Radiation of an electron in a Lorentz-violating vacuum

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1. INTRODUCTION

The Standard Model (SM) is complete but it is not a complete theory due to a number of fundamental problems that cannot be solved in its framework: it does not include gravity; no explanation of charge quantization; too many input parameters; a huge hierarchy of particle masses and energy scales of interactions; a generation problem; no solutions to dark matter and dark energy, baryon asymmetry in the Universe; etc. These problems stimulate the development of theories generalizing the SM. Some of these theories include violation of Lorentz invariance, among which we single out the effective field theory, which is called the Standard Model Extension (SME) (D. Colladay, V. A. Kostelecký, PRD (1997, 1998); V. A. Kostelecký, N. Russell, RMP (2011), arXiv (2022)).

The SME Lagrangian is the sum of the SM Lagrangian and additional terms representing various combinations of SM fields with free Lorentzian indices (this violates Lorentz invariance), which are convoluted with constant tensors of the corresponding ranks and mass dimensions. Such a structure of the Lagrangian expands the concept of effective field theory [S. Weinberg, 1979], and the indicated tensors, considered as constant background fields, simulate the complex structure of the vacuum induced by the new physics beyond the SM (in particular, the effects of quantum gravity).

Various effects have been investigated within the SME framework, and we note only a few works, limited to the case of an electron interacting with an axial-vector background field (AVBF): production of an electron-positron pair by a photon and emission of a photon by an electron and a positron [V. Ch. Zhukovsky, A. E. Lobanov, E. M. Murchikova (2006, 2007)], synchrotron radiation of an electron taking into account its anomalous magnetic moment and interaction with the AVBF [I. E. Frolov, V. Ch. Zhukovsky (2007)], effect of the AVBF on the radiation of a hydrogen-like atom [O. G. Kharlanov, V. Ch. Zhukovsky (2007)], generation of a vacuum current by the AVBF [A. F. Bubnov, N. V. Gubina, V. Ch. Zhukovsky (2017)].

In the present talk, based on the following publications:
A. V. Borisov, T. G. Kiril'tseva, Mosc. Univ. Phys. Bull. **75**, 10 (2020),
A. V. Borisov, Eur. Phys. J. C **82**, 460 (2022),
I consider the electromagnetic radiation of an electron moving in a tensor background field of a quasi-magnetic type with use of the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{T}}, \tag{1}$$

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(\gamma^{\mu}\left(i\partial_{\mu} + eA_{\mu}\right) - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_{\mu}A^{\mu})^2 \tag{2}$$

is the Lagrangian of the standard QED in the Lorentz gauge, ψ is the electron-positron field (m and $-e < 0$ are the electron mass and charge), A^{μ} and $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ — 4-potential and tensor of the electromagnetic field strength;

$$\mathcal{L}_{\mathrm{T}} = -\frac{1}{2}\bar{\psi}\sigma^{\mu\nu}H_{\mu\nu}\psi \tag{3}$$

is the Lagrangian of interaction with a tensor constant background field $H_{\mu\nu}$.

2. ELECTRON WAVE FUNCTIONS IN A BACKGROUND FIELD

The wave function of an electron in a tensor background field satisfies the Dirac equation:

$$\left(i\gamma^\mu \partial_\mu - m - \frac{1}{2}\sigma^{\mu\nu} H_{\mu\nu}\right)\psi = 0. \tag{4}$$

We consider the case of a background field of the quasi-magnetic type for which

$$H^{\mu\nu} H_{\mu\nu} > 0, \; H^{\mu\nu} \widetilde{H}_{\mu\nu} = 0,$$

$\widetilde{H}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} H^{\alpha\beta}/2$ ($\varepsilon_{0123} = -\varepsilon^{0123} = -1$). In a special reference frame, the nonzero components of the tensors are as follows:

$$H_{21} = -H_{12} = h, \; \widetilde{H}_{03} = -\widetilde{H}_{30} = h, \tag{5}$$

the tensor field is equivalent to the axial vector (we put $h > 0$)

$$\mathbf{h} = h\mathbf{e}_z, \; h = \left[H^{\mu\nu} H_{\mu\nu}/2\right]^{1/2}. \tag{6}$$

The Dirac equation in the Hamiltonian form:

$$i\frac{\partial\psi}{\partial t} = \widehat{H}\psi,\ \widehat{H} = \boldsymbol{\alpha}\cdot\widehat{\mathbf{p}} + m\beta - \beta\Sigma_3 h, \tag{7}$$

$\widehat{\mathbf{p}} = -i\nabla$, $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, $\beta = \gamma^0$, $\Sigma_3 = i\gamma^1\gamma^2$.
The corresponding wave function:

$$\psi_{\mathbf{p}\zeta}(t,\mathbf{r}) = \frac{1}{\sqrt{V}} u(\mathbf{p},\zeta)\exp\left(-iEt + i\mathbf{p}\cdot\mathbf{r}\right),$$

$$u(\mathbf{p},\zeta) = 2^{-3/2}\begin{pmatrix} A_+\left(B_+ + \zeta B_-\right) \\ -\zeta A_-\left(B_+ - \zeta B_-\right)e^{i\phi} \\ A_+\left(B_+ - \zeta B_-\right) \\ \zeta A_-\left(B_+ + \zeta B_-\right)e^{i\phi} \end{pmatrix}. \tag{8}$$

