Particle shadows of black holes and new characteristic surfaces

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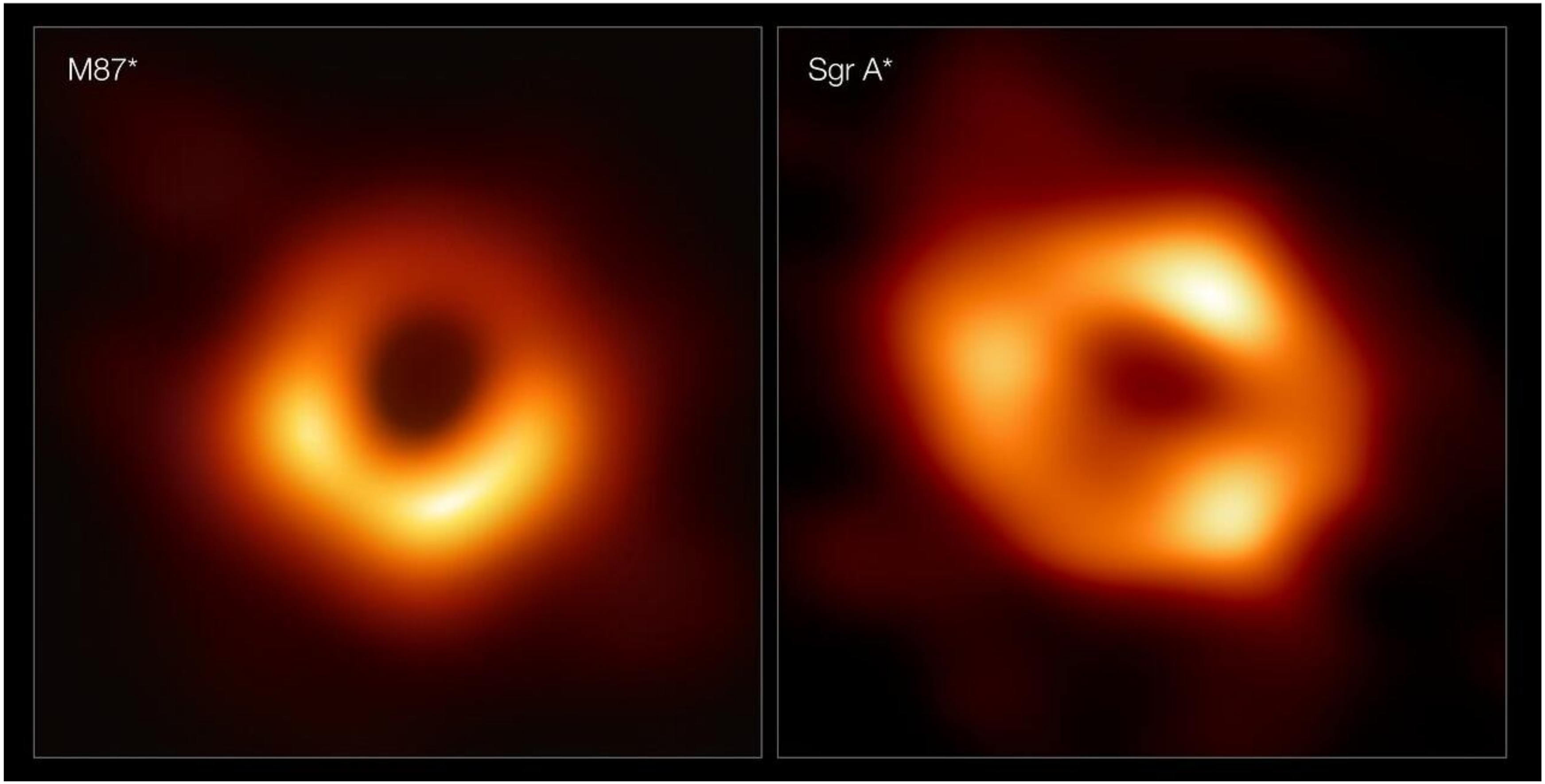
Black hole astronomy breakthrough



[K. Akiyama et al. [Event Horizon Telescope], Astrophys. J. Lett. 875, L1 (2019) [arXiv:**1906.11238**]]
[P. Kocherlakota et al. [Event Horizon Telescope], Phys. Rev. D 103, no.10, 104047 (2021) [arXiv:**2105.09343**]]

Photon surface

Definition. A *photon surface* of (M, g) is an immersed, nowhere-spacelike hypersurface S of (M, g) such that, for every point $p \in S$ and every null vector $\boldsymbol{k} \in T_pS$, there exists a null geodesic $\gamma\colon (-\epsilon, \epsilon) \to M$ of (M, g) such that $\dot{\gamma}(0) = \boldsymbol{k}$, $\gamma \subset S$.

