# Improved cosmological bounds for a fine-tuned seesaw mechanism of keV sterile neutrinos

**Mikhail Dubinin**

**database reference: M.D., Dmitri Kazarkin, arXiv:2206.05186 [hep-ph]**

SINP MSU

QFTHEPC, JINR, Dubna, 20 July 2022 г.

# Outline

**– Features of the neutrino minimal standard model ($\nu$MSM). Representation of experimental constraints on the** *mixing – heavy neutral lepton (HNL) mass* **plane.**

**– Diagonalization of 6$\times$6 mixing matrix in the extended leptonic sector. HNL mass states in the see-saw type I. Active – sterile neutrino mixing. $R$-matrix.**

**– HNL dark matter in the $\nu$MSM model. Improved cosmological restrictions from the HNL lifetime and the HNL DM energy.**

**– Summary**

sterile neutrino DM

S. Dodelson, L. M. Widrow, PRL 1994 (hep-ph/9303287)

neutrino minimal standard model ($\nu$MSM)

T.Asaka, S.Blanchet, M.Shaposhnikov, PLB 2005 (hep-ph/0503065)

– $N_1$ sterile neutrino is a dark matter (DM) candidate
– explains the data on neutrino oscillations and DM without introducing new physics above the EW scale
– ensures generation of baryon asymmetry.

Includes 18 weakly constrained parameters (3 Dirac mass term parameters, 3 Majorana mass term parameters, 6 mixing angles and 6 phases), so any exclusion contours are related to their convolutions.
**Experimental collaborations are using $|U|^2 - mass$ contour**,
where $|U| = M_D/M_N$ is somehow taken in the 'model-independent phenomenological approach', assuming that only a single HNL is phenomenologically accessible while other (heavy) do not affect the analysis. There are two parameters in such a scheme: $M_N$ and $M_D = Y_{\alpha I} v$ for a given flavor $\alpha$, assuming that mixing of other flavors does not contribute. However, for a toy-model with only one generation of fermions we have

# see-saw with one generation of fermions

$$\mathcal{L}_M = -m_D \bar{\nu_L} N_R - \frac{m_N}{2} \bar{N^c_R} N_R = -\frac{1}{2} \begin{pmatrix} \bar{\nu_L} & \bar{N^c_R} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \begin{pmatrix} \nu^c_L \\ N_R \end{pmatrix}$$

$$\nu' = \begin{pmatrix} 1 - \frac{m_D^2}{2m_N^2} \\ -\frac{m_D}{m_N} \end{pmatrix}, \quad N' = \begin{pmatrix} \frac{m_D}{m_N} \\ 1 - \frac{m_D^2}{2m_N^2} \end{pmatrix}$$

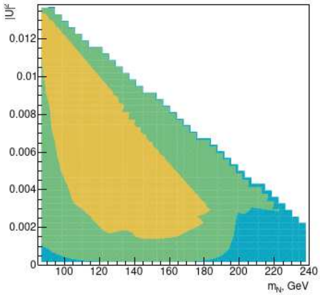
flavor state

$$\nu = \nu'(1 - \frac{U^2}{2}) + UN'$$

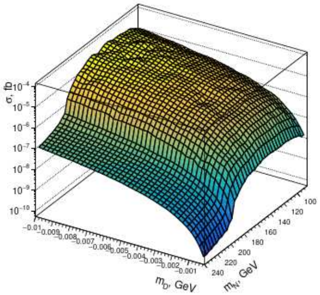
mixing parameter

$$U = -\frac{m_D}{m_N} = -\frac{\sqrt{-m_{ne} m_{Ne}}}{m_{ne} + m_{Ne}} \sim -\sqrt{\frac{m_{ne}}{m_{Ne}}}$$

where $m_{ne}, m_{Ne}$ are the mass eigenvalues. $m_{ne}, m_{Ne} \longrightarrow m_{ne}, U$ gives a rectangle $\rightarrow$ triangle, when $U$ is mass dependent. Even in the toy model with one generation 'model independent approach' does not work.



|U|²
0.012
0.01
0.008
0.006
0.004
0.002
0
100
120
140
160
180
200
220
240
m_N, GeV

