Electromagnetic conductivity of quark-gluon plasma under extreme conditions

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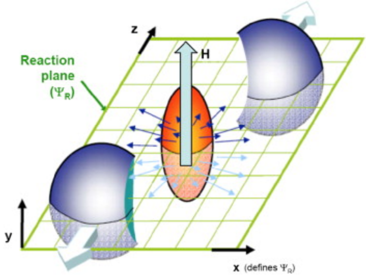
Motivation

"A system with a nonzero chirality responds to a magnetic field by inducing a current along the magnetic field. This is the Chiral Magnetic Effect."

[K. Fukushima, D. Kharzeev, H.J. Warringa, 2008]



$$\vec{J}_{CME} = \frac{e^2}{2\pi^2}\mu_5\vec{B}$$

Dynamical CME is manifested through electromagnetic conductivity

Motivation

- $\vec{E}, \vec{B}$
- $\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2}(\vec{E}, \vec{B}) - \frac{\rho_5}{\tau}$, τ - chirality-changing scattering time
- $\rho_5 = \frac{e^2\tau}{4\pi^2}(\vec{E}, \vec{B})$
- $\rho_5 \sim \mu_5 B \Rightarrow \mu_5 \sim \frac{e^2\tau}{4\pi^2}\frac{(\vec{E},\vec{B})}{B}$
- $\vec{J} = \sigma\vec{E} + \frac{e^2}{2\pi^2}\vec{B} \times \mu_5$
- $\sigma_{\|}^{CME} \sim eB\tau$
- Manifestation of CME: rise of the conductivity with B
- Anomaly related quantum phenomenon (classically $\sigma_{\|}^{CME} = 0$)
- Observed in experiment (Dirac and Weyl semimetals)

  Q. Li et al., Nature Phys. 12 (2016) 550-554

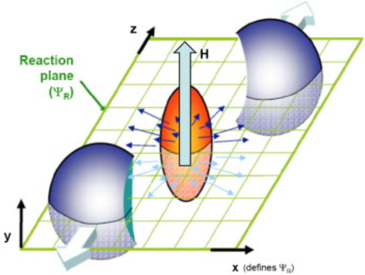
  H. Li et al., Nat. Comm. 7, 10301 (2016)

  ...
- Observed in heavy-ion collision experiments(?)

  B. I. Abelev et al. (STAR), Phys.Rev.Lett. 103, 251601 (2009)

  B. Abelev et al. (ALICE), Phys.Rev.Lett. 110, 012301 (2013)

Motivation



- Charge transport of QGP is important for dynamics of QGP
- QGP in heavy-ion collisions may have non-zero baryon density
- Baryon density introduces addition fermion states to QGP
- Baryon density might change σ significantly

Lattice studies of electromagnetic conductivity

- H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch,E. Laermann, and W. Soeldner, Phys. Rev. D83, 034504(2011)
- A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands,and J.-I. Skullerud, Phys. Rev. Lett.111, 172001 (2013)
- G. Aarts, C. Allton, A. Amato, P. Giudice, S. Hands, and J.-I. Skullerud, JHEP02, 186 (2015)
- B. B. Brandt, A. Francis, B. Jager, and H. B. Meyer,Phys. Rev. D93, 054510 (2016)
- H.-T. Ding, O. Kaczmarek, and F. Meyer, Phys. Rev. D94, 034504 (2016)
- P.V. Buividovich, D. Smith, L. von Smekal, Phys.Rev.D 102 (2020) 9, 094510

