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# Polarization of gravitational waves in hybrid metric-Palatini f(R)-gravity

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# Gravitational wave detectors

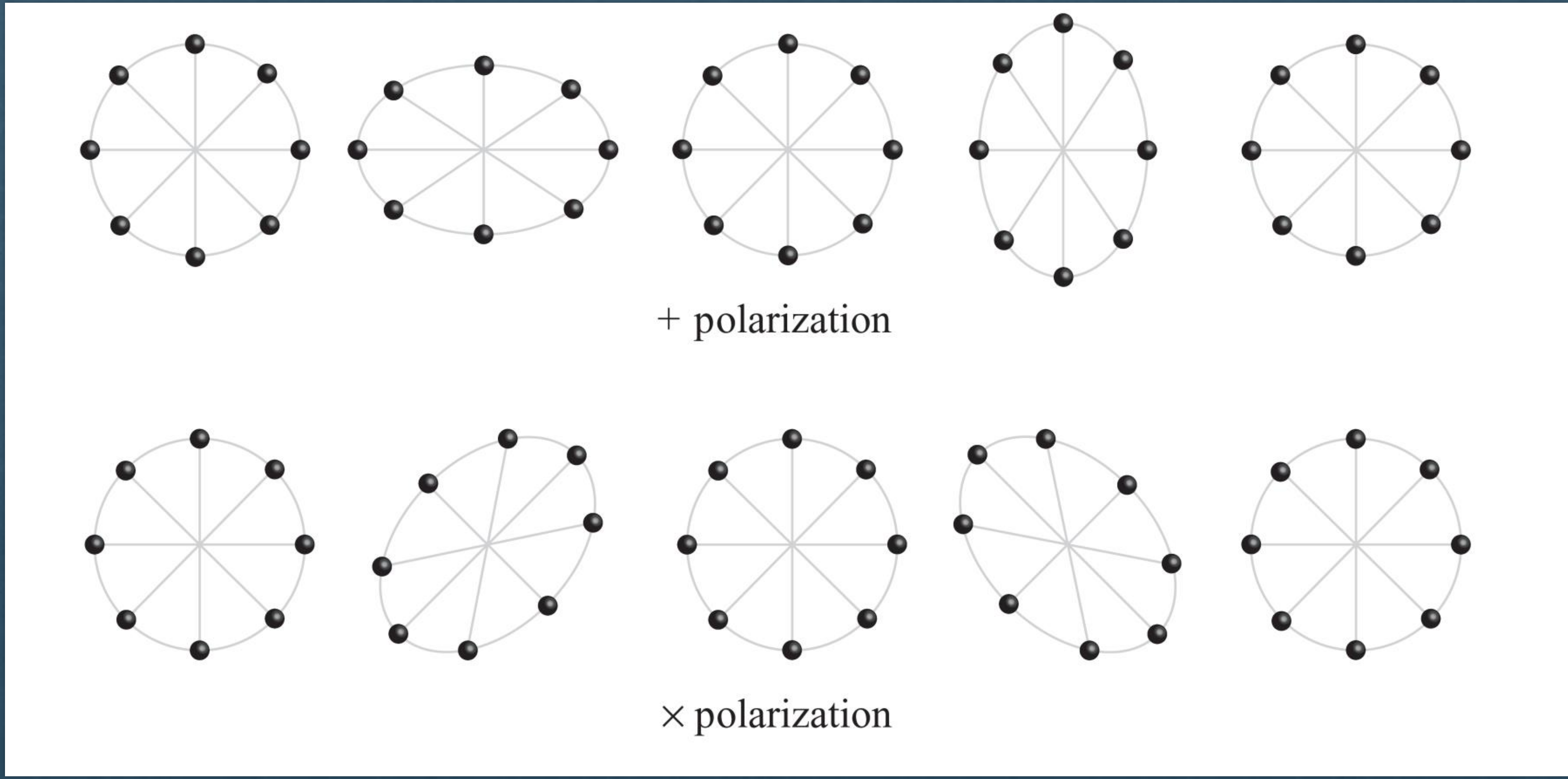
LIGO detector

VIRGO detector

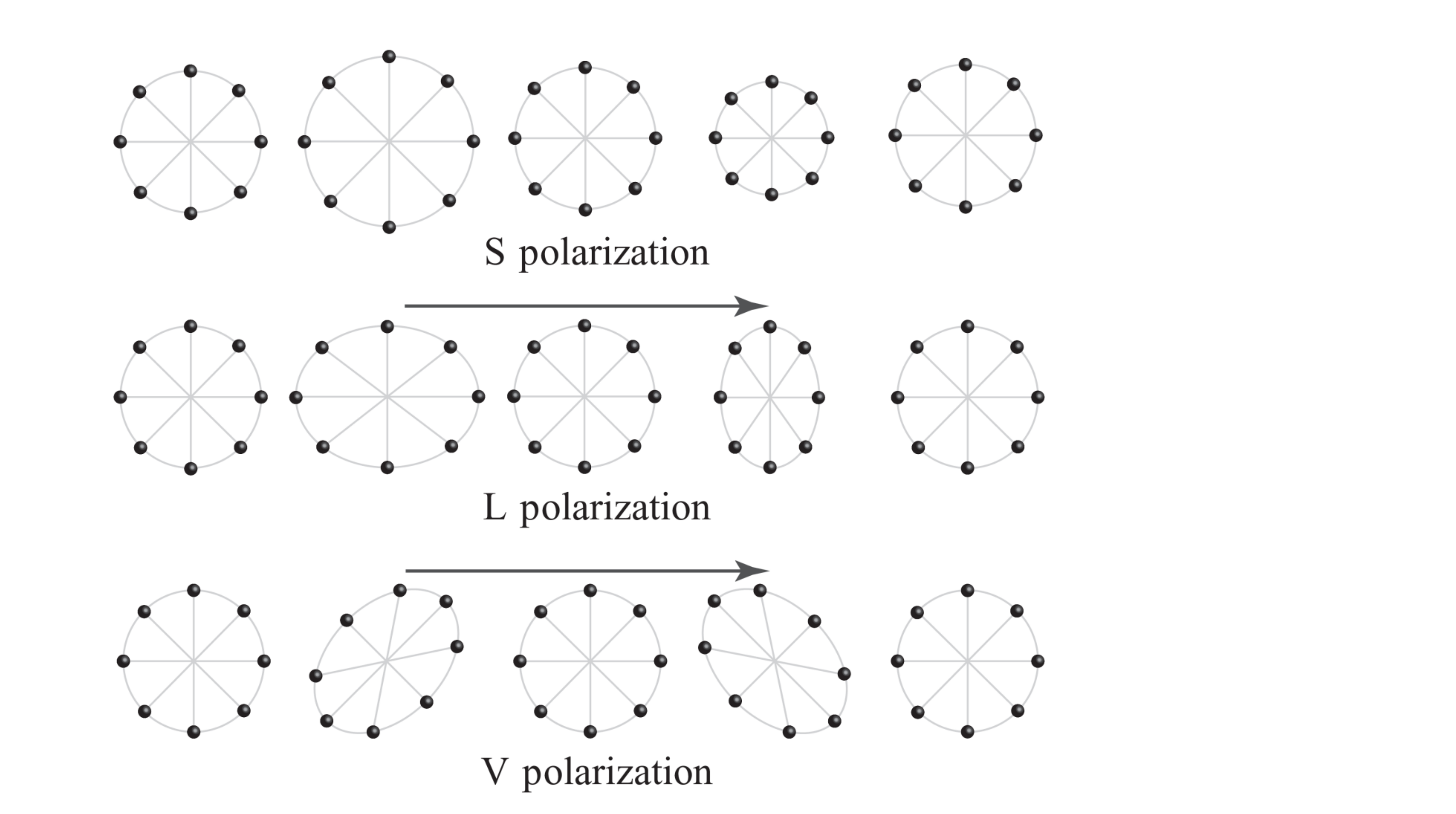
# Polarizational modes

- Tensor “+”-mode
- Tensor “×”-mode
- Scalar transverse breathing mode
- Scalar longitudinal mode
- 2 Vector modes

# “+”and “×” polarizations



# Scalar and vector modes



# f(R)-gravity

## Metric f(R)-gravity

Variables: metric $g_{\alpha\beta}$

## Palatini f(R)-gravity

Variables: metric $g_{\alpha\beta}$, independent affine connection $\Gamma^{\beta}{}_{\alpha\gamma}$

# Hybrid f(R)-gravity

$$S = \frac{1}{2k^2} \int d^4x \sqrt{-g}[R + f(\Re)] + S_m,$$

- $R$ – Ricci scalar
- $\Re$ - Palatini curvature
- $k^2=8\pi G/c^4$

*T. Harko et al., Phys. Rev. D 85, 084016 (2012)*

# Scalar-tensor representation

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}\left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m,$$