σ, fb
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
m_D, GeV
m_N, GeV

# Collider exclusion contours for the three lepton events

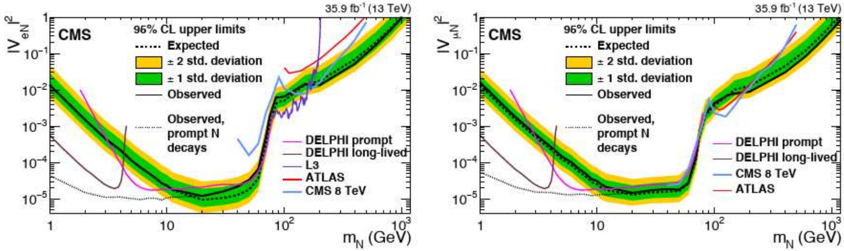



Figure 2: Exclusion region at 95% CL in the $|V_{\mathrm{eN}}|^2$ vs. $m_{\mathrm{N}}$ (left) and $|V_{\mu\mathrm{N}}|^2$ vs. $m_{\mathrm{N}}$ (right) plane

Рис.: CMS Collaboration, CERN-EP-2018-006

# Three generations of fermions

Sterile Majorana neutrinos $N_1$, $N_2$, $N_3$ with the see-saw mechanism type I

$$\mathcal{L}_\nu = i\overline{\mathcal{N}_a}_R \gamma^\mu \partial_\mu \mathcal{N}_{aR} - \left( Y_{la} \overline{L_l} \mathcal{N}_{aR} \tilde{\Phi} + \frac{1}{2} M_{N\,ab} \overline{\mathcal{N}_a^C}_R \mathcal{N}_{bR} + h.c. \right)$$

$L_l^T = (\nu_l, l)_L$, $l = e, \mu, \tau$, $\tilde{\Phi}_m = \epsilon_{mn} \Phi_n^*$, $a, b = 1, 2, 3$,
$Y_{la}$ is the Yukawa matrix 3×3.
$SU(2)$ states $\nu_l$ and mass states $\nu_i$: $\nu_{l\,L} = (U_{PMNS})_{li} \nu_{i\,L}$.

The mass matrix of active neutrinos

$$(m_\nu)_{lk} = -(M_D)_{la} (M_N^{-1})_{ab} (M_D^T)_{bk}$$

$(M_D)_{la} = Y_{la} v / \sqrt{2}$ $\quad$ $3 \times 3$ matrix of Dirac mass term
$M_N$ $\quad$ $3 \times 3$ mass matrix of the three HNL, $v = \sqrt{2} \langle \Phi \rangle$, $l, k = e, \mu, \tau$.

DEF: Mixig matrix *active – sterile* $\quad \theta = M_D \, M_N^{-1}$
Flavor states and mass states: $\begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}_l = \mathcal{U} \begin{pmatrix} \nu_L \\ N \end{pmatrix}_{Li}$; $\quad \mathcal{U} = \mathcal{W} \cdot diag(U, V^*)$

$U, V$ – 3×3 matrices diagonalizing active and sterile sectors

$$\mathcal{W}^T \mathcal{M} \mathcal{W} = \begin{pmatrix} U^* \hat{m} U^\dagger & 0 \\ 0^T & V^* \hat{M} V^\dagger \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix}$$

$\hat{M} = diag(M_1, M_2, M_3)$ mass matrix for the HNL states $N_i$,
$\hat{m} = diag(m_1, m_2, m_3)$ mass matrix for active neutrinos, $\mathcal{W}$-matrix can be expressed as an exponent of the anti-hermitian matrix

$$\mathcal{W} = exp \begin{pmatrix} 0 & R \\ -R^\dagger & 0 \end{pmatrix} = \begin{pmatrix} I - \frac{1}{2} R R^\dagger & R \\ -R^\dagger & I - \frac{1}{2} R^\dagger R \end{pmatrix} + O(R^3),$$

with $O(R^2)$ accuracy $\theta = R$

$$\begin{aligned} M_D - R^* M_N &\simeq 0 \\ -M_D R^\dagger - R^* M_D^T + R^* M_N R^\dagger &\simeq U^* \hat{m} U^\dagger = m_\nu \\ M_N + R^T M_D + M_D^T R &\simeq V^* \hat{M} V^\dagger \end{aligned}$$

so $R^* \simeq M_D M_N^{-1}$, $m_\nu = U^* \hat{m} U^\dagger = -R^* M_N R^\dagger$