[C.-M. Claudel, K.S. Virbhadra, G.F.R. Ellis, J.Math.Phys. 42 (2001) 818-838, arXiv:**gr-qc/0005050**]

- That is, if a null geodesic is tangent to the photon surface, it completely belongs to the photon surface.

- Photon surface is umbilic:

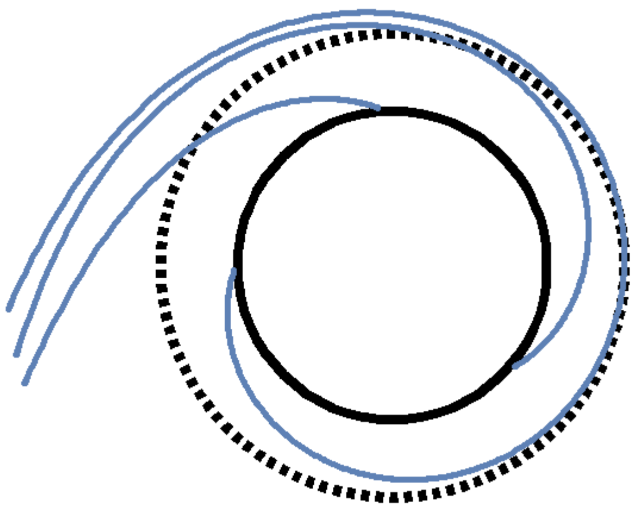
$\chi_{\mu\nu} = \left(\nabla_\mu n_\nu\right)_\parallel$ is a second fundamental form of S.

$$k^\mu \chi_{\mu\nu} k^\nu = 0,$$

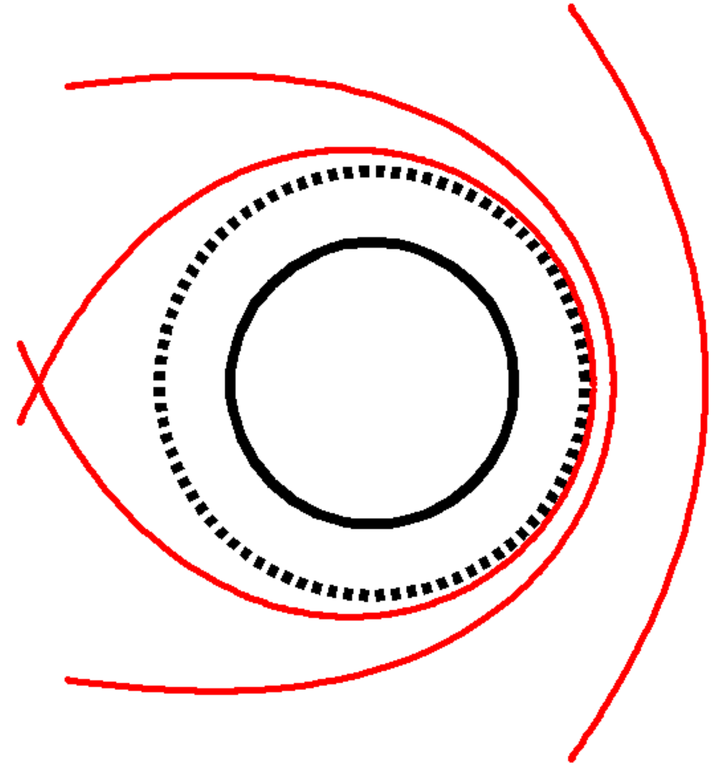
for any null $k^\mu \in T_pS$ and $p \in S$

Photon surfaces

Null geodesics intersecting PS



Null geodesics approaching PS



See for other generalization and application in shadow construction:

[Igor Bogush et. al., “Photon surfaces, shadows and accretion disks in gravity with minimally coupled scalar field,” arXiv: **2205.01919**]

Charged massive particle surfaces

How to generalize photon surfaces to the surfaces for charged massive particles?