Conductivity in lattice simulations

- $J_i = \sigma_{ij} E_j$
- Electromagnetic conductivity
$\sigma_{ij} = \lim_{\omega\to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x \; e^{i\omega t} \langle [J_i(x), J_j(0)] \rangle$
$\rho_{ij} = -\frac{1}{\pi} Im G_R^{ij}(\omega, \vec{k} = 0)$
$\sigma_{ij} = \pi \lim_{\omega\to 0} \frac{1}{\omega} \rho_{ij}(\omega)$
- Analytic continuation
$G_E(\omega, \vec{p}) = -G_R(i\omega, \vec{p}), \quad \omega > 0$
- On lattice we measure
$C_E(\tau) = \int d^3x \langle J_i(\tau, \vec{x}) J_j(0, \vec{0}) \rangle$
$C_E(\tau) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega\tau\right)}{sh\left(\frac{\omega}{2T}\right)}, \quad \tau \in \left(0, \frac{1}{T}\right)$

Conductivity with staggered fermions

- We account only connected diagrams
- Correlation function for staggered fermions

$$C_{ij}(\tau) = \frac{1}{L_s^3} \langle J_i(\tau) J_j(0) \rangle,$$

$$J_i(\tau) = \frac{1}{4} e \sum_f q_f \sum_{\vec{x}} \eta_i(x) \left(\bar{\Psi}^f_x U_{x,i} \Psi^f_{x+i} + \bar{\Psi}^f_{x+i} U^+_{x,i} \Psi^f_x \right)$$

- Two branches of staggered correlator

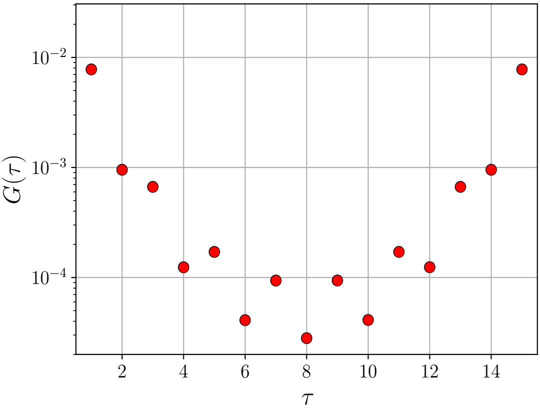
$$C^e_{ij}(\tau = 2n \times a) = \int d^3y \left(\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle - \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle \right)$$

$$C^o_{ij}(\tau = (2n+1) \times a) = \int d^3y \left(\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle + \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle \right)$$

$$A_i = e \sum_f q_f \bar{\psi}^f \gamma_i \psi^f, \quad B_i = e \sum_f q_f \bar{\psi}^f \gamma_5 \gamma_4 \gamma_i \psi^f$$

Conductivity with staggered fermions

- Typical plot for the staggered correlation function



Conductivity with staggered fermions

The strategy of the calculation

- Measure $C_E^{even,odd}(\tau)$ on two branches
- Reconstruct the $\rho^{even,odd}(\omega)$ (Backus-Gilbert method)

$$C_E^{even,odd}(t) = \int_0^\infty d\omega \rho^{even,odd}(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Calculate $\rho(\omega) = \frac{1}{2}(\rho^{even}(\omega) + \rho^{odd}(\omega))$

  (what corresponds to the $\langle J_{el}(\tau) J_{el}(0) \rangle$)
- Calculate the conductivity $\sigma = \pi \frac{\rho(\omega)}{\omega}\Big|_{\omega \sim 0}$

Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i,\omega), \quad K(x_i,\omega) = \frac{ch\left(\frac{\omega}{2T} - \omega x_i\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega},\omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega},\omega)\rho(\omega)$$

- Let us expand $\delta(\bar{\omega},\omega)$ as

$$\delta(\bar{\omega},\omega) = \sum_i b_i(\bar{\omega}) K(x_i,\omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

- Goal: minimize the width of the resolution function

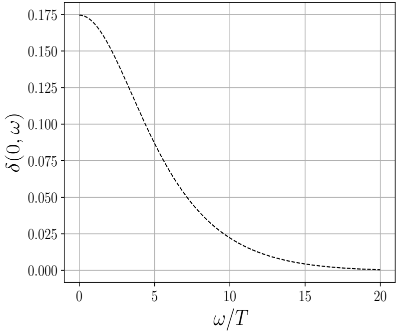
$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i,\omega)(\omega - \bar{\omega})^2 K(x_j,\omega), R_i = \int d\omega K(x_i,\omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1-\lambda) S_{ij}, \quad 0 < \lambda < 1$$