A. Ibarra, E. Molinaro, S. Petcov, JHEP 09 (2010) 108

# Active neutrino and HNL currents in the mass basis

with $O(R^2)$ accuracy

$$
\begin{aligned}
(\nu_{Lk})_{flavor} &= (1-\frac{1}{2}RR^\dagger)U\nu_L + RV^* N_L \\
(N^c_{Rk})_{flavor} &= -R^\dagger U\nu_L + (1-\frac{1}{2}RR^\dagger)V^* N_L
\end{aligned}
$$

Lagrangian terms with active neutrino currents, $U_{PMNS} = (1 - RR^\dagger/2)\,U$

$$
\mathcal{L}^{\nu}_{CC} = -\tfrac{g}{\sqrt{2}}\bar{l}\gamma_\mu \nu_{lL} W^\mu + h.c. = -\tfrac{g}{\sqrt{2}}\bar{l}\gamma_\mu (U_{PMNS})_{li}\nu_{iL} W^\mu + h.c.
$$

$$
\mathcal{L}^{\nu}_{NC} = \tfrac{g}{2c_W}\bar{\nu}_{iL} U^\dagger \gamma_\mu (I - \tfrac{1}{2}RR^\dagger - \tfrac{1}{2}R^\dagger R) U \nu_{jL} Z^\mu + h.c.
$$

$\nu_{iL}$ are the mass states of active neutrinos. Lagrangian terms with HNL currents

$$
\mathcal{L}^{N}_{CC} = -\tfrac{g}{2\sqrt{2}}\bar{l}\gamma_\mu (RV^*)_{lk}(1-\gamma_5) N_k W^\mu + h.c.
$$

$$
\begin{aligned}
\mathcal{L}^{N}_{NC} = & \left(-\tfrac{g}{2c_w}\bar{\nu}_{iL}\, U^\dagger\, \gamma_\mu (I - \tfrac{1}{2}RR^\dagger)(RV^*) N_{jL} Z^\mu + h.c.\right) \\
& + \tfrac{g}{c_W}\overline{N}_{iL}\gamma^\nu V^T(-\tfrac{1}{2}RR^\dagger)V^* N_{jL} Z^\mu
\end{aligned}
$$

Mixing of HNL and left active neutrino is characterized by the matrix $RV^*$.

Using $M_N = -M_D^T m_\nu^{-1} M_D$ in the basis where $V$ is a unit matrix,
$M_N = \hat{M} = diag(M_1, M_2, M_3)$

$$\hat{M} = -M_D^T m_\nu^{-1} M_D \simeq -M_D^T U^*_{PMNS} \hat{m}^{-1} U^\dagger_{PMNS} M_D.$$

which gives the orthogonality condition $\Omega^T \Omega = I$

$$I = [-i\sqrt{\hat{m}^{-1}} U^\dagger_{PMNS} M_D \sqrt{\hat{M}^{-1}}]^T [-i\sqrt{\hat{m}^{-1}} U^\dagger_{PMNS} M_D \sqrt{\hat{M}^{-1}}]$$

for the general form of $\Omega$-matrix

$$\Omega = -i\sqrt{\hat{m}^{-1}} U^\dagger_{PMNS} M_D \sqrt{\hat{M}^{-1}}$$

and it follows that

$$M_D = i U^*_{PMNS} \sqrt{\hat{m}}\, \Omega \sqrt{\hat{M}}.$$

If $V$ is not a unit matrix, $RV^* = M_D \hat{M}^{-1}$

$$(RV^*)_{\alpha I} = -\frac{\sum_k \sqrt{m_k}\ \Omega^*_{kI} (iU_{PMNS})_{\alpha k}}{\sqrt{M_I}}.$$

