The Hubble stream near a galaxy group: the exact analytical solution for the spherically-symmetric case and the group mass determination

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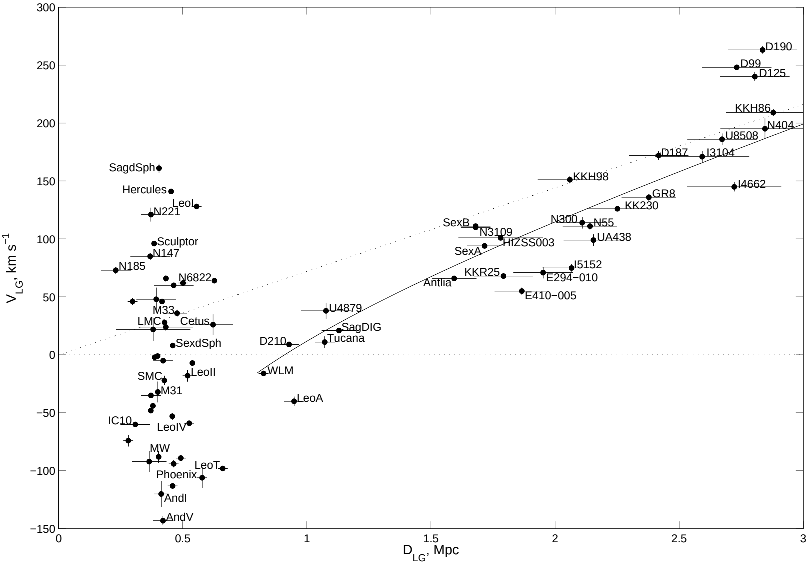
Local Group (Makarov et al. 2009)



Undisturbed Universe

$$\left(\frac{da}{adt}\right)^2 = \frac{8\pi}{3} G \left[\rho_{M,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\gamma,0}\left(\frac{a_0}{a}\right)^4 + \rho_{\Lambda,0} + \rho_{a,0}\left(\frac{a_0}{a}\right)^2\right]$$

The curvature density $\frac{8\pi}{3c^2} G \rho_{a,0} = \frac{k}{a_0^2}$.

For our Universe $\Omega_{a,0} = 0$, $\Omega_{\gamma,0} \simeq 0$. Therefore, $\Omega_{\Lambda,0} + \Omega_{M,0} = 1$.

The age of the Universe

$$t_0 = H_0^{-1} \frac{2}{3\sqrt{\Omega_{\Lambda,0}}} \ln\left[\frac{\sqrt{1-\Omega_{M,0}}+1}{\sqrt{\Omega_{M,0}}}\right] = $$
$$= H_0^{-1} \frac{2}{3\sqrt{\Omega_{\Lambda,0}}} \operatorname{arcosh}\left(1/\sqrt{\Omega_{M,0}}\right)$$

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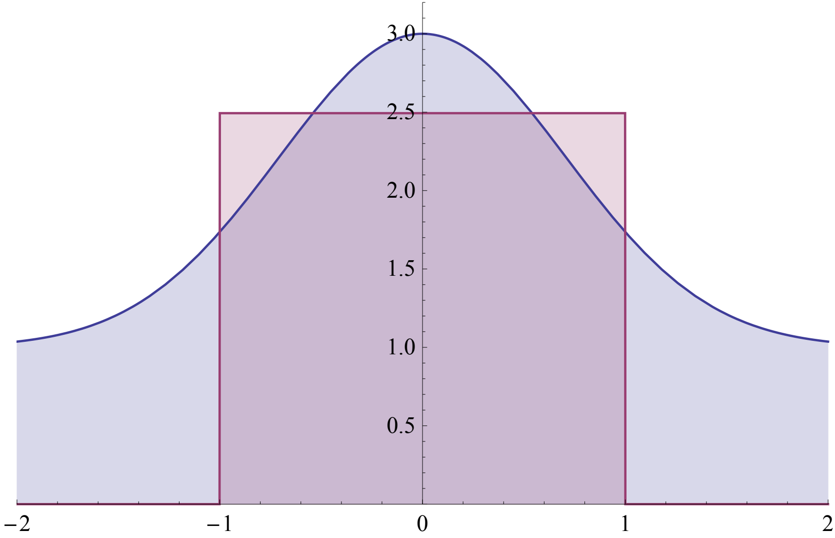
Suppositions

1. The Universe is flat ($\Omega_{a,0} = 0$) in absence of structures, the dark energy is the cosmological constant, we neglect the radiation term $\Omega_{\gamma,0} \sim 10^{-4}$.
2. The size of the group is large with respect to its gravitational radius $R_0 \gg R_g \equiv 2GM/c^2$ and small with respect to c/H_0. For real groups $R_0/R_g > 10^4$, $R_0 H_0/c < 10^{-3}$, i.e., both the conditions are well satisfied. The significance of this assumption will be explained below.
3. The group of galaxies is spherically symmetric and does not experience any tidal perturbations. Typically, this assumption is not quite valid for real galaxy groups. However, it allows to find a precise analytical solution even in the nonlinear regime.