Backus-Gilbert method for the spectral function



- We calculate the estimator of the spectral function

$$\bar{\rho}(\bar{\omega}) = \int d\omega \delta(\omega, \bar{\omega}) \rho(\omega)$$

- Average spectral function(conductivity) over the width $\sim$ *few* $\times$ T

Backus-Gilbert method for the spectral function

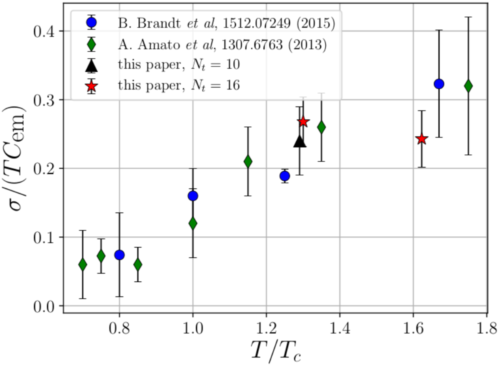
- Width of the resolution function is $\sim (3-4) \times T$
- For very narrow spectral density BG method underestimates conductivity
- But lattice studies give the width $\sim 4T$ or larger
    - G. Aarts et al, JHEP02, 186 (2015)
    - B. B. Brandt et al,Phys. Rev.D93, 054510 (2016)
    - H.-T. Ding, et al, Phys. Rev.D94, 034504 (2016)

Details of lattice simulations

- Stout smeared staggered $2+1$ fermions
- Physical pion m_π and strange m_s quark masses
- $T \approx 200, 250$ MeV
- $\mu_u = \mu_d = \mu_B/3$, $\mu_s = 0$
- Bacause of the sign problem the simulations are carried out at imaginary $\mu_B = I\mu_I$
- Lattice parameters:

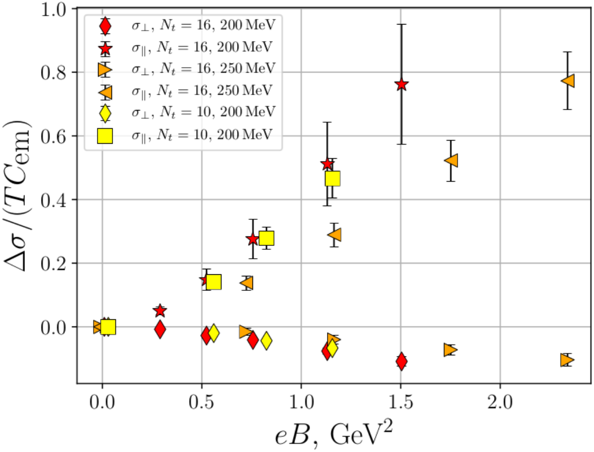
a, fm	L_s	N_t	T, fm
0.0988	48	10	200
0.0788	48	10	250
0.0820	48	12	200
0.0657	48	12	250
0.0618	64	16	200
0.0493	64	16	250

Conductivity at zero magnetic field $eB = 0$



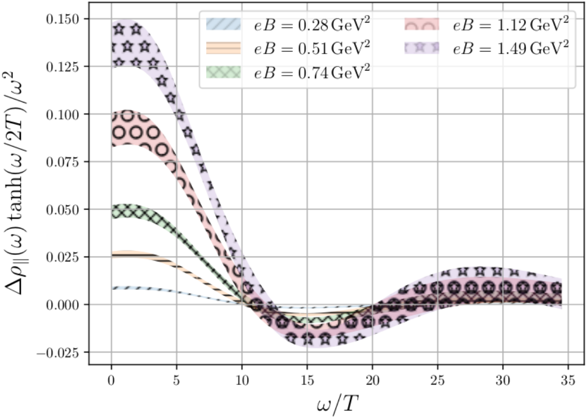
- **First calculation of the conductivity at physical pion mass**
- Agreement with previous papers
- Discretization effects are under control

