Regge cuts in QCD

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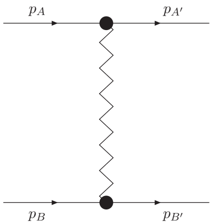
QCD can be called an unique theory due to Reggeization of all its elementary particles – quarks and gluons– in perturbation theory. The Reggeization is very important and widely used for theoretical description of high energy processes.
The gluon Reggeization is especially valuable because gluon exchanges in cross-channels provide non-decreasing with energy cross sections. In particular, the gluon Reggeization appears to be the basis of the BFKL
(Balitskii-Fadin-Kuraev-Lipatov) equation, which was first derived in non-Abelian theories with spontaneously broken symmetry
F. V.S., Kuraev E.A., Lipatov L.N., 1975
and whose applicability in QCD was then shown
Balitsky I.I., Lipatov L.N., 1978.
The equation was derived using unitarity and analyticity.

For elastic scattering processes $A + B \to A' + B'$ in the Regge kinematical region: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s) the Reggeization means that scattering amplitudes with the gluon quantum numbers in the t-channel and negative signature (symmetry with respect to $s \leftrightarrow u$) can be presented as



$$\mathcal{A}_{AB}^{A'B'} = \Gamma^{c}_{A'A} \left[\left(\frac{-s}{-t} \right)^{\omega(t)} - \left(\frac{s}{-t} \right)^{\omega(t)} \right] \Gamma^{c}_{B'B} \; ;$$

Introduction

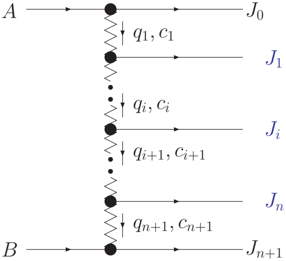
$\Gamma^{c}_{P'P}$–particle-particle-Reggeon (PPR) vertices or scattering vertices ("c" are colour indices); $j(t) = 1 + \omega(t)$ – Reggeon trajectory.

The Reggeization means definite form not only of elastic amplitudes, but of inelastic amplitudes in the multi-Regge kinematics (MRK) as well.

MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets.

The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.

MRK amplitudes can be presented by the picture



and their real parts have a simple factorized form

$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

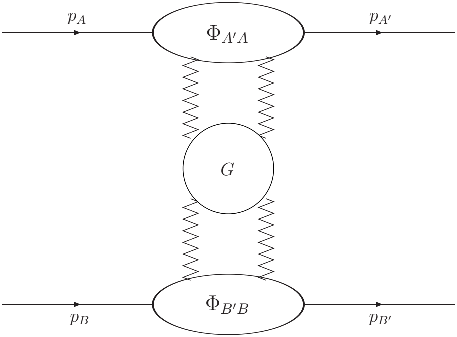
Here $\gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1})$ – the Reggeon-Reggeon-particle (RRP) or production vertices.

Two-Reggeon cuts

It is well known that there is no consistent theory in which all singularities in j plane (plane of complex angular momenta) are poles.
Regge poles in the j plane generate Regge cuts.
In QCD we know only one Regge pole – the pole with the Reggeized gluon trajectory $j(t) = 1 + \omega(t)$.
Clearly, the Reggeized gluon can not appear in amplitudes with different from adjoint representations.
In these representations Regge cuts appear already in the LLA.
In particular, the BFLK Pomeron is a two-Reggeon cut.
It appears that in amplitudes with positive signature, where the real parts of the leading logarithmic terms cancel out, so that remaining piece is pure imagine in the LLA.
From the unitarity relation, using the pole Regge form of elastic and MRK amplitudes, we obtain that the s channel imaginary parts of elastic amplitudes are presented in the form:

Two-Reggeon cuts

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}$$



where Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$
$B \to B'$
G – Green's function for two interacting Reggeized gluons,

Two-Reggeon cuts

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

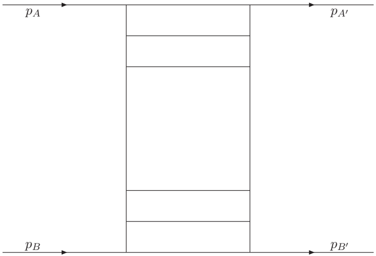
$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.
The kernel is universal (process independent).