Here V is the normalization volume;

$$A_\pm = \left(1 \pm \zeta\frac{m}{\varepsilon_\perp}\right)^{1/2}, \quad B_\pm = \left(1 \pm \frac{p_z}{E}\right)^{1/2}. \tag{9}$$

This wave function describes the stationary state of an electron in the background field and is an eigenfunction of the Hamiltonian $\widehat{H}$, the momentum operator $\widehat{\mathbf{p}}$, and the operator of the spin projection onto the direction $\mathbf{h}$ (Oz axis),

$$\widehat{\Pi} = \gamma^5 \gamma^\mu \widetilde{H}_{\mu\nu} p^\nu / h = \gamma^5 \left(\gamma^0 p_z - \gamma^3 E \right), \tag{10}$$

where $p^\nu = (E, \mathbf{p})$; the spin quantum number $\zeta = \pm 1$ is related to the eigenvalue of the operator (10) by the relation

$$\widehat{\Pi} \psi_{\mathbf{p}\zeta} = (\zeta \varepsilon_\perp - h) \psi_{\mathbf{p}\zeta};$$

the electron energy

$$E = \left[(\varepsilon_\perp - \zeta h)^2 + p_z^2 \right]^{1/2} \tag{11}$$

depends on ζ, the longitudinal p_z and transverse $p_\perp = \sqrt{p_x^2 + p_y^2}$ (via $\varepsilon_\perp = \sqrt{m^2 + p_\perp^2}$) components of the momentum $\mathbf{p}$; the angle ϕ in (8) specifies the direction of the transverse momentum $\mathbf{p}_\perp = (p_x, p_y) = p_\perp(\cos\phi, \sin\phi)$.

Note that the function (8) can be obtained from the wave function of a neutron moving in an external constant magnetic field [I. M. Ternov, V. G. Bagrov, A. M. Khapaev, Sov. Phys. JETP **21**, 613 (1965)] by replacing

$$\mu_n F_{\mu\nu} \to H_{\mu\nu},$$

where μ_n is the anomalous magnetic moment of the neutron, with the corresponding change in notations.

3. RADIATIVE TRANSITION AMPLITUDE

Let us consider electron transition $|i\rangle = |\mathbf{p},\zeta\rangle \rightarrow |f\rangle = |\mathbf{p}',\zeta'\rangle$ with the emission of a photon with 4-momentum $k^\mu = (\omega, \mathbf{k})$ and polarization vector $\mathbf{e}_\lambda$. The amplitude of this radiative transition:

$$S_{fi} = \frac{ie}{\sqrt{2\omega V}} \frac{(2\pi)^4}{V} \delta\left(E' + \omega - E\right) \delta^{(3)}\left(\mathbf{p}' + \mathbf{k} - \mathbf{p}\right) \mathbf{e}_\lambda^* \cdot \langle \boldsymbol{\alpha} \rangle\,, \qquad (12)$$

where

$$\langle \boldsymbol{\alpha} \rangle = u^+\left(\mathbf{p}',\zeta'\right) \boldsymbol{\alpha} u\left(\mathbf{p},\zeta\right). \qquad (13)$$

The frequency of a photon emitted in the direction $\mathbf{n} = \mathbf{k}/\omega$ ($|\mathbf{n}| = 1$) is derived from the energy and momentum conservation law:

$$\omega = \frac{2h\sqrt{1 - v_z^2}}{1 - \mathbf{n}\cdot\mathbf{v}} \delta_{\zeta',1}\delta_{\zeta,-1}. \qquad (14)$$

This frequency is nonzero only in the case of an electron spin-flip transition: $\zeta = -1 \rightarrow \zeta' = 1$.

Expression (14) was obtained in the leading (linear) approximation in the background field strength h in view of the rigid constraint [V. A. Kostelecký, N. Russell, RMP (2011), arXiv (2022)]:

$$h \lesssim 10^{-17}\ \mathrm{eV}. \tag{15}$$

In this approximation, $\mathbf{v} = \mathbf{p}/\varepsilon$ is the velocity of a free electron:

$$\varepsilon = E\,(h = 0) = \sqrt{m^2 + \mathbf{p}^2}, \tag{16}$$

v_z is its projection onto $\mathbf{h}$.

Using transition amplitude (10) we find radiation power

$$W^{(\lambda)} = \frac{\alpha}{2\pi} \int d^3 k \delta \left(E' + \omega - E \right) \left| \mathbf{e}_\lambda^* \cdot \langle \boldsymbol{\alpha} \rangle \right|^2 , \qquad (17)$$

where $\mathbf{p}' = \mathbf{p} - \mathbf{k}$.
The quasi-magnetic background field configuration is invariant with respect to Lorentz transformations (boosts) along the field direction (axis Oz). Therefore, we limit ourselves to the analysis of two different orientations of the initial electron momentum with respect to Oz: (i) $\mathbf{p} \perp Oz$ ($p_z = 0$) and (ii) $\mathbf{p} \parallel Oz$ ($p_\perp = 0$).
As in the synchrotron radiation theory, we introduce unit vectors of σ and π components of linear polarization to characterize the radiation polarization:

$$\mathbf{e}_\sigma = \frac{\mathbf{e}_z \times \mathbf{n}}{|\mathbf{e}_z \times \mathbf{n}|}, \quad \mathbf{e}_\pi = \mathbf{n} \times \mathbf{e}_\sigma , \qquad (18)$$

where $\mathbf{n} = \mathbf{k}/\omega$.