Simple special choice $\Omega =$I gives

$$R = (M_D \hat{M}^{-1})^* = \begin{pmatrix} iU_{e1}\sqrt{\frac{m_1}{M_1}} & iU_{e2}\sqrt{\frac{m_2}{M_2}} & iU_{e3}\sqrt{\frac{m_3}{M_3}} \\ iU_{\mu 1}\sqrt{\frac{m_1}{M_1}} & iU_{\mu 2}\sqrt{\frac{m_2}{M_2}} & iU_{\mu 3}\sqrt{\frac{m_3}{M_3}} \\ iU_{\tau 1}\sqrt{\frac{m_1}{M_1}} & iU_{\tau 2}\sqrt{\frac{m_2}{M_2}} & iU_{\tau 3}\sqrt{\frac{m_3}{M_3}} \end{pmatrix}.$$

In the following formulae of Section 3 this explicit form is denoted as $\overline{RV^*}$

$$\Omega(\theta_1,\theta_2,\theta_3)=\begin{pmatrix} c_1c_3-s_1s_2s_3 & -s_1c_2 & -s_1s_2-c_3-s_3c_1 \\ s_1c_3+s_2s_3c_1 & c_1c_2 & s_2c_1c_3-s_1s_3 \\ s_3c_2 & -s_2 & c_2c_3 \end{pmatrix}$$

$c_i=\cos\theta_i$ and $s_i=\sin\theta_i$, $i=1,2,3$. If $\hat{M}\approx diag(0,0,M_3)$ and $\hat{m}\approx diag(0,m_2,m_3)$

$$M_D\approx\begin{pmatrix} 0 & 0 & \sqrt{M_3}\big(\sqrt{m_2}\Omega_{23}U^*_{e2}+\sqrt{m_3}\Omega_{33}U^*_{e3}\big) \\ 0 & 0 & \sqrt{M_3}\big(\sqrt{m_2}\Omega_{23}U^*_{\mu 2}+\sqrt{m_3}\Omega_{33}U^*_{\mu 3}\big) \\ 0 & 0 & \sqrt{M_3}\big(\sqrt{m_2}\Omega_{23}U^*_{\tau 2}+\sqrt{m_3}\Omega_{33}U^*_{\tau 3}\big) \end{pmatrix},\Omega_1=\begin{pmatrix} -s_1 & 0 & -c_1 \\ c_1 & 0 & -s_1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$M_D=iU^*_{PMNS}\sqrt{\hat{m}}\left(\Omega V^\dagger\right)\left(V^*\sqrt{\hat{M}}V^\dagger\right)=iU^*_{PMNS}\sqrt{\hat{m}}\Omega'\sqrt{\hat{M}'},$$

$$V_1=\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix},\quad \Omega'_1=\Omega_1V_1^T=\begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

one can restore the case $\Omega=I$ by taking $\theta_1=0$ and changing the mass ordering $\hat{M}=diag(M_1,M_2,M_3)$ to the mass ordering $\hat{M}=diag(M_3,M_1,M_2)$.

# The case of discrete nonabelian group $S_3$

.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

unitary transformation $U^+ \, x \, U$ with $U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$

$$m_\nu = U^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^+ =$$

$$= \frac{m_1 + m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2 - m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1 - m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Tri-bimaximal mixing, P.Harrison, R.Scott, PLB, 2002

# $N_1$ lifetime restrictions

$$\Gamma\Big(N_1 \to \sum_{\alpha,\beta} \nu_\alpha, \nu_\beta, \overline{\nu_\beta}\Big) = \frac{G_F^2 M_1^5}{192\pi^3} \sum_\alpha |(RV^*)_{\alpha 1}|^2, \quad (\alpha, \beta = e, \mu, \tau).$$

$$\Gamma\Big(N_1 \to \gamma, \nu\Big) = \frac{9\alpha_{EM} G_F^2 M_1^5}{256\pi^4} \sum_\alpha |(RV^*)_{\alpha 1}|^2$$

$\Gamma_{N\to\nu\nu\nu}/\Gamma_{N\to\gamma\nu} \equiv k_{rad} = \frac{8\pi}{27\alpha_{EM}} \approx 127.5$. Radiative decay does not contribute significantly.