Peculiarity of Regge cuts in perturbative QCD

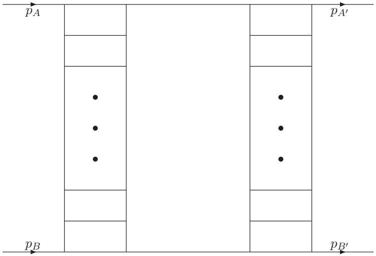
There is a significant distinction between Regge cuts in perurbative QCD and in former (before the advent of QCD) complex angular momentum theory.
Actually, it is due to different ideas about the Regge poles. In the conventional complex angular momentum theory the pole conceived as an infinite series of ladder diagrams depicted below



Assuming the Regge pole behaviour $A(s,t) \sim s^{\alpha(t)} \ln s$

Peculiarity of Regge cuts in perturbative QCD

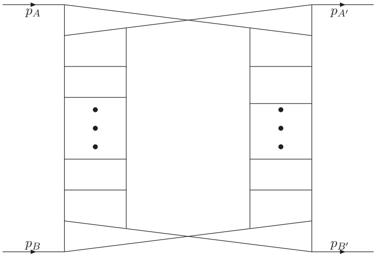
of this series and using only the two-particle intermediate state in the *s*-channel unitarity relation for the amplitude corresponding to the diagram



D. Amati, S. Fubini and A. Stanghellini, 1962
obtained the Regge cut behaviour for this diagram.
But in
S. Mandelstam, 1963

Peculiarity of Regge cuts in perturbative QCD

the cuts in the angular-momentum plane proposed by Amati, Fubini and Stanghellini were examined further and it was shown that though such cuts exist in each of the dispersion diagrams they are not present in the complete diagram, due to the cancellation between various diagrams.
More complicated diagrams of the type



were proposed and it was shown that they lead to the Regge cuts. Pay attention that they are non-planar.

Peculiarity of Regge cuts in perturbative QCD

Since then, the AFS-type diagrams have been rejected and it was believed that only non-planar diagrams could lead to Regge cuts.
But the situation with cuts in perturbative QCD is completely different.
It differs already in the formulation of the problem.
In the old theory of complex angular momenta the main subject of research was the asymptotic behaviour of the amplitudes at high energies and fixed momentum transfers.
In perturbative QCD we try to calculate leading terms in each order of perturbation series, generally speaking, having no idea what this summation will result in. We then try to interpret the results obtained as the contributions of the Regge poles and cuts.
The big difference lies in structure of Reggeons and how these Reggeons form cuts.

Peculiarity of Regge cuts in perturbative QCD

Reggeons in the old theory of complex angular momenta do not have one-particle t-channel states.

The Reggeized gluon in QCD begin with one-gluon state.

Due to this Mandelstam arguments do not work.

In the lowest (g^4) order the BFKL Pomeron is two-gluon t-channel exchange, which is evidently planar.

In the old theory of complex angular momenta Regge cuts are given by exchange of non-interacting Reggeons in t-channel.

In QCD t-channel Reggeons interact. Their interaction is described by the BFKL kernel.

In the old theory of complex angular momenta Regge cuts are moving.

The BFKL Pomeron has fixed branch point.

Regge cuts in QCD are neither Mandelstam, nor he BFKL Pomeron is in fact AFS cut.

Even what little is known about cuts in the old theory of complex angular momentum is inapplicable in QCD.

Three-Reggeon cuts

In the LLA and in the NLLA elastic and MRK amplitudes with gluon quantum numbers and negative signature are given by the pole Regge contribution.
But the pole Regge form is violated in the NNLLA.
The first observation of the violation was done
Del Duca V., Glover E.W.N., 2001
at consideration of the high-energy limit of the two-loop amplitudes for gg, gq and qq scattering. The discrepancy appears in non-logarithmic terms.
If the pole Regge form would be correct in the NNLLA, they should satisfy the factorization condition. However, it is not the case.
Using the infrared factorization techniques, consideration of the terms responsible for breaking of the pole Regge form in amplitudes of elastic scattering in QCD was performed by
Del Duca V., Falcioni G., Magnea L., Vernazza L., 2013-2015.

Three-Reggeon cuts

The observed violation of the pole Regge form should be explained by Regge cut contributions.

Indeed, all known cases of breaking of the pole Regge form were explained by such contributions.
F. V.S., 2016;
F. V.S., Lipatov L.N., 2017

The cut contributions were calculated using the Feynman diagramm approach.
Due to the signature conservation the cut with negative signature must be the three-Reggeon one.
Since our Reggeon is the Reggeized gluon, the three-Reggeon cut first appears in Feynman diagrams with three-gluon exchanges in the t-channel.