Transverse Motion

Assuming that $p_z = 0$ and $\mathbf{p} = p\mathbf{e}_x$ (i.e., $\varphi = 0$), which does not lead to loss of generality, since the background field is axially symmetric. Taking only the first order of expansion in background field h into account we find (for $\zeta' = -\zeta = 1$):

$$\langle \alpha_1 \rangle = \frac{1}{2} \frac{k_z}{\gamma \varepsilon}, \quad \langle \alpha_2 \rangle = \frac{i}{2} \frac{k_z}{\varepsilon}, \quad \langle \alpha_3 \rangle = -\frac{1}{2} \left(\frac{k_x}{\gamma \varepsilon} + i \frac{k_y}{\varepsilon} \right), \qquad (19)$$

where the Lorentz factor $\gamma = \varepsilon / m = 1/\sqrt{1 - v^2}$.
Note that matrix elements satisfy the relation

$$\omega \langle \alpha_0 \rangle - \mathbf{k} \cdot \langle \boldsymbol{\alpha} \rangle = 0, \qquad (20)$$

which follows ($\alpha_0 = I$ and $\langle \alpha_0 \rangle = 0$ in the present case) from the conservation of electromagnetic current in the momentum representation.

Assuming that $d^3k = \omega^2 d\omega d\Omega$, we find the angular distribution of power of polarized radiation:

$$\frac{dW^{(\lambda)}}{d\Omega} = \frac{\alpha}{2\pi}\frac{\omega^2}{1-\mathbf{n}\cdot\mathbf{v}}\left|\mathbf{e}^*_\lambda \cdot \langle\boldsymbol{\alpha}\rangle\right|^2 . \tag{21}$$

Summing over polarizations, $\sum_\lambda e^{*i}_\lambda e^k_\lambda = \delta^{ik} - n^i n^k$, we find

$$\frac{dW}{d\Omega} = \frac{\alpha}{2\pi}\frac{\omega^2}{1-\mathbf{v}\cdot\mathbf{n}}\left|\langle\boldsymbol{\alpha}\rangle\right|^2 , \tag{22}$$

where it is taken into account that $\langle\boldsymbol{\alpha}\rangle \cdot \mathbf{n} = 0$. Substitution here expressions for $\langle\alpha_k\rangle$ gives

$$\frac{dW}{d\Omega} = W_0\left(1-v^2\right)\frac{1-n_x^2+\left(1-v^2\right)\left(1-n_y^2\right)}{\left(1-vn_x\right)^5}, \tag{23}$$

$$d\Omega = \sin\theta d\theta d\varphi; \quad n_x = \sin\theta\cos\varphi, n_y = \sin\theta\sin\varphi, n_z = \cos\theta. \tag{24}$$

$$W_0 = \frac{2\alpha}{\pi}\frac{h^4}{m^2}. \tag{25}$$

It follows from (23) that an electron at rest (v = 0) also emits radiation, since this emission is induced by the electron spin flip:

$$\frac{dW}{d\Omega}(v=0) = W_0\left(1+n_z^2\right). \tag{26}$$

Figures 1 and 2 show the normalized angular distribution of radiation $R(v,\mathbf{n}) = W_0^{-1}dW/d\Omega$ for an electron at rest ($v=0$) and a relativistic electron ($v=0.85$, respectively. The well-known "projector" effect is seen in the latter case.

Fig. 1. Angular radiation distribution at $v = 0$

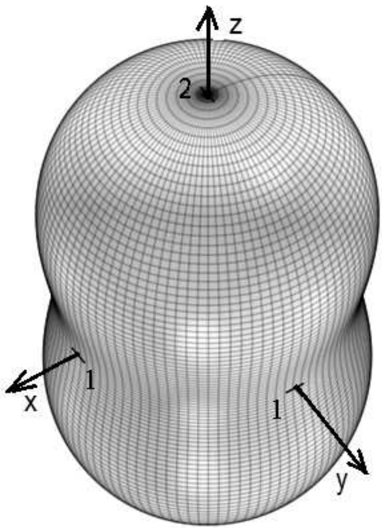
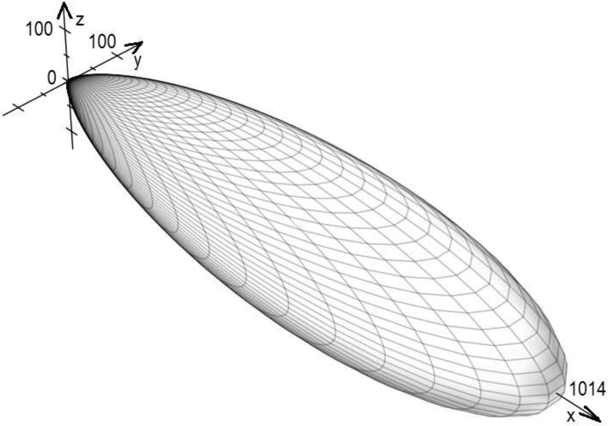