Conductivity at nonzero magnetic field $eB \neq 0$



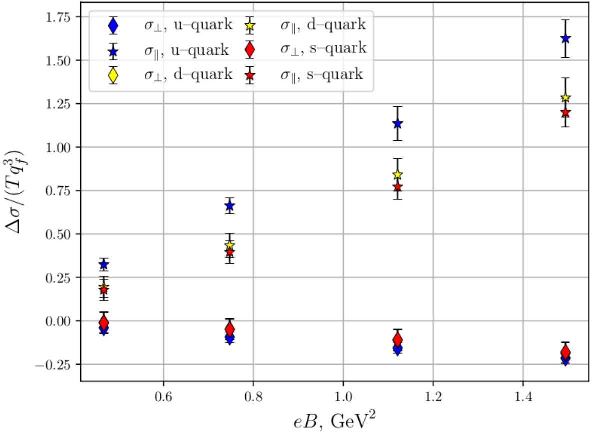
- $\Delta\sigma = \sigma(B) - \sigma(B = 0)$
- We observe CME and magnetoresistance in QGP
- Discretization effects are under control

The BG reconstructed spectral function



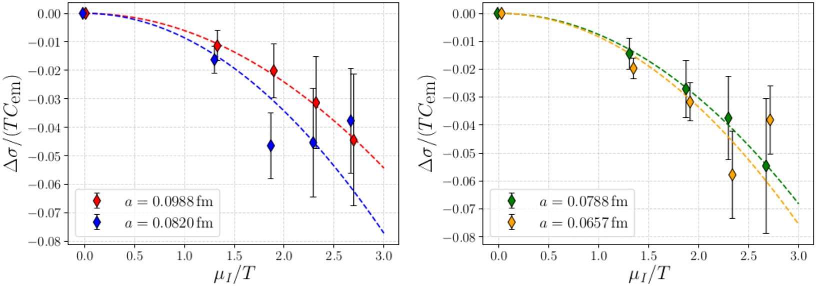
- $\Delta\rho_{\|} = \rho_{\|}(B) - \rho_{\|}(B=0)$
- Considerable rise of spectral density in the infrared region

The contribution of different quarks



- The conductivity scale as q_f^3
- $\sigma_d/q_d^3 \simeq \sigma_s/q_s^3$, $\sigma_u/q_u^3 > \sigma_{d,s}/q_{d,s}^3$ ($|q_u| = \frac{2}{3}, |q_d| = |q_s| = \frac{1}{3}$)
- Large mass of s-quark does not influence the conductivity

E.m. conductivity at finite baryon density



- $\Delta\sigma = \sigma(\mu_I) - \sigma(\mu_I = 0)$ **(to subtract UV contribution)**
- Discretization effects are under control
- Our results can be well described by

$$\frac{\Delta\sigma}{TC_{em}} = -c(T)\left(\frac{\mu_I}{T}\right)^2 \Rightarrow \frac{\Delta\sigma}{TC_{em}} = c(T)\left(\frac{\mu_B}{T}\right)^2, \quad C_{em} = e^2 \sum_f q_f^2$$

- $c(T) \sim 0.007 \Rightarrow$ **BARYON DENSITY ENHANCES E.M. CONDUCTIVITY**
- $c(T)$ weakly depends on temperature
- At $\mu_q \sim T \quad \frac{\Delta\sigma}{\sigma} \sim 30\%$
  reasonable agreement with Phys. Rev. C 89, 035203 (2014), Phys. Rev. C 91, 044903 (2015)

Conclusion:

- E.m. conductivity at finite baryon density and strong magnetic field was calculated
- **We observe CME and magnetoresistance in QGP**
- **Baryon density enhances e.m. conductivity**