Principle idea of the solution



The Friedmann equation for R_0

$$\left(\frac{dR}{Rdt}\right)^2 = \frac{8\pi}{3}G\left[\sigma_{M,0}\left(\frac{R_0}{R}\right)^3 + \sigma_{\Lambda,0} + \sigma_{a,0}\left(\frac{R_0}{R}\right)^2\right]$$

Since R_0 is the stop-radius, the right part of this equation should be zero: $\sigma_{M,0} + \sigma_{\Lambda,0} + \sigma_{a,0} = \sigma_{M,0} + \rho_{\Lambda,0} + \sigma_{a,0} = 0$.

We may introduce

$$\alpha = \frac{\sigma_{M,0}}{\sigma_{\Lambda,0}} = \frac{\sigma_{M,0}}{\rho_{\Lambda,0}} = \frac{\sigma_{M,0}}{\rho_{c,0}\,\Omega_{\Lambda,0}}$$

and rewrite the Friedmann equation as

$$\left(\frac{dR/R_0}{R/R_0 dt}\right)^2 = \frac{8\pi}{3}G\rho_{\Lambda,0}\left[\alpha\left(\frac{R_0}{R}\right)^3 + 1 - (\alpha+1)\left(\frac{R_0}{R}\right)^2\right].$$

The age of the 'R_0-universe'

$$t_0 = \frac{1}{H_0\sqrt{\Omega_{\Lambda,0}}}\int_0^1 \frac{dx}{x\sqrt{\alpha/x^3+1-(\alpha+1)/x^2}}$$

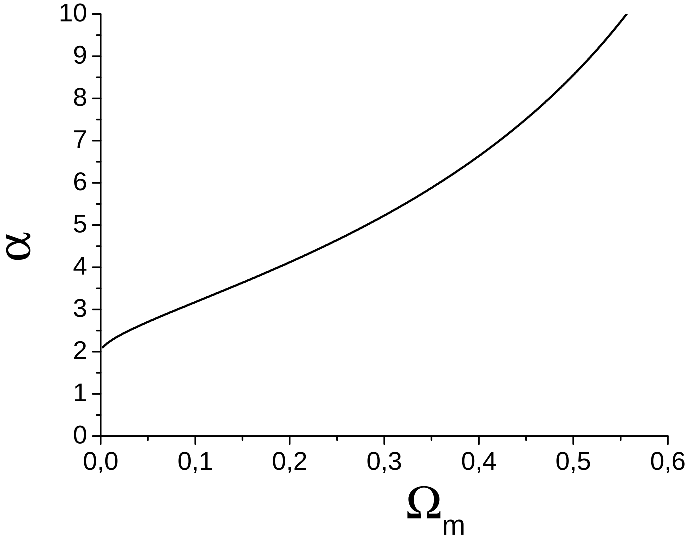
Of course, the ages of the Universe and the 'R_0-universe' should be the same!

$$\frac{2}{3}\ln\left[\frac{\sqrt{1-\Omega_{M,0}}+1}{\sqrt{\Omega_{M,0}}}\right] = \int_0^1 \frac{\sqrt{x}dx}{\sqrt{\alpha+x^3-x(\alpha+1)}}$$

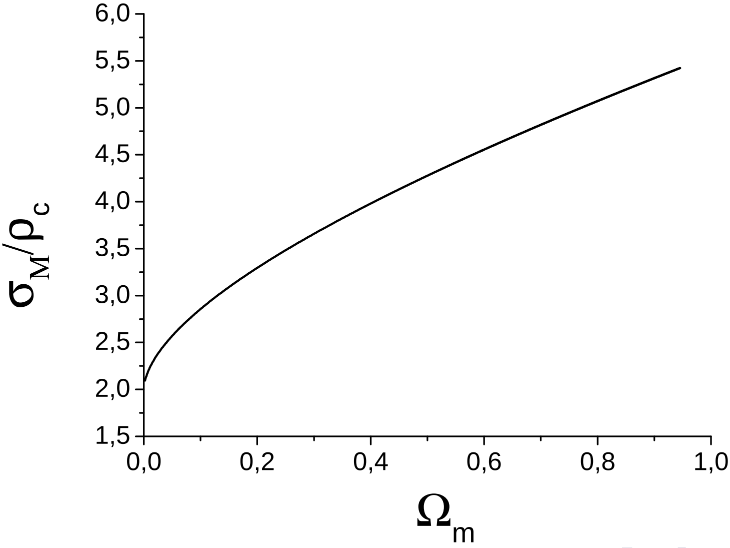
If we introduce a function

$$I(\alpha) \equiv \int_0^1 \frac{\sqrt{x}dx}{\sqrt{\alpha+x^3-x(\alpha+1)}} \qquad \Omega_{M,0} = \frac{1}{\cosh^2(3I(\alpha)/2)}$$