Fig. 2. Angular radiation distribution at $v = 0.85$



Let us consider the polarization properties of radiation. The linear polarization vectors:

$$\mathbf{e}_\sigma = -\sin\varphi\,\mathbf{e}_x + \cos\varphi\,\mathbf{e}_y,$$
$$\mathbf{e}_\pi = -\cos\theta\cos\varphi\,\mathbf{e}_x - \cos\theta\sin\varphi\,\mathbf{e}_y + \sin\theta\,\mathbf{e}_z. \tag{27}$$

The angular distributions of radiation powers of linear polarization components:

$$\frac{dW^{(\sigma)}}{d\Omega} = W_0\left(1-v^2\right)\frac{\cos^2\theta\left(1-v^2\sin^2\varphi\right)}{\left(1-v\sin\theta\cos\varphi\right)^5},$$
$$\frac{dW^{(\pi)}}{d\Omega} = W_0\left(1-v^2\right)\frac{1-v^2\cos^2\varphi}{\left(1-v\sin\theta\cos\varphi\right)^5}. \tag{28}$$

Their sum is given by (23), as must be the case.

The total radiation powers of linear polarization components:

$$W^{(\sigma)} = \frac{\pi}{3} W_0 \gamma^4 (4 - 2v^2 + 3v^4 - v^6),$$
$$W^{(\pi)} = \frac{\pi}{3} W_0 \gamma^4 (12 + 18v^2 - 3v^4 + v^6). \tag{29}$$

The total power:

$$W = W^{(\sigma)} + W^{(\pi)} = \frac{16\pi}{3} W_0 \gamma^4 (1 + v^2), \tag{30}$$

The radiation is linearly polarized (π component is predominant) and the polarization degree:

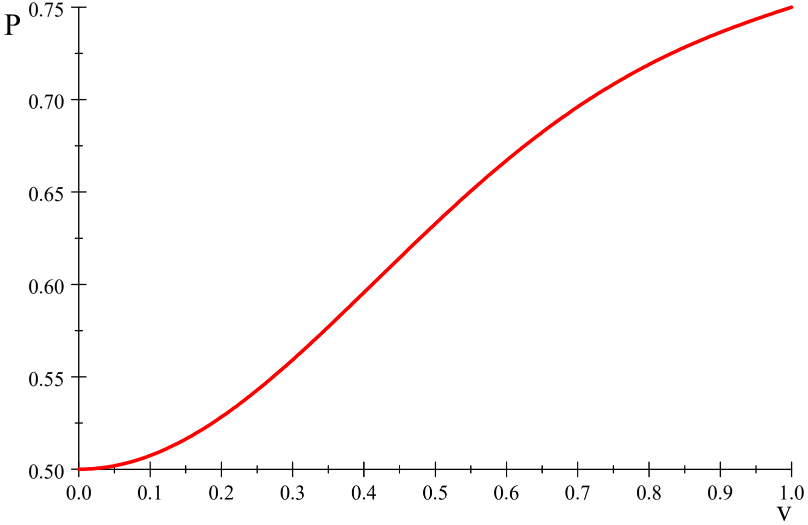
$$P = \frac{W^{(\pi)} - W^{(\sigma)}}{W} = \frac{4 + 10v^2 - 3v^4 + v^6}{8\,(1 + v^2)}. \tag{31}$$

The polarization degree increases monotonically with velocity (see Fig. 3) from $1/2$ at $v = 0$ to $3/4$ at $v \to 1$, i.e. for a high-energy electron

$$W^{(\pi)} = \frac{7}{8} W, \quad W^{(\sigma)} = \frac{1}{8} W,$$

and the same is true for synchrotron radiation [A. A. Sokolov and I. M. Ternov, *Radiation from Relativistic Electrons* (AIP, 1986)].

Fig. 3. Degree of radiation polarization



Longitudinal Motion

For $p_\perp = 0$ and $\mathbf{p} = p\mathbf{e}_z$, the matrix elements (in leading order in h)

$$\langle\alpha_0\rangle = \frac{vk_\perp}{2m}, \langle\alpha_1\rangle = e^{-i\varphi}\frac{\omega}{2m}(v - n_z), \langle\alpha_2\rangle = i\langle\alpha_1\rangle, \langle\alpha_3\rangle = \frac{k_\perp}{2m}, \quad (32)$$

and due to the conservation of electromagnetic current

$$\omega\langle\alpha_0\rangle - \mathbf{k}\cdot\langle\boldsymbol{\alpha}\rangle = 0. \quad (33)$$

The angular distribution of radiation power is axially symmetric (does not depend on the azimuth angle angle φ):

$$\frac{dW}{d\Omega} = W_0\frac{(1-v^2)^2}{(1-vn_z)^5}\left[\left(1+v^2\right)\left(1+n_z^2\right) - 4vn_z\right], \quad (34)$$

where $n_z = \cos\theta$. The total radiation power:

$$W = \frac{16\pi}{3}W_0, \quad (35)$$

which is equal to the radiation power of an electron at rest ($v = 0$).

