The equation for the probability of quantum transitions in the method of path integrals and stochastic processes in the space of joint events.

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TASK: To construct the equation for probabilities of quantum transitions on the basis of probability theory axioms.

HYPOTHESIS: The equation for probabilities of quantum process can be constructed on Kolmogorov's axioms if they are supplemented with a new axiom for joint events.

Interpretation of the equation for the probability of quantum transitions in probability theory.

We are investigating a quantum system

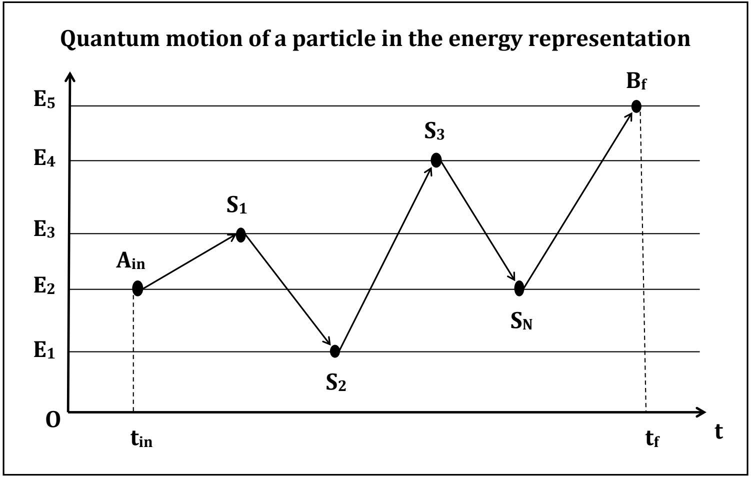
The Hamiltonian of the system has a discrete spectrum of eigenvalues (energies)

$$\hat{H}_{syst}|n\rangle = n|n\rangle, \tag{4.1}$$

where $n = 1, 2, 3, ...$ - the number of the quantum state (energy level); $|n\rangle$ - the vector of states (corresponding to these levels).
The system at the moment of time $t = 0$ begins to interact with the electromagnetic field. The interaction is characterized by a Hamiltonian $\hat{V}_{int}(\tau)$. The interaction Hamiltonian can have the form

$$\hat{V}_{int}(\tau) = E_0 \cos(\Omega\tau). \tag{4.2}$$

The system begins to make quantum transitions between states. These transitions are shown in the figure.
We will describe the state of the system by a statistical operator $\hat{\rho}(t)$.


Quantum motion of a particle in the energy representation
E5
E4
E3
E2
E1
O
Bf
S3
S1
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The equation of quantum motion of a particle in the energy representation

Equation of evolution of a statistical operator:

$$\hat{\rho}(t) = \hat{U}_D(t)\hat{\rho}(0)\hat{U}_D^+(t), \tag{6.1}$$

where $\hat{\rho}(t)$, $\hat{\rho}(0)$ — Statistical operators of the system, respectively, at time moment t and $t = 0$,

$$\hat{U}_D(t, t_0) = T \exp[-\frac{\imath}{\hbar} \int\limits_{t_0}^{t} \hat{V}_D(\tau) d\tau], \tag{6.2}$$

$$\hat{V}_D(\tau) = \exp[\frac{\imath}{\hbar} \hat{H}_{syst}\tau]\hat{V}_{int}(\tau) \exp[-\frac{\imath}{\hbar} \hat{H}_{syst}\tau]. \tag{6.3}$$

In energy representation

$$\rho_{n_f m_f}(t) = \sum_{n_0, m_0} \langle n_f|\hat{U}_D(t)|n_0\rangle \rho_{n_0, m_0} \langle m_0|\hat{U}_D^+(t)|m_f\rangle, \tag{6.4}$$

where

$$\rho_{n_f m_f}(t) = \langle n_f|\hat{\rho}(t)|m_f\rangle, \qquad \rho_{n_0, m_0} = \langle n_0|\hat{\rho}(0)|m_0\rangle. \tag{6.5}$$