Diagrammatic approach

The amplitude of the process $\mathcal{A}_{AB}^{A'B'}$ can be written as the sum over permutations σ of products of colour factors and colour-independent matrix elements:

$$\mathcal{A}_{AB}^{A'B'} = \sum_{\sigma} \left(C_{AB}^{(0)\sigma} \right)_{\alpha\beta}^{\alpha'\beta'} M_{AB}^{(0)\sigma}(s,t) \ ,$$

where α and β (α' and β') are colour indices of incoming (outgoing) projectile A and target B respectively.
The colour factors can be decomposed into irreducible representations $\mathcal{R}$ of the colour group in the t-channel:

$$\left(C_{AB}^{(0)\sigma} \right)_{\alpha\beta}^{\alpha'\beta'} = \sum_{R} [\mathcal{P}_{AB}^{R}]_{\alpha\beta}^{\alpha'\beta'} \sum_{\sigma} \mathcal{G}(R)_{AB}^{(0)\sigma},$$

where $\hat{\mathcal{P}}^{R,n}$ is the projection operator on the state n in the representation $\mathcal{R}$,

Diagrammatic approach

In contrast to the Reggeon, which contributes only to amplitudes with the adjoint representation of the colour group (colour octet in QCD) in the t-channel, the cut can contribute to various representations.

With account of Bose statistic for gluons, possible representations of the colour group in the t-channel and their symmetry, in the amplitudes with negative signature besides the adjoint representation there are there are **1** for quark-quark-scattering and **10** and $\mathbf{10}^*$ for the gluon-gluon scattering.

It turns out that for the representations R different from the Reggeized gluon one the colour coefficients $\mathcal{G}(R)_{AB}^{(0)\sigma}$ do not depend on σ, so that momentum dependent factors for them summed up to the eikonal amplitude

$$\sum_{\sigma} M_{AB}^{(0)\sigma}(s,t) = A^{eik} = g^6 \frac{s}{t}\left(\frac{-4\pi^2}{3}\right)\vec{q}^{\,2} A_2(q_\perp)\ ,$$

Diagrammatic approach

where $A_2(q_\perp)$ is depicted by the diagram

and is written as

$$A_2(q_\perp) = \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^2} \; .$$

Note that the "infrared" ϵ is used, $\epsilon = (D-4)/2$, D is the space-time dimension.

This result is very important, because contribution of the cut must be gauge invariant, whereas $M_{AB}^{(0)\sigma}$ taken separately are gauge dependent.

Diagrammatic approach

It occurs that in the Reggeized gluon channel the terms violating the pole factorization also have σ-independent colour coefficients, so that momentum factors for them summed up to the gauge invariant eikonal amplitude.
But separation of the pole and cut contributions is impossible in the two-loop approximation because of the ambiguity of the allocation of the part of the amplitudes violating the factorization. The separation becomes possible in higher loops, due to different energy dependence of the pole and cut contributions. Energy dependence of the pole contribution is determined by the Regge factor of the Reggeized gluon $\exp(\omega(t)\ln s)$, where $\omega(t)$ is the gluon trajectory, whereas for the three-Reggeon cut it is

$$e^{[(\hat{\omega}_1+\hat{\omega}_2+\hat{\omega}_3+\hat{\mathcal{K}}_r(1,2)+\hat{\mathcal{K}}_r(1,3)+\hat{\mathcal{K}}_r(2,3))\ln s]} ,$$

where $\hat{\mathcal{K}}_r(m,n)$ is the real part of the BFKL kernel describing interaction between Reggeons m and n.

Diagrammatic approach

With the help of the integral representation of the trajectory

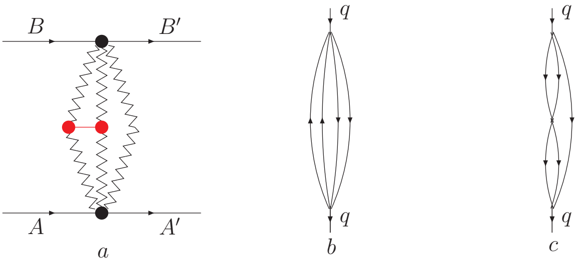
$$\omega(t) = -g^2 N_c \vec{q}^{\,2} \int \frac{d^{2+2\epsilon} l}{2(2\pi)^{(3+2\epsilon)} \vec{l}^{\,2} (\vec{q}-\vec{l})^2}$$

and the explicit form of the real part of the kernel describing interaction between two Reggeons with transverse momenta $\vec{l}_1$ and $\vec{l}_2$ and colour indices c_1 and c_2

$$\left[\mathcal{K}_r(\vec{q}_1, \vec{q}_2; \vec{k})\right]_{c_1 c_2}^{c_1' c_2'} = -T^a_{c_1 c_1'} T^a_{c_2 c_2'} \frac{g^2}{(2\pi)^{D-1}} \left[\frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\,2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right],$$

Diagrammatic approach

the first order corrections are expressed through the diagrams b and c.