This is explained by the fact that the total radiation power is Lorentz invariant, and the configuration of the quasi-magneticthe background field is invariant under boosts along the axis Oz.

The polarization properties of radiation:

$$\frac{dW^{(\sigma)}}{d\Omega} = W_0 \left(1 - v^2\right)^2 \frac{(v - n_z)^2}{(1 - v n_z)^5},$$

$$\frac{dW^{(\pi)}}{d\Omega} = W_0 \left(1 - v^2\right)^2 \frac{1}{(1 - v n_z)^3},$$

$$W^{(\sigma)} = \frac{4\pi}{3} W_0 = \frac{1}{4} W, \quad W^{(\pi)} = 4\pi W_0 = \frac{3}{4} W, \tag{36}$$

$$P = \frac{W^{(\pi)} - W^{(\sigma)}}{W} = \frac{1}{2}.$$

5. APPLICATION OF THE OPTICAL THEOREM

In the previous part of the present talk, the standard calculation method is used based on the amplitude of the radiative transition in the first order in the electromagnetic coupling, here we consider the imaginary part of the one-loop radiative shift of the electron energy ΔE in the initial state (see Fig. 4), which, according to the optical theorem, determines the radiation probability w:

$$w = -2\mathrm{Im}\Delta E, \tag{37}$$

$$\Delta E = -\frac{ie^2}{T}\int d^4x d^4x' \bar{\psi}_{\mathbf{p}\zeta}(x)\gamma^\mu G(x,x')\gamma^\nu \psi_{\mathbf{p}\zeta}(x') D_{\mu\nu}(x,x'), \tag{38}$$

where $T(\to\infty)$ is the interaction time, the photon propagator in the Lorentz gauge

$$D_{\mu\nu}(x,x') = g_{\mu\nu}\int \frac{d^4k}{(2\pi)^4} D(k) e^{-ik\cdot(x-x')}, \tag{39}$$

$$D(k) = (k^2 + i0)^{-1}. \tag{40}$$

Fig. 4. The one-loop radiative shift of the electron energy

The electron propagator in the background field

$$G(x, x') = \int \frac{d^4q}{(2\pi)^4} G(q) e^{-iq\cdot(x-x')}, \tag{41}$$

where $G(q)$ satisfies the equation

$$\left(\gamma^\mu q_\mu - m - \frac{1}{2}\sigma^{\mu\nu} H_{\mu\nu}\right) G(q) = 1. \tag{42}$$

The explicit form of the propagator $G(q)$ follows from the expression obtained in [V. Egorov, I. Volobuev, arXiv:2107.11570v2 [hep-ph] (2022)] for the propagator of a neutrino moving in a constant magnetic field by the obvious renaming:

$$
\begin{aligned}
G(q) &= \hat{Q}(q)R(q), \\
\widehat{Q}(q) &= \Big\{\left(q^2 - m^2\right)(\gamma\cdot q + m) \\
&- h^2(\gamma\cdot q - m) - 2H_{\mu\nu}(Hq)^{\nu}\gamma^{\mu} + 2m(\widetilde{H}q)_{\mu}\gamma^{\mu}\gamma^5 \\
&+ \sigma^{\mu\nu}\left[\frac{1}{2}\left(q^2 + m^2 - h^2\right)H_{\mu\nu} - 2(Hq)_{\mu}q_{\nu}\right]\Big\}, \\
R(q) &= \Big[\left(q^2 - m^2\right)\left(q^2 - m^2 + i0\right) - 2h^2\left(q^2 + m^2\right) \\
&+ 4(Hq)^2 + \frac{1}{2}h^2\Big]^{-1},
\end{aligned} \tag{43}
$$

where $(Hq)^{\mu} = H^{\mu\nu}q_{\nu}$, $(\widetilde{H}q)^{\mu} = \widetilde{H}^{\mu\nu}q_{\nu}$.

After integrating over x and x' in (38), we obtain:

$$\Delta E = -\frac{ie^2}{(2\pi)^4}\int d^4q D(p-q)R(q)\bar{u}(\mathbf{p},\zeta)\gamma^\mu \widehat{Q}(q)\gamma_\mu u(\mathbf{p},\zeta). \qquad (44)$$

According to the Cutkosky rules, the imaginary part of ΔE is determined by the following replacement in the integrand of the right-hand side of Eq. (44):

$$2i\mathrm{Im}\Delta E = \Delta E\Big(D(p-q) \to -2\pi i\delta(D^{-1}(p-q)), R(q) \to +2\pi i\delta(R^{-1}(q))\Big). \qquad (45)$$

Note that the "$+$" sign in front of the second delta function is due to the additional factor $q^2 - m^2$ in the denominator of the electron propagator $G(q)$ and the well-known relation $\delta(x)/a = \mathrm{sgn}(a)\delta(ax)$.