The equation for the probability of quantum transitions is constructed from the equation for the statistical density matrix in the energy representation. We assume in the equation $n_f = m_f = b, n_0 = m_0 = a, \rho_{n_0 n_0} = 1, \rho_{n_0, m_0} = 0$, we get

$$P(b, t|a.t = 0) = \rho_{n_f n_f}(t) = \langle n_f|\hat{U}_D(t)|n_0\rangle\langle n_0|\hat{U}_D^+(t)|n_f\rangle, \tag{7.1}$$

The core of the evolution operator of a quantum system can be represented as a sum along trajectories from a functional:

$$\langle n_f|\hat{U}_D(t)|n_0\rangle = \sum_{n_1=1}^{N} \int_0^1 \int_0^1 \int_0^1 \int_0^1 P_f \exp[iS[n_f; ...n_1, \xi_1; n_0, \xi_0]]d\xi_0 d\xi_1 d\xi_2 ..., \tag{7.2}$$

$$S[n_f; ...n_1, \xi_1; n_0, \xi_0] = \sum_{n_1=1}^{N} S[n_k, t_k; n_{k-1}, t_{k-1}; \xi_{k-1}]; \tag{7.3}$$

$$S[n_k, t_k; n_{k-1}, t_{k-1}; \xi_{k-1}] = 2\pi(n_k - n_{k-1})\xi_{k-1} + \Omega^R_{n_k n_{k-1}}[\cos(2\pi(n_k - n_{k-1})\xi_{k-1} +$$

$$+(\Omega + \omega_{n_k, n_{k-1}})\frac{t_k + t_{k-1}}{2}) + \cos(2\pi(n_k - n_{k-1})\xi_{k-1} - (\Omega - \omega_{n_k, n_{k-1}})\frac{t_k + t_{k-1}}{2}))](t_k - t_{k-1}).$$

P_f is normalizing constant.

The probability of a quantum transition of a system can be represented as a sum along trajectories from a real functional:

$$P(n_f,t_f|n_{in},t_{in}) = \sum_{n_1>m_1=1}^{N} \int_0^1\int_0^1 P_f \exp[i(S[n_f;n_1,\xi_1;n_{in},\xi_0] - S[n_f;m_1,\zeta_1;n_{in},\zeta_0])] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1,$$

$$P(n_f,t_f|n_{in},t_{in}) = \sum_{n_1>m_1=1}^{N} \int_0^1\int_0^1 P_f \cos[S[n_f;n_1,\xi_1;n_{in},\xi_0] - S[n_f;m_1,\zeta_1;n_{in},\zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 +$$

$$+ \sum_{n_1>m_1=1}^{N} \int_0^1\int_0^1 P_f i \sin[S[n_f;n_1,\xi_1;n_{in},\xi_0] - S[n_f;m_1,\zeta_1;n_{in},\zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1. \qquad (8.1)$$

$$\sum_{n_1>m_1=1}^{N} \int_0^1\int_0^1 P_f i \sin[S[n_f;n_1,\xi_1;n_{in},\xi_0] - S[n_f;m_1,\zeta_1;n_{in},\zeta_0]] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 = 0.$$

P_f is normalizing constant.(8.2)

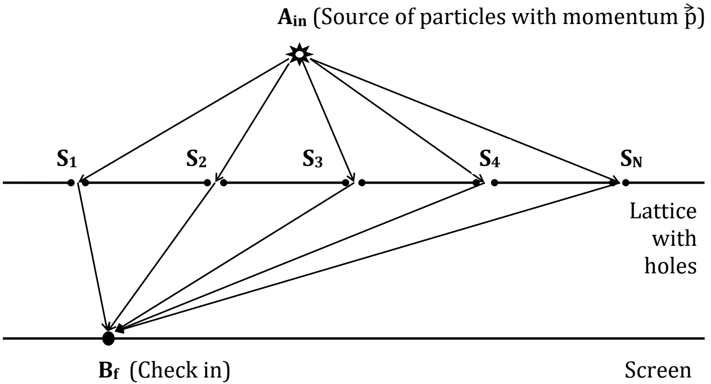
The equation is represented in the form

$$P(n_f, t_f | n_{in}, t_{in}) = \sum_{n_1, m_1 = 1}^{N} \int_0^1 \int_0^1 \int_0^1 \int_0^1 d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 \times$$

$$\times P_f \cos[S[n_f; n_1, \xi_1; n_{in}, \xi_0] - S[n_f; m_1, \zeta_1; n_{in}, \zeta_0]], \tag{9.1}$$

