\kappa H(t_e)^2)^{-\frac{1}{3}})$$

Case with $\kappa < 0$. Initial conditions

- Solution of inequality: $H \in (H_{sol}; \frac{1}{3\sqrt{\kappa}}) \quad \phi \in (\phi_{sol}, \infty)$

$$H_{sol}^2 = \Bigg[\Bigg(\Bigg(\Bigg(36\sqrt{\frac{972k^2 - 27e^{6(q-N)}\kappa + 8e^{12(q-N)}}{e^{6(q-N)}\kappa}}\kappa - 324\kappa - 8e^{6(q-N)}\Bigg)e^{12(q-N)}\Bigg)^{2/3} - 4e^{6(q-N)}(27\kappa - e^{6(q-N)})\Bigg] \Bigg/ \Bigg[18e^{6(q-N)}\kappa \times$$

$$\times\Bigg(\Bigg(36\sqrt{\frac{972\kappa^2 - 27e^{6(q-N)}\kappa + 8e^{12(q-N)}}{e^{6(q-N)}\kappa}}\kappa - -324\kappa - 8e^{6(q-N)}\Bigg)e^{12(q-N)}\Bigg)^{1/3}\Bigg] +$$

$$+\frac{2}{9\kappa} \approx \frac{1}{9\kappa}\left(1 - \frac{13e^{6(q-N)}}{324\kappa}\right)$$

$$\dot{\phi}_{sol} = \sqrt{\frac{3H_{sol}^2}{4\pi(1 - 9\kappa H_{sol}^2)}} \sim \frac{3\sqrt{3}}{\sqrt{13\pi}} \cdot e^{3(N-q)}$$

for $N = 60$ and $\kappa = 1$ $\quad \dot{\phi} \sim 4.97 \cdot 10^{77}$
H differs from $\frac{1}{3\sqrt{\kappa}}$ by $1.1 \cdot 10^{-79}$

Case with $\kappa < 0$. Initial conditions.

- Case with $V(\phi) = V_0\phi^{1.5}$. Areas of initial values of $\phi, \dot{\phi}, H$ leading to different regimes of inflation