- Let κ^μ is a Killing vector (timelike at infinity $r \to \infty$)
- Let γ is a worldline for a particle with mass m and charge q
- Let $\mathcal{E} = \dot{\gamma}^\mu \kappa_\mu + q A^\mu \kappa_\mu$ is an integral of motion

κ^α may be just a projection of the Killing vector onto the surface $\Rightarrow$ dynamical spacetimes

- **Definition**. A *charged particle surface* of (M, g) is an immersed, timelike, hypersurface S of M such that, for every point $p \in S$ and every vector $v^\alpha|_p \in T_p S$ such that

$$v^\alpha \kappa_\alpha + q A^\mu \kappa_\mu \Big|_p = \mathcal{E} \text{ and } v^\alpha v_\alpha = -m^2,$$

there exists a worldline γ of M for a particle with mass m, electric charge q and total energy $\mathcal{E}$ such that $\dot{\gamma}^\alpha(0) = v^\alpha|_p$ and $\gamma \subset S$.

Charged massive particle surfaces

- A charged particle surface is characterized by
  - mass m
  - charge q
  - total energy $\mathcal{E}$ defined with respect to Killing vector κ^α
- Worldline equation

$$\dot{\gamma}^\mu \nabla_\mu \dot{\gamma}^\nu = qF^\nu{}_\mu \dot{\gamma}^\mu$$

- No normal acceleration with respect to the surface

$$v^\mu \chi_{\mu\nu} v^\nu = -qn^\nu F_{\nu\mu} v^\mu$$

n^μ is a unit normal to the surface

Charged massive particle surfaces

- This implies a condition on the second fundamental form

$$\chi_{\alpha\beta} = \frac{\chi_0}{n-2} H_{\alpha\beta} - \frac{q}{\mathcal{E}_k} \mathcal{F}_{\alpha\beta},$$

$n = \dim M$ is a dimension of the spacetime

where

$$H_{\alpha\beta} = h_{\alpha\beta} + \frac{m^2}{\mathcal{E}_k^2} \kappa_\alpha \kappa_\beta,$$

$$\mathcal{E}_k = \mathcal{E} - q A^\mu \kappa_\mu,$$

$$\mathcal{F}_{\alpha\beta} = \frac{1}{2} \kappa_{(\alpha} n^\mu F_{\mu\nu} h^\nu_{\beta)},$$

$$\chi_0 = \frac{n-2}{H^\alpha_\alpha} \left(\chi^\alpha_\alpha + \frac{q \mathcal{F}^\alpha_\alpha}{\mathcal{E}_k} \right).$$

$h_{\alpha\beta}$ is an induced metric on S

No weight in symmetrization

A closer look at CMPS

- Component along the Killing vector

$$\kappa^\alpha \chi_{\alpha\beta} \kappa^\beta = \kappa^2 (\chi^\alpha_\alpha - \chi_0)$$

- Components along the orthogonal directions

$$\tau_i^\alpha \chi_{\alpha\beta} \tau_j^\beta = \frac{\chi_0}{n-2} \tau_{(i)}^\alpha h_{\alpha\beta} \tau_j^\beta$$

Partially umbilic along orthogonal directions τ_i^α

- Mixed components

$$\tau_{(i)}^\alpha \chi_{\alpha\beta} \kappa^\beta = -\frac{q}{2\mathcal{E}_k} \kappa^2 n^\mu F_{\mu\nu} \tau_{(i)}^\nu$$

If $n^\mu F_{\mu\nu}$ is parallel to the Killing vector κ_ν, the Maxwell field enters the equations through $\mathcal{E}_k = \mathcal{E} - q\kappa^\alpha A_\alpha$ only, which is the case for most of the interesting solutions with CMPSs.

A closer look at CMPS

- Mixed components

$$\tau^{\alpha}_{(i)}\chi_{\alpha\beta}\kappa^{\beta} = -\frac{q}{2\mathcal{E}_k}\kappa^2 n^{\mu}F_{\mu\nu}\tau^{\nu}_{(i)}$$

If $n^{\mu}F_{\mu\nu}$ is parallel to the Killing vector κ_{ν}, the Maxwell field enters the equations through $\mathcal{E}_k$=$\mathcal{E} - q\kappa^{\alpha}A_{\alpha}$ only.