Biryukov A. A.,Degtyareva Y.V., Shleenkov M.A., Calculating the Probabilities of Quantum Transitions in Atoms and Molecules Numerically through Functional Integration, Bulletin of the Russian Academy of Sciences: Physics. 2018, **8**2, 12, p.1565-1569.

Diffraction of particles



Drawing 1. Particle diffraction

The amplitude of the particle transition from the source a_{in} to the point $_f$ of the screen is represented by the expression

$$\psi(b_f,t|a_{in}) = \sum_{n=1}^{N} \sqrt{P_f}\exp[\imath S[b_f,t;n,a_{in}]], \tag{11.1}$$

Where
$S[b_f,t;n,a_{in}] = \omega t - \mathbf{k}(\mathbf{r}_{an} + \mathbf{r}_{nb})$ — Dimensionless action of a particle along line passing through the points $\mathbf{a}, \mathbf{n}, \mathbf{b}$; $\mathbf{r}_{an}$ - line from source to the coordinates of the center of the hole of the diffraction grating; $\mathbf{r}_{nb}$—line from the coordinates of the center of the hole of the diffraction grating to the point on the screen ; $\mathbf{k}$—the wave vector of the particle; $\sqrt{P_f}$ -is the amplitude of the wave function.
The probability of a particle transition is determined by the formula:

$$P(b_f,t|a_{in}) = \psi(b_f,t|a_{in})\psi^*(b_f,t|a_{in}) = \sum_{m,n=1}^{N} P_f \exp[\imath(S[b_f,t;n,a_{in}] - S[b_f,t;m,a_{in}])]. \tag{11.2}$$

P_f is normalizing constant.

The equation takes the form

$$P(b_f,t|a_{in}) = \sum_{m,n=1}^{N} P_f \cos[(S[b_f,t;n,a_{in}] - S[b_f,t;m,a_{in}])]+$$

$$+\imath \sum_{m,n=1}^{N} P_f \sin[(S[b_f,t;n,a_{in}] - S[b_f,t;m,a_{in}])]. \tag{12.1}$$

$$\sum_{m,n=1}^{N} P_f \sin[(S[b_f,t;n,a_{in}] - S[b_f,t;m,a_{in}])] = 0 \tag{12.2}$$

Therefore

$$P(b_f,t|a_{in}) = \sum_{m,n=1}^{N} P_f \cos[(S[b_f,t;n,a_{in}] - S[b_f,t;m,a_{in}])]. \tag{12.3}$$

Transition probability density in coordinate representation.

The amplitude of the transition in space in the trajectory integration method is represented by the expression

$$\psi(b_f, t|a_{in}) = \int_{-\infty}^{\infty} \sqrt{P_f} \exp[\imath S[b_f, t; \mathbf{r}, a_{in}]] d[b_f, t; \mathbf{r}, a_{in}], \tag{13.1}$$

Where $S[b_f, t; \mathbf{r}, a_{in}]$ — dimensionless action of a particle along trajectory $[b_f, t; \mathbf{r}, a_{in}]]$ through the points $a, \mathbf{r}, b$; integration is carried out along all trajectories $[b_f, t; \mathbf{r}, a_{in}]]$. $\sqrt{P_f}$ -is the amplitude of the wave function.

The probability density of a particle transition is determined by the formula:

$$P(b_f, t|a_{in}) = \psi(b_f, t|a_{in})\psi^*(b_f, t|a_{in}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \exp[\imath(S[b_f, t; \mathbf{r}, a_{in}] - S[b_f, t; \mathbf{r}', a_{in}])] \times$$

$$\times d[b_f, t; \mathbf{r}, a_{in}] d[b_f, t; \mathbf{r}', a_{in}]. \tag{13.2}$$

P_f is normalizing constant.