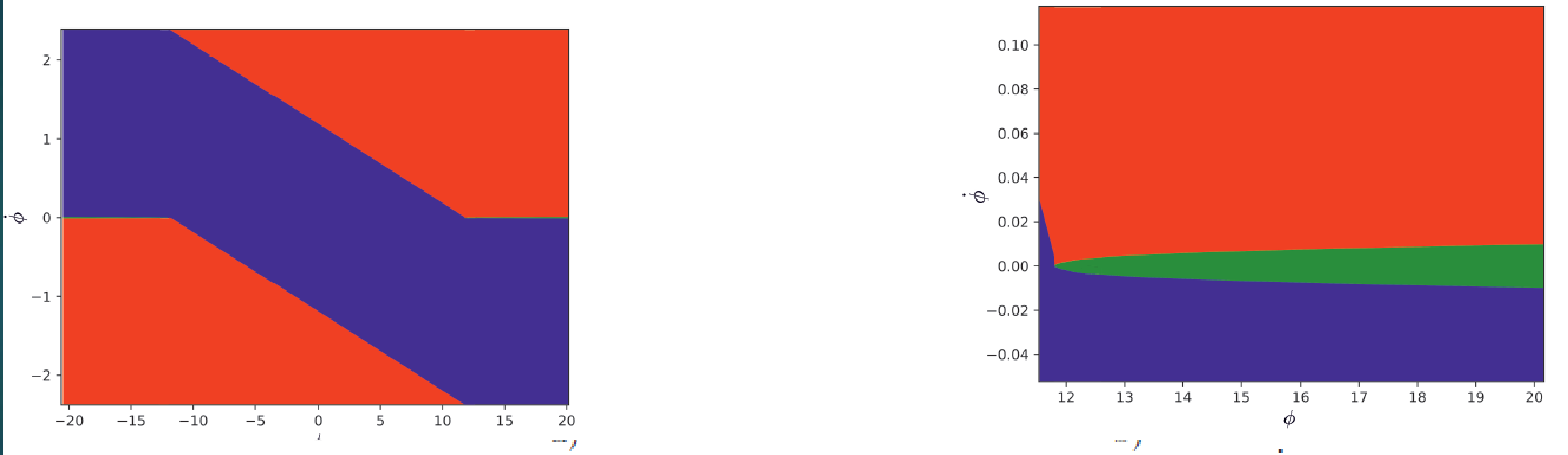


Figure 1: The diagrams are plotted in the coordinates ϕ and $\dot{\phi}$. The red and green areas show initial conditions for which the eternal inflation scenario is realized,the blue zone indicates the initial conditions leading to sufficient inflation. Here $\kappa = 100$, $V_0 = 10^{-5}$. The right panel is zoomed part of the left plot showing the region with the green area.

Case with $\kappa < 0$. Initial conditions.

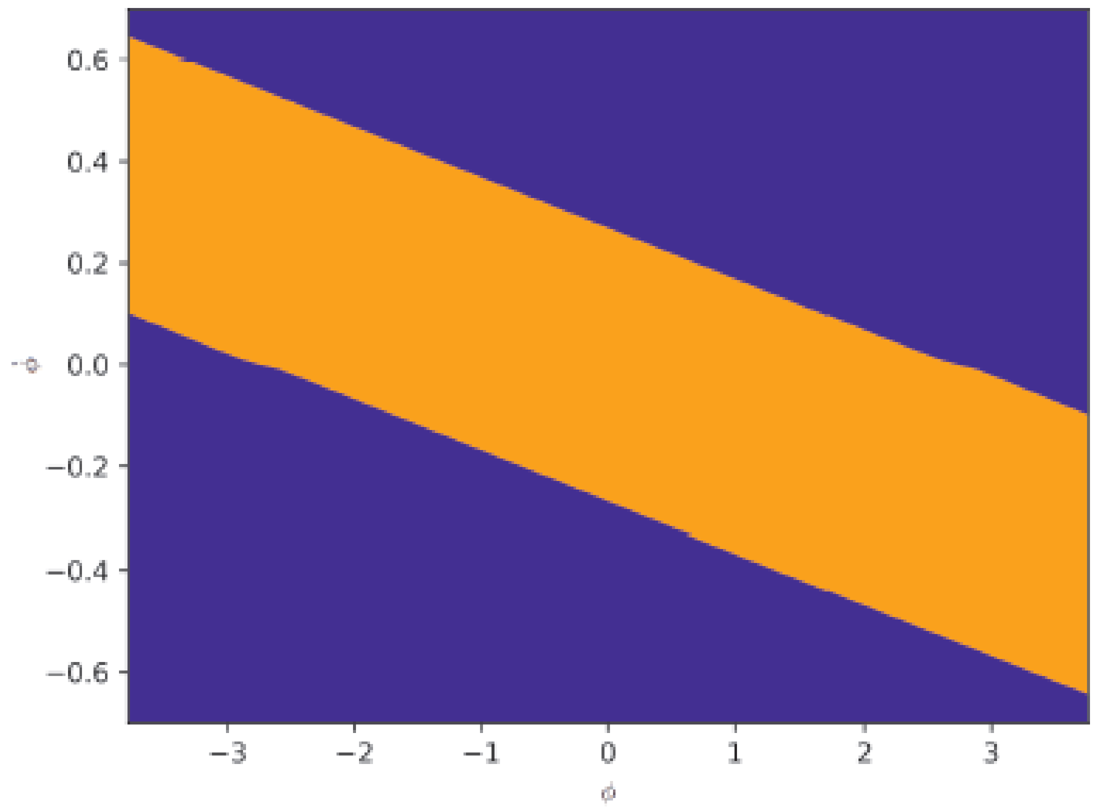


Figure 2: The blue zone represent initial condition giving sufficient inflation, in the orange zone inflation is absent or insufficient.

Case with $\kappa < 0$. Initial conditions.