- $\phi$ – scalar field
- $V(\phi)$ – scalar potential

*S. Capozziello et al., Universe 1 (2015) 2, 199-238*

# Field equations

$$R_{\mu\nu} = \frac{1}{1+\phi}\Big(k^2(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$$

$$+ \frac{1}{2}g_{\mu\nu}(V(\phi) + \nabla_\alpha\nabla^\alpha\phi) + \nabla_\mu\nabla_\nu\phi - \frac{3}{2\phi}\partial_\mu\phi\partial_\nu\phi\Big),$$

$$-\nabla_\mu\nabla^\mu\phi + \frac{1}{2\phi}\partial_\mu\phi\partial^\mu\phi + \frac{\phi[2V(\phi) - (1+\phi)V_\phi]}{3} = \frac{\phi k^2}{3}T,$$

$$V_\phi = \frac{dV(\phi)}{d\phi}.$$

*S. Capozziello et al., Universe 1 (2015) 2, 199-238*

# Perturbations of a scalar field and metric tensor

$\varphi = \varphi_0 + \phi,$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.$

- $\varphi_0$ is the asymptotic background value of the scalar field,
- $\eta_{\mu\nu}$ is the Minkowski background,
- $h_{\mu\nu}$ and $\phi$ are small perturbations of tensor and scalar fields respectively.

# Linearized field equations

$$\overline{G}_{\mu\nu} = \frac{1}{1+\phi_0}(\nabla_\mu \nabla_\nu \varphi - \eta_{\mu\nu} \Box \varphi),$$

$$\left(\nabla^2 - m_\varphi^2\right) \varphi = 0,$$

$$m_\varphi^2 = [2V_0 - V' - (1+\phi_0)\phi_0 V'']/3$$

where $\overline{G}_{\mu\nu}$ is the linearized Einstein tensor:

$$\overline{G}_{\mu\nu} = \overline{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \overline{R},$$

$$\overline{R}_{\mu\nu} = \frac{1}{2}(\partial_\mu \partial_\alpha h^\alpha_\nu + \partial_\nu \partial_\alpha h^\alpha_\mu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu}),$$

$$\overline{R} = \partial_\mu \partial_\nu h^{\mu\nu} - \Box h.$$

# New variables

$$\theta_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h - \frac{1}{1+\phi_0}\eta_{\mu\nu}\varphi,$$

$$\theta = -h - 4\frac{1}{1+\phi_0}\varphi.$$

$$\partial_\mu \theta^{\mu\nu} = 0$$

# Linearized field equations in new variables

$$\left(\nabla^2 - m_\varphi^2\right)\varphi = 0,$$

$$\Box\theta_{\mu\nu} = 0.$$

# Plane wave solutions

$$\theta_{\mu\nu} = q_{\mu\nu} \exp^{-ik_\alpha x^\alpha},$$

$$\varphi = \phi_0 \exp^{-ip_\alpha x^\alpha},$$

$$k^\mu = (\Omega, 0, 0, \Omega), \qquad p^\mu = (p_t, 0, 0, p_z)$$

$$p_t^2 - p_z^2 = m^2$$

# Polarizations

$$h_{ij} = \begin{pmatrix} h_b + h_+ & h_\times & h_x \\ h_\times & h_b - h_+ & h_y \\ h_x & h_y & h_L \end{pmatrix},$$

$$\partial^2 h_{ij} / \partial t^2 = -2 R_{0i0j}.$$

# Electric part of Riemann tensor

$$R_{0i0j} = \begin{pmatrix} -\frac{1}{2(1+\phi_0)} p_t^2 \varphi + \frac{1}{2}\Omega^2 \theta_{xx} & \frac{1}{2}\Omega^2 \theta_{xy} & 0 \\ \frac{1}{2}\Omega^2 \theta_{xy} & -\frac{1}{2(1+\phi_0)} p_t^2 \varphi - \frac{1}{2}\Omega^2 \theta_{xx} & 0 \\ 0 & 0 & -\frac{1}{2(1+\phi_0)} m^2 \varphi \end{pmatrix} .$$

# Eardley classification

$$R_{tjtk} = \begin{pmatrix} -\frac{1}{2}(\Re\Psi_4 + \Phi_{22}) & \frac{1}{2}\Im\Psi_4 & -2\Re\Psi_3 \\ \frac{1}{2}\Im\Psi_4 & \frac{1}{2}(\Re\Psi_4 - \Phi_{22}) & 2\Im\Psi_3 \\ -2\Re\Psi_3 & 2\Im\Psi_3 & -6\Psi_2 \end{pmatrix}$$

*D.M. Eardley, D.L. Lee, A.P. Lightman, Phys. Rev. D 8, 3308 (1973).*

# Eardley classification

Class $\mathrm{II}_6$ $\Psi_2 \neq 0$. All observers measure the same nonzero amplitude of the $\Psi_2$ mode, but the presence or absence of all other modes is observer-dependent.