Diagrammatic approach

$$\mathcal{G}(\mathbf{8_a})^{(cut)}_{AB} g^6 \frac{s}{t}\left(\frac{-4\pi^2}{3}\right)\vec{q}^{\,2}$$

$$\times\left(A_2(q_\perp)+g^2N_c\ln s\left(\frac{1}{2}A_3^b(q_\perp)-A_3^c(q_\perp)\right)\right)\;,$$

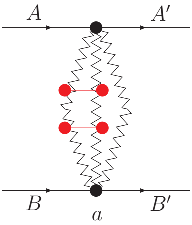
$$A_3^b(q_\perp)=-\int\frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2 d^{2+2\epsilon}l_3}{(2\pi)^{3(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}\vec{l}_3^{\,2}(\vec{q}-\vec{l}_1-\vec{l}_2-\vec{l}_3)^2}\;,$$

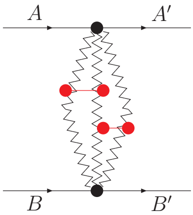
$$A_3^c(q_\perp)=\int\frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2 d^{2+2\epsilon}l_3(\vec{q}-l_1)^2}{(2\pi)^{3(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}\vec{l}_3^{\,2}(\vec{q}-\vec{l}_1-\vec{l}_2)^2(\vec{q}-\vec{l}_1-\vec{l}_3)^2}\;.$$

$$\mathcal{G}(\mathbf{8_a})^{(cut)}_{gg}=-\frac{3}{2}\,,\quad \mathcal{G}(\mathbf{8_a})^{(cut)}_{gq}=-\frac{3}{2}\,,\mathcal{G}(\mathbf{8_a})^{(cut)}_{qq}=\frac{3(1-N-c^2)}{4N_c^2}\;.$$

The calculation of the three-loop corrections shows that the restrictions imposed by the infrared factorization can be fulfilled in the NNLLA at two and three loops if besides the Regge pole contribution there is the Regge cut contribution.

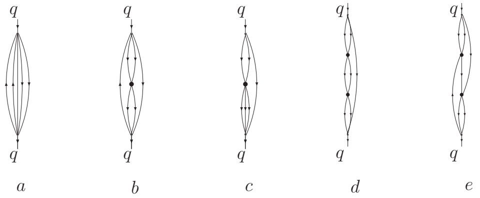
Four loops calculations
F.V.S. 2019-2021
There are three types of corrections. The first (simplest) ones come from account of the Regge factors of each of three Reggeons. The second type of the corrections are given by the products of the trajectories and real parts of the BFKL kernel, and the third come from account of Reggeon-Reggeon interactions.





Diagrammatic approach

All the types of the corrections are expressed through the integrals in the transverse momentum space corresponding to the diagrams



Diagrammatic approach

$$I_i = \int \frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2\, d^{2+2\epsilon}l_3}{(2\pi)^{3(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}\vec{l}_3^{\,2}} F_i \delta^{3+2\epsilon}(\vec{q}-\vec{l}_1-\vec{l}_2-\vec{l}_3)\,,$$

$$F_a = f_1(\vec{l}_1)f_1(\vec{l}_2)\,,\quad F_b = f_1(\vec{l}_1)f_1(\vec{l}_1)\,,\quad F_c = f_2(\vec{l}_1+\vec{l}_2)\,,$$

$$F_d = f_1(\vec{l}_1+\vec{l}_2)f_1(\vec{l}_1+\vec{l}_2)\,,\quad F_e = f_1(\vec{q}-\vec{l}_1)f_1(\vec{q}-\vec{l}_3)\,,$$

$$f_1(\vec{k}) = \vec{k}^{\,2}\int \frac{d^{2+2\epsilon}l}{(2\pi)^{(3+2\epsilon)}\vec{l}^{\,2}(\vec{l}-\vec{k})^2}\,,\quad f_2(\vec{k}) = \int \frac{d^{2+2\epsilon}l f_1(\vec{l})}{(2\pi)^{(3+2\epsilon)}\vec{l}^{\,2}(\vec{l}-\vec{k})^2}\,.$$

The difficult problem here is the calculation of color coefficients

It shows that as well as in the two and three loops the terms violating the pole factorization have σ-independent colour coefficients.

It provides their gauge invariance.