We obtain the representation of the total radiation probability in the form

$$w = \frac{2\alpha}{\pi} \int d^4 q \delta(X_\gamma) \delta(X_e) F(q), \tag{46}$$

$$X_\gamma = D^{-1} = (p - q)^2,$$

$$X_e = R^{-1} = (q^2 - m^2 - h^2)^2 - 4h^2(q_\perp^2 + m^2),$$

$$F(q) = (q^2 - m^2)\langle 2m - \gamma \cdot q \rangle + h^2 \langle 2m + \gamma \cdot q \rangle$$

$$+2h^2 \langle \gamma^1 q_x + \gamma^2 q_y \rangle + 2hm \langle (\gamma^0 q_z - \gamma^3 q_0)\gamma^5 \rangle$$

with $\langle \cdots \rangle = \bar{u}(\mathbf{p}, \zeta)(\cdots)u(\mathbf{p}, \zeta)$.

Note that the energy spectrum (the eigenvalues of the Hamiltonian) is determined by the poles of the electron propagator $G(q)$ with respect to the variable q_0 , i.e., by the roots of the equation $X_e(q_0) = 0$:

$$q_0 = \pm \left[\mathbf{q}^2 + m^2 + h^2 \pm 2h\sqrt{q_\perp^2 + m^2} \right]^{1/2}, \qquad (47)$$

which agrees with (11): $E = \left[(\varepsilon_\perp - \zeta h)^2 + p_z^2 \right]^{1/2}$, and negative values of q_0 correspond to the positron.

Let us derive the angular distribution of the radiation probability. Having made in (46) the change of integration variables, $k = p - q$ (it is the photon 4-momentum),

$$d^4q = dk_0 d^3k, \quad \delta\left(X_\gamma\right) \to \tfrac{1}{2\omega}\delta\left(k_0 - \omega\right), \\ \omega = \left|\mathbf{k}\right|, \mathbf{k} = \omega\mathbf{n}, \left|\mathbf{n}\right| = 1, \tag{48}$$

and after trivial integration over k_0 we obtain

$$\frac{dw}{d\Omega} = \frac{\alpha}{\pi}\int d\omega\omega\delta\left(X_e\right)F(q), \quad q = (E - \omega, \mathbf{p} - \omega\mathbf{n}), \tag{49}$$

where $d\Omega$ is the solid angle element in the $\mathbf{n}$ direction. Next, we transform the argument of the delta function:

$$X_e = 4\left(E_{\mathbf{n}}^2 - h^2 n_\perp^2\right)\omega(\omega - \omega_{\mathbf{n}}), n_\perp^2 = 1 - n_z^2, \\ E_{\mathbf{n}} = E - \mathbf{n}\cdot\mathbf{p}, \quad \omega_{\mathbf{n}} = \frac{2h(\varepsilon_\perp E_{\mathbf{n}} - hn_x p_x)}{E_{\mathbf{n}}^2 - h^2 n_\perp^2}. \tag{50}$$

Using (50), we integrate over ω in (49) and obtain the angular distribution of the radiation probability:

$$\frac{dw}{d\Omega} = \frac{\alpha F(q)}{4\pi(E_{\mathbf{n}}^2 - h^2 n_\perp^2)}, \quad q = (E - \omega_{\mathbf{n}}, \mathbf{p} - \omega_{\mathbf{n}}\mathbf{n}). \tag{51}$$

The radiation frequency $\omega_{\mathbf{n}}$ is determined by the radiation direction $\mathbf{n}$, and the radiative transition is due to the electron spin flip: $\zeta = -1 \to \zeta' = +1$.
Expression (51) is exact in terms of the background field strength h, the value of which is strictly limited from above: $h \lesssim 10^{-17}\ \mathrm{eV}$.
Therefore, in what follows, we restrict ourselves to taking into account only the leading terms in the expansion with respect to the parameter h.

In this approximation, for the function F we obtain

$$F = 4h^3\sqrt{1-v_z^2}\,\frac{f(\mathbf{v},\mathbf{n})}{(1-\mathbf{v}\cdot\mathbf{n})^2},$$

$$f(\mathbf{v},\mathbf{n}) = (1-n_z v_z)^2\left(1+\frac{1-v^2}{1-v_z^2}\right) - (1-v^2)(1-n_z^2) - v_x^2 n_x^2, \quad (52)$$

and the angular angular probability distribution

$$\frac{dw}{d\Omega} = w_0\sqrt{1-v_z^2}\,\frac{1-v^2}{(1-\mathbf{v}\cdot\mathbf{n})^4} f(\mathbf{v},\mathbf{n}), \quad w_0 = \frac{\alpha h^3}{\pi m^2}, \quad (53)$$

where $\mathbf{v} = \mathbf{p}/\varepsilon = (v_x, 0, v_z)$ is the velocity of a free electron.

Multiplying (53) by the photon energy

$$\omega_{\mathbf{n}} = \frac{2h\sqrt{1 - v_z^2}}{1 - \mathbf{v} \cdot \mathbf{n}}, \tag{54}$$

we obtain the angular distribution of the radiation power

$$\frac{dW}{d\Omega} = \omega_{\mathbf{n}} \frac{dw}{d\Omega} = W_0(1 - v_z^2) \frac{1 - v^2}{(1 - \mathbf{v} \cdot \mathbf{n})^5} f(\mathbf{v}, \mathbf{n}), \quad W_0 = \frac{2\alpha h^4}{\pi m^2}. \tag{55}$$