$\alpha(\Omega_{M,0}) = \sigma_{M,0}/\rho_{\Lambda,0}$



$\sigma_{M,0} = \alpha(\Omega_{M,0})\Omega_{\Lambda,0}\rho_{c,0}$



The group mass

$$M = \frac{4}{3}\pi R_0^3 \alpha(\Omega_{M,0})\Omega_{\Lambda,0}\rho_{c,0} = \frac{\alpha(\Omega_{M,0})}{2G}(1-\Omega_{M,0})R_0^3 H_0^2$$

For our Universe $\alpha(\Omega_{M,0} = 0.306) \simeq 5.30$, and we obtain:

$$M = 2.278 \cdot 10^{12} M_{\odot} \times \left(\frac{R_0}{1\text{Mpc}}\right)^3 \left(\frac{H_0}{73\frac{\text{km/s}}{\text{Mpc}}}\right)^2$$

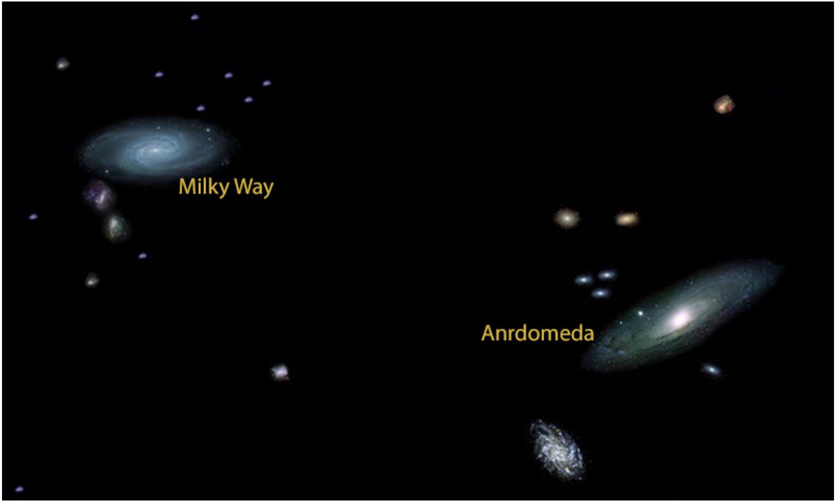
The Local Group has $R_0 \simeq 0.9$ Mpc, i.e., $M \simeq 2.05 \cdot 10^{12} M_{\odot}$

The case of arbitrary z

$$\Omega_{M,z} = \frac{\Omega_{M,0}(z+1)^3}{\Omega_{M,0}(z+1)^3 + \Omega_{\Lambda,0}} \qquad H_z^2 = H_0^2\left(\Omega_{M,0}(z+1)^3 + \Omega_{\Lambda,0}\right)$$

If $z \gg 1$, then $\Omega_{M,z} \simeq 1$, and $\frac{\sigma_{M,0}}{\rho_{c,0}} = \left(\frac{3\pi}{4}\right)^2$

The Local Group



The *point* model: the cluster has a central point mass M_c and surrounded by relatively small constant density $\rho_M = \Omega_{M,0}\rho_{c,0}$ (which is equal to the average matter density in the Universe).

$$\Sigma_{M,0}(r_0) = \left(\Sigma_{M,0}(L_0) - \Omega_{M,0}\right)\left(L_0/r_0\right)^3 + \Omega_{M,0}$$

The *halo* model also has two components: the uniform distribution of matter with the density $\Omega_{M,0}\rho_{c,0}$ and a large halo with $\rho \propto r^{-2}$, the factor being chosen so that $L_0 = 1$ Mpc.

$$\Sigma_{M,0}(r_0) = \left(\Sigma_{M,0}(L_0) - \Omega_{M,0}\right)\left(L_0/r_0\right)^2 + \Omega_{M,0}$$

The *empty* model supposes that all the matter is concentrated in the cluster center, and the space around is empty (i.e., contains only dark energy).

$$\Sigma_{M,0}(r_0) = \Sigma_{M,0}(L_0)\left(L_0/r_0\right)^3$$

Contrary to the first two models, the *empty* one has incorrect asymptotic behavior: it does not transform into the undisturb Universe at large distances, and its average density tends to $\Omega_{\Lambda,0}\rho_{c,0}$, and not to $\rho_{c,0}$, as $r_0 \to \infty$.

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