For $A_{\mu}dx^{\mu} = A_t(r)dt + A_{\phi}(\theta)d\phi$ and $r = const$ surfaces it is always the case.

- Traceless part

$$\sigma_{\alpha\beta} = \chi_{\alpha\beta} - h_{\alpha\beta}\frac{\chi^{\gamma}_{\gamma}}{n-1} \sim \frac{\chi_0 m}{(n-2)\mathcal{E}_k}\kappa_{\lambda} - \frac{q}{m}n^{\mu}F_{\mu\lambda}$$

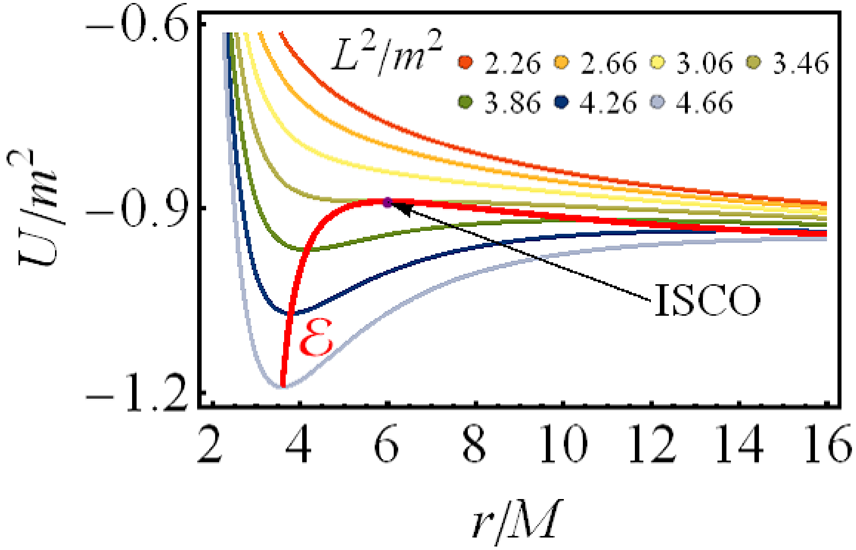
For charged particles it can be zero for some $\mathcal{E}$ $\Rightarrow$ the surface coincide with a photon surface.

Stability

- $\mathcal{E}$ is a function of the surface
- **Claim:** $\frac{\mathrm{d}\mathcal{E}}{dr} = 0$ gives marginally stable orbits, such as innermost stable circular orbits (ISCO)
- **Example:** Schwarzschild

$$\left(\frac{dr}{ds}\right)^2 = \mathcal{E}^2 - U(r),$$

$$U(r) = \frac{(r-2M)(L^2 + m^2 r^2)}{r^3}$$



Example 1: Schwarzschild and Fisher

Fisher metric:

$$ds^2 = -f^{\sigma} dt^2 + f^{-\sigma} dr^2 + f r^2 d\Omega_2^2,$$

$$F = 1 - \frac{2M}{\sigma r}, \quad 0 < \sigma \leq 1$$

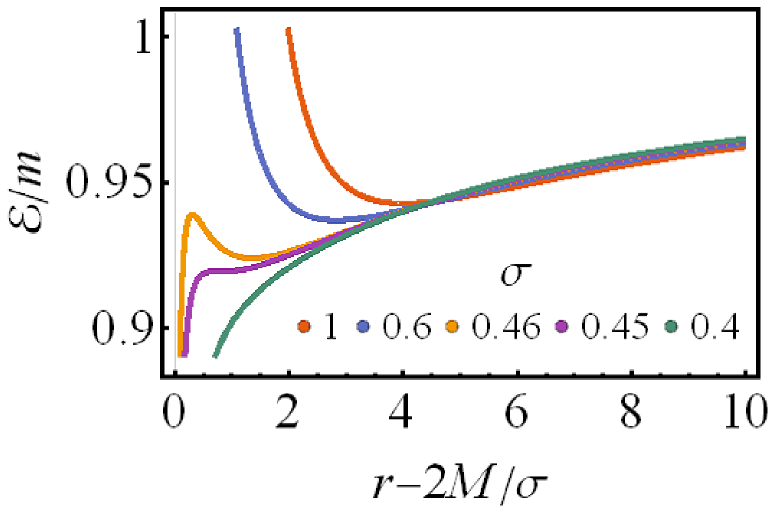
Energy of the particle surface:

$$\frac{\mathcal{E}^2}{m^2} = \left(1 - \frac{2\,M}{r\,\sigma}\right)^{\sigma} \frac{M + M\,\sigma - r\,\sigma}{M + 2\,M\,\sigma - r\,\sigma}$$