The equation takes the form

$$P(b_f,t|a_{in}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \cos[(S[b_f,t;\mathbf{r},a_{in}] - S[b_f,t;\mathbf{r}',a_{in}])]d[b_f,t;\mathbf{r},a_{in}]d[b_f,t;\mathbf{r}',a_{in}]+$$

$$+\imath \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \sin[(S[b_f,t;\mathbf{r},a_{in}] - S[b_f,t;\mathbf{r}',a_{in}])]d[b_f,t;\mathbf{r},a_{in}]d[b_f,t;\mathbf{r}',a_{in}] \qquad (14.1)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \sin[(S[b_f,t;\mathbf{r},a_{in}] - S[b_f,t;\mathbf{r}',a_{in}])]d[b_f,t;\mathbf{r},a_{in}]d[b_f,t;\mathbf{r}',a_{in}] = 0, \qquad (14.2)$$

$$P(b_f,t|a_{in}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \cos[(S[b_f,t;\mathbf{r},a_{in}] - S[b_f,t;\mathbf{r}',a_{in}])]d[b_f,t;\mathbf{r},a_{in}]d[b_f,t;\mathbf{r}',a_{in}]. \qquad (14.3)$$

Probabilistic interpretation of the equation for the transition probability

We represent the equation for the transition probability in the form

$$P(b_f,t|a_{in}) = \sum_{m,n=1}^{N} g_{n,m} P(S[b_f,t;n,a_{in}],S[b_f,t;m,a_{in}]), \qquad (15.1)$$

$$where\ P(S[b_f,t;n,a_{in}],S[b_f,t;m,a_{in}]) = P_f \mid \cos[S[b_f,t;n,a_{in}]-S[b_f,t;m,a_{in}] \mid,$$

$$g_{n,m} = g(S[b_f,t;n,a_{in}],S[b_f,t;m,a_{in}]) =$$

$$= \cos[S[b_f,t;n,a_{in}]-S[b_f,t;m,a_{in}]| \cos[S[b_f,t;n,a_{in}]-S[b_f,t;m,a_{in}] \mid^{-1}$$

is the function takes the value +1 or -1, or 0 depending on the action values $S[b_f,t;n,a_{in}], S[b_f,t;m,a_{in}]$. We represent the equation in the form

$$P(b_f,t|a_{in}) = \sum_{n=1}^{N} P(b_f,n,a_{in}) + 2 \sum_{m<n=1}^{N} g_{n,m} P(S[b_f,t;n,a_{in}],S[b_f,t;m,a_{in}]), \qquad (15.2)$$

where $P(S[b_f,t;n,a_{in}],S[b_f,t;m,a_{in}])$- the probability of a couple of joint events. They are characterized by numbers $S[b_f,t;n,a_{in}], S[b_f,t;m,a_{in}]$. Summation is carried out for all pairs of joint trajectories.

Hypothesis.

There are systems whose transition probabilities between states in the trajectory integration method are expressed in terms of the probabilities of compatibility of three trajectories, four trajectories, etc.

$$P(n_f \mid n_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S[n_{in};n;n_f], S[n_{in};m;n_f]) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S[n_{in};n;n_f], S[n_{in};m;n_f], S[n_{in};k;n_f]) + \ldots +$$

$$+2^{N-1} g_{12\ldots N} P(S[n_{in};1;n_f], S[n_{in};2;n_f], ..., S[n_{in};N;n_f]). \qquad (16.1)$$

$$g_{n,m} = +1 \ or \ -1, or \ 0, n \neq m; \ g_{n,n} = +1, n = m; \ g_{nmk} = +1 \ or \ -1, or \ 0.$$

Where $P(S[n_{in};n;n_f], S[n_{in};m;n_f], S[n_{in};k;n_f])$ - the probabilities of compatibility of three trajectories.
The equation is constructed in the theory of stochastic processes in the space of joint events.
Biryukov A. A.,Equations of quantum theory in the space of randomly joint quantum events. EPJ Web of Conferences 222, year 2019, pages 03005, https://doi.org/10.1051/epjconf/201922203005

Model of the space of joint random events with symmetric difference and sum of events

KOLMOGOROV'S AXIOMS:

INITIAL CONCEPTS:

Ω — a set of elementary events ω, space of elementary events.