Class $\mathrm{III}_5$ $\Psi_2 = 0$, $\Psi_3 \neq 0$. All observers measure the absence of the $\Psi_2$ mode and the presence of the $\Psi_3$ mode, but the presence or absence of $\Psi_4$ and $\Phi_{22}$ is observer-dependent.

Class $\mathrm{N}_3$ $\Psi_2 = \Psi_3 = 0$, $\Psi_4 \neq 0 \neq \Phi_{22}$. The presence or absence of all modes is observer-independent.

Class $\mathrm{N}_2$ $\Psi_2 = \Psi_3 = \Phi_{22} = 0$, $\Psi_4 \neq 0$. The presence or absence of all modes is observer-independent.

Class $\mathrm{O}_1$ $\Psi_2 = \Psi_3 = \Psi_4 = 0$, $\Phi_{22} \neq 0$. The presence or absence of all modes is observer-independent.

Class $\mathrm{O}_0$ $\Psi_2 = \Psi_3 = \Psi_4 = \Phi_{22} = 0$. No wave is observed.

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# Detection strategies for scalar gravitational waves with interferometers and resonant spheres

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We compute the response and the angular pattern function of an interferometer for a scalar component of gravitational radiation in Brans-Dicke theory. We examine the problem of detecting a stochastic background of scalar GWs and compute the scalar overlap reduction function in the correlation between an interferometer and the monopole mode of a resonant sphere. While the correlation between two interferometers is maximized taking them as close as possible, the interferometer-sphere correlation is maximized at a finite value of $f\times d$, where $f$ is the resonance frequency of the sphere and $d$ the distance between the detectors. This defines an optimal resonance frequency of the sphere as a function of the distance. For the correlation between the VIRGO interferometer located near Pisa and a sphere located in Frascati, near Rome, we find an optimal resonance frequency $f\simeq 590$ Hz. We also briefly discuss the difficulties in applying this analysis to the dilaton and moduli fields predicted by string theory.

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## I. INTRODUCTION

A number of interferometers for gravitational wave (GW) detection are presently under construction and are expected to be operating in the next few years. In particular, VIRGO is being built near Pisa, the two Laser Interferometric Gravitational Wave Observatory (LIGO) interferometers are being built in the US, GEO600 near Hannover, and TAMA300 in Japan. These interferometers are in principle sensitive also to a hypothetical scalar component of gravitational radiation. Scalar GWs appear already in the simplest generalization of general relativity, namely Brans-Dicke theory, whose action reads

$$\mathcal{S}_{\rm BD}=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\varphi R-\frac{\omega_{\rm BD}}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi\right]+\mathcal{S}_{\rm matter}, \quad (1)$$

In this paper we investigate whether it is possible to search for such scalar particles using the GW interferometers under construction, as well as the resonant spheres which are under study. We start from the Brans-Dicke theory and, in Sec. II, we discuss the response of an interferometer to a GW with a scalar component: in particular, we find that such a scalar component creates a transverse (with respect to the direction of propagation of the GW) stress in the detector; we compute the phase shift $\Delta\varphi$ measured in the interferometer and derive the angular pattern function, i.e. the dependence of the signal on the direction $(\theta,\phi)$ of the impinging GW (see Fig. 1). We find $\Delta\varphi\propto\sin^2\theta\cos 2\phi$. We also show the physical (and formal) equivalence of two different gauges used to describe scalar radiation.