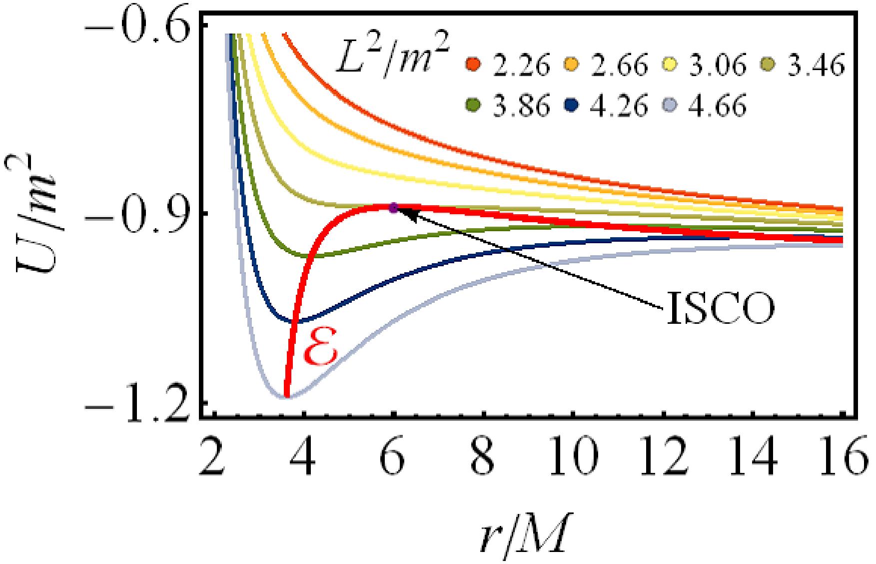
ISCO:

$$\frac{r_{ISCO}}{M} = \sigma^{-1} + 3 + \sqrt{5 - \sigma^{-2}}$$

Example 1: Schwarzschild and Fisher



Energy of the particle surface for different values of σ in Fisher metric.

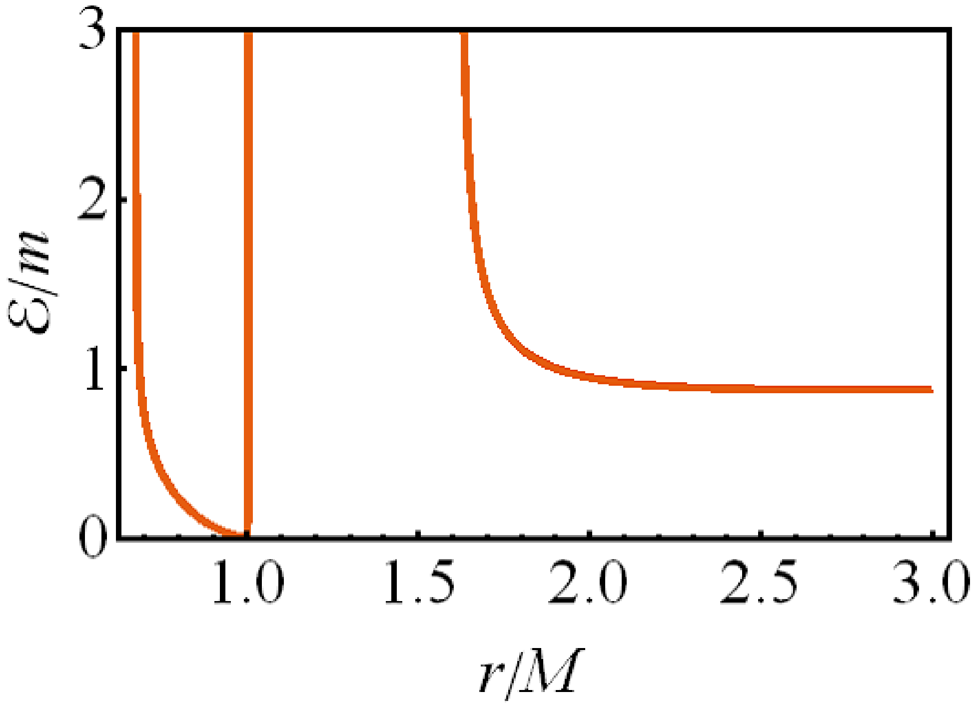


Radial potential in Schwarzschild metric for different angular momentum L. Red curve is the energy of the particle surface.