$\mathfrak{F}$ — a set of subsets from Ω, set of casual events A, B, C, ...

AXIOM I:

$\mathfrak{F}$ — is algebra of sets (exist $A \cup B, A \cap B, A \setminus B, \ldots$, which belong $\mathfrak{F}$).

AXIOM II:

The probability of an event (a set measure) is entered $P(A) \geq 0$.

AXIOM III:

$P(\Omega) = 1$.

AXIOM IV:

not joint events if follows $A \cap B = 0, P(A + B) = P(A) + P(B)$.

Joint events in Kolmogorov's axiomatics

Definition: two events S_1, S_2 are joint, if $S_1 \cap S_2 \neq 0$;
probability of association of joint events

$$P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2) \tag{19.1}$$

Definition: events $S_1, S_2, ..., S_N$ are joint, if $S_n \cap S_m \neq 0, S_n \cap S_m \cap S_k \neq 0, ..., S_1 \cap S_2 \cap ... \cap S_N \neq 0$, where $n, m, k, ... = 1, 2, ..., N$.
Probability of association of N joint events

$$P(\bigcup_{n=1}^{N} S_n) = \sum_{n=1}^{N} P(S_n) - \sum_{n<m=1}^{N} P(S_n \cap S_m) + \sum_{n<m<k=1}^{N} P(S_n \cap S_m \cap S_k) + \ldots + (-1)^{N-1} P(S_1 \cap ... \cap S_N). \tag{19.2}$$

Rozanov Yu. A.,Lectures on probability theory, 1968,Science,Moscow,p.120.

Symmetric difference joint events in Kolmogorov's axiomatics

Symmetric difference of two joint events

$$S_1^- = S_1 \setminus (S_1 \cap S_2), \;\; , \;\; S_2^- = S_2 \setminus (S_1 \cap S_2), \;\; P(S_1^- \cup S_2^-) = P(S_1) + P(S_2) - 2P(S_1 \cap S_2). \quad (20.1)$$

Symmetric difference of N joint events

$$P(\bigcup_{n=1}^{N} S_n^-) = \sum_{n=1}^{N} P(S_n) - 2 \sum_{n<m=1}^{N} P\big(S_n \cap S_m\big) +$$

$$+4 \sum_{n<m<k=1}^{N} P\big(S_n \cap S_m \cap S_k\big) + \ldots + (-2)^{N-1} P(S_1 \cap S_2 \cap \ldots \cap S_N). \quad (20.2)$$

Prohorov A. B., Ushakov V.G., Ushakov N.G., Problems in the theory of probability, 1986, Science,Moscow,p.328.

In addition to Kolmogorov's axioms. Let's define quantum joint events.

Two events S_1^q, S_2^q are quantum joint, if $S_1^q \cap S_2^q \neq 0$, probability of association of events

$$P(S_1^q \cup S_2^q) = P(S_1^q) + P(S_2^q) + P(S_1^q \cap S_2^q) \tag{21.1}$$

Symmetric sum of two quantum joint events is a postulate

$$S_1^+ = S_1^q \cup (S_1^q \cap S_2^q), \quad , \quad S_2^+ = S_2^q \cup (S_1^q \cap S_2^q) \tag{21.2}$$

$$P(S_1^+ \cup S_2^+) = P(S_1^q) + P(S_2^q) + 2P(S_1^q \cap S_2^q) \tag{21.3}$$

Symmetric association of N quantum events S_n^+ is sum:

$$P(\bigcup_{n=1}^{N} S_n^+) = \sum_{n=1}^{N} P(S_n^q) + 2 \sum_{n<m=1}^{N} P\big(S_n^q \cap S_m^q\big) -$$

$$-4 \sum_{n<m<k=1}^{N} P\big(S_n^q \cap S_m^q \cap S_k^q\big) + \ldots + (-2)^{N-1} P(S_1^q \cap S_2^q \cap \ldots \cap S_N^q). \tag{21.4}$$