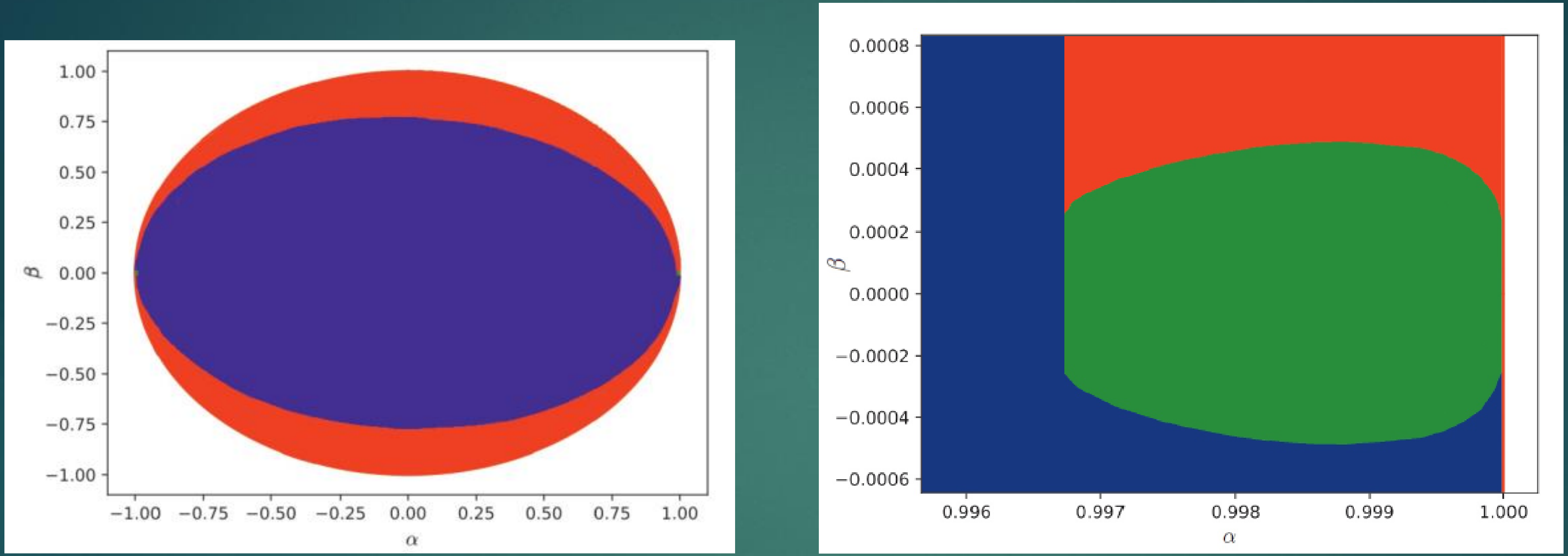


Figure 3: The diagrams are plotted in the coordinates $\alpha = \frac{\phi}{\sqrt{1+\phi^2+\dot{\phi}^2}}$ and $\beta = \frac{\dot{\phi}}{\sqrt{1+\phi^2+\dot{\phi}^2}}$. The red and green areas show initial conditions for which the eternal inflation scenario is realized,the blue zone indicates the initial conditions leading to sufficient inflation. Here $\kappa = 100$, $V_0 = 10^{-5}$. The right panel is zoomed part of the left plot showing the region with the green area.

Results

- As we saw, the model with non-minimally kinetically coupled scalar field solely can be already ruled out for any power law potential

- Though in principle inflation in the model under investigation can be driven by kinetic coupling term only, the adequate inflation (no less than 60 e-folds) requires either exponentially large initial values of $\dot{\phi}$ or exponentially large values of the coupling constant κ. Initial value of H must lie in a very narrow range of values $(H_{sol}, \frac{1}{3\sqrt{\kappa}})$

- On the contrary, non-zero scalar field potential naturally leads to a successful inflation without strong fine-tuning.

N. Avdeev and A. Toporensky, [arXiv:2203.14599 [gr-qc]]
N. Avdeev and A. Toporensky, Grav. Cosmol. 27, 269–274 (2021).