Example 2: EMD

EMD naked singularities can have stable photon orbits.

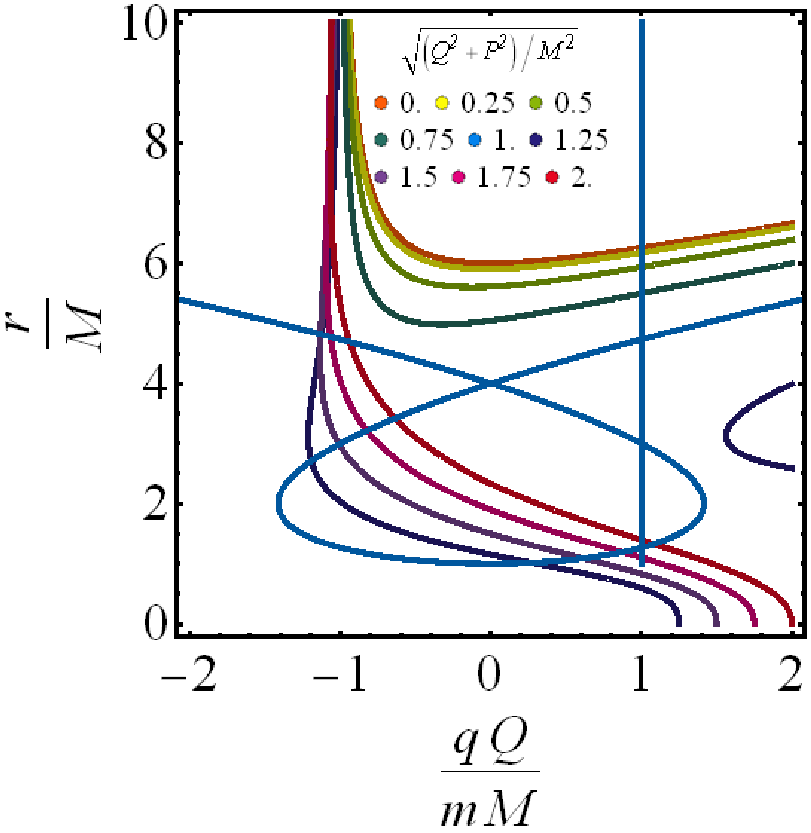
[I. Bogush, G. Clément, D. Gal'tsov, D. Torbunov, Phys. Rev. D 103, 064045 (2021), arXiv:**2009.07922**]



$M = 1,\qquad N = 0,\qquad Q = -1.49,\qquad P = 0.15,\qquad D = -0.65$

Example 3: Reissner-Nordström

- Structure of ISCO dramatically differs for the RN black hole, extreme RN black hole and RN naked singularity.
- For the supersymmetric state $M^2 = Q^2 + P^2$, and the no-force condition $\frac{qQ}{mM} = 1$, energy of the CMPSs is constant. The test particle does not interact with the black hole.
- Repulsive particles can lay on the photon sphere.
- Marginally stable orbits may be degenerate $\frac{d^2\mathcal{E}}{dr^2} = 0$.



Marginally stable orbits as a function of qQ/mM for different $\sqrt{\frac{P^2+Q^2}{M^2}}$

Conclusions

- We have suggested a generalization of the photon surfaces to the charged massive particle surfaces. Geometrical conditions for the existence of such surfaces are given. Nonstationary spacetimes can be described by them as well.
- Such surfaces are partially umbilic.
- Each surface is characterized by the energy, mass and charge of the corresponding particles.
- Surfaces with the locally extremal energy contain marginally stable orbits.
- In Fisher metric, marginally stable orbits exist for $\sigma > 1/\sqrt{5}$.
- Photon surfaces correspond to the singularity of the $\mathcal{E}/m$ function.
- There are degenerate marginally stable orbits in Reissner-Nordström metric.
- No-force condition can be formulated as $\mathcal{E} = \text{const}$.

Thank you
for your attention!