$$\tau_{N_1} = 2.88 \times 10^{19} \times \left(\frac{M_1}{1\ \text{keV}}\right)^{-5} \left(\sum_\alpha |(RV^*)_{\alpha 1}|^2\right)^{-1} \text{sec.} \qquad (1)$$

explicit form of mixing

$$\sum_\alpha |\overline{RV^*}_{\alpha 1}|^2 = \sum_\alpha \frac{m_1}{M_1} |(U_{PMNS})_{\alpha 1}|^2 = \frac{m_1}{M_1} < \frac{\sum_{k=1}^3 m_k}{M_1} < \overline{\Sigma m} \cdot 10^{-3} \left(\frac{M_1}{1\ \text{keV}}\right)^{-1} \qquad (2)$$

general form of mixing

$$\sum_\alpha |RV^*_{\alpha 1}|^2 < \sum_\alpha \left(\frac{\sum_k \sqrt{\Sigma_i m_i}}{\sqrt{M_I}}\right)^2 = 27 \frac{\sum_i m_i}{M_I} < 27\ \overline{\Sigma m} \cdot 10^{-3} \left(\frac{M_1}{1\ \text{keV}}\right)^{-1} \qquad (3)$$

Planck 2015, arXiv:1502.01589[astro-ph]

# DM energy restrictions

Non-resonant production, WMAP, Lyman alpha-forest, PRD 2001,2005 (astro-ph/0101524, astro-ph/0501562)

$$\Omega_N h^2 \sim 0.1 \sum_{I=1}^{3} \sum_{\alpha=e,\nu,\tau} \left( \frac{|(RV^*)_{\alpha I}|^2}{10^{-8}} \right) \left( \frac{M_I}{1 \text{ keV}} \right)^2$$

using $RV^* = M_D^* M_N^{-1}$ a universal estimate in the basis where $V = I$ and $M_N = \hat{M} = diag(M_1, M_2, M_3)$

$$\sum_{I,\alpha} |RV^*_{\alpha I}|^2 = Tr \left( M_D^* \hat{M}^{-1} (\hat{M}^{-1})^\dagger M_D^T \right) = \sum_{I,\alpha} |(M_D)_{I\alpha}|^2 \cdot \frac{1}{M_I^2}$$

so for $\mathcal{K}$ DM neutrino

$$\sum_{I=1}^{\mathcal{K}\leq 3} |(M_D)_{I\alpha}|^2 = m_0^2 \quad , \text{ where } m_0 = \mathcal{O}(0.1 \text{ eV}) \qquad (4)$$

# DM energy restrictions

Important result by T.Asaka, S.Blanchet, M.Shaposhnikov, PLB 2005 (hep-ph/0503065)
for the three generations only one HNL can be DM particle, $\mathcal{K}$ =1, the lightest active neutrino mass $m_1 < m_\nu^{dm} = \mathcal{O}(10^{-5}\text{eV})$, so (2) can be written as

$$\sum_\alpha |\overline{RV^*}_{\alpha 1}|^2 < \frac{m_\nu^{dm}}{M_1} < 10^{-8} \left( \frac{M_1}{1 \text{ keV}} \right)^{-1}$$

if $\mathcal{K} = 1$ $(M_D)_{\alpha I} = i(U^*_{PMNS})_{\alpha I} \sqrt{m_I} \sqrt{M_I}$ $(\Omega = 1)$ so from (4) $m_1 M_1 = m_0^2$ and additional power is acquired

$$\sum_\alpha |\overline{RV^*}_{\alpha 1}|^2 = \frac{m_1}{M_1} = \frac{m_0^2}{M_1^2} < 10^{-8} \left( \frac{M_1}{1\text{keV}} \right)^{-2}$$