In Sec. III we consider the detection of a stochastic background of scalar GWs. In this case it is necessary to correlate

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# Polarizations of gravitational waves in Horndeski theory

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**Abstract** We analyze the polarization content of gravitational waves in Horndeski theory. Besides the familiar plus and cross polarizations in Einstein's General Relativity, there is one more polarization state which is the mixture of the transverse breathing and longitudinal polarizations. The additional mode is excited by the massive scalar field. In the massless limit, the longitudinal polarization disappears, while the breathing one persists. The upper bound on the graviton mass severely constrains the amplitude of the longitudinal polarization, which makes its detection highly unlikely by the ground-based or space-borne interferometers in the near future. However, pulsar timing arrays might be able to detect the polarization excited by the massive scalar field. Since additional polarization states appear in alternative theories of gravity, the measurement of the polarizations of gravitational waves can be used to probe the nature of gravity. In addition to the plus and cross states, the detection of the breathing polarization means that gravitation is mediated by massless spin 2 and spin 0 fields, and the detection of both the breathing and longitudinal states means that gravitation is propagated by the massless spin 2 and massive spin 0 fields.

VIRGO, the future space-borne Laser Interferometer Space Antenna (LISA) [7] and TianQin [8], and pulsar timing arrays (e.g., the International Pulsar Timing Array and the European Pulsar Timing Array [9,10]). In fact, in the recent GW170814 [4], the Advanced VIRGO detector joined the two aLIGO detectors, so they were able to test the polarization content of gravitational waves for the first time. The result showed that the pure tensor polarizations were favored against pure vector and pure scalar polarizations [4,11]. Additionally, GW170817 is the first observation of a binary neutron star inspiral, and its electromagnetic counterpart, GRB 170817A, was later observed by the Fermi Gamma-ray Burst Monitor and the International Gamma-Ray Astrophysics Laboratory [5,12,13]. The new era of multi-messenger astrophysics comes.

In GR, the gravitational wave propagates at the speed of light and it has two polarization states, the plus and cross modes. In alternative metric theories of gravity, there may exist up to six polarizations, so the detection of the polarizations of gravitational waves can be used to distinguish different theories of gravity and probe the nature of gravity [14,15]. For null plane gravitational waves, the six polarizations are classified by the little group $E(2)$ of the Lorentz

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# Ghosts in pure and hybrid formalisms of gravity theories: A unified analysis

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In the first-order formalism of gravitational theories, the spacetime connection is considered as an independent variable to vary together with the metric. However, the metric still generates its Levi-Civita connection that turns out to determine the geodesics of matter. Recently, "hybrid" gravity theories have been introduced by constructing actions involving both the independent Palatini connection and the metric Levi-Civita connection. In this study a method is developed to analyze the field content of such theories, in particular to determine whether the propagating degrees of freedom are ghosts or tachyons. New types of second-, fourth- and sixth-order derivative gravity theories are investigated and the so-called $f(X)$ theories are singled out as a viable class of "hybrid" extensions of general relativity. These are the theories in which the corrections to Einstein's theory are written in terms of the deviations from the usual trace equation $X = R - \kappa^2 T$.

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## I. INTRODUCTION

Motivated by the oddities discovered by recent astronomical observations, a plethora of modified theories of gravity has been advanced during recent years. Their main scope is to reproduce the observed behavior of our Universe without invoking any undetected entity, such as dark matter, dark energy or the inflaton field. Although some of them may succeed in reaching the goal, some others give rise to unphysical features such as, for example, the appearance of ghosts.

Among all these modified theories, the most considered

other theories, such as $f(R)$ gravity, it produces completely new dynamical field equations, resulting in a different phenomenology; see [1,3] for reviews.

The metric and Palatini approaches have been recently combined[1] to give birth to a new class of modified gravitational theories, which has been named the hybrid metric-Palatini or $f(X)$ gravity principle [8]. The action is taken to depend linearly on the metric curvature scalar $R$ but nonlinearly on the Palatini curvature scalar $\hat{R}$, which is modulated by an arbitrary function in analogy with Palatini $f(R)$ theories. Some cosmological and astrophysical applica-

# Rest frame of the massive scalar field

$$p_t^2 = m^2,$$

$$-\frac{1}{2(1+\phi_0)}m^2\varphi = -\frac{1}{2(1+\phi_0)}p_t^2\varphi.$$

# Conclusion

Both scalar modes are manifestations of the same scalar degree of freedom and represent a mixture of polarization states of a gravitational wave. Therefore, there is no contradiction between the number of degrees of freedom and the number of polarization states in hybrid f(R)-gravity.


Thank you for
your attention!