The uniform equation for a symmetric difference and symmetric sums of random joint events

Let's designate joint events S_n, S_n^q one symbol S_n.
Let's designate events S_n^-, S_n^+ one symbol $\tilde{S}$.
These designations give the chance to write down the equations for $P(\bigcup_{n=1}^N S_n^-)$, $P(\bigcup_{n=1}^N S_n^+)$ in the form of one equation

$$P(\bigcup_{n=1}^{N} \tilde{S}_n) = \sum_{n=1}^{N} P(S_n) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_n \cap S_m) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_n \cap S_m \cap S_k) + \ldots + 2^{N-1} g_{12\ldots N} P(S_1 \cap S_2 \cap \ldots \cap S_N), \quad (22.1)$$

$$g_{n,m} = +1 \; or \; -1, g_{nmk} = -g_{n,m}, \ldots$$

The equation for for probability of transition between the random joint events

The equation for for probability of transition between the random not joint events

The event $A_{in}(t_{in})$ is realized at the moment time t_{in}. The event $B_f(t_f)$ is realized at the moment $t_f > t_{in}$. One of the events $S_n, n = 1, 2, ...N$, is realized at the moment t $(t_f > t > t_{in})$. Equation for events:

$$B_f \cap A_{in} = B_f \cap (\bigcup_{n=1}^{N} S_n) \cap A_{in} = \bigcup_{n=1}^{N} (B_f \cap S_n \cap A_{in}) = \bigcup_{n=1}^{N} S_{fni} \qquad (24.1)$$

The probability of transition from an event $A_{in}(t_{in})$ to an event $B_f(t_f)$ is defined equation

$$P(B_f \cap A_{in}) = P(B_f \cap (\bigcup_{n=1}^{N} S_n) \cap A_{in}) = P(\bigcup_{n=1}^{N} (B_f \cap (S_n) \cap A_{in})) = \sum_{n=1}^{N} P(B_f \cap S_n \cap A_{in}) \qquad (24.2)$$

It is possible to present in a look of Markov equation.

$$P(B_f \mid A_{in}) = \sum_{n=1}^{N} P(B_f \mid S_n) P(S_n \mid A_{in}) \qquad (24.3)$$

It is possible to write down in a look $B_f \cap S_n \cap A_{in} = S_{fni}$, $P(f \mid i) = \sum_{n=1}^{N} P(S_{fni})$

The equation for probabilities of transition in space of the random joint events

The equation for the random joint events we will write down in a look

$$B_f \cap A_{in} = B_f \cap (\bigcup_{n=1}^{N} \tilde{S}_n) \cap A_{in} = \bigcup_{n=1}^{N} (B_f \cap \tilde{S}_n \cap A_{in}) = \bigcup_{n=1}^{N} \tilde{S}_{fni}. \tag{25.1}$$

We will designate $B_f \cap \tilde{S}_n \cap A_{in} = \tilde{S}_{fni}$, $B_f \cap S_n \cap A_{in} = S_{fni}$.
It is possible to present the offered equation in a look

$$P(B_f \mid A_{in}) = P(\bigcup_{n=1}^{N} \tilde{S}_{fni}) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}) +$$
$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots +$$
$$+2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}), \tag{25.2}$$
$$g_{n,m} = +1 \; or \; -1, or 0.$$

The equation for probabilities of transition in space of the random joint events

It is possible to present the offered equation in a look

$$P(B_f \mid A_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots + 2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}), \tag{26.1}$$

$$g_{n,m} = +1 \ or \ -1, or \ 0, n \neq m; \ g_{n,n} = +1, n = m; \ g_{nmk} = +1 \ or \ -1, or \ 0.$$

Biryukov A. A.,Equations of quantum theory in the space of randomly joint quantum events.
EPJ Web of Conferences 222, year 2019, pages 03005, https://doi.org/10.1051/epjconf/201922203005

Interpretation of the stochastic process equation for joint events

Model of physical system

Let's consider system which can stay in states $n = 1, 2..., N$.
The system eventually makes transitions between states.
The transition of the system from the state n_{in} at the time t_{in} to the state n_f at the time $t_f > t_{in}$ is carried out along a certain random trajectory determined by the set of numbers $n_{in}, n_1, n_2, ...n_f$.
Action of system is determined for each trajectory:

$$S[n_{in}, n_1, n_2, ..., n_N, n_f]$$

.
Action of system we shall present as: $S[n_{in}, n, n_f]$ where $n(n_1, n_2, ..., n_N)$ a multiindex.