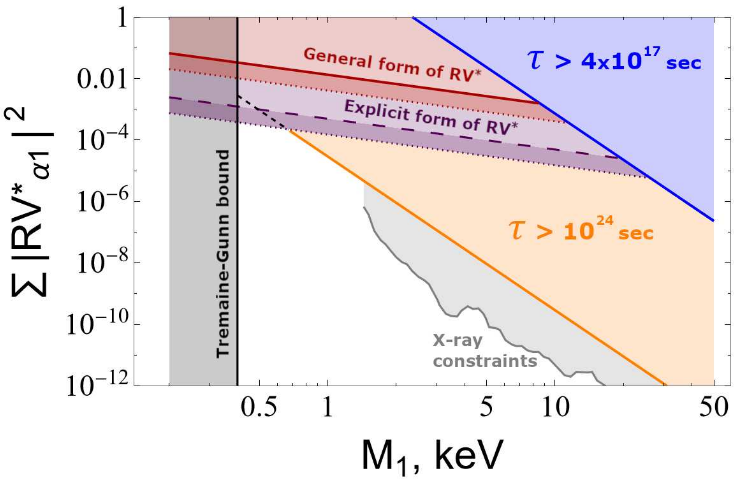



Рис.: Constraints for $|RV^*|^2$ summed by the flavors of active neutrinos versus mass of the lightest sterile neutrino $N_1$. Blue and orange lines correspond to the lifetime bounds $\tau_{N_1} > \tau_0$ (1) and gamma-ray general estimate $\tau_{N_1} > 10^7 \times \tau_0$, respectively. Dark red solid line and purple dashed line are the constraints given by (3) and (2). Wide stripes of purple and dark red color show uncertainty in the choice of the parameter $\overline{\Sigma m}$ from 0.15 eV (Lyman-$\alpha$) to 0.49 eV (Planck). The gray line shows model-independent constraints from non-observation of X-rays from radiative decay $N_1 \to \gamma, \nu$ (arXiv:0811.2385 [astro-ph]).

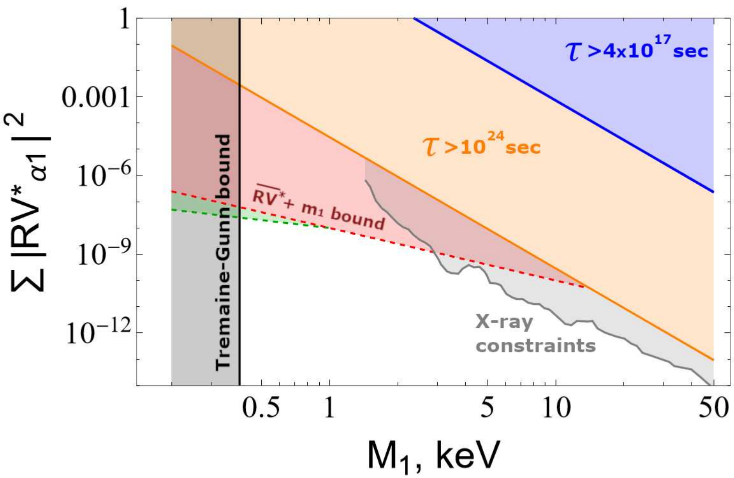



Рис.: Constraints for $|RV^*|^2$ summed by the flavors of active neutrinos versus mass of the lightest sterile neutrino $N_1$. Orange, blue and grey lines are the same as in figure above. The red and green lines correspond to the restriction for *explicit* form of $RV^*$ with the following estimates from cosmological energy fraction: $m_1 = \mathcal{O}(10^{-5}$ eV) (green) and $m_1 = \frac{m_0^2}{M_1}$, ($m_0 \sim 0.1$ eV) (red). The constraints in the area 0.5 keV$< M_1 <$5 keV are most sensitive to a specified type of the mixing matrix.

# Summary

- HNL mixing matrix for the case $\Omega = 1$ is the simplest, demonstrates a visual mass hierarchy, gives the strongest restriction of the sort mass – mixing and diagonalizes the mass term in a numerically stable way
- numerical improvement of the lifetime and energy restrictions is achieved. Improvement of the mass – mixing bound behaviour is most essential for the interval 1 – 10 keV
- the case $\Omega = 1$ defines flavor basis where the HNL and neutrino sectors can be simultaneously diagonal. Charged leptons are rotated by $U_{PMNS}$, leading to LFV in the charged lepton sector. Another choice of $\Omega$ gives the basis where LFV takes place in the HNL sector, leaving charged leptons and active neutrino flavor diagonal.