To calculate the total probability and power of radiation, it is convenient to express the angles in (53) and (55) in terms of the angles (marked with the index 0) in the reference frame moving with the velocity v_z along the axis Oz (as in the theory of synchrotron radiation), using the corresponding boost, which does not change the configuration of the quasi-magnetic background field:

$$n_z = \frac{n_{0z} + v_z}{1 + v_z n_{0z}},\ n_x = \frac{\sqrt{1 - v_z^2}\, n_{0x}}{1 + v_z n_{0z}},$$

$$v_x = v_{0x}\sqrt{1 - v_z^2},\ d\Omega = \frac{1 - v_z^2}{(1 + v_z n_{0z})^2} d\Omega_0. \quad (56)$$

Using (56), we obtain:

$$\frac{dw}{d\Omega_0} = \sqrt{1 - v_z^2}\frac{dw^{(0)}}{d\Omega_0} = w_0\sqrt{1 - v_z^2}\frac{1 - v_0^2}{(1 - v_0 n_{0x})^4} f_0,$$

$$\frac{dW}{d\Omega_0} = (1 + v_z n_{0z})\frac{dW^{(0)}}{d\Omega_0} = W_0\frac{(1 + v_z n_{0z})(1 - v_0^2)}{(1 - v_0 n_{0x})^5} f_0,$$

$$f_0 = 1 - v_0^2 n_{0x}^2 + (1 - v_0^2) n_{0z}^2. \quad (57)$$

Here $v_0 \equiv v_{0x}$ is invariant under boosts along the axis Oz:

$$v_0 = \frac{p_\perp}{\varepsilon_\perp} = \frac{v_\perp}{\sqrt{1 - v_z^2}} \quad (58)$$

with $v_\perp = \sqrt{v^2 - v_z^2}$.

From (57) we get relations for total probability and power of radiation

$$w = \sqrt{1 - v_z^2}\, w^{(0)}, \; W = W^{(0)} \tag{59}$$

in agreement with the special relativity. We emphasize that the total radiation power is a Lorentz invariant.

The integration of the angular distributions is greatly simplified if we choose Ox as the polar axis (in the reference frame, where $v_z = 0$). Then $n_{0x} = \cos\alpha$, $n_{0z} = \sin\alpha \sin\beta$, which allows independent integration over α and β. As a result, we obtain explicit expressions for w and W:

$$w = \frac{8\alpha h^3}{3m^2}\sqrt{1 - v_z^2}\,\frac{2 + v_0^2}{1 - v_0^2}, \qquad W = \frac{32\alpha h^4}{3m^2}\,\frac{1 + v_0^2}{(1 - v_0^2)^2}. \tag{60}$$

For an unpolarized electron, it is necessary to introduce an additional factor $1/2$ into the right-hand sides of (60).

6. DISCUSSION

Expressions (59) and (60) are valid for an arbitrary angle between the electron momentum **p** and the direction of the background field **h**.
Since an electron has both an electric charge ($-e$) and magnetic moment (we take only the standard Dirac moment into account):

$$\mu_e = \frac{\sqrt{\alpha}}{2m}, \tag{61}$$

it is of some interest to separate their contributions to the radiative transition amplitude and the radiation power and compare the above results with the results obtained in [I. M. Ternov, V. G. Bagrov, and A. M. Khapaev, Sov. Phys. JETP 21, 613 (1965)] for the neutron radiation in a constant magnetic field. For this purpose, we use use the generalized Gordon identity:

$$\bar{u}(\mathbf{p}', \zeta')\gamma^{\mu}u(\mathbf{p}, \zeta) = \bar{u}(\mathbf{p}', \zeta')\left[\frac{p'^{\mu} + p^{\mu}}{2m} + \frac{i}{2m}\sigma^{\mu\nu}(p'^{\mu} - p^{\mu}) + \frac{1}{m}\widetilde{H}^{\mu\nu}\gamma_{\nu}\gamma^{5}\right]u(\mathbf{p}, \zeta). \quad (62)$$

The first, the second, and the third terms at the righthand side characterize the contributions of the electric charge (e), the magnetic moment (μ), and the background field (h), respectively. In the case of transverse electron motion, the contributions of different mechanisms to the radiative transition amplitudes (in the leading order in h) are separated as follows:

$$\langle\alpha_1\rangle = \langle\alpha_1\rangle_e + \langle\alpha_1\rangle_\mu = \tfrac{\omega}{2m}n_z\left[(-v^2)_e + (1)_\mu\right],$$

$$\langle\alpha_2\rangle = \langle\alpha_2\rangle_\mu = i\tfrac{\omega}{2m}n_z\sqrt{1 - v^2},$$

$$\langle\alpha_3\rangle = \langle\alpha_3\rangle_\mu + \langle\alpha_3\rangle_h = \tfrac{\omega}{2m}\left[(v - n_x - in_y\sqrt{1 - v^2})_\mu + (-v + v^2 n_x)_h\right]. \quad (63)$$

The total radiation power in the form of a sum of contributions of different emission mechanisms:

$$W = W_e + W_\mu + W_h + W_{e\mu} + W_{eh} + W_{\mu h}, \tag{64}$$

where

$$W_e = \pi W_0 \left[L(v) - 2\gamma^4 \left(1 - \frac{5}{3} v^2 \right) \right], \tag{65}$$

$$W_\mu = \frac{16\pi}{3} W_0 \gamma^4, \tag{66}$$

$$W_h = \pi W_0 \left[L(v) - 2\gamma^4 \left(1 - 3v^2 \right) \right], \tag{67}$$

$$W_{e\mu} = -\frac{8\pi}{3} W_0 \gamma^4 v^2, \tag{68}$$

$$W_{eh} = -2\pi W_0 \left[L(v) - 2\gamma^4 \left(1 - \frac{5}{3} v^2 \right) \right], \tag{69}$$

$$W_{\mu h} = \frac{16\pi}{3} W_0 \gamma^4 v^2 \tag{70}$$

with $L(v) = \frac{1}{v} \ln \frac{1+v}{1-v}$.