Interpretation of the stochastic process equation for joint event

To events B_f, A_{in} we put in conformity of the states n_f, n_{in}.
To events S_{fni} we put in conformity the action of system $S[n_{in}; n; n_f]$.
The equation in space of the states of system becomes

$$P(n_f \mid n_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S[n_{in}; n; n_f], S[n_{in}; m; n_f]) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S[n_{in}; n; n_f], S[n_{in}; m; n_f], S[n_{in}; k; n_f]) + \ldots +$$

$$+2^{N-1} g_{12\ldots N} P(S[n_{in}; 1; n_f], S[n_{in}; 2; n_f], ..., S[n_{in}; N; n_f]). \qquad (29.1)$$

$$g_{n,m} = +1 \; or \; -1, or \; 0, n \neq m; \; g_{n,n} = +1, n = m; \; g_{nmk} = +1 \; or \; -1, or \; 0.$$

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Probabilities of transition for in pairs joint events

The equation in space of the states of system becomes

$$P(n_f \mid n_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S[n_{in}; n; n_f], S[n_{in}; m; n_f]) \tag{30.1}$$

.

$$g_{n,m} = +1 \ or \ -1, or \ 0, n \neq m; \ g_{n,n} = +1, n = m$$

Probabilities of transition for in pairs joint events

We do a postulate

$$P(S[n_{in};n;n_f],S[n_{in};m;n_f]) = g_{nm}P_f \mid \cos[S_{fni} - S_{fmi}] \mid \tag{31.1}$$

$$g_{nm} = \cos[S_{fni} - S_{fmi}] | \cos[S_{fni} - S_{fmi}] \mid^{-1} \tag{31.2}$$

Therefore

$$P(n_f \mid n_{in}) = \sum_{n,m=1}^{N} P_f \cos[S_{fni} - S_{fmi}]. \tag{31.3}$$

Results

1. The system of axioms of probability theory is supplemented by a new axiom on joint quantum random events.

2. In the new set of general events we have the equation for stochastic process

$$P(f \mid i) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_{fni} \cap S_{fm}) +$$
$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots + 2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}), \quad (32.1)$$

3. The equation in space of the states of system becomes

$$P(n_f \mid n_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S[n_{in}; n; n_f], S[n_{in}; m; n_f]) +$$
$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S[n_{in}; n; n_f], S[n_{in}; m; n_f], S[n_{in}; k; n_f]) + \ldots +$$
$$+2^{N-1} g_{12\ldots N} P(S[n_{in}; 1; n_f], S[n_{in}; 2; n_f], \ldots, S[n_{in}; N; n_f]). \quad (32.2)$$

Results

4. For the case when events in the process are only joint in pairs

$$P(S_{fni} \cap S_{fmi} \cap S_{fki}) = 0, \;\; P(S_{f1i} \cap S_{f2i} \cap ... \cap S_{fNi}) = 0$$

The equation has the form

$$P(B_f \cap A_i) = \sum_{n=1}^{N} P(S_n) + 2 \sum_{n>m=1}^{N} g(S_n, S_m) P(S_n \cap S_m) \qquad (33.1)$$

The equation in space of the states of system becomes

$$P(n_f \mid n_{in}) = \sum_{n,m=1}^{N} P_f \cos[S_{fni} - S_{fmi}]. \qquad (33.1)$$

This equation describes real quantum processes (particle diffraction, quantum transitions, and others).

5. There is a prospect for exploring a new space, a new equation.

Thanks for your attention!