# Advantages of hybrid f(R)-gravity

*S. Capozziello et al.*
*Astroparticle Physics*
*50-520 (2013), pp. 65-75*

*I. Leanizbarrutia et al.*
*Phys. Rev. D 95,*
*084046 (2017)*

*S. Capozziello et al.*
*JCAP 07 (2013) 024*

## 2.1. Action and gravitational field equations

The action is specified as [28,54]

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,[R + f(\mathcal{R})] + S_m\,, \tag{1}$$

where $S_m$ is the matter action, $\kappa^2 \equiv 8\pi G$, $R$ is the Einstein-Hilbert term, $\mathcal{R} \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu}$ is the Palatini curvature, defined in terms of an independent connection $\hat{\Gamma}^{\alpha}_{\mu\nu}$ as

$$\mathcal{R} \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu} \equiv g^{\mu\nu}\left(\hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\lambda}\hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\alpha\nu}\right)\,, \tag{2}$$

that generates the Ricci curvature tensor $\mathcal{R}_{\mu\nu}$ as

$$\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\lambda}\hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\alpha\nu}\,. \tag{3}$$

Varying the action given by Eq. (1) with respect to the metric, one obtains the following gravitational field equation

$$G_{\mu\nu} + F(\mathcal{R})\mathcal{R}_{\mu\nu} - \frac{1}{2}f(\mathcal{R})g_{\mu\nu} = \kappa^2 T_{\mu\nu}\,, \tag{4}$$

where the matter stress-energy tensor is defined as usual,

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta(g^{\mu\nu})}. \tag{5}$$

Varying the action with respect to the independent connection $\hat{\Gamma}^{\alpha}_{\mu\nu}$, one then finds as the solution to the resulting equation of motion that $\hat{\Gamma}^{\alpha}_{\mu\nu}$ is compatible with the metric $F(\mathcal{R})g_{\mu\nu}$, conformally related to the physical metric $g_{\mu\nu}$, with the conformal factor given by $F(\mathcal{R}) \equiv df(\mathcal{R})/d\mathcal{R}$. This implies that

$$\mathcal{R}_{\mu\nu} \;=\; R_{\mu\nu} + \frac{3}{2}\frac{1}{F^2(\mathcal{R})}F(\mathcal{R})_{,\mu}F(\mathcal{R})_{,\nu} - \frac{1}{F(\mathcal{R})}\nabla_{\mu}F(\mathcal{R})_{,\nu} - \frac{1}{2}\frac{1}{F(\mathcal{R})}g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}F(\mathcal{R})\,. \tag{6}$$

### 2.2. Scalar-tensor representation

Like in the pure metric and Palatini cases [55,56], the action (1) for the hybrid metric-Palatini theory can be turned into that of a scalar-tensor theory by introducing an auxiliary field $A$ such that

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\Omega_A R + f(A) + f_A(\mathcal{R} - A)\right] + S_m\,, \tag{11}$$

where $f_A \equiv df/dA$ and we have included a coupling constant $\Omega_A$ for generality. Note that $\Omega_A = 1$ for the original hybrid metric-Palatini theory [28]. Rearranging the terms and defining $\phi \equiv f_A$, $V(\phi) = Af_A - f(A)$, Eq. (11) becomes

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\Omega_A R + \phi\mathcal{R} - V(\phi)\right] + S_m\,. \tag{12}$$

It is easy to see that this action is equivalent to our original starting point (1). Variation of the above action with respect to the metric, the scalar $\phi$ and the connection leads to the field equations

$$\Omega_A R_{\mu\nu} + \phi\mathcal{R}_{\mu\nu} - \frac{1}{2}\left(\Omega_A R + \phi\mathcal{R} - V\right)g_{\mu\nu} = \kappa^2 T_{\mu\nu}\,, \tag{13}$$

$$\mathcal{R} - V_\phi = 0\,, \tag{14}$$

$$\hat{\nabla}_\alpha\left(\sqrt{-g}\,\phi g^{\mu\nu}\right) = 0\,, \tag{15}$$

respectively.

The solution of Eq. (15) implies that the independent connection is the Levi-Civita connection of a metric $h_{\mu\nu} = \phi g_{\mu\nu}$. This means that the relation (6) between the tensors $\mathcal{R}_{\mu\nu}$ and $R_{\mu\nu}$ can be now rewritten as

$$\mathcal{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2\phi^2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\nabla_\alpha\nabla^\alpha\phi\right) , \tag{16}$$

which can be used in the action (12) to get rid of the independent connection and to obtain the following scalar-tensor representation that belongs to the "Algebraic Family of Scalar-Tensor Theories" [57], so that one finally arrives at the following action

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[(\Omega_A + \phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m . \tag{17}$$