All contributions apart from W_μ are monotonic functions of velocity v that turn to zero at $v = 0$. At $v > 0$,

$$W_e + W_h + W_{e\mu} + W_{eh} = 0.$$

Thus, the total radiation power

$$W = W_\mu + W_{\mu h} = \frac{16\pi}{3} W_0 \gamma^4 (1 + v^2). \quad (71)$$

Tthe contribution of the magnetic moment is dominant at $v \ll 1$. At $v \to 1$ (high-energy region: $\gamma \gg 1$), both contributions become almost equal. In the case of longitudinal motion of an electron, our matrix elements after the corresponding rescaling coincide with those obtained in [I. M. Ternov, V. G. Bagrov, and A. M. Khapaev, Sov. Phys. JETP 21, 613 (1965)], so that the radiation power of a neutral fermion with a magnetic moment is reproduced (the other contributions in this case are equal to zero).

After substitutions $\alpha \to (2m\mu_n)^2$ and $h \to \mu_n H$ in W_μ, we obtain for the radiation power of a neutron with the magnetic moment μ_n:

$$W_n = \frac{128}{3}\mu_n^6 H^4 \begin{pmatrix} \gamma^4 \\ 1 \end{pmatrix} \text{ for } \begin{pmatrix} \mathbf{v} \perp \mathbf{H} \\ \mathbf{v} \parallel \mathbf{H} \end{pmatrix}. \tag{72}$$

Consider the average emitted energy of an electron, i.e., the average photon energy

$$\langle\omega\rangle = \frac{\int \omega dw}{\int dw} = \frac{W}{w}. \tag{73}$$

During a time interval

$$\tau_R = 1/w, \tag{74}$$

an electron emits a photon, having made a spin-flip transition to a state that is radiatively stable: a radiative transition from it is forbidden. Consequently, if the electron beam is initially unpolarized, then as a result of radiation it becomes completely polarized along the direction of the background field **h**, and the characteristic polarization time is equal to τ_R.

A similar effect of polarization due to a radiative transition with spin flip was noted for neutrons moving in a magnetic field [I. M. Ternov, V. G. Bagrov, and A. M. Khapaev, Sov. Phys. JETP 21, 613 (1965)], as well as for neutrinos in a magnetic field and matter (neutrino spin light): [A. Lobanov, A. Studenikin, Phys. Lett. B **564**, 27 (2003); A. E. Lobanov, Phys. Lett. B **619**, 136 (2005)].
For the average photon energy, we obtain

$$\langle\omega\rangle = \frac{4h(1+v_0^2)}{\sqrt{1-v_z^2}(1-v_0^2)(2+v_0^2)}, \quad \langle\omega\rangle_\perp = 4h\gamma^2\frac{1+v^2}{2+v^2}, \; \langle\omega\rangle_\parallel = 2h\gamma. \tag{75}$$

The effects of Lorentz violation increase with increasing electron energy, and are much more noticeable for the transverse motion. For this case, for $\gamma \gg 1$ and $h\gamma \ll 1$, we find the radiative polarization length $L_R = v\tau_R$ in ordinary units:

$$L_R \simeq \frac{c}{w_\perp} \simeq \frac{\lambdabar_e}{8\alpha}\left(\frac{m}{h}\right)^3\gamma^{-2}, \tag{76}$$

where $\lambdabar_e$ is the Compton wavelength of the electron.

For a numerical estimation, we set $h = 10^{-17}$ eV and $\varepsilon = 10^{16}$ GeV (the energy scale of the Grand Unification of the three fundamental interactions). Then we obtain $\langle\omega\rangle_\perp \simeq 10^{13}$ GeV (this is two orders of magnitude greater than the maximum registered energy of particles in cosmic rays $\simeq 10^{11}$ GeV) and $L_R \simeq 2.3 \times 10^{20}$ cm (for comparison, the distance from the Sun to the nearest star $\simeq 4 \times 10^{18}$ cm, and from the Sun to the center of the Galaxy $\simeq 2.5 \times 10^{22}$ cm).

7. CONCLUSION

We calculated the probability and power of electromagnetic radiation by an electron in a constant background field of the quasi-magnetic type simulating a Lorentz-violating vacuum.

It is found that the radiation has an appreciable linear polarization, the degree of which reaches 75 % for high-energy electrons.

The radiative transition due to spin flip leads to complete polarization of the initially unpolarized electron beam along the direction of the background field.

We have shown that the considered radiative effect can be noticeable under astrophysical conditions for ultrahigh-energy electrons.

Thank you for attention!