Wilson line approach

There is an another approach to three-Reggeon cuts
Caron-Huot S., Gardi E., Vernazza L., 2017
based on representation of scattering amplitudes by Wilson lines.
In this approach, no connection of the three-Reggeon cuts with Feynman diagrams is traced. The colour coefficients $\mathcal{G}(\mathbf{R})^{(0)C}_{AB}$ for the cut contributions are taken as

$$\mathcal{G}(\mathbf{R})^{(0)C}_{AB} = \frac{\left(\mathcal{T}_A^{c_1}\mathcal{T}_A^{c_2}\mathcal{T}_A^{c_3}\right)^{\alpha'}{}_{\alpha}}{3N_{\mathbf{R}}\ T_A\ T_B}\left(\sum_{\sigma}\mathcal{T}_B^{c_1^\sigma}\mathcal{T}_B^{c_2^\sigma}\mathcal{T}_B^{c_3^\sigma}\right)^{\beta'}{}_{\beta}[\mathcal{P}^{\mathbf{R}\,*}_{AB}]^{\alpha\beta}{}_{\alpha'\beta'}\,.$$

As for the momentum dependent part, it is taken equal to A^{eik}. For the representations different from the Reggeized gluon one it agrees with the diagrammatic approach, since the colour coefficients $\mathcal{G}(\mathbf{R})^{(0)\sigma}_{AB}$ do not depend on σ for such representations. Therefore, the cut contributions in both approaches are the same for these representations.

Wilson line approach

But it is not so for the adjoint representation, where the colour coefficients $\mathcal{G}(\mathbf{8})^{(0)C}_{AB}$ turn out to be

$$\mathcal{G}(\mathbf{8})^{(0)C}_{AB} = \mathcal{G}(\mathbf{8})^{(0)}_{AB} + \frac{N_c^2}{24} \ ,$$

As for the momentum dependent part, it is also taken equal to A^{eik}.

It looks strange from the point of view of the diagrammatic approach, because appearance of A^{eik} requires equality of all terms in the sum over σ. In two loops difference Δ_{AB} between two approaches is such that

$$\Delta_{gg} + \Delta_{qq} = 2\Delta_{gq}$$

and therefore it can be attributed to the pole contribution. To do this, it is sufficient to change the two-loop contributions to the gluon-gluon-Reggeon and quark-quark-Reggeon vertices. But in three loops it becomes necessary to introduce the Reggeon-cut mixing.

Wilson line approach

It is necessary to note that in this approach
the cut contribution is not suppressed at large N_c,
i.e. it exists in the planar $N = 4$ SYM, in contradiction with the common opinion, that in the high energy limit the four-point amplitudes in this theory are given by the Reggeized gluon contribution.

Recently the three-Reggeon cut contributions to elastic amplitudes were calculated in four loops
G. Falcioni, E. Gardi, N. Maher, C. Milloy and L. Vernazza, 2022
To perform calculations these authors used the same technique as in Caron-Huot S., Gardi E., Vernazza L., 2017, But to escape the contradiction with the planar $N = 4$ SYM these authors suggested another scheme for separating the pole and cut contribution, so called Regge-cut scheme, where the planar part of the three-Reggeon exchanges is adsorbed by the pole contribution.

But it makes problem with gauge invariance more evident

Regge cuts in MRK amplitudes

The study of Regge cuts in QCD is interesting in itself, but from the point of view of applications, it is necessary to develop the BFKL approach in the NNLLA.

The unitarity relations used in this approach require knowledge of multi-particle production amplitudes in the MRK.

In the LLA and NLLA only real parts of the these amplitudes contribute.

But in the NNLLA imaginary parts become also important.

It can come from imaginary parts of the Reggeized gluon contributions,

which, in principle, are known, though have complicated (not factorized) form,

and from two-Reggeon cuts, which are not known yet, as well.

In the NLLA real parts of MRK amplitudes coming from three-Reggeon cuts become also important. They are definitely exist, although their investigation is practically absent.

Summary

- The gluon Reggeization is one of remarkable properties of QCD, which is very important for theoretical description of high energy processes. In particular, it is the basis of the BFKL equation.
- In the LLA and NLLA QCD amplitudes with adjoint representation of the colour group in cross channels and negative signature have the pole Regge form.
- In channels with positive signature there are Regge cuts already in the LLA.
- There is a big difference between Regge cuts in the old theory of angular momentum in complex plane and in QCD. In particular, in QCD they are neither Mandelstam, nor AFS cuts.
- The pole Regge form is violated in QCD in the NNLLA. It means that along with the pole there is the three-Reggeon cut contribution.

Summary

- Separation of the pole and the three-Reggeon cut contributions is not unique.
- There are different approaches to definition of three-Reggeon cut contributions. In two of them these contributions to elastic amplitudes are calculated at four loops. Unfortunately, the results are not compared.
- Till now calculation of Regge cut contributions to MRK amplitudes, which play an important role in the BFKL approach, is absent.